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Assessment of analytical formulations for the ULS resistance verification of structural glass elements accounting for the effects of different load durations

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ABSTRACT

Glass is an increasingly used material in construction and buildings. Despite its large application as load-bearing construction material, several aspects related to safe and optimal design are still under investigation and exploration, towards the full implementation of standardized and conservative rules of practical use. One of the main concerns in the design of this typically tensile brittle material is given by the rational estimation of static fatigue phenomena, under the effects of multiple design loads. In this paper, careful consideration is paid to the Ultimate Limit State (ULS) resistance verification of structural glass elements under a combination of variable loads with assigned duration. Taking advantage of past literature contributions and existing design standards for glass, an assessment of several available formulations to account for the cumulative stress effects of combined design loads is first carried out by means of selected case studies and extended parametric analytical calculations. Based on the obtained results, an alternative, simplified but practical and rather accurate - compared to the exact theoretical model - linear formulation is also finally proposed to account for static fatigue phenomena in load-bearing glass elements.

1. Introduction and research objectives

Taking advantage of a multitude of aspects, the use of structural glass as construction material in buildings is rapidly increasing in the last years. Major applications of load-bearing glass components can be found in the form of roofs, facades, stairs, columns, etc., including a wide range of possible loading and boundary configurations, as well as un-conventional loading conditions like dynamic loads or impacts. In most of the cases, uncertainties in their design can also arise from the combined use of glass together with other bearing components, typically including steel, aluminum, timber of fiber-reinforced polymers.

As in the case of traditional construction materials, several design standards and guidelines have been proposed in the last years, with the objective of implement appropriate methods and formulations voted to fail-safe design principles [1–8]. Research efforts are also currently ongoing, aimed to provide harmonization and refinement of existing design recommendations (see for example [9,10]).

In this regard, several researchers explored specific aspects related to the structural performance of glass load-bearing elements, acting both at the material as well as at the component and assembly level (i.e [11–23].). In a large number of cases, research studies have been spent for the implementation of appropriate formulations to account for the cumulative stress effects in glass elements subjected to a combination of variable loads. So far (see for example [17–23]), several approaches have been in fact proposed to take into account the effects of static fatigue phenomena on the actual design strength of glass, being this latter parameter highly sensitive to the loading conditions (i.e. characteristic duration) as well as to the actual sequence of multiple variable loads (with specific stress ratio effects) which should occur during the life-time of a structural glazing element to verify.

In this paper, taking advantage of recent literature efforts and existing design standards for structural glass elements, different formulations are compared with an exact theoretical model, aimed to check the level of approximation and their possible criticalities. The linear approach suggested by the pr-EN European standard provisions is first considered ([2], 'pr-EN' in the following). Despite the simplicity of application of the method, its main limitation - as also discussed in [23] - is given by fully neglecting the magnitude and sequence of each simultaneous action.

The Palmgren-Miner based method implemented in the CNR-DT

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210/2013 Italian code ([3], 'CNR' in the following) is then taken into account. Taking inspiration from fatigue models for metals, the maximum stress effect deriving from a combination of multiple actions is calculated as the sum of the effects due to each design load, compared to the corresponding design strength. A totally different formulation is indeed implemented in American national standard for glass, the ASTM E1300 code ([8], 'ASTM'), where cumulative effects due to combined design loads are calculated on the base of an equivalent distributed load. Later on, Haldimann proposed a further correction to the ASTM formulation ([24], 'ASTM-H' in the following).

The aforementioned standardized methods are hence assessed in this paper towards an accurate theoretical model, proposed by Franco and Royer-Carfagni ([23], 'F & R' in the following) and taking advantage of the subcritical crack growth model originally proposed by Wiederhorn and Bolz [17].

In this paper, based on a critical discussion of two extended case studies and a wide parametric analytical investigation, the level of approximation of the European, Italian and American standardized approaches are first highlighted, compared to the exact analytical solution taken from [23].

A novel, alternative verification approach taking advantage of a linear cumulative damage model ('WA', in the following) is also finally proposed. As shown, as far as the pre-stressing contribution is accounted on the side of actions within the 'WA' method, according to the 'F & R' formulation, interesting predictions can be generally obtained for a generic loading condition and stress ratio. At the same time, the simplicity and practicality of linear cumulative approaches can be preserved, hence resulting in a possible suitable tool for designers. At the current stage of the research study, assessment and discussion of comparative results is provided in this paper in the form of analytical calculations only. It is thus expected that further validation of the examined methods could later derive from extended experimental testing.

2. Theoretical background and overview of a selection of existing analytical formulations

The resistance verification of structural glass elements represents an open question for researchers and designers, aimed to provide sufficiently wide safety margins on one side and to maximum optimize the weight and cost of load-bearing elements, compared to their structural performance.

Following the European standardized design approach, given a structural glass element and its overall design life-time t_F, the effects of $j = 1, \ldots N$ design actions with different characteristic duration t_j and magnitude should be properly estimated and compared to the design resistance of the material itself. In accordance with most of the European regulations for structural glass elements, a conventionally accepted expression for the calculation of its design strength f_{gd} takes the form [2]:

$$f_{gd} = f_{gd,b} + f_{gd,p} = \frac{k_{\mathrm{mod}} k_{sp} f_{gk}}{\gamma_{MA}} + \frac{k_v (f_{bk} - f_{gk})}{\gamma_{Mv}}, \tag{1}$$

where f_{gd} is given by the well-known sum of two main contributions representative of the tensile resistance of annealed (AN) float glass only ($f_{gd,b}$) and of possible toughening treatments ($f_{gd,p}$), respectively.

In Eq. (1), $f_{gd,b}$ is primarily dependent on the k_{mod} load duration factor, being this latter coefficient able to take into account the material strength reduction due to static fatigue phenomena. A practical and generally accepted expression for the estimation of k_{mod} is given by [3]:

$$k_{\mathrm{mod}} = 0.585 \cdot t^{-1/16}, \tag{2}$$

where t is expressed in hours for each of the assigned j-th design loads.

According to Eq. (2), typical values of k_{mod} corresponding to ordinary design actions for construction elements lie in a range of 0.26 (permanent loads, 50 years), 0.36 (mid-term loads, 3 months) and 0.88–0.91 (instantaneous loads, 3–5 s).

In Eq. (1), f_{gk} and f_{bk} denote respectively the characteristic tensile bending strength of AN glass (nominal value in the order of 45 MPa) and pre-stressed glass (nominal values in the order of 70 MPa for heat-strengthened (HS) and 120 MPa for fully-tempered (FT) glass), while k_{sp} and k_v are coefficients accounting for the surface finishing and tempering process respectively. γ_{MA} and γ_{Mv}, finally, are partial safety factors for material and pre-stressing treatments.

For design purposes, given a single design action with characteristic duration t, the conventional resistance verification would require the satisfaction of the condition:

$$\sigma_{\max} \le f_{gd}, \tag{3}$$

with σ_{max} the maximum stress and $f_{gd} = f(k_{mod})$ given by Eq. (1).

As far as a glass elements is subjected to a combination of N design actions with specific time loading t_j and variably spanning over the full life-time t_F, however, the design strength f_{gd} should properly calculated.

According to the Eurocode design approach [26], as well as to other design standards for structural elements in general (i.e [27].), the resistance verification of a given glass member should be in fact traditionally carried out at the Ultimate Limit State (ULS) by taking into account a load combination in the type of:

$$F_{d,ULS} = \gamma_G G + \gamma_Q Q_{k,1} + \gamma_Q \sum_i \Psi_{0,i} Q_{k,i}, \tag{4}$$

where G is representative of permanent loads, while $Q_{k,1}$ and $Q_{k,i}$ denote respectively the dominant and i-th variable actions; $\gamma_G = 1.35$ and $\gamma_Q = 1.5$ are partial factors and $\Psi_{0,i}$ is the combination factor, depending on the assigned variable actions (comprised in the range between 0.7 and 0, see [26]).

2.1. Load duration effects

Following Eqs. (1)–(4), one of the major design problems for glass elements arises from the estimation of the maximum effects due to N combined actions (see Fig. 1), due to the brittle nature of glass.

