A Robust Nonlinear Observer-based Approach for Distributed Fault Detection of Input-Output Interconnected Systems ⋆

Christodoulos Keliris [a], Marios M. Polycarpou [a], Thomas Parisini [b]

[a] *KIOS Research Center for Intelligent Systems and Networks, Department of Electrical and Computer Engineering, University of Cyprus, Nicosia 1678, Cyprus*

[b] *Dept. of Electrical and Electronic Engineering, Imperial College London, UK, and also with the Dept. of Engineering and Architecture, University of Trieste, Italy*

Abstract

This paper develops a nonlinear observer-based approach for distributed fault detection of a class of interconnected input-output nonlinear systems, which is robust to modeling uncertainty and measurement noise. First, a nonlinear observer design is used to generate the residual signals required for fault detection. Then, a distributed fault detection scheme and the corresponding adaptive thresholds are designed based on the observer characteristics and, at the same time, filtering is used in order to attenuate the effect of measurement noise, which facilitates less conservative thresholds and enhanced robustness. Finally, a fault detectability condition characterizing quantitatively the class of detectable faults is derived.

Key words: Fault detection; Nonlinear observers; Nonlinear uncertain systems; Filtering

1 Introduction

Many of the vital services of everyday life depend on highly complex and interconnected engineering systems, where potential faults could lead to performance degradation, or even trigger a chain of failing subsystems, which may cause major catastrophes. The safe and reliable operation of such systems through the early detection of a "small" fault before it becomes a serious failure is a crucial component of the overall system performance and sustainability.

Over the last two decades the fault detection and isolation (FDI) problem has been examined intensively. In most real world applications the presence of modeling uncertainty and measurement noise may influence significantly the performance of fault detection schemes. In addition, recent advances in distributed sensing and communications motivated the investigation of not only centralized fault diagnosis approaches but also the development of hierarchical, decentralized and distributed schemes, most of which assume the availability of all state variables [6,12,14,15]. In many cases, a distributed FDI framework is not an option but a necessity, since many factors contribute to this formulation such as the large scale nature of the system to be monitored, its spatial distribution, the inability to access certain parts of the system from a remote monitoring component and therefore local diagnosis should be performed.

In the case of input-output nonlinear systems, several papers dealing with the fault diagnosis problem have appeared but, as in the full-state measurement case, the vast majority of them address the problem in a centralized framework [4,21,23,24]. Lately, special attention has been given to decentralized and distributed fault detection approaches [3,12,13,19,22]. One of the key methodologies for fault diagnosis of input-output systems is the observer-based approach. As pointed out in [7], the observers used in fault diagnosis are primarily output observers which simply reconstruct the measurable part of the state variables, rather than state observers which are required for control purposes. The use of state observers for nonlinear systems has not been used extensively for the FDI problem, even though analytical results regarding the stability of the nonlinear observers and design procedures have been established [2, 9, 17, 18, 25]. The

⋆ This paper was not presented at any IFAC meeting. Corresponding author T. Parisini. Tel. +44 (0) 20 75946240.

Email addresses: ckeliris@ucy.ac.cy (Christodoulos Keliris), mpolycar@ucy.ac.cy (Marios M. Polycarpou), t.parisini@gmail.com (Thomas Parisini).

main issue with the observer approach is that the design of observers for nonlinear systems with asymptotically stable error dynamics is not an easy task even when the nonlinearities are fully known. As a result, the research in fault diagnosis for nonlinear systems utilizing state observers is more limited [1, 8, 10, 22].

In this paper, we propose a nonlinear observer-based approach for distributed fault detection of a class of interconnected input-output nonlinear systems, which is robust to modeling uncertainty and measurement noise. The use of nonlinear observer design allows for a particular class of input-output systems to be considered. Specifically, in order to deal with the fault detection task, a nonlinear observer is designed for each subsystem that guarantees that the state estimation error, for the nominal nonlinear system, converges to zero. Then, the designed observer is combined with filtering for attenuating measurement noise and is used in a novel way for the derivation of suitable adaptive thresholds for the filtered state estimation error of the uncertain system, which guarantee no false alarms. Therefore, the fault detection scheme is inherently tied with the nonlinear observer design. In addition, a general class of filters is integrated in the design for the purpose of attenuating the measurement noise and hence it facilitates the design of tight, adaptive detection thresholds. This filtering approach for nonlinear fault diagnosis was first developed in [12] where the case of full-state measurement was considered and a rigorous investigation of the filtering impact (according to the poles' location and filters' order) on the detection time was presented. In this work, we extend significantly the approach given in [12] by relaxing the assumption of the availability of all the state measurements (through the design of a nonlinear observer) whilst maintaining the use of filters for dampening the uncertainty effects. Due to the lack of full-state measurement, the analytical treatment of the filtering design in this paper is different than the one in [12]. The novelty in the filtering approach in this work stems from its treatment as a linear state transformation, which allows a more general class of filters to be considered. It must be pointed out, that this paper extends significantly the work by the same authors in [13], where a simplified problem formulation for the input-output case is investigated in which the nonlinear observer design is not needed for fault detection purposes, since the nonlinearity term in [13] contains only terms that can be measured with some uncertainty. The nonlinear observer design was also not required in [6, 12] since full state measurement was considered. Finally, the distributed fault detection scheme is based on local fault filtering schemes with each one assigned to monitor one subsystem and provide a decision regarding its health.

The paper is organized as follows: in Section 2, the problem formulation for distributed fault detection of a class of input-output nonlinear dynamical systems with modeling uncertainty and measurement noise is presented and in Section 3, details regarding the nonlinear observer design are given. In Section 4, the design of the distributed fault detection scheme based on Lyapunov analysis combined with a filtering approach is presented in detail and, in Section 5, a fault detectability condition is derived. In Section 6, a simulation example illustrates the concepts presented and finally, Section 7 provides some concluding remarks.

2 Problem Formulation

Consider a large-scale distributed nonlinear dynamic system, which is comprised of N subsystems Σ_I, $I \in \{1, ..., N\}$, described by:

$$\Sigma_I : \begin{cases} \dot{x}_I(t) = A_I x_I(t) + f_I\big(x_I(t), \bar{C}_I \bar{x}_I(t), u_I(t)\big) \\ \qquad\quad + \eta_I\big(x_I(t), \bar{x}_I(t), u_I(t), t\big) \\ \qquad\quad + \beta_I(t - T_0)\phi_I\big(x(t), u_I(t)\big) & (1) \\ y_I(t) = C_I x_I(t) + \xi_I(t), & (2) \end{cases}$$

