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MATHEMATICS ANXIETY AND WORKING MEMORY

What is the relationship?

Maria Chiara Passolunghi, Marija Živković and Sandra Pellizzoni

Mathematics is all around us. Mathematical competence is one of the most important abilities that a person needs to master in life both for professional choices – especially for careers in science, technology and engineering (Dougherty, 2003; Rivera-Batiz, 1992) – and for properly mastering everyday activities, (e.g., cooking, banking, shopping; Mazzocco, 2008) as well as personal wellbeing (e.g., Reyna & Brainerd, 2007; Reyna, Nelson, Han, & Dieckmann, 2009). Further, mathematical abilities not only affect the individual's life but are also crucial for the entire society given their effects on economic opportunity and on meaningful participation in the community (Moses & Cobb, 2001; Peterson, Woessmann, Hanushek, & Lastra-Anadòn, 2011). In fact, there is a correlation between the development of society and mathematics: the more complex a society is, the more it requires advanced mathematical abilities.

Recent progress in research on mathematical performance has given the professionals that work with mathematical abilities the opportunity to evaluate different areas that underline mathematical achievement, including both cognitive and emotional aspects. The role of these two factors is extensively studied in literature even though there is still lack of consensus concerning the beginning of the influence of math anxiety on mathematical achievement. Moreover, the reciprocal influence of emotional and cognitive aspects of mathematical task performance is far from clear. Scientific studies of the topic are scarce, provide limited information and are not conclusive, especially with regard to young children. It is crucial to investigate this topic, from both a developmental and a clinical point of view, in order to understand the developmental trajectories of these factors and to gain important information that may facilitate work with children with mathematical difficulties. Therefore, the chapter will provide an update and critical revision of the state of the art concerning the reciprocal effects between working memory (WM) and math anxiety (MA).

To provide a clear overview of the theme, the chapter will be organized in different sections specifically addressing the different factors (mathematical abilities, WM and MA) that we are going to consider and then the specific relations between these aspects. In particular:

1 The first part will provide a description of WM processes;
2 The second part will provide a short description of how different mathematical abilities are influenced by cognitive factors, with particular focus on WM;
3 The third part will focus on the affective factors that influence mathematical learning. It also examines the reciprocal interplay between MA and WM, evaluating this relationship from a developmental perspective.

I. Working memory

WM is a limited-capacity system that holds information for a brief period while simultaneously manipulating it (Baddeley & Hitch, 1974). Baddeley and Hitch (1974) proposed an explanation for WM as a three-way system composed of the central executive (assumed to be an attention-controlling system), the visuo-spatial sketchpad, which manipulates visual images, and the phonological loop, which stores and rehearses speech-based information. Baddeley (2000) added a fourth element: the episodic buffer, a limited-capacity system that integrates and provides temporary storage of information from the two subsystems and long-term memory (see Figure 6.1). Studies of this element are very limited, especially within developmental psychology. For this reason, our analysis will not focus on this component.

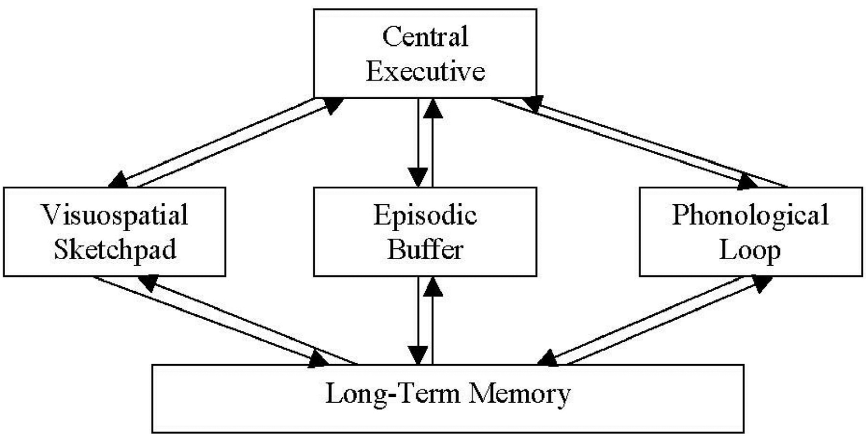

FIGURE 6.1 Working memory as a four-way system

(Baddeley, 2000)

The distinctions between the central executive system and specific memory storage systems (i.e., the phonological loop and visuo-spatial sketchpad) in some ways parallel the distinction between working memory and short-term memory. Working memory is a term that refers to a processing resource of limited capacity that is involved in the preservation of information while simultaneously processing the same or other information (Baddeley & Logie, 1999).