Despite the final expressions of formulations available in the literature, most of them are implicitly or explicitly based on the assumption that the crack growth and the related probability of failure can be described using the risk integral (see also [25]). In this regard, an exponential theoretical approach has been for example proposed by Franco and Royer-Carfagni [23] to provide an exact analytical formulation of the problem (see Section 2.5). This is not the case of most of the existing verification approaches currently in use in design standards for glass structures, where linear cumulative models are implemented for simplicity (see for example [2,3]). As also partly observed in [23], however, the effect of improper assumptions of existing linear cumulative models for the definition of the resulting stress level due to N actions typically leads to approximate estimations, both on the safe and unsafe side. A totally different formulation is then taken into account by the ASTM national standard, see [8], together with its later revision suggested by Haldimann [24].

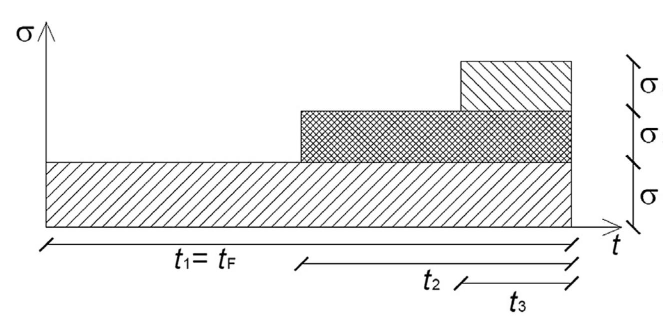

Fig. 1. Stress effects deriving from $j = 3$ concurrent design actions with different characteristic duration t_j and spanning over the design life-time t_F.

In this context, the current research study aims first to assess the level of approximation for some cumulative based approaches in use for the design of structural glass elements, as compared to the exact predictions provided by the exponential analytical model presented in [23]. An alternative method, still based on a linear cumulative approach as in the case of the pr-EN or CNR provisions, is then also proposed (see Section 4). As shown, the so called weighted approach ('WA') preserves the typical simplicity and intuitivism of linear methods. At the same time, thanks to the weighted average calculation of an equivalent $k_{mod,w}$ coefficient accounting both for the characteristic duration t_j and stress ratio $R_{\sigma j}$ of each j-th action (i.e. stress vs. design strength due to the j-th action), the WA approach allows to obtain an accurate estimation of the failure configuration for a given glass element, both in the case of AN glass only as well as in presence of possible pre-stressing treatments.

2.2. European standard prescriptions (pr-EN 16612:2013)

According to the pr-EN (pr-EN 16612:2013 [2]), given a combination of N design actions, the maximum stress calculated as the sum of the $j = 1, \ldots N$ actions spanning over the life-time t_F of a glass element in the most onerous combination (i.e. Eq. (4)) should not exceed the design value of glass strength f_{gd}, where f_{gd} can be evaluated by means of Eq. (1).

In other words, the possible failure of a given glass member under a ULS combination of N design actions can be prevented as far as the condition:

$$\frac{\sum_{j=1}^{N} \sigma_j}{\max(f_{gd,j})} \leq 1 \tag{5}$$

is satisfied.

The pr-EN linear approach is simple in use, but a strong approximation derives from the calculation of the design resistance value f_{gd} in presence of multiple loads. The effects of N actions with different time loading t_j and magnitude are in fact taken into account by assuming for f_{gd} (see Eq. (1)) the highest $k_{mod,j}$ coefficient (Eq. (2)) due to the j-th action (i.e. the shortest time loading t_j). In doing so, the corresponding stress ratio $R_{\sigma,j}$ is fully disregarded. Possible pre-stressing treatments in glass, finally, are considered in the verification condition given by Eq. (5) on the side of the design resistance only, according to Eq. (1). Following the earlier pr-EN standard recommendations, over the past years several European codes have been implemented over the past years on the base of the same design provisions, see for example NEN 2608 [5] as well as the German regulations DIN 18008 [6]. As a result, comparative calculations labelled in this paper as 'pr-EN' can be considered representative of the actual design approach for glass structures in Europe.

2.3. Italian code prescriptions (CNR-DT 210/2013)

A Palmgren-Miner based, linear cumulative damage approach is implemented in the Italian CNR document for the design of structural glass elements (CNR-DT 210/2013 [3]). In it, the condition preventing possible failure for a given glass member under a ULS combination of N actions is given by:

$$\sum_{j=1}^{N} \frac{\sigma_j}{f_{gd,j}} \leq 1, \tag{6}$$

that is by the sum, for each j-th action, of the corresponding σ_j stress effect divided by the design strength $f_{gd,j}$, to be separately calculated by means of Eq. (1).

Differing from the pr-EN approach, the CNR cumulative method is still linear, but each one of the N actions is accounted in the form of its own reduction strength effect, via the corresponding $k_{mod,j}$ coefficient (Eq. (2)). Moreover, as in the case of the pr-EN approach, possible pre-stressing treatments in glass are then taken into account on the side of the design strengths $f_{gd,j}$ (Eq. (1)), rather than on the side of stress effects in glass.

2.4. American national standard (ASTM E1300)

A totally different verification approach is proposed for glass plates in the American national standard ASTM E1300 [8]. The ASTM approach, in particular, based on the glass failure prediction model by Beason and Morgan [28], basically takes the forms of charts provided to derive the minimum glass thickness for the glass element to verify.

For a glass element subjected to a single design load, reference values of allowable surface stress are provided (3 s duration load), depending on the glass type (i.e. AN, HS or FT). As a major discrepancy from the European design approach, the collapse verification under multiple design loads requires that the assigned uniform load q should not exceed the design 'load resistance' LR of the glass plate, being LR representative of the 'non-factored load' NFL multiplied by a 'glass type factor' GLF:

$$q \leq LR = NFL \times GTF, \tag{7}$$

hence the resistance verification is based on loads rather than principal stresses.

As such, given a glass plate, possible compressive residual stresses are accounted in terms of GFT values. As far as a given plate is subjected to $j = 1, \ldots N$ distributed loads, the resistance verification is carried out by taking into account an equivalent, 3 s load q_3 given by:

$$q_3 = \sum_{j=1}^{N} q_j \left[, \frac{d_j}{3} \right]^{\frac{1}{n}}, \tag{8}$$

with d_j in seconds for each q_j load and $n = 16$ representing the static fatigue exponent for AN glass.

Following the original ASTM provisions [8], Haldimann highlighted in [24] the inconsistency of Eq. (8), and suggested to replace it with (labelled as 'ASTM-H', in this paper):

$$q_3 = \left(\frac{1}{3} \sum_{j=1}^{j} [q_j^n \times d_j] \right)^{\frac{1}{n}}. \tag{9}$$

In the following sections, both the ASTM and ASTM-H analytical estimations are presented for the examined case study (AN glass only). In order to establish a direct correlation between cumulative damage models considered in this paper, the ASTM and ASTM-H combination formulae given in Eqs. (8) and (9) are then applied to combination of stresses rather than loads.

2.5. Subcritical cracks growth-based formulation

The aforementioned pr-EN, CNR and ASTM derived methods are assessed in this paper towards the theoretically exact verification approach proposed by Franco and Royer-Carfagni in [23]. The 'F & R' formulation takes advantage from a consolidated model of subcritical crack growth (i.e. static fatigue) and consists in an exponential analytical expression for the safety domain of structural glass elements under a generic combination of N design actions.

Two key aspects are introduced with this formulation. Any possible pre-stressing effect is in fact accounted on the side of the actions, rather on the side of the material design strength. In other words, the design strength of AN glass only (i.e. first term of Eq. (1)) is calculated. This basic assumption is also in agreement with an earlier analytical model proposed by Siebert [29], where a further coefficient (f_{tS}) was proposed to account for relative magnitude, load duration and environmental conditions in a combination of N design loads.