where $x_I \in \mathbb{R}^{n_I}$, $u_I \in \mathbb{R}^{m_I}$ and $y_I \in \mathbb{R}^{p_I}$ are the state, input and measured output vectors of the I-th subsystem respectively and $x \triangleq \left[x_1^\top, x_2^\top, \dots, x_N^\top\right]^\top \in \mathbb{R}^n$ is the state vector of the overall system. The vectors $\bar{x}_I \in \mathbb{R}^{\bar{n}_I}$ and $\bar{C}_I \bar{x}_I \in \mathbb{R}^{\bar{p}_I}$ denote the state variables and the corresponding output variables, respectively, of neighboring subsystems that affect the I-th subsystem. Specifically, the interconnection variables $\bar{C}_I \bar{x}_I$ are a subset of noiseless output variables of neighboring subsystems and, this special form is required for the design of the nonlinear observer. The matrix $A_I \in \mathbb{R}^{n_I} \times \mathbb{R}^{n_I}$ and the function $f_I : \mathbb{R}^{p_I} \times \mathbb{R}^{\bar{p}_I} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ characterize the known nominal function dynamics of the I-th subsystem and, are derived from the linearization at the origin of the I-th nominal nonlinear subsystem. Note that the function f_I, which contains only terms strictly higher than a linear function with respect to x_I [18], contains also the known part of the interconnection function between the I-th and its neighboring subsystems, and moreover, note that the influence of the interconnected subsystems is known with some uncertainty (measurement noise). The matrix $C_I \in \mathbb{R}^{p_I} \times \mathbb{R}^{n_I}$ is the known nominal output matrix of the I-th subsystem. The vector function $\eta_I : \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{m_I} \times \mathbb{R}^+ \mapsto \mathbb{R}^{n_I}$ denotes the modeling uncertainty associated with the nominal dynamics and $\xi_I \in \mathcal{D}_{\xi_I} \subset \mathbb{R}^{p_I}$ ($\mathcal{D}_{\xi_I}$ is a known compact set) represents the measurement noise. The term $\beta_I(t - T_0)\phi_I(x, u_I)$ characterizes the time-varying fault function dynamics affecting the I-th subsystem. More specifically, the term $\phi_I : \mathbb{R}^n \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ represents the unknown fault function and the term $\beta_I(t - T_0) : \mathbb{R} \mapsto \mathbb{R}^+$ models the time evolution of the fault, where T_0 is the unknown time of the fault occurrence. Note that the fault function ϕ_I may depend on the global state variable vector x and not only on the local state vector x_I. From a practical perspective, this allows for propagative fault effects to be transferred across neighboring subsystems (as it is the case in real networks such as electric power systems, transportation systems, etc). In this work, no particular modeling is considered for the time profile $\beta_I(t - T_0)$

which can be used to model both abrupt and incipient faults. Instead, we simply consider it to be zero prior to the fault occurrence, i.e. $\beta_I(t - T_0) = 0$, for all $t < T_0$.

In this paper, we do not deal explicitly with the control problem. Therefore, it is assumed that there exist feedback controllers for selecting u_I such that some desired control objectives are achieved.

The notation $\|\cdot\|$ used in this paper denotes the Euclidean 2-norm for vectors and, the matrix norm induced by the 2-norm for matrices. The following assumptions are used throughout the paper:

***Assumption* 1.** *For each subsystem Σ_I, $I \in \{1, ..., N\}$, the pair (A_I, C_I) is observable.*

***Assumption* 2.** *For each subsystem Σ_I, $I \in \{1, ..., N\}$ the local state variables $x_I(t)$ and the local inputs $u_I(t)$ remain bounded before and after the occurrence of a fault (well-posedness).*

***Assumption* 3.** *The modeling uncertainty η_I in each subsystem is an unstructured and possibly unknown nonlinear function of x_I, $\bar{x}_I$, u_I and t but whose norm is bounded by a known positive function $\bar{\eta}_I$:*

$$\|\eta_I(x_I, \bar{x}_I, u_I, t)\| \leq \bar{\eta}_I(y_I, \bar{y}_I, u_I), \tag{3}$$

for all $t \geq 0$ and for all $(x_I, \bar{x}_I, u_I) \in \mathcal{D}_I$, where $\bar{y}_I \in \mathcal{D}_{\bar{y}_I} \subset \mathbb{R}^{\bar{p}_I}$ is the measurable noisy counterpart of $\bar{C}_I\bar{x}_I$, i.e. $\bar{y}_I = \bar{C}_I\bar{x}_I + \bar{\xi}_I$, $\bar{\xi}_I \in \mathcal{D}_{\bar{\xi}_I} \subset \mathbb{R}^{\bar{p}_I}$ and $\bar{\eta}_I(y_I, \bar{y}_I, u_I) \geq 0$ is a known bounding function in some region of interest $\mathcal{D}_I = \mathcal{D}_{x_I} \times \mathcal{D}_{\bar{x}_I} \times \mathcal{D}_{u_I} \subset \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{m_I}$. The regions $\mathcal{D}_{\bar{\xi}_I}$, $\mathcal{D}_{\bar{y}_I}$ and $\mathcal{D}_I$ are known compact sets.

Assumption 1 is required for the design of a nonlinear observer to allow estimation of the unmeasurable state variables, while Assumption 2 is needed for well-posedness, since we do not consider the fault accommodation problem in this paper. Assumption 3 characterizes the class of modeling uncertainties being considered.

***Assumption* 4.** *The function $f_I(x_I, \bar{y}_I, u_I)$ satisfies the generalized Lipschitz condition:*

$$\|f_I(z_1, \bar{y}_I, u_I) - f_I(z_2, \bar{y}_I, u_I)\| \leq \|G_{f_I}(z_1 - z_2)\|, \tag{4}$$

for $z_1, z_2 \in \mathcal{D}_{x_I}, \bar{y}_I \in \mathcal{D}_{\bar{y}_I}, u_I \in \mathcal{D}_{u_I}$ where $G_{f_I} \in \mathbb{R}^{n_I \times n_I}$ is a known constant matrix.

The condition stated in Assumption 4 is less conservative than the standard Lipschitz condition which uses a Lipschitz constant $\gamma_{f_I} > 0$ instead of the matrix G_{f_I} [16]. Note that the standard Lipschitz condition can also be used by replacing G_{f_I} with $\gamma_{f_I} I$ in what follows.

In general, the distributed fault detection scheme is composed of N local filtered fault detection modules Ω_I, one for each subsystem Σ_I. Each local fault detection module Ω_I requires the input and output measurements of the subsystem Σ_I that is monitoring and also the measurements of all interconnecting subsystems Σ_J that are influencing Σ_I. Note that these latter measurements are communicated by neighboring fault detection modules Ω_J, and not by the subsystems Σ_J. Therefore, there is the need of communication between the fault detection modules depending on their interconnections. Note that, the interconnection variables $\bar{C}_I\bar{x}_I$ are measurable with some uncertainty as $\bar{y}_I = \bar{C}_I\bar{x}_I + \bar{\xi}_I$. Hence, the fault detection modules exchange these measurements and are used by the nonlinear observer for the state estimation and for the generation of the residual and threshold signals. Therefore, the distributed nature of the scheme stems from the fact that there is communication between the fault detection modules. An example of the distributed fault detection scheme is shown in Figure 1.

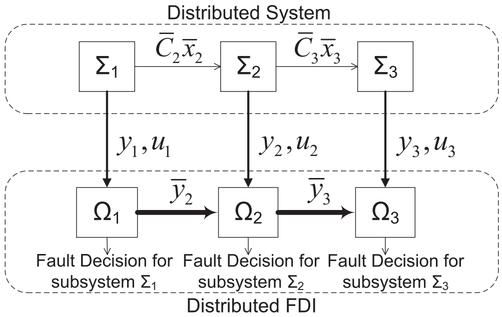


Fig. 1. Distributed fault detection scheme for the case of 3 subsystems Σ_1 - Σ_3, where Σ_1 affects Σ_2 and Σ_2 affects Σ_3.

To dampen the effects of measurement uncertainties $\xi_I(t)$, each measured variable $y_I^{(k)}(t)$ (k-th component of y_I) is filtered by $H_p(s)$, where $H_p(s)$ is a p-th order, strictly proper, asymptotically stable transfer function

$$H_p(s) = \frac{d_{p-1}s^{p-1} + d_{p-2}s^{p-2} + \ldots + d_0}{s^p + c_{p-1}s^{p-1} + \ldots + c_1 s + c_0}. \tag{5}$$

The filter $H_p(s)$ is asymptotically stable and hence BIBO stable. Therefore, for bounded measurement noise $\xi_I(t)$, the filtered measurement noise $\epsilon_{\xi_I}(t) \triangleq H_p(s)[\xi_I(t)]$ (filtering each component of $\xi_I(t)$) is bounded, i.e. $\|\epsilon_{\xi_I}(t)\| \leq \bar{\epsilon}_{\xi_I}(t), \quad \forall t \geq 0$, where $\bar{\epsilon}_{\xi_I}(t)$ is assumed to be a known bounding function. Note that, a different filter $H_p(s)$ can be used by each fault detection module Ω_I, but in this work, without loss of generality, we consider the same filter for simplicity.