Another model of WM, named the continua model (Cornoldi & Vecchi, 2000, 2003), proposes that WM consists of two dimensions: the horizontal continuum and the vertical continuum. The horizontal continuum is related to verbal, visual and spatial material and receives sensory information from the outside world, but is simultaneously connected with the representations stored in the long-term memory. The vertical continuum involves the processes that require active elaboration between information from various sources and is defined by the level of processing information needed ("active" or "passive"). The conical structure represents several subsystems at the "passive" dimension of the horizontal continuum, completely independent of each other. Along the vertical continuum, it can be observed that the processes become progressively more independent of the form in which the information is presented. It will be much easier to understand this model through examples: a forward span task as an example of a "passive" task and a listening span task as an example of an "active" task. In the forward span task the participant is asked to remember increasingly longer series of words, figures or syllables (usually from 2 to 7 for children), and to reproduce them from memory in the same order of presentation. These tasks can be considered as "passive" because the material must be reproduced in the same format in which it was presented, and does not need to be manipulated or transformed to any significant degree. The listening span task is requires the participant to judge the truth or falsity of a series of sentences immediately after hearing them. The participant has to reproduce the last word of the sentence, and of each previous sentence in order, as far back as they can remember. The task is composed of an increasing number of sentences. The listening span task considered "active" because it requires *both* storage *and* a high level of control and processing of information. Some experimental data show that the level of difficulty of both types of task can be manipulated and that "passive" tasks can be made significantly more difficult than active ones (see Figure 6.2, Cornoldi & Vecchi, 2000, 2003; Passolunghi & Siegel, 2001, 2004).

II. Mathematical abilities

Research has extensively studied the different components that characterize mathematical ability. For example, Geary (2004) listed the following components: word problems, calculation procedures, number fact retrieval and visuo-spatial abilities. Karagiannakis and colleagues (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014) proposed a different way to approach mathematical abilities (and disabilities) proposing core numbers, memory, mathematical reasoning and

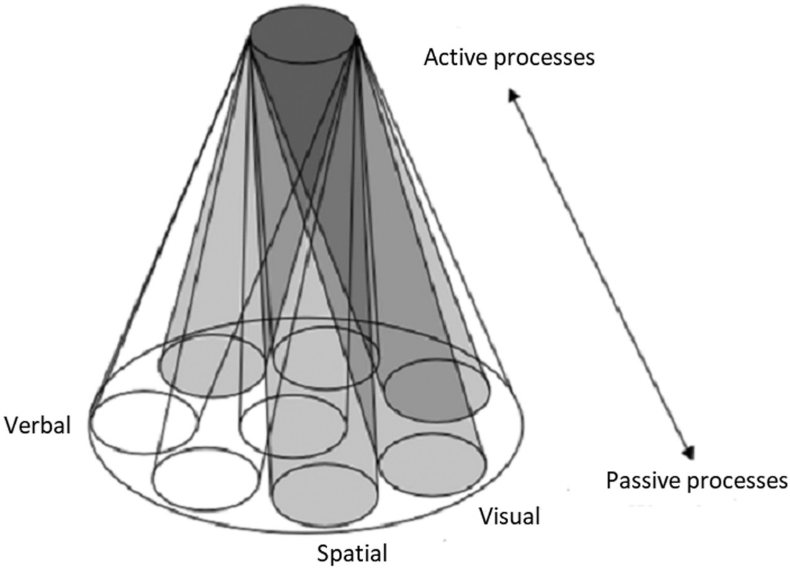

FIGURE 6.2 Continua model
(Cornoldi & Vecchi, 2003)

visual spatial abilities as the foundations of mathematical development. Geary's approach (2000, 2004) mainly focuses on domain-specific abilities as the basis of the different mathematical domains, while Karagiannakis's model puts more emphasis on domain-general cognitive abilities (Karagiannakis et al., 2014) at the base of mathematical learning. Though the debate over whether domain-general vs. domain-specific abilities are the main foundations of mathematical development is still ongoing, the literature agrees that mathematical learning requires a variety of different abilities in relation to the different topics. In this chapter we describe some specific aspects of mathematical learning that have been extensively studied in the literature. These aspects are strongly related to WM and recent studies suggest that they may be affected by MA: 1) the approximate number system, 2) calculation skills and 3) problem-solving.

The approximate number system

Numbers represent a basic way of describing a world and are abstractions that can be applied in different real or imaginary situations (four pieces of cake, four peers, four windows, four trees, four times, four minutes, four unicorns) (see, e.g., Spelke, 2017). Despite the ubiquity and simplicity of natural number concepts, their origins are debated and their cognitive foundations are still unclear. A wealth of empirical studies have suggested that both human and many non-human animal species share an intuitive "number sense", that comprises a variety of competencies, such as the ability to subitize and count to distinguish number patterns, to discriminate quantities and to switch between different numerical formats (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Libertus, Feigenson, & Halberda, 2011). One central component of this number sense is the approximate number system (ANS) (Feigenson, Dehaene, & Spelke, 2004). This begins as a primitive, non-verbal and noisy cognitive system that allows the representation, estimation and discrimination of numerical quantities in an imprecise and intuitive way, without using counting or numerical symbols (Gilmore et al., 2013). More specifically, ANS can be described as an innate, preverbal system, which is independent of language, and remains active during the entire lifespan (Dehaene, 1997; Feigenson, Libertus, & Halberda, 2013). Several studies have investigated this topic through research on specific characteristics of the ANS. First of all, ANS-acuity involves the degree of accuracy of the internal quantity representation, and there are significant interindividual differences in ANS precision (Park & Starns, 2015). Many studies have found correlations between this aspect of ANS and mathematical abilities (Geary, Hoard, Nugent, & Rouder, 2015; Libertus, Feigenson, & Halberda, 2011, 2013; Soto-Calvo, Simmons, Willis, & Adams, 2015; Van Marle, Chu, Li, & Geary, 2014).