Beside this similarity, the relevant aspect of the F & R approach is that it allows to accurately take into account the non-linearity of the design problem, i.e. by properly combining the σ_j stress effects of multiple actions with time intervals t_j on the overall life-time t_F of a given structural element to verify. The general expression of the safety domain for a glass elements under N design loads takes in fact the form:

$$\sum_{j=1}^{N} \frac{\left[\left\langle\left(\sum_{i=1}^{j}\sigma_i\right)-\sigma_p\right\rangle^+\right]^n-\left[\left\langle\left(\sum_{i=1}^{j-1}\sigma_i\right)-\sigma_p\right\rangle^+\right]^n}{(f_{gd,b})_i^n} \le 1. \tag{10}$$

In Eq. (10), given a time interval $0 \le t \le t_F$, the positive part of the resisting domain function $F(t)$ is only taken into account, i.e.:

$$\langle F(t)\rangle^+ = \max\{F(t), 0\}. \tag{11}$$

In the same equation, where $n = 16$, $\sigma_p = f_{gd,p}$ (see Eq. (1)) represents the effect of possible toughening processes (when present), σ_i is the tensile stress due to each one of the assigned N actions, while $(f_{gd,b})_i$ denotes the corresponding design resistance, inclusive of the AN glass contribution only (Eq. (1)).

Based on this formulation and Eq. (10), since possible compressive stresses σ_p deriving from toughening processes are implemented on the side of the actions, it is clear that any possible crack propagation and failure mechanism in glass can only occur as far as the total tensile stresses due to the applied N actions exceed the initial compressive state σ_p.

3. Assessment of existing formulations via extended parametric analyses

In order to establish a general relationship between the pr-EN, CNR, ASTM and F & R approaches earlier described, i.e. quantifying the level of approximation of the pr-EN, CNR and ASTM cumulative methods compared to the exact F & R solution, an extended investigation was first carried out, with up to 400 the total number of examined cases.

A wide range of configurations, including variation of geometrical properties (i.e. overall dimensions $b \times L$ for the examined glass panes, aspect ratio $\alpha = L/b$, thickness of glass panes h, thickness of interlayer h_{int}), boundary and loading conditions (i.e. number N of design actions, amplitude and duration t of the design loads, etc.), type of interlayer (i.e. PVB (Polyvinyl Butyral) or SG (SentryGlas®)) and type of glass (i.e. pre-stressed or simple AN float glass), was taken into account. For all the examined geometries, b was set to denote the size of restrained edges, with L being representative of the bending length.

For clarity of presentation, two full examples are first discussed in detail in Sections 3.1 and 3.2, giving evidence of the observed analytical results. A general presentation and comparative discussion is then provided in Section 5 for further extended parametric calculation.

For simplicity and accuracy of collected predictions, all the comparative examples were carried out by taking into account the 'equivalent thickness formulation' proposed in [30]. The used equations represent an improved calculation method for laminated glass elements, compared to earlier European provisions related to equivalent thickness formulations, see also [30]. Beside the equivalent thickness formulation considered in this paper, however, as well as in accordance with the purpose of the current research study, the combination of stress effects due to multiple actions is here first emphasized, rather than the accuracy of stress predictions for glass panels under a single design action.

In accordance with [30], see Fig. 2, given a laminated glass section composed of two h-thick glass panes and a middle interlayer, h_{int} its nominal thickness, rational estimations under a certain boundary and loading configuration can be in fact carried out by considering:

- an equivalent thickness for deflections

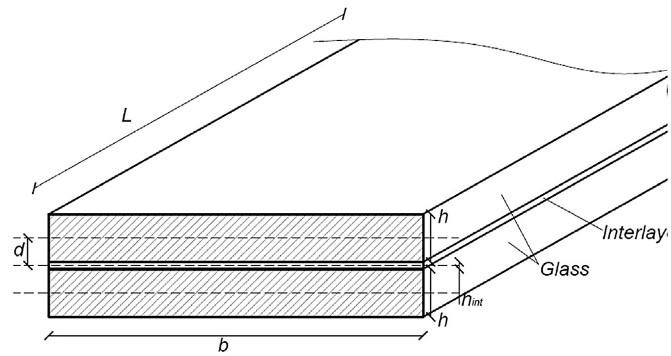

Fig. 2. Reference geometrical configuration for a symmetrical laminated glass panel (b = supported edge).

$$\hat{h}_w = \sqrt[3]{\frac{1}{\frac{\eta}{2\cdot h^3+24\cdot hd^2}+\frac{1-\eta}{2\cdot h^3}}} \tag{12}$$

- an equivalent thickness for stresses (identical for both the glass panes, assuming a symmetrical cross-section)

$$\hat{h}_\sigma = \sqrt{\frac{1}{\frac{2\eta d}{2\cdot h^3+24\cdot hd^2}+\frac{2\cdot h}{h_w^3}}} \tag{13}$$

where

$$0 \le \eta = \frac{1}{1+\frac{Eh_{\mathrm{int}}}{G_{\mathrm{int}}b}\frac{I_{abs}}{I_{full}}A^*\Psi} \le 1 \tag{14}$$

$$I_{abs} = 2{\cdot}bh^3/12 \tag{15}$$

$$I_{full} = I_{abs} + 2Ad^2 \tag{16}$$

$$A^* = A/2 \tag{17}$$

In the above equations, according to Fig. 2, $A = b \times h$ and $d = (h + h_{int})/2$, while E and G_{int} denote respectively the nominal Young modulus of glass and the secant modulus of the interlayer. The Ψ coefficient value, finally, can be derived from [30], based on the examined loading and boundary conditions.

3.1. Case study 1: Annealed glass roof panel

The first worked example takes inspiration from the case study proposed in [23] for the verification of a given AN glass panel. For comparative purposes only, basic input geometrical and mechanical data were directly derived from [23]. Following [23], in particular, the reference pane has overall dimensions $b = 1000 \times L = 655\,\mathrm{mm}^2$, is simply supported on the b longest edges only and subjected to the combined action of dead loads G (self-weight, calculated on the base of the panel geometry, with 2500 kg/m^3 and 1100 kg/m^3 the density of mass of glass and interlayer respectively), as well as two variable loads representative of snow Q_s (0.8kN/m^2) and maintenance Q_m (0.50kN/m^2) loads respectively.

The resisting cross-section consists of a laminated glass unit, where the connection between the AN glass layers is provided by a 1.52 mm thick PVB foil. Its secant stiffness modulus G_{int}, as required in Eq. (14), is estimated by taking into account the reference time loading t and temperature T for each design action, see Table 1 [23]. For glass, a nominal modulus elasticity $E = 70$GPa was considered [31].

As a first attempt of design of the pane, based on Eqs. (12)–(17), the minimum glass thickness h_{min} required to satisfy the ULS resistance verification was calculated, following the pr-EN (Eq. (5)), CNR (Eq. (6)) and F & R approaches (Eq. (11)). The same calculations were also carried out by taking in account a combination of stresses in accordance with Eqs. (8) and (9).

Table 1
Input parameters for the case study 1 [23].

		Design action		
		G	Q_s	Q_m
t	–	50 years	3 months	3 s
T	[°C]	50	30	30
k_{mod}	–	0.26	0.36	0.91
G_{int}	[MPa]	0.052	0.57	0.85
$f_{gd,b}$	[MPa]	6.75	9.43	20.50

Due to the assigned boundary and loading condition (i.e. simply supported beam-like element under uniformly distributed loads), the Ψ coefficient of Eq. (14) was set equal to $168/17L^2$ [30]. Based on the worst ULS combination of design loads given by Eq. (1), the maximum expected tensile stress due to each j-th action was then calculated as [23]:

$$\sigma_{\max,j} = 0.75\frac{L^2}{\widehat{h}_{\sigma}^{2}}F_{d,ULS,j}, \tag{18}$$

with $F_{d,ULS,j}$ denoting the j-th design action value for the reference ULS combination.

In Fig. 3(a), the so achieved analytical predictions are compared, in terms of expected damage level D, as a function of the assigned glass thickness h. For comparative purposes, the non-dimensional damage level D was defined as the left term of Eqs. (5), (6) and (11), or equivalent stress vs. allowable surface value in accordance with Eqs. (8) and (9).

Discussion of the full parametric results is then given towards the F & R predictions.

From Fig. 3(a) it can be seen that the pr-EN approach seems to provide always unsafe estimations, compared to the exponential F & R approach. While the minimum theoretical glass thickness able to optimize the pane (i.e. $D = 1$) would be equal to $h_{min} = 3.1$ mm, the pr-EN approach would suggest in fact a minimum thickness $h_{min} = 2.7$ mm, with an underestimation in the order of ≈ 14%.