The local fault detection module Ω_I is made of the local nonlinear observer, the local residual signal and the corresponding detection threshold. The local nonlinear observer is given by

$$\begin{aligned}\dot{\hat{x}}_I(t) =& A_I\hat{x}_I(t) + f_I\big(\hat{x}_I(t), \bar{y}_I(t), u_I(t)\big) \\ &+ L_I\big(y_I(t) - \hat{y}_I(t)\big) \qquad (6)\\ \hat{y}_I(t) =& C_I\hat{x}_I(t), \qquad (7)\end{aligned}$$

where $L_I \in \mathbb{R}^{n_I \times p_I}$ is the observer gain matrix (to be designed) with $\hat{x}_I(0) = 0$ for simplicity. The local residual vector signal $r_I(t)$ is generated according to

$$r_I(t) \triangleq H_p(s)\big[y_I(t) - \hat{y}_I(t)\big]. \tag{8}$$

The decision regarding the detection of a fault is made when $\|r_I(t)\| \geq \bar{r}_I(t)$ in any local fault detection module Ω_I, where $\bar{r}_I(t)$ is the detection threshold (to be specified later on). To summarize, for each subsystem Σ_I the proposed fault filtered scheme Ω_I relies on designing a nonlinear observer gain matrix L_I (Section 3), selecting a filter with transfer function $H_p(s)$ of the form of (5) and obtaining a computable adaptive detection threshold $\bar{r}_I(t)$ (Section 4).

3 Nonlinear Observer Design

In this section, we deal with the problem of the design of the local nonlinear observer so that the state estimation error for the nominal part of the nonlinear subsystem Σ_I under healthy mode of operation converges to zero exponentially fast.

First, we rewrite the dynamics of Σ_I given in (1) as:

$$\begin{aligned}\dot{x}_I(t) =& A_I x_I(t) + f_I\big(x_I(t), \bar{y}_I(t), u_I(t)\big) + \Delta f_I(t) \\ &+ \eta_I\big(x_I(t), \bar{x}_I(t), u_I(t), t\big) \\ &+ \beta_I(t - T_0)\phi_I\big(x(t), u_I(t)\big) \end{aligned} \tag{9}$$

where

$\Delta f_I(t) \triangleq f_I\big(x_I(t), \bar{C}_I\bar{x}_I(t), u_I(t)\big) - f_I\big(x_I(t), \bar{y}_I(t), u_I(t)\big)$.

It is worth noting that in (9), the functions Δf_I, η_I and ϕ_I are in general unknown. By excluding these functions, we define the known nominal system as follows:

$$\dot{x}_{I,N}(t) = A_I x_{I,N}(t) + f_I\big(x_{I,N}(t), \bar{y}_I(t), u_I(t)\big) \tag{10}$$

$$y_{I,N}(t) = C_I x_{I,N}(t). \tag{11}$$

The nonlinear observer given in (6), (7) is constructed based on the dynamics described by (10), (11). By using (10) and (6), the nominal state estimation error $\tilde{x}_{I,N} \triangleq x_{I,N} - \hat{x}_I$ (based on the nominal system so that $y_I = y_{I,N}$ in (6)) satisfies the nominal error dynamics

$$\begin{aligned}\dot{\tilde{x}}_{I,N}(t) =& A_{I,0}\tilde{x}_{I,N}(t) + f_I\big(x_{I,N}(t), \bar{y}_I(t), u_I(t)\big) \\ &- f_I\big(\hat{x}_I(t), \bar{y}_I(t), u_I(t)\big). \end{aligned} \tag{12}$$

where $A_{I,0} \triangleq A_I - L_I C_I$. The proposed fault detection scheme (to be presented in Section 4) relies on designing a nonlinear observer such that the nominal error dynamics satisfy

$$\dot{V}_I(\tilde{x}_{I,N}) < -\mu_I V_I(\tilde{x}_{I,N})$$

for some given $\mu_I > 0$, where the Lyapunov function candidate $V_I(\tilde{x}_{I,N}) = \tilde{x}_{I,N}^\top P_I \tilde{x}_{I,N}$ with $P_I > 0$ and $P_I \in \mathbb{R}^{n_I \times n_I}$ is considered. It must be pointed out though, that any nonlinear observer of the form (6), (7) satisfying the aforementioned condition can be used for the fault detection task. Below, we propose such an observer by modifying the one described in [16] by adapting it to our problem formulation and by adding one extra degree of freedom (μ_I), which is required for the fault detection task. For brevity, the proof of the following Lemma 1 is omitted, since it follows along the lines of [16].

Lemma 1. *Consider the nominal nonlinear system described in (10), (11) which satisfies the generalized Lipschitz condition (4). Then, there exists a nonlinear observer as described in (6), (7) such that the state estimation error $\tilde{x}_{I,N}$ is quadratically stabilized if and only if there exist $\mu_I \geq 0$, $\epsilon_I > 0$ and $\delta_I \in \mathbb{R}$ such that the following Riccati inequality has a symmetric, positive definite solution P_I:*

$$\bar{A}_I^\top P_I + P_I\bar{A}_I + \epsilon_I G_{f_I}^\top G_{f_I} + \frac{P_I P_I}{\epsilon_I} - \delta_I^2 C_I^\top C_I < 0 \tag{13}$$

where $\bar{A}_I \triangleq A_I + \frac{1}{2}\mu_I I$. The observer gain matrix can then be chosen as $L_I = \frac{\delta_I^2}{2} P_I^{-1} C_I^\top$.

According to this design, the nominal error satisfies $\dot{V}_I(\tilde{x}_{I,N}) < -\mu_I V_I(\tilde{x}_{I,N})$ for some given $\mu_I \geq 0$ and hence it converges to zero. For fault detection purposes it is required that $\mu_I > 0$; after some mathematical manipulations the nominal state estimation error satisfies:

$$\|\tilde{x}_{I,N}(t)\| < \alpha_I e^{-\frac{\mu_I}{2}t}\|x_{I,N}(0)\| \tag{14}$$

where $\alpha_I \triangleq \sqrt{\frac{\lambda_{max}(P_I)}{\lambda_{min}(P_I)}}$. Note that $\tilde{x}_{I,N}(0) = x_{I,N}(0)$ because $\hat{x}_I(0)$ is considered zero for simplicity. The requirement $\mu_I > 0$ is needed to guarantee the exponential convergence to zero of the nominal state estimation error and, to allow for the filters' stability in the detection threshold (to be designed). Additionally, the extra degree of freedom μ_I gives greater flexibility in solving (13), in the sense that various solutions can be obtained by varying its value.

4 Distributed Fault Detection

In this section, we exploit the nonlinear observer properties derived in the previous section for the task of fault detection. The observer design is based on the known nominal system dynamics, not taking into account the unknown components; i.e., the uncertainty term η_I, the function discrepancy $\Delta f_I(t)$ and the measurement noise ξ_I, which affect the performance of the observer. In particular, the measurement noise term may create significant challenges in fault diagnosis since it is difficult to strike a balance between conservative detection thresholds and avoiding the presence of false alarms.

In the sequel we will use the following Lemmas.