Other characteristics of ANS are the distance and size effect. The distance effect means that it is easier to discriminate numbers that are further apart in the numerical distance (2 vs. 8 is easier than 3 vs. 5); and the size effect means that it is easier to discriminate smaller numbers compared to larger ones with the same distance between them (3 vs. 5 is easier than 33 vs. 35). However, not all studies have shown a link between the ANS and mathematical achievement in children (Sasanguie, De Smedt, Defever, & Reynvoet, 2012a; Sasanguie, Van den Bussche, & Reynvoet, 2012b; Soltész, Szúcs, & Szúcs, 2010), and the evidence for this relationship in adults is mixed (Castronovo & Göbel, 2012; Feigenson et al., 2013; Price, Palmer, Battista, & Ansari, 2012).

The direction of causation between ANS and mathematical ability is still unclear. In particular, some studies suggest that ANS is a precursor of later mathematical abilities (Gilmore et al., 2010; Lyons & Beilock, 2011; Park & Brannon, 2013). According to this view, more precise approximate number representations might make some individuals better at mathematics. Conversely, other research has found that stronger mathematical abilities (maybe due to differences in the quality and/or quantity of the mathematics education that individuals receive) sharpen ANS representations (Mussolin, Nys, Content, & Leybaert, 2014; Pica, Lemer, Izard, & Dehaene, 2004). A third possibility is that the relationship between the ANS and mathematical abilities found in some studies could be mediated by domain-general cognitive abilities (Fuchs & McNeil, 2013; Hyde, Khanum, & Spelke, 2014), such as inhibition (Gilmore et al., 2013) or working memory skills. The association between ANS and mathematics could also be moderated by participant age (Inglis, Attridge, Batchelor, & Gilmore, 2011; Rousselle & Noël, 2008) or by the specific mathematics measures used in the studies (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013).

Recent studies have investigated the relation between ANS and MA. Some studies indicate lower performance in symbolic magnitude processing in adults with high MA (Maloney, Risko, Ansari, & Fugelsang, 2010; Maloney, Ansari, & Fugelsang, 2011; Núñez-Peña & Suárez-Pellicioni, 2014) while no significant association with MA was found in the few studies that adopted non-symbolic tasks to assess magnitude processing in children (Gómez-Velázquez, Berumena, & González-Garrido, 2015; Hart, Logan, Thompson Kovas, McLoughlin, & Petrill, 2016). A recent study Cargenlutti, Tomasetto, and Passolunghi (2017b), which will be described in the next section, proposed an interesting model of the relationships between WM, MA, ANS and mathematical achievement.

Calculation skills

Arithmetic calculation is an important academic skill that children learn when they start formal education. Gelman and Gallistel (1978) proposed five implicit counting principles at the base of calculation skills. The principles include one-to-one correspondence (only one word is assigned for each counted object: "one", "two", "three", . . .), the stable order principle (the order of the words must be invariant across counted sets), the cardinality principle (the value of the final word represents the quantity of the items in the counted set), the abstraction principle (any kind of objects can be collected together and counted) and the order irrelevance principle (items in a set can be tagged in any sequence).

Basic addition skills are a fundamental prerequisite for the development of multiplications skills and increasingly complex arithmetic abilities. A substantial body of research has focused on identifying the cognitive processes that underlie arithmetic calculation and has demonstrated the important role of working memory. For instance, to perform a mental calculation (e.g., 57 + 8), it is necessary to temporarily retain the phonological representations of the numbers. The next step may be to employ one or more procedures (e.g., counting) to combine the numbers and produce an answer. Alternatively, employing carrying or regrouping strategies will maintain recently processed information while performing other mental operations. For example 57 + 10 – 2, or 55 + 8 + 2. Finally, we would need to add the products held in working memory, resulting in the correct solution.