From Fig. 3(a) it can be also noticed that the Palmgren-Miner based method (CNR), conversely, overestimates the minimum required thickness ($h_{min} = 4.2$ mm), hence resulting in largely conservative predictions (≈ +24% the scatter for the proposed worked example). The ASTM calculations, despite the different assumptions for their reference cumulative approach, are indeed in rather close correlation with the CNR predictions. This is not the case of the modified ASTM formulation proposed by Haldimann (see the 'ASTM-H' plots), where severely unsafe results were generally obtained, compared to the aforementioned approaches.

Despite the theoretical accuracy of the F & R method, based on the comparative calculations collected in Fig. 3(a), a general observation on the same method could be related to some implicit risks in its use for design. Since implemented on the base of an exponential formulation for the resisting domain, the F & R approach provides in fact an accurate estimation of the theoretically required minimum thickness h_{min} able to fully optimize the design calculations themselves (i.e. leading the designer to assume a minimum glass thickness h_{min}: $= D \rightarrow 1$). Nevertheless, the same calculations are very sensitive to even small variations in the glass thickness h, hence resulting in possible hazardous assumptions for design practitioners in general. For the worked example

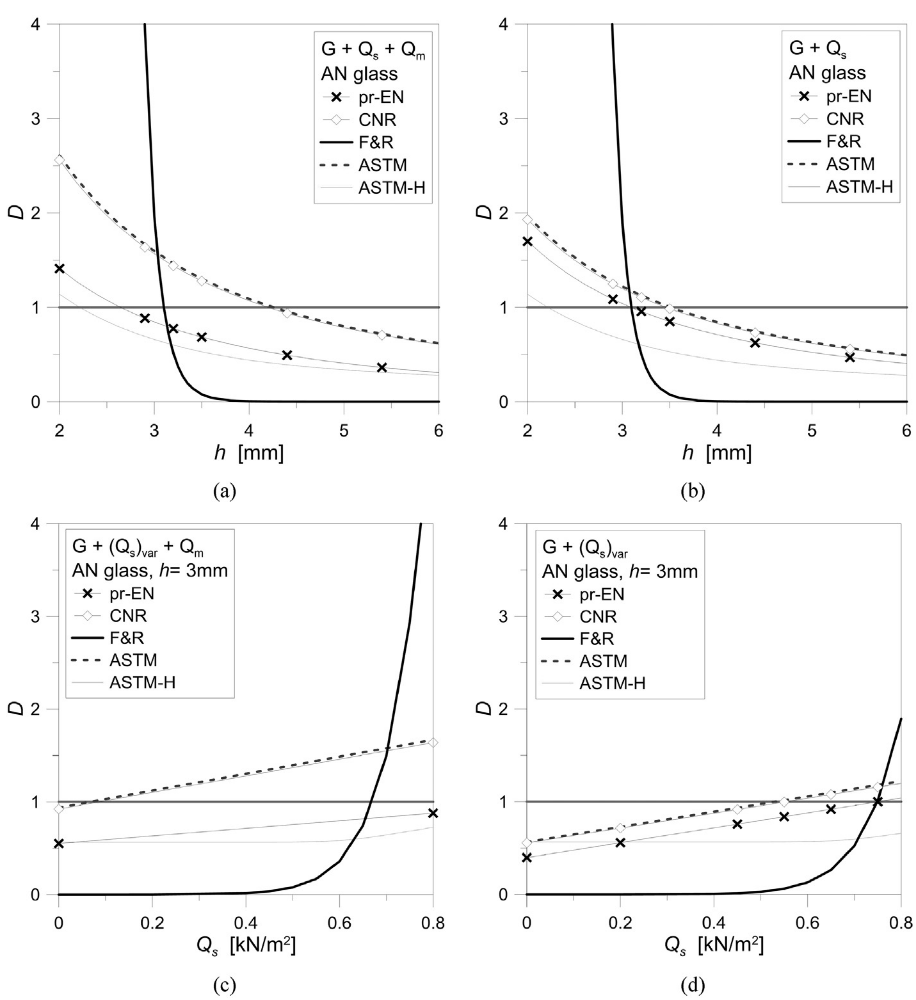

Fig. 3. Comparative analytical calculations for an AN laminated glass panel, as a function of (a), (b) the assigned glass thickness h and (c), (d) as a function of the assigned variable load Q_s.

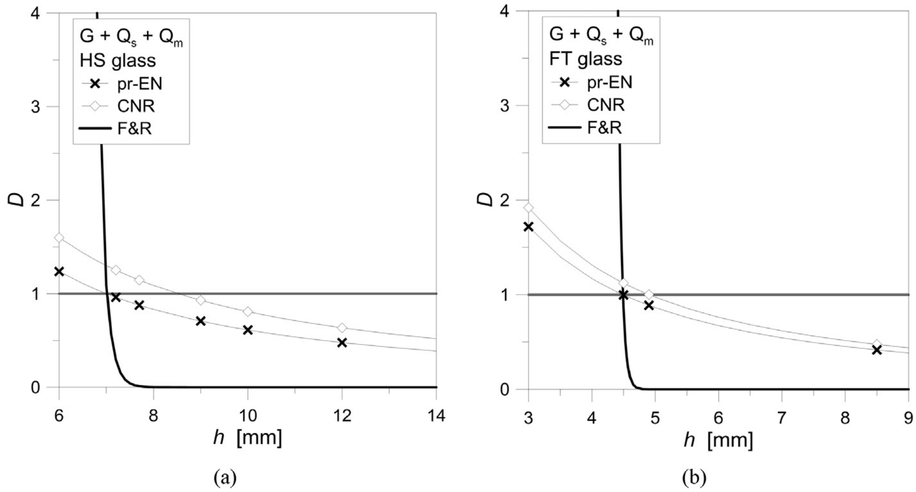

Fig. 4. Comparative analytical calculations for an (a) HS and (b) FT laminated glass panel, as a function of the assigned glass thickness h.

of Fig. 3(a), it is possible to notice that for a glass thickness h higher or at least equal to 3.8 mm, a null crack propagation is estimated by the F & R method (i.e. $D = 0$). An abrupt increase in the expected D value is instead observed when reducing the minimum thickness from 3.5 mm to 3.1 mm.

This is not the case of the pr-EN, CNR or ASTM approaches, where – despite their intrinsic approximations – the expected level of damage D progressively increases as far as the glass thickness h reduces, hence providing designers a certain sensitivity towards the possible failure of the panel object of verification.

In order to assess the effects of different distribution of applied loads on the same case study roof, a second parametric investigation was then carried out by taking into account – for identical $b \times L$ geometry and boundary condition – the simultaneous action of two loads only, i.e. the self-weight load G and the assigned snow load $Q_s = 0.8\text{kN/m}^2$. As in the case of Fig. 3(a), an iterative analytical calculation was hence performed by means of Eqs. (12)–(17), by changing the input glass thickness h, and taking into account the resisting domain equations provided by the pr-EN, CNR, F & R and ASTM design approaches.

Compared to Fig. 3(a), some further interesting aspects were observed, see Fig. 3(b). Following the F & R approach, almost the same analytical curve of Fig. 3(a) was found, with negligible variations in a (h, D) point-by-point comparison. This effect was found to primarily derive from the magnitude of the imposed snow load Q_s on the assigned glass pane geometry and boundaries, as well as on the glass type itself (see Table 1).

The characteristic duration t_j of each design action, as known, does not exhaustively suggest which action is the predominant one, among the overall ULS combination (i.e. in terms of stress-to-strength ratio $R_{\sigma j}$). The assigned snow load Q_s, based on the given input parameters for the worked example, resulted in fact associated to the highest R_σ ratio, both in the case of two variable loads (Fig. 3(a), where $(D = 1)_{\text{F \& R}}$ when $h = 3.1$ mm, with $R_\sigma = 0.62$, 0.53 and 0.35 for snow load Q_s, self-weight G and maintenance load Q_m respectively) as well as in presence of a single variable load (Fig. 3(b)).