Lemma 2. *[5]. The impulse response $h_p(t)$ of a strictly proper and asymptotically stable transfer function $H_p(s)$ decays exponentially; i.e., $|h_p(t)| \leq \kappa e^{-\upsilon t}$ for some $\kappa > 0$, $\upsilon > 0$, for all $t > 0$.*

Lemma 3. *[20]. For any $z_1, z_2 \in \mathbb{R}^n$ and any positive definite matrix $P \in \mathbb{R}^{n\times n}$ the following inequality holds:*

$$2z_I^\top z_2 \leq z_1^\top P z_1 + z_2^\top P^{-1} z_2.$$

Lemma 4. *[11]. Let $w(t), V(t) : [0, \infty) \mapsto \mathbb{R}$. Then, for any finite α, if $\dot{V}(t) \leq -\alpha V(t) + w(t)$ for all $t \geq t_0 \geq 0$ then*

$$V(t) \leq e^{-\alpha(t-t_0)}V(t_0) + \int_{t_0}^{t} e^{-\alpha(t-\tau)} w(\tau)\,\mathrm{d}\tau, \quad \forall t \geq t_0 \geq 0.$$

In the following, $z_f(t) \triangleq H_p(s)\big[z(t)\big]$ denotes the filtered version of any signal $z(t)$ which is passed through a filter with transfer function $H_p(s)$. Let $h_p(t)$ be the impulse response associated with $H_p(s)$; i.e. $h_p(t) \triangleq \mathcal{L}^{-1}\left[H_p(s)\right]$. Then $z_f(t)$ can be written as $z_f(t) = \int_0^t h_p(\tau) z(t-\tau)\,\mathrm{d}\tau$. By taking the derivative of $z_f(t)$ and using the Leibniz integral rule, we obtain

$$\dot{z}_f(t) = H_p(s)\left[\dot{z}(t)\right] + h_p(t) z(0). \tag{15}$$

By using (12) and (15) with $z = \tilde{x}_{I,N}$, the filtered nominal estimation error $\tilde{x}_{I,N,f}$ satisfies:

$$\begin{aligned}\dot{\tilde{x}}_{I,N,f}(t) &= H_p(s)\big[\dot{\tilde{x}}_{I,N}(t)\big] + h_p(t)\tilde{x}_{I,N}(0)\\ &= A_{I,0}\tilde{x}_{I,N,f}(t) + H_p(s)\big[f_I\big(x_{I,N}(t), \bar{y}_I(t), u_I(t)\big)\\ &\quad - f_I\big(\hat{x}_I(t), \bar{y}_I(t), u_I(t)\big)\big] + h_p(t) x_{I,N}(0).\end{aligned} \tag{16}$$

It is worth noting that the term $x_{I,N}(0)h_p(t)$ in (16) converges to zero because of Lemma 2.

In the following analysis we prove that the filtered nominal state estimation error $\tilde{x}_{I,N,f}$ converges to zero with exponential speed. This will be used as the basis for the derivation of an adaptive detection threshold for the filtered state estimation error $\tilde{x}_{I,f}(t) = H_p(s)[\tilde{x}_I(t)]$, where $\tilde{x}_I \triangleq x_I - \hat{x}_I$. By using Lemma 2 and (14), the norm of $\tilde{x}_{I,N,f}(t)$ satisfies:

$$\begin{aligned}\|\tilde{x}_{I,N,f}(t)\| &= \|H_p(s)\big[\tilde{x}_{I,N}(t)\big]\|\\ &\le \int_0^t |h_p(\tau)|\|\tilde{x}_{I,N}(t-\tau)\|\,\mathrm{d}\tau \qquad (17)\\ &< \int_0^t \kappa e^{-\upsilon\tau}\alpha_I e^{-\frac{\mu_I}{2}(t-\tau)}\|x_{I,N}(0)\|\,\mathrm{d}\tau. \qquad (18)\end{aligned}$$

By solving the integral we obtain:

$$\|\tilde{x}_{I,N,f}(t)\| < \begin{cases}\alpha_I\kappa\|x_{I,N}(0)\| t e^{-\frac{\mu_I}{2}t} & \text{if } \upsilon = \frac{\mu_I}{2},\\ \alpha_I\kappa\|x_{I,N}(0)\|\frac{e^{-\upsilon t} - e^{-\frac{\mu_I}{2}t}}{\frac{\mu_I}{2} - \upsilon} & \text{if } \upsilon \neq \frac{\mu_I}{2}.\end{cases}$$

Note that $\lim_{t\to\infty}\|\tilde{x}_{I,N,f}(t)\| = 0$ for any $\mu_I, \upsilon > 0$ and in fact the filtered nominal state estimation error $\tilde{x}_{I,N,f}$ converges to zero exponentially fast; i.e. there exists a sufficiently small $\rho_I > 0$ such that

$$\|\tilde{x}_{I,N,f}(t)\| < \alpha_I e^{-\frac{\rho_I}{2}t}\|x_{I,N}(0)\|. \tag{19}$$

In other words, since the nominal state estimation error dynamics $\tilde{x}_{I,N}$ converge to zero exponentially fast with a convergence speed of $\frac{\mu_I}{2}$ (see (14)), the filtered nominal state estimation error $\tilde{x}_{I,N,f}$ still converges to zero exponentially fast and we need to find a sufficient convergence rate $\frac{\rho_I}{2}$, something that is dealt with later. In addition, note that since the filtered nominal state estimation error $\tilde{x}_{I,N,f}$ decays exponentially, it satisfies a differential inequality of the form

$$\dot{V}_I(\tilde{x}_{I,N,f}) < -\rho_I V_I(\tilde{x}_{I,N,f}) \tag{20}$$

with initial condition $\tilde{x}_{I,N,f}(0) = x_{I,N}(0)$. This is similar to the case of the nominal state estimation error $\tilde{x}_{I,N}$ which satisfies $\dot{V}_I(\tilde{x}_{I,N}) < -\mu_I V_I(\tilde{x}_{I,N})$ and (14).

Therefore, by combining (17), (14) and (19), a sufficient condition for (20) to be satisfied for all $t > t^*$ is:

$$\int_0^t |h_p(\tau)| e^{-\frac{\mu_I}{2}(t-\tau)}\,\mathrm{d}\tau < e^{-\frac{\rho_I}{2}t} \quad \forall t > t^*. \tag{21}$$

Thus, ρ_I must be selected so that (21) is satisfied. The time $t^* \ge 0$ is selected by the designer and constitutes a time instant for which the detection results in the time interval $[0, t^*]$ are ignored and its purpose is to allow greater flexibility in choosing ρ_I. As it is clearly indicated by (21), the selection of a suitable value for ρ_I depends on the filters' impulse response $h_p(t)$ and the nonlinear observer design (due to μ_I).

In order to select ρ_I, we first select a time $t^* \ge 0$. Then, for a given filter (hence $h_p(t)$) and a given nonlinear observer design that satisfies the required condition $\dot{V}_I(\tilde{x}_{I,N}) < -\mu_I V_I(\tilde{x}_{I,N})$ (hence given μ_I), we verify for various values of ρ_I that (21) holds for all $t > t^*$ through numerical methods by trying to solve (21) as equality and verifying that no solution exists; i.e. the two sides of (21) do not cross for $t > t^*$. Actually, the left part of (21) can be bounded by an exponentially decaying function due to the asymptotic stability of the filter (i.e. see Lemma 2) and hence, there always exists a sufficiently small exponential decay speed ρ_I. Having (21) satisfied for all $t > t^*$ rather than $t > 0$ provides greater flexibility in choosing ρ_I since the left and right side of (21) are allowed to cross at time t^* and hence (20) is satisfied for all $t > t^*$. This comes at the mere cost of having to initiate the fault detection scheme at time zero but ignoring the detection results in the time interval $[0, t^*]$ since false alarms may occur during this time (because (20) might not hold for $t \le t^*$). In any case, no false alarms are guaranteed and the only downside of using t^* is the possibility of delayed fault detection at a time after t^* in the case that a fault occurs before t^*.