This example clearly shows how the cognitive processes involved in performing arithmetic calculations are embedded within the working memory system. Even the simplest mathematical calculations require the temporary storage of problem information, the retrieval of relevant procedures and the processing of operations to convert the information into a numerical output. Studies have also shown that the different working memory components (e.g., visuo-spatial sketchpad, phonological loop and central executive) play specialized and unique roles in the arithmetic calculation (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Higher working memory capacity is associated with higher accuracy in solving complex arithmetic calculation in adults as well as in children. In particular, children with higher working memory abilities tend to use more sophisticated strategies such as decomposition in preference to less sophisticated strategies such as finger counting (Geary, Hoard, Byrd-Craven, & Catherine DeSoto, 2004; Caviola, Mammarella, Pastore, & LeFevre, 2018).

Studies indicate that the central executive plays a greater role in mental calculation than the phonological loop (e.g., De Rammelaere, Stuyven, & Vandierendonck, 2001). The phonological loop does play a major role when calculation involves storing temporary information, while carrying operations put a major demand on the central executive processes (Furst & Hitch, 2000). Only a limited number of studies have examined the role of the visuo-spatial component of the WM model (Passolunghi & Mammarella, 2010, 2012; Mammarella, Caviola, Giofré, & Szűcs, 2018; Szűcs, 2016; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013). These studies have shown that visuo-spatial WM is related to performance in written calculation. In particular, it is important during the initial stages of arithmetic calculation for encoding arithmetic problems presented visually.

In relation to the possible interplay between calculation, MA and WM, some authors (Ashcraft, 2002; Ashcraft & Kirk, 2001) have proposed that, due to the effect of anxiety on the central executive component of WM, MA will be associated with lower performance, especially in more complex calculations. On the other hand, other accounts propose that MA only affects calculation performance in individuals with high working memory (Beilock, Holt, Kulp, & Carr, 2004). In the next section, the relationships between MA, WM and mathematical performance will be discussed in more detail with a specific focus on studies with young students.

Mathematical problem-solving

Mathematical problem-solving is a core aspect of mathematics acquisition (Pimta, Tayruakham, & Nuangchalerm, 2009), crucial for analyzing and interpreting the world in mathematical terms and applying mathematics to everyday life to use their mathematical knowledge in solving daily problems. The word problems have been described as instruments that develop the students' abilities and talents in solving mathematical problems (De Corte, Vershaffel, & De Win, 1989). It is for this reason that word problems are typically introduced in the earliest stages of mathematical instruction (Cummins, 1991). Word problems are defined as verbal descriptions of problem situations, which raise one or more questions, that can be answered by the application of mathematical operations to the numerical data available in the problem statement (Verschaffel, Greer, & De Corte, 2000). An example of such a task is the following: "John bought 4 pizzas with 8 slices each. He and his friend Bruce ate 12 slices of pizza. How many slices were left?"Geary (1995)states that children make more errors when solving word problems than solving comparable number problems. The reason for such inability is the fact that solving such word problems requires children both to perform mathematical computations and possess the linguistic knowledge necessary to understand the problems (Cummins, Kintsch, Reusser, & Weimer, 1988). Word problem-solving, in fact, depends on several factors: linguistic comprehension, understanding the situation, selecting relevant information while ignoring irrelevant information, working out the right formal procedure to be applied and the appropriate arithmetical operations to be carried out to reach the solution and, finally, computational ability (Fuchs et al., 2010; Mayer, 1992; Mayer, Larkin, & Kadane, 1984). From a domain-general cognitive point of view, there are also different factors involved in mathematical problem-solving, for example, intelligence (Giofré, Borella, & Mammarella, 2017), WM, specifically dual tasks (e.g., Cornoldi & Giofrè, 2014; Cornoldi, Drusi, Tencati, Giofrè, & Mirandola, 2012) and visuo-spatial WM (e.g., Bizzaro, Giofrè, Girelli, & Cornoldi, 2018), as well as processing speed (Demetriou et al., 2014; Passolunghi, 1999; Salthouse, 1996; Salthouse & Meinz, 1995). Several results have suggested that complex working memory skills are impaired in children with mathematical learning disabilities. In particular the findings suggest a working memory deficit in children with mathematical difficulties, mainly in the central executive component of Baddeley's model, and, specifically, in the inhibitory and updating processes (Passolunghi, Cornoldi, & De Liberto, 1999; Passolunghi & Siegel, 2001, 2004; Passolunghi & Pazzaglia, 2005). Besides cognitive characteristics, an inverse correlation between MA and arithmetical problem abilities was found in a study by Passolunghi, Cargnelutti, & Pellizzoni (under review). Strikingly, this study found the correlation to be present even in tasks without a time limit.

Despite the cognitive and emotional profiles characteristic of students with mathematical difficulties, studies of mathematical problem-solving have highlighted a complex interplay of variables contributing to the development of this skill and, in particular, to students' abilities to make flexible choices in solving mathematical problems (Verschaffel & De Corte, 1997; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009). Notably, the characteristics of the tasks itself (e.g., amount of unnecessary information in the text, the verbal complexity of the problem, familiarity with the situation described), the specific abilities of the student (especially in relation to domain-general cognitive abilities), and contextual characteristics (e.g., educational methods, socioeconomic status) seem to be crucial to mathematical problem-solving performance. This approach to the topic of mathematical problem-solving implies that it is important to study not only the ways in which tasks are solved but also student and contextual characteristics (Acevedo Nistal, Van Dooren, Clarebout, Elen, & Verschaffel, 2009; Verschaffel, De Corte, de Jong, & Elen, 2011).