In this regard, Fig. 3(c) and (d) present a different series of comparative calculations, where differing from Fig. 3(a) and (b) the glass thickness of the examined panel was kept fix to $h = 3$ mm. The magnitude of the medium-term variable load $(Q_s)_{var.}$ was then progressively modified, aiming to find the load value $(Q_s)_{var.} = Q_{s,collapse}$ able to lead the assigned glass element to collapse ($D = 1$). In doing so, all the other input parameters required for the analytical check were kept equal to the previous calculations, see also Table 1.

In the case two variable loads are considered ($G + (Q_s)_{var.} + Q_m$, with $Q_m = 0.5\text{kN/m}^2$), see Fig. 3(c), the pr-EN method seems totally unsafe, compared to F & R predictions ($(D = 1)_{\text{pr-EN}}$ for $Q_{s,collapse} = 0.67\text{kN/m}^2$), with up to ≈63% the level of underestimation. The CNR approach, on the other hand, appears strongly conservative ($(D = 1)_{\text{CNR}}$ when $Q_{s,collapse} = 0.09\text{kN/m}^2$), with ≈83% the amount of discrepancy (on the safe side) compared to the exact solution.

Also in this case, however, despite the strong limitations of the pr-EN and CNR linear approaches, the exact F & R method would suggest the possible introduction of an additional partial safety factor. From Fig. 3(c) it is in fact possible to perceive how a very small variation in the assigned $(Q_s)_{var.}$ magnitude could result in opposite estimations – both conservative and non-conservative – in terms of D calculations for the assigned geometry and loading condition.

In presence of one variable load only ($G + (Q_s)_{var}$), see Fig. 3(d), minor discrepancies were found for the collapse configuration of the examined panel, as given by the examined formulations. The expected $Q_{s,collapse}$ magnitude leading the glass layers to failure at the ULS, based on the pr-EN approach, was in fact found to fully agree with the F & R formulation ($Q_{s,collapse} = 0.75\text{kN/m}^2$). The CNR method still appeared markedly conservative, with up to ≈26% the scatter from the exact solution ($Q_{s,collapse} = 0.56\text{kN/m}^2$).

3.2. Case study 2: Pre-stressed glass

A second case study was then also taken into account, in order to draw some further analytical comparisons in the specific case of pre-stressed glass panes. As highlighted in Section 2, one of the major limits of the linear pr-EN and CNR formulations is given by the calculation of the stress effects due to toughening processes on the side of the design strength, rather than on the side of actions.

Both the options of HS and FT glass were hence separately considered. In terms of total design resistance f_{gd}, the corresponding value was hence calculated by means of Eq. (1), assuming $f_{bk} = 70$ MPa and 120 MPa respectively. Due to the additional and beneficial pre-stressing contribution, differing from the case study 1, a total span $L = 2100$ mm was taken into account for the bending length of the examined glass pane, with $Q_s = 1.20\text{kN/m}^2$ the snow load, and keeping fix all the other geometrical and mechanical input data (see also Table 1 and Fig. 2).

Parametric results derived from iterative analytical calculations carried out by changing the glass thickness h in Eqs. (12)–(17) are proposed in Fig. 4(a) and (b), as obtained for the HS and FT panels respectively, under the ULS combined action of self-weight G, snow load Q_s and maintenance load Q_m. For clarity of presentation, based on the outcomes of the ASTM calculations provided in Section 3.1 (case study 1), the estimations provided by the latter approach were omitted from Fig. 4.

As shown, the pr-EN approach gives always failure predictions (i.e.

$D = 1$) that coincide with the F & R exponential calculations. The CNR verification approach, on the other hand, is still conservative, compared to the F & R method, with scatter lying in the range of ≈ 22% and ≈ 9% for the HS and FT panels respectively.

The F & R method, finally, as far as the pre-stressing contribution magnifies for the HS ($\sigma_p = 20.83$ MPa) and FT glass panels ($\sigma_p = 55.55$ MPa), is progressively more sensitive to thickness variations h, even in the order of decimal or centesimal part of millimetres respectively. Also in this last example, in conclusion, it can be noticed that as far as any pre-stressing treatment is applied to glass, the expected level of damage D provided by the F & R method is null up to few decimal parts of the h_{min}: $= D = 1$ limit thickness. A direct effect of this finding is a potential lack of safety margins in design calculations, hence requiring careful attention on the designer side, as well as the possible implementation – on the design standards side – of additional partial safety factors able to provide appropriate safety margins in the case of possible calculation errors.

4. Alternative proposal (WA formulation)

Following the theoretical background recalled in Section 2 and the observed trends of case studies proposed in Section 3, some first considerations were derived.

In general, it is well-known that design formulations should be as much as possible practical in use and safely calibrated. The linear cumulative damage models, on one side, are usually preferred by designers for their simplicity, as well as for their typically higher perception of stress ratios for a given structural member.

The worked examples proposed in Section 3, in this regard, highlighted the strong limits and criticisms of the pr-EN approach, where for a given combination of $j = 1, \ldots, N$ design actions, a key role is assigned to the minimum time loading $t_{j,\min} = \min(t_j)$ only. The Palmgren-Miner based formulation implemented in the CNR code, at the same time, proved to be roughly approximate – even always on the safe side, compared to the exact F & R formulation – due to the basic assumptions of the approach. On the other hand, the F & R exponential approach suggested a high sensitivity to even small variations in the input data, hence suggesting the potential lack of safety margins in design calculations.

In this regards, an alternative, linear but rather accurate for failure detection analytical formulation is here proposed for the resistance verification of a given structural glass elements under a ULS combination of N design actions. The proposed weighted approach (WA, in the following), differing from the pr-EN and CNR methods, is based on the use of an equivalent $k_{mod,w}$ coefficient calculated as the weighted average of all the N imposed actions. In the case of AN glass, in accordance with Fig. 1, $k_{mod,w}$ is given by:

$$k_{\mathrm{mod,w}} = \frac{\sum_{j=1}^{N} \sigma_j k_{\mathrm{mod,j}}}{\sum_{j=1}^{N} \sigma_j}. \tag{19a}$$

For pre-stressed glass, since the additional compressive state σ_p must be also taken into account, $k_{mod,w}$ can be calculated in accordance with Fig. 5, that is:

$$k_{\mathrm{mod,w}} = \frac{\sum_{j=1}^{N} \sigma'_j k_{\mathrm{mod,j}}}{\sum_{j=1}^{N} \sigma'_j} \tag{19b}$$

In the case that any toughening treatment is applied to glass, see Eq. (19b) and Fig. 5, σ_j' represents in fact for a given j-th time interval t_j the increment of tensile stress, deprived of the maximum term between the total (j-1) tensile stresses or possible pre-stressing compressive terms,

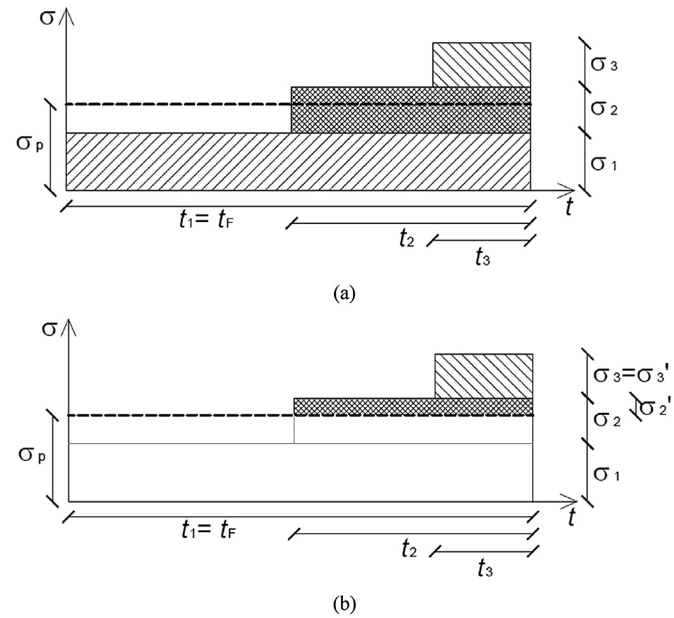

Fig. 5. Stress effects deriving from $j = 3$ concurrent design actions with different characteristic duration t_j and spanning over the design life-time t_F of a pre-stressed glass element ($\sigma_p \neq 0$). Charts with evidence of (a) separate stress effects σ_j and (b) tensile stress increment σ_j' over each j-th time interval.

that is:

$$\sigma'_j = \left(\left(\sum_{i=1}^{j} \sigma_i\right) - \max\left(\left(\sum_{i=1}^{j-1} \sigma_i\right), \sigma_p\right)\right)^3 0. \tag{20}$$

Once $k_{mod,w}$ is known, the resistance verification under a given ULS combination of N actions can be rationally carried out, both in the cases of AN or pre-stressed glass, by checking the satisfaction of the following condition:

$$\frac{\left(\sum_{j=1}^{N} \sigma_j\right) - \sigma_p}{f_{gd,b}^*} \leq 1, \tag{21a}$$

where $f_{gd,b}{}^*$ is still given by the first term of Eq. (1), with $k_{mod} = k_{mod,w}$ (Eq. 19a and 19b)).