Note that, by using $V_I(\tilde{x}_{I,N,f}) = \tilde{x}_{I,N,f}^\top P_I \tilde{x}_{I,N,f}$, its time derivative along the trajectories of the filtered nominal error dynamics (16) satisfies:

$$\begin{aligned}\dot{V}_I(\tilde{x}_{I,N,f}) &= 2\tilde{x}_{I,N,f}^\top P_I\Big[H_p(s)\big[f_I\big(x_{I,N}(t), \bar{y}_I(t), u_I(t)\big)\\ &\qquad - f_I\big(\hat{x}_I(t), \bar{y}_I(t), u_I(t)\big)\big] + h_p(t) x_{I,N}(0)\Big]\\ &\qquad + \tilde{x}_{I,N,f}^\top\big[A_{I,0}^\top P_I + P_I A_{I,0}\big]\tilde{x}_{I,N,f} \qquad (22)\\ &< -\rho_I V_I(\tilde{x}_{I,N,f}),\end{aligned}$$

where the last inequality was derived by using (20).

Now, let's consider the uncertain nonlinear system (9), (2) with the designed nonlinear observer given in (6), (7). In the absence of a fault, the error $\tilde{x}_I$ satisfies:

$$\begin{aligned}\dot{\tilde{x}}_I(t) =& A_{I,0}\tilde{x}_I(t) + \tilde{\Delta} f_I(t) + \Delta f_I(t)\\ &+ \eta_I\big(x_I(t), \bar{x}_I(t), u_I(t), t\big) - L_I\xi_I(t), \qquad (23)\end{aligned}$$

where $\tilde{\Delta} f_I(t) \triangleq f_I\big(x_I(t), \bar{y}_I(t), u_I(t)\big)$ $- f_I\big(\hat{x}_I(t), \bar{y}_I(t), u_I(t)\big)$. By using (23) and (15) with $z = \tilde{x}_I$, the filtered state estimation error $\tilde{x}_{I,f}$ satisfies:

$$\begin{aligned}\dot{\tilde{x}}_{I,f}(t) &= H_p(s)\big[\dot{\tilde{x}}_I(t)\big] + h_p(t)\tilde{x}_I(0)\\ &= A_{I,0}\tilde{x}_{I,f}(t) + \tilde{\Delta} f_{I,f}(t) + h_p(t)x_I(0) + \chi_I(t) \quad (24)\end{aligned}$$

where $\chi_I(t) \triangleq H_p(s)\left[\eta_I(x_I, \bar{x}_I, u_I, t)\right] + \epsilon_{\Delta_I}(t) - L_I\epsilon_{\xi_I}(t)$, $\epsilon_{\Delta_I}(t) \triangleq H_p(s)\left[\Delta f_I(t)\right]$. Therefore, the time derivative of $V_I(\tilde{x}_{I,f})$ along the trajectories of the filtered error dynamics (24) satisfies:

$$\begin{aligned}\dot{V}_I(\tilde{x}_{I,f}) &= 2\tilde{x}_{I,f}^\top P_I\big(\tilde{\Delta} f_{I,f}(t) + h_p(t)x_I(0)\big)\\ &+ \tilde{x}_{I,f}^\top\big[A_{I,0}^\top P_I + P_I A_{I,0}\big]\tilde{x}_{I,f} + 2\tilde{x}_{I,f}^\top P_I\chi_I(t). \quad (25)\end{aligned}$$

Note that, the first two terms in (25) are as in (22) and hence, (25) can be re-written as

$$\dot{V}_I(\tilde{x}_{I,f}) < -\rho_I V_I(\tilde{x}_{I,f}) + 2\tilde{x}_{I,f}^\top P_I\chi_I(t) \qquad (26)$$

with initial condition $\tilde{x}_{I,f}(0) = x_I(0)$.

For the derivation of the detection threshold, we will use the following assumption.

Assumption 5. *The norm of the filtered function mismatch term $\epsilon_{\Delta_I}(t)$ is bounded by a computable positive function $\bar{\epsilon}_{\Delta_I}(t)$; i.e., $\|\epsilon_{\Delta_I}(t)\| \le \bar{\epsilon}_{\Delta_I}(t), \quad \forall t \ge 0$.*

Assumption 5 is based on the observation that filtering dampens the error effect of measurement noise present in the function mismatch term $\Delta f_I(t)$. A suitable selection of the bound $\bar{\epsilon}_{\Delta_I}$ can be made through the use of simulations by filtering the function mismatch term $\Delta f_I(t)$ using the known nominal function dynamics and the available noise characteristics.

Now, consider the term $\chi_I(t)$ which satisfies:

$$\begin{aligned}\|\chi_I(t)\| &= \|H_p(s)\left[\eta_I(x_I, \bar{y}_I, u_I, t)\right] + \epsilon_{\Delta_I}(t) - L_I\epsilon_{\xi_I}(t)\|\\ &\le \int_0^t |h_p(t-\tau)|\|\eta_I(x_I(\tau), \bar{y}_I(\tau), u_I(\tau), \tau)\|\,\mathrm{d}\tau\\ &+ \bar{\epsilon}_{\Delta_I}(t) + \|L_I\|\bar{\epsilon}_{\xi_I}(t). \qquad (27)\end{aligned}$$

Based on (27), the designed bounding function $\bar{\chi}_I(t)$ such that $\|\chi_I(t)\| \le \bar{\chi}_I(t)$ is given by:

$$\begin{aligned}\bar{\chi}_I(t) \triangleq& \bar{H}_p(s)\left[\bar{\eta}_I(y_I(t), \bar{y}_I(t), u_I(t))\right] + \|L_I\|\bar{\epsilon}_{\xi_I}(t)\\ &+ \bar{\epsilon}_{\Delta_I}(t), \qquad (28)\end{aligned}$$

where $\bar{H}_p(s)$ is a transfer function (with impulse response $\bar{h}_p(t)$) given by $\bar{H}_p(s) = \frac{\kappa}{s+\upsilon}$ (determined using Lemma 2). Note that if $h_p(t)$ is non-negative, i.e. $h_p(t) \ge 0$, for all $t \ge 0$, then the calculation of $\bar{H}_p(s)$ can be omitted since $|h_p(t-\tau)| = h_p(t-\tau)$ (see (27)).

In the following, we proceed to "decouple" the last term of (26). Using Lemma 3, for any positive scalar γ_I we can write the sign-indefinite term $2\tilde{x}_{I,f}^\top P_I\chi_I(t)$ in (26) using $z_1 = \sqrt{\gamma_I}\tilde{x}_{I,f}$, $z_2 = \frac{1}{\sqrt{\gamma_I}}P_I\chi_I(t)$ and $P = P_I$ as follows:

$$\begin{aligned}2\tilde{x}_{I,f}^\top P_I\chi_I(t) \le& \gamma_I\tilde{x}_{I,f}^\top P_I\tilde{x}_{I,f} + \gamma_I^{-1}\chi_I^\top(t)P_I\chi_I(t)\\ \le& \gamma_I V_I(\tilde{x}_{I,f}) + \gamma_I^{-1}\lambda_{max}(P_I)\bar{\chi}_I^2(t). \quad (29)\end{aligned}$$

Therefore, (26) can be written as

$$\dot{V}_I(\tilde{x}_{I,f}) + (\rho_I - \gamma_I)V_I(\tilde{x}_{I,f}) < \gamma_I^{-1}\lambda_{max}(P_I)\bar{\chi}_I^2(t). \qquad (30)$$