III. The relation between math anxiety and working memory

Anxiety and math anxiety

Anxiety, defined as a dispositional dysfunctional response to a situation perceived as threatening (Lewis, 1970), has a variety of repercussions on achievement, as well as on mental health and well-being. In the school context, studies show that clinical levels of anxiety are present in 10% of children and are recorded even in kindergarten (Egger & Angold, 2006). Anxiety has been observed especially often in students with learning difficulties, typically described as more anxious than their peers (Fisher, Allen, & Kose, 1996). The American Psychiatric Association (2013) defines anxiety as "an emotion characterized by feelings of tension, worried thoughts and physical changes like increased blood pressure". Recent studies indicate relationships between anxiety and complex cognitive performance (Arnsten, 2009; Diamond, Campbell, Park, Halonen, & Zoladz, 2007). The studies showed that emotional factors have an impact on mathematical performance (Ashcraft, Kirk, & Hopko, 1998; Ashcraft, 2002; Ho et al., 2000) and that a form of anxiety specifically relates to mathematics (Ashcraft, 2002; Hembree, 1990; Maloney & Beilock, 2012). Richardson and Suinn (1972) defined MA as "a feeling of tension, and anxiety that interferes with manipulation of numbers and the solving of mathematical problems in ordinary life and academic situations". Ashcraft (2002) suggested that individuals with high MA avoid situations in which they have to perform mathematical calculations. Unfortunately, avoidance of mathematical results in less exposure and practice in mathematics, resulting in lower achievement, which, in turn, makes the students more anxious. In college and university, students with higher mathematics anxiety take fewer mathematics courses and have more negative attitudes toward mathematics. MA can range from mild to severe, from seemingly minor frustration to overwhelming emotional (and physiological) disruption. There are some practices in the traditional mathematics classroom that cause great anxiety in many students, for example, authoritarian practices by teachers and school leaders, public exposure (e.g., being asked to perform a calculation in front of the class or to solve a problem at the blackboard) and time limits.

As shown in the sections above, MA and WM are among the two factors that most influence mathematics achievement. In the next section, the reader will be introduced to the analysis of the literature that investigates the reciprocal effects of MA and WM on mathematics performance. The research will first describe the studies that evaluate the effects of these two factors on adults and will then discuss the issues from a developmental perspective.

Reciprocal effects of MA and WM on mathematical performance

For decades the effects of MA and WM (considered respectively as emotional and cognitive factors) on mathematical performance have been studied separately. More recently, researchers have tried to evaluate the reciprocal effects of WM and MA on mathematical abilities, and they agree that WM resources, usually devoted to solving mathematical tasks resolution, are decreased by the presence of MA and that this detrimentally impacts performance (Ashcraft & Kirk, 2001; Beilock & Carr, 2005; Calvo & Eysenck, 1992; Miller & Bichsel, 2004; Young, Wu, & Menon, 2012). Ashcraft and Faust (1994) observed that a high level of MA did not affect the execution of easy mathematical tasks such as single-digit addition problems (e.g., 2 + 3) but that it negatively impacted the execution of more complex and less automatic tasks, such as those with operations requiring carrying e.g., 46 + 25). To explain this experimental evidence, Ashcraft (Ashcraft, 2002; Ashcraft & Kirk, 2001) proposed a significant role for the central executive: if this component of WM does not work properly because of anxiety, it prevents an adequate processing of the information relevant to the task along with concurrent inhibition of irrelevant information (Ashcraft et al., 1998).

As to the question of who is most susceptible to WM disruption as result of high MA, studies show contrasting results. On the one hand, some authors posit that the better an individual's WM, the better they can manage mathematical performance in the face of anxiety-driven thoughts (Ashcraft & Kirk, 2001). An opposite account, on the other hand, indicates that individuals with higher WM are more susceptible than others to performance failures caused by anxiety (Beilock & Carr, 2005). With regard to the first hypothesis, the authors proposed that a higher level of WM leads to reduced susceptibility to the detrimental effects of anxiety, given that the greater WM capacity permits simultaneous attention to both the tasks and to the anxiety-driven thoughts. Miller and Bichsel (2004) found such effects especially with regard to spatial WM in both mathematical calculation skills and problem-solving. An alternative hypothesis suggests that high WM individuals are more affected by MA. High WM individuals, in fact, seem to devote fewer resources to the task execution when anxiety-related thoughts intrude on mathematical tasks (Beilock & Carr, 2005; Mattarella-Micke, Mateo, Kozak, Foster, & Beilock, 2011). The authors observed such an effect especially when they asked participants to solve tasks in a stressful situation, and they referred to it as "choking under pressure" (Beilock & Carr, 2005).