As far as the pre-stressing effects are accounted on the side of the design resistance rather than on the side of the external actions, the condition to satisfy for resistance verification takes the form:

$$\frac{\sum_{j=1}^{N} \sigma_j}{f_{gd}^*} \leq 1, \tag{21b}$$

where $f_{gd}{}^*$ is given by both the terms of Eq. (1), with $k_{mod} = k_{mod,w}$ Eq. 19a and 19b). In the latter case, it should be noticed that the assumption of Eq. (21b) fully coincides with the verification condition provided by the pr-EN method (with the only exception that $k_{mod} = k_{mod,w}$ according to the novel WA formulation).

From Eqs. (21a) and (21b), some further interesting aspects can be also pointed out. When $\sigma_p = 0$, Eqs. (21a) and (21b) are obviously identical, for a given ULS combination of N actions and $0 \leq D \leq 1$. This is not the case of pre-stressed glass ($\sigma_p \neq 0$), where Eqs. (21a) and (21b) are identical in terms of failure configuration only ($D = 1$). Major effects deriving from their basic assumptions on the σ_p term take the form of substantial variations in the estimations of the expected level of damage, as far as $D < 1$. This difference is highlighted in Fig. 6, where the expected damage D for the 1000 × 2100 mm panel ($h = 5$ mm) already examined in Section 3.2 is again proposed, by changing the magnitude of Q_s.

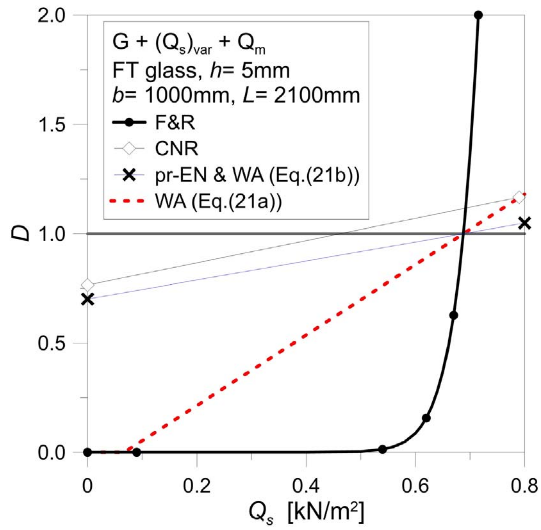

Fig. 6. Comparative analytical calculations for an FT laminated glass panel, as a function of the assigned variable load Q_s.

Eq. (21b), as shown, coincides with the pr-EN results. Eq. (21a), on the other hand, still offers the practical design benefits deriving from linear formulations, but at the same time is less conservative than Eq. (21b) for loading configurations such that $D < 1$.

Some further comparative plots are also proposed in Fig. 7, for the two worked examples of Section 3. In the case of the AN glass panes, see Fig. 7(a), it can be seen that Eqs. (21a) and (21b) are identical and provide for the failure configuration ($D = 1$) a rather good estimation, compared to the exact F & R approach, with substantial improvements to the pr-EN and CNR simplified formulations. The margin of approximation – always on the safe side – was found in fact to be equal to ≈ 13% the h_{min} value given by the F & R method.

It is important to further highlight, on the other hand, that as far as any toughening process is applied to glass, the scatter of Eqs. (21a) and (21b) at failure ($D = 1$) tends to 0 and lies always on the safe side, compared to the F & R solution, with an average magnitude of discrepancy (depending on the stress ratio and number of design actions) in the order of ≈ 1 ÷ 2% for HS glass and ≈ 0 ÷ 0.3% for FS glass respectively. A totally different damage evolution is obtained, however, for intermediate damage configurations in which $D < 1$ (see for example Fig. 7(b)).

From Fig. 7(b), it is possible to notice that the four analytical curves (i.e. F & R, the pr-EN and the WA (both Eqs. (21a) and (21b) methods) provide the same failure condition ($D = 1$). This happens when, for the assigned ULS combinations, the tensile stresses σ_j due to most of the N applied loads do not exceed the compressive state of stress σ_p. As a result, the shortest design action (i.e. the highest $k_{mod,j}$ coefficient, as given by Eq. (2) still represents, for a large scenario of loading configurations of practical interest in the structural glass design field, the most influencing loading contribution among the full combination of N imposed loads.

For a general case study object of verification, according to Eqs. (21a), (21b) and to the discussed comparisons, the first of them should be preferred. Its intrinsic advantage is in fact given by the pre-stressing effect accounted on the side of actions rather than on the material design strength, and this is in line with the design philosophy presented in [23].

5. Extended parametric investigation

Based on Sections 3 and 4 (i.e. Eqs. (21a) and (21b)), a further extended parametric study was finally carried out in order to draw some general considerations and suggestions about the explored design approaches, as well as to verify the accuracy of Eqs. (21a) and (21b) for the collapse detection ($D = 1$), compared to the exact F & R formulation. For this purpose, a sufficiently wide set of configurations of practical interest was taken into account. Assuming a rectangular shape for the reference glass pane object of analysis, with $b \times L$ the assigned dimensions (b the restrained edges), the parametric study was carried out by keeping fix some input geometrical/mechanical properties and giving evidence, step by step, of a set of key aspects for the design of the reference pane. These variations included and properly combined all together:

- different composition (PVB, SG) and thickness for the interlayer (h_{int} = 0.38 mm, 0.76 mm, 1.52 mm);
- number of variable design loads Q_{ki} (one or two), acting together with permanent loads G;
- duration of each variable load Q_{ki} (spanning in the range comprised between 3 s and 3 months).

In particular, the medium-term variable load (Q_1, 3 months, 30°) was varied in magnitude in the range 0–1.2 kN/m^2, while for the instantaneous variable load (Q_2, 3 s, 30°), the reference amplitudes of 0.25, 0.5 and 1.0 kN/m^2 were taken into account for all the geometrical configurations.

Assuming a symmetrical resisting cross-section for all the examined panels (with h the thickness of both the panes, see Fig. 2), the minimum glass thickness h_{min} required to satisfy the ULS resistance verification according to the pr-EN, CNR, F & R approaches was separately

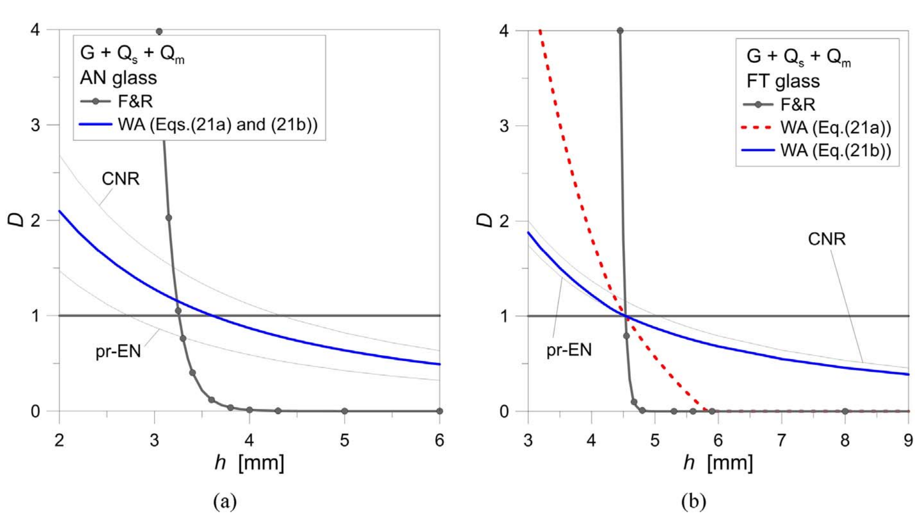

Fig. 7. Comparative analytical calculations for (a) AN (L = 655 m, Q_s = 0.80kN/m^2) and (b) FT (L = 2100 mm, Q_s = 1.20kN/m^2) laminated glass panels, as a function of the assigned glass thickness h.