Therefore, by using Lemma 4, (30) becomes

$$\begin{aligned}V_I(\tilde{x}_{I,f}) <& e^{-(\rho_I-\gamma_I)t}V_I(\tilde{x}_{I,f}(0))\\ &+ \int_0^t e^{-(\rho_I-\gamma_I)(t-\tau)}\gamma_I^{-1}\lambda_{max}(P_I)\bar{\chi}_I^2(\tau)\,\mathrm{d}\tau.\end{aligned}$$

By utilizing linear filtering techniques, the previous inequality can be written as

$$\begin{aligned}V_I(\tilde{x}_{I,f}) <& e^{-(\rho_I-\gamma_I)t}V_I(\tilde{x}_{I,f}(0))\\ &+ \frac{1}{s + (\rho_I - \gamma_I)}\left[\gamma_I^{-1}\lambda_{max}(P_I)\bar{\chi}_I^2(t)\right].\end{aligned}$$

Hence, by using $\lambda_{min}(P_I)\|\tilde{x}_{I,f}\|^2 \le V_I(\tilde{x}_{I,f}) \le \lambda_{max}(P_I)\|\tilde{x}_{I,f}\|^2$ we obtain an adaptive bound $\tilde{x}_{I,f}^{max}(t)$ such that $\|\tilde{x}_{I,f}(t)\| < \tilde{x}_{I,f}^{max}(t)$ and is given by

$$\tilde{x}_{I,f}^{max}(t) \triangleq \alpha_I\Big(e^{-(\rho_I-\gamma_I)t}\bar{x}_{I,d}^2 + \frac{\gamma_I^{-1}}{s + (\rho_I - \gamma_I)}\left[\bar{\chi}_I^2(t)\right]\Big)^{\frac{1}{2}}, \qquad (31)$$

where $\bar{x}_{I,d}$ is a bounding estimate of $x_I(0)$ such that $\|x_I(0)\| \le \bar{x}_{I,d}$ and γ_I is selected so that $0 < \gamma_I < \rho_I$ so that the filter is stable.

Using (2) and (7), the residual (8) satisfies

$$\begin{aligned}\|r_I(t)\| &= \|H_p(s)\big[C_I\tilde{x}_I(t) + \xi_I(t)\|\big]\\ &\le \|C_I\|\|\tilde{x}_{I,f}(t)\| + \|\epsilon_{\xi_I}(t)\|.\end{aligned}$$

Finally, by using the bound on $\|\tilde{x}_{I,f}(t)\|$, the detection threshold $\bar{r}_I(t)$ so that $\|r_I(t)\| < \bar{r}_I(t)$ is given by:

$$\bar{r}_I(t) \triangleq \|C_I\|\tilde{x}_{I,f}^{max}(t) + \bar{\epsilon}_{\xi_I}. \qquad (32)$$

The following theorem summarizes the findings.

Theorem 1. *Consider the nominal nonlinear system described in (10), (11) satisfying the generalized Lipschitz assumption (4) and the nonlinear observer described in (6), (7) that is designed such that the nominal state estimation error $\tilde{x}_{I,N}$ is quadratically stabilized satisfying $\dot{V}_I(\tilde{x}_{I,N}) < -\mu_I V_I(\tilde{x}_{I,N})$ for some $\mu_I > 0$, where $V_I(\tilde{x}_{I,N}) = \tilde{x}_{I,N}^\top P_I \tilde{x}_{I,N}$, $P_I > 0$ and $P_I \in \mathbb{R}^{n_I \times n_I}$. Then, for the uncertain nonlinear system given in* (1), (2) *the residual $r_I(t)$ in* (8) *which is implemented using a filter $H_p(s)$ of the form given in (5), satisfies $\|r_I(t)\| < \bar{r}_I(t), \forall t \in [t^*, T_0)$, where $\bar{r}_I(t)$ is the detection threshold given by* (32) *and* (31), *$\bar{\chi}_I(t)$ is given by* (28), *ρ_I and t^* are selected so that* (21) *is satisfied and γ_I (in* (31)*) is a scalar selected such that $0 < \gamma_I < \rho_I$.*

Remark 1. In the particular nonlinear observer approach (see Lemma 1) the constants ϵ_I, δ_I, μ_I appear. Note that, the parameters ϵ_I, δ_I affect only the nonlinear observer design and can be chosen arbitrarily as long as (13) can be solved whereas the choice of μ_I is more important because it affects both the nonlinear observer design and the fault detection scheme. Qualitatively, we would like μ_I to have a sufficiently large value to allow for more flexibility in choosing ρ_I later through (21). The fault detection scheme also depends on the choice of the parameters ρ_I and γ_I. Qualitatively, it is preferable that ρ_I is chosen sufficiently large so that γ_I can be chosen more freely according to $0 < \gamma_I < \rho_I$ in order to guarantee filter stability (in (31)). Although the choice of μ_I is fixed according to the nonlinear observer design, one can find a range of values for the parameters ρ_I and γ_I so that these two can be changed online.

Remark 2. In the proposed scheme, the filters are used primarily to mitigate the effects of noise and their use allows less conservative detection thresholds to be obtained. Following a similar procedure as before, it can be shown that in the case where no filtering is used, and hence the residual in this case is simply $r_I(t) = y_I(t) - \hat{y}_I(t)$, a suitable detection threshold is given by $\bar{r}_I(t) \triangleq \|C_I\|\tilde{x}_I^{max}(t) + \xi_{I,b}$, where

$$\tilde{x}_I^{max}(t) \triangleq \alpha_I \Big(e^{-(\mu_I - \theta_I)t} \bar{x}_{I,d}^2 + \frac{\theta_I^{-1}}{s + (\mu_I - \theta_I)} \left[\bar{v}_I^2(t) \right] \Big)^{\frac{1}{2}}$$

is the bound on $\|\tilde{x}_I(t)\|$ prior to the fault occurrence, the scalar θ_I is selected so that $0 < \theta_I < \mu_I$, $\bar{v}_I(t) \triangleq \bar{\eta}_I(y_I, \bar{y}_I, u_I, t) + \overline{\Delta f}_I + \|L_I\|\xi_{I,b}$, $\|\xi_I(t)\| \leq \xi_{I,b}$ and

$$\overline{\Delta f}_I \triangleq \sup_{\substack{(x_I, \bar{x}_I, u_I) \in \mathcal{D}_I \\ \bar{\xi}_I \in \mathcal{D}_{\bar{\xi}_I}}} \|f_I\big(x_I(t), \bar{C}_I \bar{x}_I(t), u_I(t)\big) - f_I\big(x_I(t), \bar{C}_I \bar{x}_I(t) + \bar{\xi}_I(t), u_I(t)\big)\|.$$

Note that, the detection threshold in this case of no filtering contains terms regarding the bounds of the measurement noise ξ_I (which is multiplied by $\|L_I\|$) and of the function discrepancy term Δf_I. Therefore, the detection threshold is more conservative and, as a result, more missed faults (false negatives) may occur in comparison to the case in which filtering is used.

Remark 3. Note that, a different bound on the filtered state estimation error $\tilde{x}_{I,f}$ (instead of (31)) is given by $\bar{H}_p(s)\left[\tilde{x}_I^{max}(t)\right]$ (where $\tilde{x}_I^{max}(t)$ is given in Remark 2). However, this bound does not exploit the filtering benefits for dampening the measurement noise and its effects in order to obtain tight detection thresholds. The treatment of the filtering in this work as a linear state transformation counteracts this problem, but at the same time creates some additional challenges, that are successfully tackled, such as to how to properly select ρ_I and "correlate" the exponential convergence to zero of $\tilde{x}_{I,N,f}$ with the boundedness of $\tilde{x}_{I,f}$ (see (26)).