The literature clearly shows the detrimental effect of MA on both WM and mathematical performance. Which individuals are most susceptible to WM disruption in high anxiety situations, is, however still a matter for debate. It should be noted that the relationships between WM and MA and the possible detrimental effect of MA on WM and mathematical performance have been less studied in young children. In the next section, we give a critical summary of developmental research on links between MA, WM and mathematical achievement.

MA, WM and children's mathematical performance

The initial attempts to investigate links between MA, WM and mathematical performance have been described in articles by Ramirez, Gunderson, Levine, and Beilock (2013) and by Vukovic and colleagues (2013). Ramirez et al. (2013) gave first- and second-grade students tests of WM problem-solving, and a newly adapted MA test. The results showed a negative correlation between MA and problem-solving, which was found only among children who scored relatively high on WM, but not among those who scored relatively low in WM. The second study (Vukovic et al., 2013) extended the research to include different cognitive functions during the second and third grade. Once again, a challenge for their research was to develop a measure for MA, because existing measures were designed for children at or above the fourth grade. They developed an ad hoc scale composed of 12 items that adapted material from the MARS-E (Suinn, Taylor, & Edwards, 1988) and MAQ (Wigfield & Meece, 1988). The reported reliability of the MA self-report was a Cronbach's alpha around .70. General control variables were also tested (e.g., reading abilities and early numeracy and visuo-spatial WM). The results showed that MA was negatively correlated with applied mathematical problems. Similarly to Ramirez et al. (2013), who found MA to predict mathematical performance in students with high WM during the same school year, Vukovic and colleague (2013) highlighted that children with a high MA score in second grade demonstrated worse performance in mathematical performance in third grade. This effect was observed only in students with high WM abilities. Moreover MA does not influence all components of mathematical performance in the same way. Calculation skills and the ability to apply mathematical knowledge, but not geometric reasoning, were affected. The authors speculated that MA may specifically influence performance on tasks that involve understanding and manipulating numbers, rather than other aspects of mathematics.

In another recent study of second and third graders (Cargnelutti, Tomasetto, & Passolunghi, 2017a), both cognitive and emotional factors were studied. In particular, the study examined the interplay between domain-general factors (e.g., WM), domain-specific cognitive factors (e.g., the approximate number system) and affective aspects, and their influences on mathematical ability. This was the first attempt to explore the interplay between general anxiety and MA, with regard both to WM and the non-symbolic approximate number system (ANS) in relation to mathematical performance. The research confirmed the significant negative relation between general anxiety and mathematical performance in young school children. Also, the negative relation between general anxiety and mathematical performance was observed to operate through the disruption of WM and ANS. Beyond providing further evidence for a direct negative relation between general anxiety and mathematical performance, the authors also investigated whether anxiety might also indirectly disrupt mathematical proficiency by negatively affecting the domain-general and the domain-specific cognitive correlates of mathematics. In fact, while the negative influence of anxiety on the execution of tasks involving WM is well established (at least in older children and adults; see Eysenck, Derakshan, Santos, & Calvo, 2007), its impact on more automatic and basic quantitative skills, such as those related to non-symbolic ANS, has so far received little study. Cargnelutti et al.'s (2017b) findings indicated that ~~– among the correlates that significantly predicted mathematical proficiency –~~ WM for digits and the approximate addition skills were both significant mediators of the relation between general anxiety and mathematical performance, whereas estimation ability was not (see Figure 6.3). AuQ2

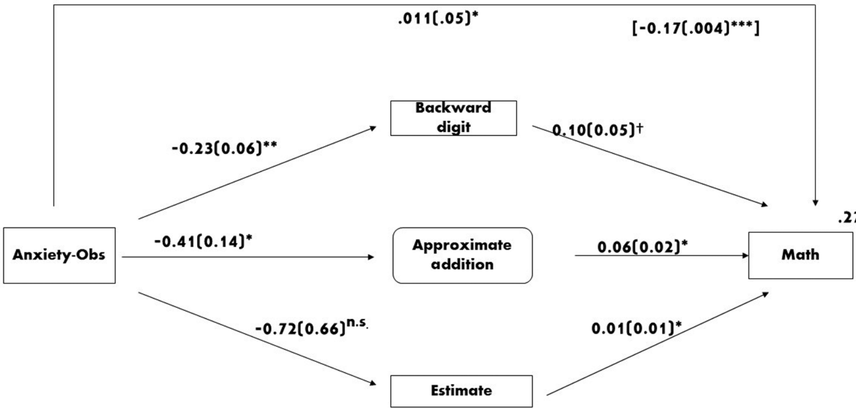

FIGURE 6.3 Multiple mediation model. Unstandardized coefficient of the mediation model between general anxiety (Anxiety-Obs) and mathematical achievement (Math)) through backward digit memory, approximate addition, and estimation (standard errors presented in parentheses). In squared parentheses, the total effect (with standard error) of anxiety observed on mathematical achievement. AuQ3

Dashed lines indicate non-significant paths. † p < .10, *p ≤ .05, **p ≤ .01, ***p ≤ .001.
Adapted by Cargnelutti et al. (2017b).