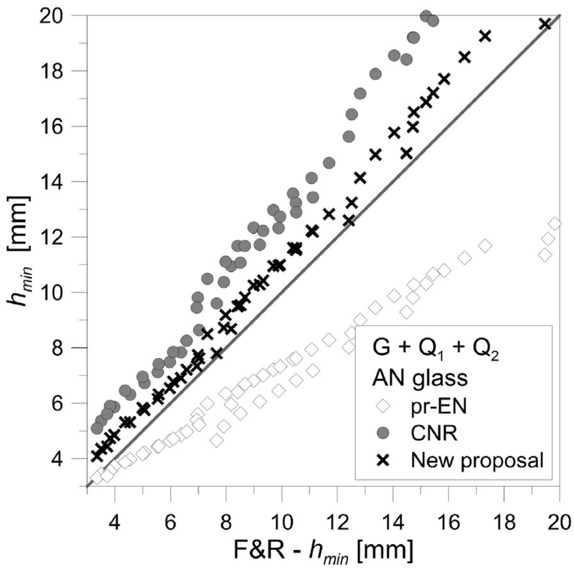

Fig. 8. Parametric ULS analytical calculations for the minimum thickness h_{min} leading the examined glass panels at failure ($D = 1$). Examples provided for AN glass panes.

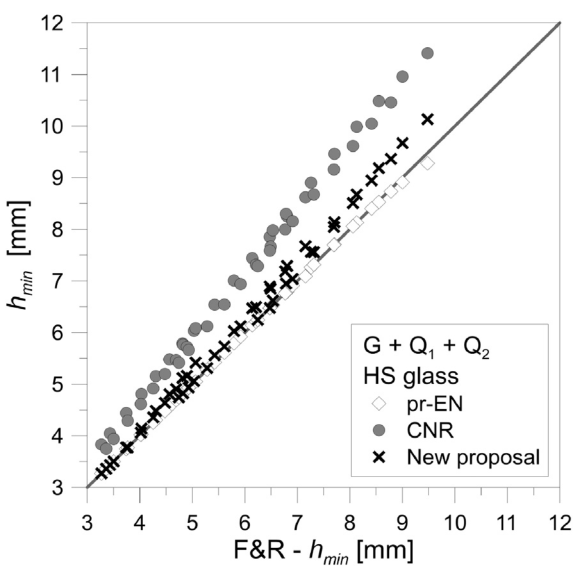

Fig. 9. Parametric ULS analytical calculations for the minimum thickness h_{min} leading the examined glass panels at failure ($D = 1$). Examples provided for HS glass panes.

calculated (Eqs. (5), (6), (17)). The same iterative calculations were then carried out by means of the alternative WA method, by means of Eqs. (21a) and (21b). The so achieved parametric results are compared in dimensional form in Figs. 8, 9 and 10 for AN, HS and FT glass respectively.

As shown in Fig. 8 for the AN glass panels, both the pr-EN and the CNR approaches are roughly approximate, compared to the F & R solution. In the case of the pr-EN calculations, all the predictions were typically found to lie on the unsafe side, with underestimations of the F & R solution typically increasing with the glass thickness h (i.e. being dependent on the stress ratios R_σ of each j-th action) and rising up to 30% of discrepancies. On the other side, the CNR formulation proved to be generally markedly conservative, compared to the F & R solutions, with a scatter typically in the order of ≈ 30%, but rising up to ≈ 60%. For the novel WA approach proposed in Eqs. (21a) and (21b), finally, safe predictions were generally found, with an average scatter (on the safe side) in the order of 10%.

The most important effects and outcomes were indeed observed from the parametric calculations carried out on the HS and FT panels.

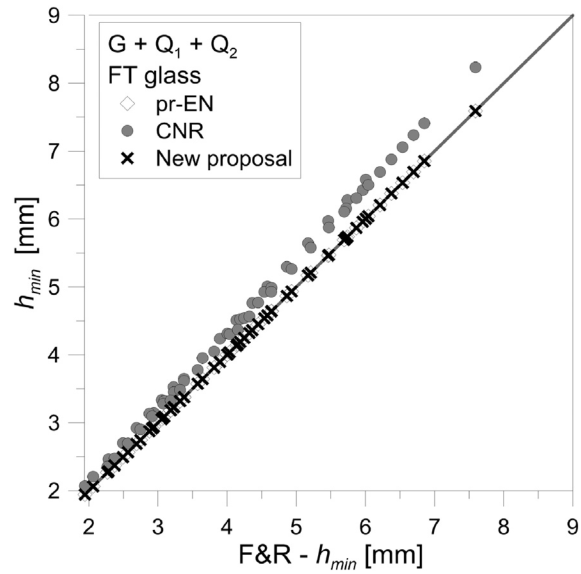

Fig. 10. Parametric ULS analytical calculations for the minimum thickness h_{min} leading the examined glass panels at failure ($D = 1$). Examples provided for FT glass panes.

In the case of HS glass, see Fig. 9, the WA approach proved in fact to offer a solution very close (on the safe side) to the exact one, for all the examined configurations. Major discrepancies, up to ≈ 5–7% the F & R method, were observed especially in presence of more than $N = 2$ design actions with typically high stress ratios R_σ (i.e. in the order of 0.7–0.8 the corresponding design strength for more than one j-th action).

The theoretically exact solution, see Fig. 9, was always captured by the pr-EN method, despite the inconsistency of its basic assumptions. In some cases only, however, unsafe predictions with ≈ 1–2% of scatter were also observed. On the other hand, the CNR approach still provided safe predictions, with average discrepancies in the order of ≈ 15–20%, compared to the F & R solution.

As far as the pre-stressing contribution was further increased, i.e. moving from the HS glass type towards the FT glass, it can be seen from Fig. 10 that Eqs. (21a) and (21b) – thanks to the weighted calculation of the equivalent $k_{mod,w}$ coefficient – proved to be able to provide exact solutions at failure, i.e. fully coincident with the F & R method. This is also the case of the pr-EN formulation, taking indirect advantage of the initial compressive state in glass. In the case of the CNR approach, conversely, safe predictions were again observed, with maximum discrepancies in the order of 5–10% the theoretically exact solutions.

6. Summary and conclusions

In this paper, an extended exploratory investigation was dedicated to the analytical verification of structural glass elements under combined design actions with specific characteristic durations and magnitudes.

Various existing analytical formulations available in the literature and design standards for the ULS resistance verification of structural glass elements were assessed via extended parametric investigations.

Careful consideration was first paid for two simplified, linear cumulative approaches currently in use within the European standard ('pr-EN' [2]) and the Italian code ('CNR' [3]) for structural glass, as well as for a theoretically exact formulation recently proposed by Franco and Royer-Carfagni ('F & R') in [23] and accounting for static fatigue effects via an exponential analytical expression. Additional assessment of the aforementioned approaches was also carried out by taking into account (for AN glass only) the American National standard provisions ('ASTM' [8]). Due to its theoretical background, the F & R approach was then taken into account as a reference model for the assessment of the pr-EN, CNR and ASTM formulations.

Based on a critical discussion of two extended case studies and a wide parametric analytical investigation, it was shown in particular that:

- For heat-strengthened (HS) or fully-tempered (FT) glass, the pr-EN formulation – despite the inconsistency of its basic assumptions – is able to provide accurate predictions for the failure configuration of a given structural glass member under a given ULS combination of actions. Compared to the F & R theoretical model, the pr-EN solution at failure was found to be always exact in the case of FT glass, and to lie in the range of a ≈ 0–2% of scatter (but always on the unsafe side) for HS glass panels.
- The same pr-EN approach, when no pre-stressing effects are considered for the glass elements to verify (i.e. AN float glass), typically provides markedly unsafe estimations, due to its wrong basic conditions, with margins of error (again on the unsafe side) up to 30–40%, compared to the F & R solution.