5 Fault detectability

So far, the design and analysis was based on devising suitable thresholds $\bar{r}_I(t)$ such that in the absence of any fault we have $\|r_I(t)\| < \bar{r}_I(t)$. In the following, a fault detectability condition of the aforementioned fault detection scheme is presented, which provides a quantitative characterization of a class of detectable faults. The following result can be easily derived.

Theorem 2. *Consider the nonlinear system (1), (2) with the distributed fault detection scheme described in (5), (6), (7), (8) and the detection threshold* (32). *A fault in the I-th subsystem occurring at $t = T_0$ is detectable if the following inequality is satisfied for some $\theta_I \in (0, \mu_I)$ at some time $t > T_0$:*

$$\Big\| \int_{T_0}^{t} C_I e^{A_{I,0}(t-\tau)} \phi_{I,f}\big(x(\tau), u_I(\tau), \tau\big)\, \mathrm{d}\tau \Big\| > \bar{r}_I(t) + \bar{\epsilon}_{\xi_I}(t) + \int_0^t \|C_I e^{A_{I,0}(t-\tau)}\| \Big(\|G_{f_I}\| \bar{H}_p(s)[\tilde{x}_I^{max,\phi}(\tau)] + |h_p(\tau)| \bar{x}_{I,d} + \bar{\chi}_I(\tau) \Big)\, \mathrm{d}\tau$$

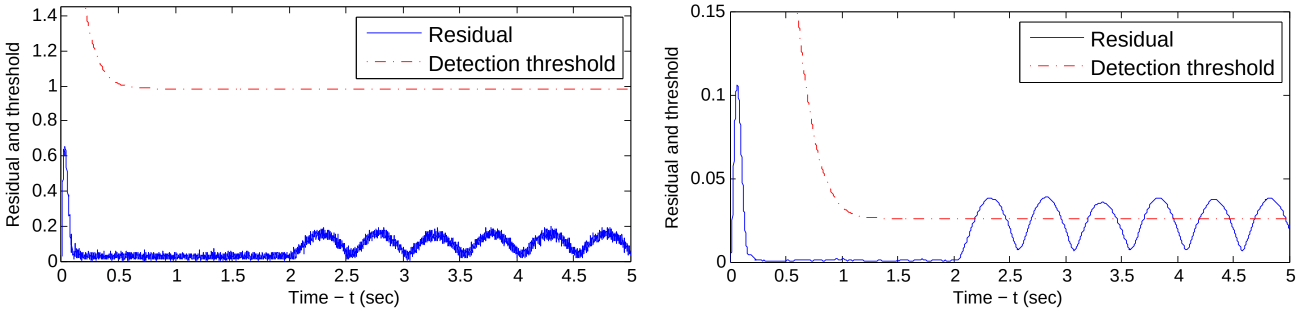
where $\phi_{I,f}\big(x(t), u_I(t), t\big) \triangleq H_p(s)[\beta_I(t - T_0)\phi_I(x(t), u_I(t))]$,

$$\tilde{x}_I^{max,\phi}(t) \triangleq \alpha_I \Big(e^{-(\mu_I - \theta_I)t} \bar{x}_{I,d}^2 + \frac{\theta_I^{-1}}{s + (\mu_I - \theta_I)} \left[(\bar{v}_I(t) + \|\phi_{I,f}\big(x(t), u_I(t), t\big)\|)^2 \right] \Big)^{\frac{1}{2}}.$$

The above fault detectability theorem implicitly characterizes the type of faults that can be detected by the proposed distributed fault detection scheme. Clearly, the fault functions $\phi_I(x, u_I)$ are typically unknown and therefore this condition cannot be checked apriori.

6 Simulation Results

In this section, we consider a numerical example to illustrate some of the concepts developed in this paper. The example is based on a system of two interconnected one-link manipulators with revolute joints actuated by a



(a) Without filtering.

(b) With filtering.

Fig. 2. Residual and detection threshold signals based on the nonlinear observer performance.

DC motor, where the elasticity of the joint can be modeled by a linear tensional spring [16] . The state variable $x_I^{(1)}$ represents the motor position, $x_I^{(2)}$ the motor velocity, $x_I^{(3)}$ the link position and $x_I^{(4)}$ the link velocity. The system dynamics of the two subsystems $I = 1, 2$, with $x_I = \left[x_I^{(1)} x_I^{(2)} x_I^{(3)} x_I^{(4)} \right]$, are given by

$$\dot{x}_I = A_I x_I + f_I + \eta_I$$
$$y_I = C_I x_I + \xi_I$$

where for the first subsystem: A_1 = [0 1 0 0 ; -48.6 -1.25 48.6 0 ; 0 0 0 1 ; 19.5 0 -19.5-3.33 0], $\eta_1 = [0; 0; 0; 0.1sin(x_1^{(3)})]$, $f_1 = [0; 43.2u; 0; -3.33sin(x_1^{(3)}) + 3.33x_1^{(3)} + 2sin(x_2^{(4)})]$, and for the second subsystem: A_2 = [0 1 0 0 ; -24.3 -0.625 24.3 0 ; 0 0 0 1 ; 9.75 0 -9.75-1.665 0], $\eta_2 = [0; 0; 0; 0.1sin(x_2^{(3)})]$, $f_2 = [0; 21.6u; 0; -1.665sin(x_2^{(3)}) + 1.665x_2^{(3)} + 2sin(x_1^{(4)})]$. The matrix C_I for both subsystems is given by $C_I = [1\ 0\ 0\ 0;\ 0\ 1\ 0\ 0;\ 0\ 0\ 0\ 1]$ and the input u_I for both subsystems is a sinusoid of magnitude 1 and frequency 1 Hz. In this simple example, we consider an abrupt multiplicative actuator fault in subsystem 1 where the input changes from $u_1 = \bar{u}_1$ to $u_1 = (1 + \psi_1)\bar{u}_1$ for some parameter $\psi_1 \in [-1, 0]$ characterizing the magnitude of the fault. In the simulation example, the fault occurs at $T_0 = 2$ sec with a magnitude $\psi_1 = -0.2$. In addition, the measurement noise is generated from a Gaussian distribution with mean $\mu_\xi = 0$, variance $\sigma_\xi^2 = 0.00025$ and it is then passed through a saturation block in order to limit its values between $[-0.05, 0.05]$ so that its maximum magnitude is 0.05.

At first, we proceed with the nonlinear observer design according to Section 3. The function f_1 satisfies the Lipschitz condition (4) with G_{f_1} = [0 0 0 0;0 0 0 0;0 0 0 0;0 0 6.66 0]. The nonlinear observer for subsystem 1 is designed so that $\dot{V}_1(\tilde{x}_1) < -\mu_1 V_1(\tilde{x}_1)$ with $\mu_1 = 30$. The specific choice for μ_1 is made because it is required that ρ_1 (which is selected based on (21)) satisfies $\rho_1 = \mu_1$ for comparison purposes between the filtering and non-filtering case. To obtain the solution of (13) we solve it as an ARE: $\bar{A}_1^\top P_1 + P_1 \bar{A}_1 + \epsilon_I G_{f_1}^\top G_{f_1} + \frac{1}{\epsilon_1} P_1 P_1 - \delta_1^2 C_1^\top C_1 = -\epsilon_1' I$. For given $\mu_1 = 30$ and $\epsilon_1' = 0.01$ (generally a small positive scalar), we try to find suitable values for ϵ_1, δ_1 that result in a symmetric, positive definite matrix P_1 solution of the previous ARE. The values that are chosen are $\epsilon_1 = 1$, $\delta_1 = 25$ and then the observer gain matrix L_1 is calculated from Lemma 1: L_1 = [22.45 -2.11 -0.11 ; -2.11 44.89 -10.86 ; 15.82 25.04 -12.36 ; -0.11 -10.86 27.13]. The eigenvalues of $(A_1 - L_1 C_1)$ are $-25.91 \pm \jmath 31.81$ and $-21.95 \pm \jmath 1.74$, and therefore they are well damped and they guarantee fast convergence to the actual states for the nominal system.