To sum up, MA, WM and mathematical achievement are already correlated in the first years of primary school. In very young students, there seems to be a detrimental effect of MA on WM and mathematical achievement, but this is only significant for children who start out with a high level of WM (Beilock & Carr, 2005; Mattarella-Micke et al., 2011). If these effects are found in the first three years of primary school, others are found during the last two years of primary school. Passolunghi, Cargnelutti, and Pellizzoni (under review) investigated the relation between cognitive and emotional factors in a group of fourth graders. This study assessed their problem-solving proficiency, their MA (MARS-R, Saccani & Cornoldi, 2005) and their WM levels. Again, cognitive and emotional factors significantly predicted problem-solving performance in a similar way to younger children (Cargnelutti et al., 2017b; Ramirez et al., 2013; Wu, Barth, Amin, Malcarne, & Menon, 2012), but another effect of MA was indicated. The results showed that children with high MA were more impaired in both problem-solving and WM tasks, and this occurred in an untimed problem-solving task. This appears to rule out the hypothesis that MA only exerts a negative role by increasing pressure during timed mathematical tasks (e.g., Faust, Ashcraft, & Fleck, 1996). Regarding the relation between WM and MA, the results of this study showed a strong negative effect of MA on both arithmetical problem-solving and WM. Children with high MA were not only more impaired in problem-solving but also in WM than those with low MA.

Going from primary to secondary school students, Passolunghi, Caviola, De Agostini, Perin, and Mammarella (2016) investigated the calculation proficiency and the cognitive profiles of secondary school students with high versus low MA. The results showed that children with high MA performed less well both in verbal short-term and working memory tasks. The effect was specific: high MA predicted poor achievement in mathematics but not in other school subjects (e.g., reading, decoding, reading comprehension and writing accuracy). Implications of this study are that high MA and low WM capacity can be seen as risk factors for poor mathematical achievement. Middle school students with high MA are at greater risk than those with a lower level of MA in performing poorly in mathematical achievement measures, and this detrimental effect is specific for mathematical abilities.

If studies of typically developing children provide interesting insights into the relation between MA and WM, so do studies of atypical development. Mammarella, Hill, Devine, Caviola, and Szucs (2015) tested verbal and visuo-spatial short-term memory and WM in children with developmental dyscalculia and high math anxiety, comparing them with typically developing children.

There were differences between children with developmental dyscalculia and those with high MA with regard to verbal and visuo-spatial short-term memory and WM. The results showed that children with developmental dyscalculia showed specific impairments in the visuo-spatial WM task. On the other hand, the MA group was particularly impaired in the verbal WM task. These data seem to show how different emotional and neurocognitive aspects can be linked to specific strengths and weaknesses related to mathematical performance.

Specific treatment for math anxiety

As seems evident from the previous sections, the role of anxiety in cognitive functions and achievement is crucial, though the literature on how to promote treatment for this condition is still scarce. The few contributions on the topic show that applying the expressive writing technique could help people control anxious feelings. This clinical approach encourages people to write freely about their thoughts and feelings concerning anxiety (Pennebaker & Beall, 1986) with the effect of increasing availability of WM for performing tasks (Klein & Boals, 2001; Yogo & Fujihara, 2008). In particular, writing about traumatic things (a mild manipulation of negative moods) appears to have an effect on cognitive WM, compared to positive or neutral writing. Thus, WM is not a static factor but can be manipulated (and increased) as a function of psychosocial manipulation, reflecting variations in intrusive thoughts related to off-task issues. For adults, expressive writing exercises help specifically to alleviate the intrusive thoughts that result from anxiety related to particular situations (Ramirez & Beilock, 2011). In a recent experiment, on the day of the final exam, Ramirez and Beilock (2011), asked half of the students to write openly for 10 minutes about their feelings toward the upcoming exam (the expressive writing group), while the other half of the participants were asked to write about a generic topic, different from their forthcoming exam, for 10 minutes (the control condition). The results indicated that the expressive writing exercise was indeed successful, as the students in the expressive writing group had higher overall scores than those who did not write about the upcoming test. While the expressive writing technique was first applied to address test anxiety, research indicates that it is also effective in increasing mathematical performance in mathematics-anxious students (Park, Ramirez, & Beilock, 2014). In particular, Park and colleagues (2014) investigated the role of expressive writing in reducing the negative impact of MA on mathematical performance. This study reported significant benefits for participants with high MA, but not for participants with low MA. Authors underlined that participants with high MA who used more anxiety-related words in writing, demonstrated better mathematical performance on the challenging problems. This was not the case with low MA participants.