Based on the generally observed trends of the collected parametric results, the pr-EN approach – although simple in use – was hence found to be not applicable in design.

In addition:

- The Palmgren-Miner based method currently implemented in the Italian CNR standard for glass structures, despite its simplicity of application, was found indeed to be markedly conservative, compared to the F & R formulation. In this latter case, the level of approximation was in fact observed to lie in a range below ≈ 10% and ≈ 20% for FT and HS glass respectively, while an average scatter of ≈ 25% (and up to 50–60%, under specific loading conditions) was generally observed in the case of AN glass, depending on several input parameters (i.e. the number and magnitude of design variable loads, compared to the material pre-stressing treatment, etc.). Despite the different formulation for cumulative damage, almost coincident results were also observed for the verification of AN glass panels, as given by the CNR as well as by the ASTM national standards.
- The theoretically exact F & R approach, finally, since formulated on the base of an exponential resisting domain, was observed to be highly non-linear (for design conditions in which $D < 1$) and markedly sensitive to even small variations in the input parameters of the design problem, including for example the thickness of glass or the magnitude of the assigned design loads. For these reasons, a possible unsafe interpretation of the formulation estimations would induce the designer to reach a damage level $D = 1$, that is to fully exploit the structural performance of a given glass element. In this regard, a lack of appropriate safety margins was in fact perceived when solving most of the typical calculations of practical interest in design of glass structures. This high sensitivity would suggest (on the side of the professional engineer) a careful and rigorous design calculation, as well as the possible implementation (on the side of the committee in charge for the proposal of standardized design recommendations) of additional partial safety factors.

Based on the commented parametric calculations, as well as on a critical assessment of possible advantages and criticisms of each one of the examined formulations, an alternative weighted approach (WA) was hence proposed for the ULS resistance verification of structural glass elements under multiple design actions.

As shown, the advantage of the novel WA formulation – compared to the exact F & R method – lies in a simple and practical linear cumulative approach, as in the case of existing pr-EN and CNR methods. Differing from the earlier pr-EN and CNR formulations, however, thanks to the calculation of an equivalent $k_{mod,w}$ coefficient, the characteristic duration as well as the stress ratio of each imposed load are properly taken into account by the WA proposal. In accordance with the F & R approach, moreover, the compressive state deriving from possible pre-stressing treatments can be accounted on the side of actions, rather than on the side of the material design strength.

In the case of FT glass, the solution at failure for the extended parametric investigation was found to always coincide with the F & R formulation. In the case of HS or AS glass respectively, safe predictions were also generally observed, with an average scatter from the F & R solution in the order of 5% and 10% respectively.

As a result, although the intrinsic non-linearity of damage propagation is not captured, the failure configuration is rationally estimated in most of the cases of practical interest for design of glass structures. Thanks to the linearity of the cumulative based model, the simplicity of calculations is also preserved, hence providing a practical tool for designers. At the current stage of the research study, in conclusion, it is expected that general observations pointed out through the wide analytical investigations and comparisons could represent a useful background for development and improvement of practical design formulations, as well as that they could be further validated and refined by extensive experimental tests.

References

[1] prEN 13474-1. Glass in building – design of glass panes – part 1: general basis of design. European Committee for Standardization; 2007.
[2] prEN 16612. CEN/TC 250. Glass in building - determination of the load resistance of glass panes by calculation and testing European Committee for Standardization. 2013; 2013. (2013).
[3] CNR-DT/210. Guide for the design, construction and control of structures made of glass structural elements – Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Costruzioni con Elementi Strutturali di Vetro. National Research Council (CNR); 2013.
[4] ÖNORM B 3716. Glass in building – structural glass construction. Osterreichisches Normungsinstitut; 2013.
[5] NEN 2608. Glass in building-requirements and determination method. Nederlands Normalisatie Instituut; 2011. (2011).
[6] DIN 18008:2010-12. Glass in building – design and construction rules. German Institute for Standardisation (Deutsches Institut für Normung); 2010.
[7] CAN/CGSB 12.20-M89. Structural design of glass for buildings. Canadian General Standards Board; 1989.
[8] ASTM E1300 – 12ae1. Standard practice for determining load resistance of glass in buildings. The American Society for Testing and Materials; 2002.
[9] Feldmann M, Kasper R, Abeln B, Cruz P, Belis J, Beyer J, et al. Dimova Pinto, Feldmann Denton, editors. Guidance for European Structural design of glass components – support to the implementation, harmonization and further development of the Eurocodes. Report EUR 26439 Joint Research Centre–Institute for the Protection and Security of the Citizen; 2014. http://dx.doi.org/10.2788/5523.
[10] Larcher M, Arrigoni M, Bedon C, van Doormaal A, Haberacker C, Hüsken G, et al. Design of blast-loaded glazing windows and façades: a review of essential requirements towards standardization. Advances in civil engineering - special issue "advancements in design, analysis, and retrofitting of structures exposed to blast" vol. 2016. 2016; 2016. p. 14. http://dx.doi.org/10.1155/2016/2604232. (Article ID 2604232).
[11] Maniatis I, Elstner M. Investigations on the mechanical strength of enamelled glass. Glas Struct Eng 2016;1(1):277–88. http://dx.doi.org/10.1007/s40940-016-0025-2.
[12] Hagl A. Development and test logics for structural silicone bonding design and sizing. Glas Struct Eng, 1(1): 131–151, doi: http://dx.doi.org/10.1007/s40940-016-0014-5
[13] Badalassi M, Biolzi L, Royer-Carfagni G, Salvatore W. Safety factors for the structural design of glass. Constr Build Mater 2014;55:114–27.
[14] Ballarini R, Pisano G, Royer-Carfagni G. New calibration of partial material factors for the structural design of float glass. Comparison of bounded and unbounded statistics for glass strength. Constr Build Mater 2016;121:69–80.
[15] Bedon C, Amadio C. Design buckling curves for glass columns and beams. Struct Build 2015;168(7):514–26.
[16] Bedon C, Amadio C. A unified approach for the shear buckling design of structural glass walls with non-ideal restraints. Am J Eng Appl Sci 2016;9(1):64–78.
[17] Wiederhorn S, Bolz L. Stress corrosion and static fatigue of glass. J Am Ceram Soc 1970;53(10):543–8.
[18] Brown W. A load duration theory for glass design. Publication no. NRC 12354. National Research Council of Canada. Division of Building Research; 1972.
[19] Beason W, Morgan J. Glass failure prediction model. J Struct Eng 1984;110(2):197–212.
[20] Sedlacek G, Blank K, Gusgen J. Glass in structural engineering. Struct Eng 1995;73(2):17–22.
[21] Fischer-Cripps A, Collins R. Architectural glazing: design standards and failure models. Build Environ 1995;30(1):29–40.
[22] Overend M. Recent developments in design methods for glass structures. Struct Eng 2010;88(14):18–26.

[23] Franco A, Royer-Carfagni G. Verification formulae for structural glass under combined variable loads. Eng Struct 2015;83:233–42.
[24] Haldimann M. Fracture strength of structural glass elements – analytical and numerical modelling, testing and design Ph.D. Dissertation, Thése EPFL no 3671 École Polytechnique Fédérale de Lausanne (EPFL); 2006.
[25] Haldimann M, Luible A, Overend M. Structural use of glass. IABSE; 2008. ISBN 978-3-85748-119-2.
[26] EN 1990. Basis of structural design. European Standard; 2002. 2002.
[27] EN 1993-1-1. Eurocode 3 – design of steel structures – part 1–1: general rules and rules for buildings. Brussels, Belgium: CEN; 2004.
[28] Beason WL, Morgan JR. Glass failure prediction model. J Struct Eng 1984;110(2):197–212.
[29] Siebert G. Beitrag zum Einsatz von Glas als tragendes Bauteil im konstruktiven Ingenieurbau PhD Dissertation Technische Universität München (TUM); 1999.
[30] Galuppi L, Royer-Carfagni G. Effective thickness of laminated glass beam. New expression via variational approach. Eng Struct 2012;38:53–67.
[31] EN 572–2. Glass in buildings-basic soda lime silicate glass products. Brussels, Belgium: CEN; 2004.