Now, we implement the residual and threshold signals without the use of filtering as described in Remark 2. The bounds used in this case are $\xi_{1,b} = 0.07$ and $\overline{\Delta f}_{1,1} = 0.1$. In addition the bound on the modeling uncertainty is $\bar{\eta}_1 = 0.1$ and the bound on the system initial conditions is $\bar{x}_{1,d} = 1$. The parameters used for the threshold are $\mu_1 = 30$ and $\theta_1 = 20$. The results are shown in Figure 2a where it is shown that the threshold is too conservative thus the fault is not detected. Next, we proceed and implement the proposed fault filtering detection scheme which allows less conservative thresholds to be obtained. The local FDI module that monitors the first subsystem computes the residual (8) by using a low-pass filter with transfer function $H_p(s) = \frac{25^2}{(s+50)^2}$ and the fault detection threshold (32) with $\rho_1 = 30$, $\gamma_1 = 20$. For comparison purposes, note that these parameters have the same values as in the no filtering case presented before ($\mu_1 = 30$ and $\theta_1 = 20$). For the calculation of the detection threshold, equations (31), (28) are used with $\bar{\epsilon}_{\xi_1} = 0.002$ and $\bar{\epsilon}_{\Delta_{1,1}} = 0.002$. Note that the transfer function $H_p(s)$ has a non-negative impulse response $h_p(t)$ and therefore the calculation of $\bar{h}_p(t)$ is not needed, but $h_p(t)$ is used instead. Figure 2b shows that the fault is successfully detected at around $t = 2.19$ sec, indicating the effectiveness of the proposed scheme.

7 Conclusion

In this paper a distributed fault detection filtering approach for a class of input-output interconnected, continuous-time, nonlinear systems with modeling uncertainties and measurement noise is presented. Utilizing nonlinear observer design and under certain assumptions, a distributed fault detection scheme is proposed which is inherently tied with the observer characteristics and the fault detectability condition is obtained which characterizes in an implicit way a class of detectable faults. The main contribution of this paper is the novel use of a nonlinear observer approach for the purpose of fault detection which is used to alleviate the lack of full state measurements and the novel use of filters as a linear state transformation that allows the use of any strictly proper and asymptotically stable filters. As a result, the fault detection is correlated with the observers' performance and tight, adaptive detection thresholds are obtained due to the noise dampening characteristics of the filters. Future research efforts will be devoted in developing a comprehensive fault isolation methodology.

Acknowledgements

This work was partially supported by funding from the European Research Council under the ERC Advanced Grant (FAULT-ADAPTIVE).

References

[1] K. Adjallah, D. Maquin, and J. Ragot. Nonlinear observer-based fault detection. In *IEEE Conference on Control Applications*, number 3, pages 1115–1120, 1994.

[2] G Besançon, editor. *Nonlinear Observers and Applications*. Springer, 2007.

[3] F. Boem, R.M.G. Ferrari, T. Parisini, and M.M. Polycarpou. Distributed Fault Diagnosis for Input-Output Continuous-Time Nonlinear Systems. In *8th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, pages 1089–1094, 2012.

[4] C. De Persis and A. Isidori. A geometric approach to nonlinear fault detection and isolation. *IEEE Transactions on Automatic Control*, 46(6):853–865, June 2001.

[5] C.A. Desoer and M. Vidyasagar. *Feedback Systems: Input-Output Properties*. Academic Press, 1 edition, 1975.

[6] R.M.G. Ferrari, T. Parisini, and M.M. Polycarpou. Distributed fault detection and isolation of large-scale nonlinear systems: an adaptive approximation approach. *IEEE Transactions on Automatic Control*, 57(2):275–290, 2012.

[7] P.M. Frank and S.X. Ding. Survey of robust residual generation and evaluation methods in observer-based fault detection systems. *Journal of Process Control*, (6):403–424, 1997.

[8] E.A. Garcia and P.M. Frank. Deterministic nonlinear observer-based approaches to fault diagnosis: a survey. *Control Engineering Practice*, 5(5):663–670, 1997.

[9] J.P. Gauthier and I.A.K. Kupka. Observability and observers for nonlinear systems. *SIAM Journal on Control and Optimization*, 32(4):975–994, 1994.

[10] H. Hammouri, M. Kinnaert, and E. El Yaagoubi. Observer-based approach to fault detection and isolation for nonlinear systems. *IEEE Transactions on Automatic Control*, 44(10):1879–1884, 1999.

[11] P.A. Ioannou and J. Sun. *Robust adaptive control*. Prentice Hall, 1996.

[12] C. Keliris, M.M. Polycarpou, and T. Parisini. A Distributed Fault Detection Filtering Approach for a Class of Interconnected Continuous-Time Nonlinear Systems. *IEEE Transactions on Automatic Control*, 58(8):2032–2047, 2013.

[13] C. Keliris, M.M. Polycarpou, and T. Parisini. A Distributed Fault Detection Filtering Approach for a Class of Interconnected Input-Output Nonlinear Systems. In *European Control Conference*, pages 422–427, Zurich, 2013.

[14] S. Klinkhieo and R.J. Patton. A Two-Level Approach to Fault-Tolerant Control of Distributed Systems Based on the Sliding Mode. In *7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, Barcelona, Spain*, pages 1043–1048, 2009.

[15] R.J. Patton, C. Kambhampati, A. Casavola, P. Zhang, S.X. Ding, and D. Sauter. A generic strategy for fault-tolerance in control systems distributed over a network. *European Journal of Control*, 13(2-3):280–296, 2007.

[16] G. Phanomchoeng and R. Rajamani. Observer design for Lipschitz nonlinear systems using Riccati equations. In *American Control Conference*, pages 6060–6065, 2010.

[17] R. Rajamani. Observers for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, 43(3):397–401, 1998.

[18] F.E. Thau. Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3):471–479, 1973.

[19] L. Wei, W. Gui, Y. Xie, and S.X. Ding. Decentralized Fault Detection System Design for Large-Scale Interconnected Systems. In *7th IFAC symposium on Fault Detection, Supervision and Safety of Technical Processes, Barcelona, Spain*, pages 816–821, 2009.

[20] S. Xu. Robust H_∞ filtering for a class of discrete-time uncertain nonlinear systems with state delay. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 49(12):1853–1859, December 2002.

[21] Q. Zhang, M. Basseville, and A. Benveniste. Fault detection and isolation in nonlinear dynamic systems: A combined input-output and local approach. *Automatica*, 34(11):1359–1373, 1998.

[22] Q. Zhang and X. Zhang. A distributed detection scheme for process faults and sensor faults in a class of interconnected nonlinear uncertain systems. In *IEEE 51st Conference on Decision and Control*, pages 586–591, 2012.

[23] X. Zhang, M.M. Polycarpou, and T. Parisini. Robust fault isolation for a class of non-linear input-output systems. *International Journal of Control*, 74(13):1295—-1310, 2001.

[24] Z. Zhang and I.M. Jaimoukha. Fault detection and isolation for linear discrete-time systems using input/output measurement analysis. In *48h IEEE Conference on Decision and Control and 28th Chinese Control Conference*, pages 4908–4913, 2009.

[25] F. Zhu and Z. Han. A note on observers for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, 47(10):1751–1754, 2002.