Thus, some studies have suggested that some clinical practices are effective in adults. To the best of our knowledge, the first study that attempted to decrease MA in children was designed by Supekar, Iuculano, Chen, and Menon (2015). The research showed for the first time that an intensive eight-week intervention involving one-to-one mathematical tutoring reduces MA, and also remediates aberrant connectivity in emotion-related circuits associated with MA in primary school children. Before tutoring, children with high MA (compared with low MA children) showed aberrant functional response in the amygdala (Phelps & LeDoux, 2005). The tutoring program was adapted from MathWise (Fuchs et al., 2008; Fuchs & McNeil, 2013; Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009) and involved 15–20 hour-long sessions, over a period of between eight and nine weeks. The first four lessons focused on making students familiar with mathematical manipulatives. The fifth and sixth lessons trained them in using "mini strategies" for addition. From the seventh until the twenty-second lesson, students practiced these strategies with more difficult problems. After the training, differences in the connectivity of the amygdala disappeared, indicating remediation of impaired functional brain circuits in the children with high MA.

In a recent study of fourth-grade students, Passolunghi and Pellizzoni (2018) compared two training programs involving eight weekly sessions. One group (anxiety training) was trained using a specific approach that attempted to reframe cognitive beliefs about math anxiety (based on Rational Emotive Behavior Therapy, proposed by Ellis (2006). The second group (mathematical training) received training specifically focused on learning to apply mathematical metacognitive strategies to mathematical exercises (Poli, Molin, Lucangeli, & Cornoldi, 2006). The third group (the control group), carried out reading activities. The data showed that both anxiety and mathematical training decreased MA (measured using AMAS, Hopko, Mahadevan, Bare, & Hunt, 2003) but only the mathematical training had an effect on mathematical achievement at the end of the school year. These data seem to show that an effective MA training has to tap mathematical materials and procedures as well as anxiety as such.

Lastly, apart from the specific cognitive and emotional characteristics of each student, recent literature shows that other factors can affect mathematical abilities: beliefs and attitudes of teachers and parents. Ramirez and colleague (2013) indicated that one of the most important ways in which we can prevent the development of MA in children is to work with teachers and parents to change their negative way of interacting with children about mathematics, to change their attitudes about mathematics and to help them conquer their own MA. In a recent study (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015), showed that parents' MA and their children's mathematical learning has a negative correlation only if the higher anxious parents help the students with mathematical homework. Previous research (Hembree, 1990) demonstrated that adults with high MA express various negative attitudes toward mathematics (e.g., they state that mathematics is not useful, they have low self-efficacy and low self-motivation). With these kinds of attitudes, they can have a negative transgenerational impact on their children, demotivating them with regard to mathematics.

Conclusions and future directions

The main aim of this chapter was to describe the interplay of WM and MA in young students in order to describe and define the different aspects of the issue, considering future research perspectives and the possibility of intervention programs. In concluding our review, we want to pinpoint three aspects that characterize the interconnection between MA and WM. First, it is possible to observe a strong relation between WM and MA even in young students, showing the early roots of the relation between the two components. Some authors have suggested that this connection only applies to high WM students (Ramirez et al., 2013; Vukovic et al., 2013). These researchers' finding could be explained in different ways: 1) negative emotions disrupt the resources that high WM children rely on to retrieve basic facts from long-term memory and to inhibit competing answers (Geary, Hoard, Byrd-Craven, & Catherine DeSoto, 2004), 2) MA may make high WM children more susceptible to interference, resulting in a slower and less efficient retrieval process or 3) among young children, those with high WM are more aware of their emotions than those with low WM. The different perspectives from which this topic has been evaluated (e.g., analyzing the relation in children with high WM vs. children with high math anxiety) have produced different results that need to be more specifically evaluated. Indeed, other findings showed that high MA and low WM capacity can be seen as risk factors for poor mathematical achievement (Cargnelutti et al., 2017b; Passolunghi et al., 2016; Mammarella et al. 2015). For this reason, this topic requires further research.

Second, a fundamental aspect to consider when analyzing the interplay between WM and MA is the involvement of the tools used during the evaluations. In particular, the reliability of the tests has to be considered carefully, given young students' limited experience with self-evaluation, the developmental trajectory of children's explicit awareness of their own thoughts and emotions and the different dimensions (e.g., performance anxiety vs. emotional response to numbers) assessed by the self-report measures (Dowker, Sarkar, & Looi, 2016). Finally, we strongly believe in the need to invest more in early interventions to prevent or ameliorate MA and to increase awareness of the condition among parents and teachers. Trying to shed light on how MA and WM interact and affect mathematical achievement in young students has great potential importance both from a scientific and an educational point of view. Identifying the risk factors for MA may enable us to develop ways of identifying and developing early interventions for at risk pupils and thus to enhance achievement and, in turn, enjoyment of mathematics.

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