Gaussian approach for the synthesis of phase-only beam-scanning linear aperiodic antenna arrays

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A B S T R A C T

This paper describes a method for the synthesis of linear aperiodic antenna arrays with beam scanning capability. The algorithm is based on a recently proposed Gaussian approach, regarding linear aperiodic arrays of isotropic elements with uniform amplitude distribution, which yields the optimal element positions of the array in such a way as to form a Gaussian beam of prescribed properties. The latter approach is here extended to solve a problem of beam scanning in an angular region of interest. Precisely, the array element positions are determined in such a way as to form a pencil beam, that can be pointed in any direction of the angular region of interest by a suitable distribution of the excitation phases. Thus, a continuous beam scanning can be performed by phase-only control. It is also shown that the alternating projection approach can mitigate the pattern degradation that arises for pointing angles near the end-fire direction.

1. Introduction

The potentialities characterizing antenna arrays have been well known for many decades [1–5], so this type of radiating structures is employed in different scenarios, ranging from remote sensing to civil and military radars, to wireless and satellite communications. A very attractive feature of antenna arrays is the capability of controlling their radiation pattern. This is known as *pattern reconfigurability*. The literature provides many synthesis methods for reconfigurable antenna arrays, as, for example, [6–8]. Here, it is worth noticing that the advantages provided by beam design are also recognized in related works of other research topics, such as, for example, imaging [9,10], remote sensing [11,12], signal recovery and reconstruction [13–15] and wireless sensor networks [16–18].

Furthermore, antenna arrays provide *beam scanning* capability, which can be regarded as a special case of pattern reconfigurability, in which all the different radiation patterns exhibit the same shape, but with the main beam pointing in different directions. So, the synthesis algorithms developed for reconfigurable arrays can also be used to achieve beam scanning. On the contrary, there are synthesis methods specifically thought of as beam-scanning arrays, that are not suitable for implementing pattern reconfigurability. A well known approach to scan the beam toward a desired direction is the so-called *progressive phase method* [19].

Of course, optimization of the element positions, in addition to that of the excitation phases, provides extra degrees of freedom to the designer, thus allowing one to obtain better performance with the same number of elements, or the same performance with fewer elements. Clearly, reducing the number of elements is certainly desirable since it reduces the complexity, the weight and the cost of the whole antenna system.

The interest in aperiodic antenna arrays (also called sparse or non-uniform arrays) appeared around the 1960s [20–23] and never expired. Nowadays, the literature offers a number of algorithms for the synthesis of aperiodic antenna arrays [24–31]. However, many of these methods can only deal with single beam antennas [24,28,30,31]. This is not the case for [25], where pattern synthesis of reconfigurable sparse arrays is performed with multiple measurement vectors, called the FOCUSS method. Precisely, the element positions, common to all patterns, and the different element excitations are simultaneously optimized. Similarly, in [26] an effective method is presented for synthesizing multiple-patterns of linear arrays with a reduced number of antenna elements. In [27], a strategy based on compressive sensing is first used to determine the element positions and, subsequently, the element excitations are optimized by an alternating projection algorithm. Also, in [29], a compressive antenna array is synthesized, for direction-of-arrival estimation, which allows reconfigurability. These approaches are interesting, but are based on a sort of thinning. In fact, in [25,27],

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the positions of the elements are chosen from a pre-defined set of candidate positions, while in [26] they are found by selecting the minimum suitable number of active elements of a periodic linear array. Moreover, the beam scanning is treated as a special case of reconfigurability. Thus, only a discrete beam scanning can be realized, and the complexity of the problem increases when the number of desired pointing directions increases.

In this paper, the recently developed Gaussian approach [32] is adopted and extended in such a way as to handle the synthesis of beam scanning linear aperiodic antenna arrays. Thanks to the formulation of the synthesis problem in terms of an auxiliary variable, the optimal element positions are simultaneously synthesized for continuous beam scanning in a desired angular region. According to the Gaussian approach, a uniform distribution of the amplitude of excitations is initially assumed, which allows one to simplify the feed network. Having uniform amplitude of the excitations has other advantages, such as increased efficiency and reduced mutual coupling. However, the degrees of freedom of the problem are reduced. So, a strategy to improve the solution may then be used, for cases in which the scanning performance obtained with the Gaussian approach is not completely satisfactory. Here, the alternating projection approach described in [33] is adopted, since it is suitable for arrays of arbitrary geometry, thus also for aperiodic arrays. Moreover, it allows one to impose a threshold on the dynamic range ratio (DRR) of the excitations, defined as the ratio between maximum and minimum amplitude of the excitations. In particular, by imposing a low threshold, the solution may be improved, preserving good performance in terms of efficiency and reduced mutual coupling. The key advantages characterizing the proposed approach with respect to the algorithms in [32,33] are due to the introduction of a suitable auxiliary variable, which allows one to perform continuous beam scanning in a very short computational time. In fact, the Gaussian approach in [32] is suitable only for fixed beam broadside arrays. On the other hand, the alternating projection approach in [33] belongs to a class of algorithms, that treats a beam scanning as a special kind of reconfigurability. This has two major disadvantages. Firstly, only discrete beam scanning can be realized. Secondly, the computational burden may become considerably higher. As an important advantage, the mutual coupling between adjacent elements can be taken into account.

The paper is organized as follows. In the next section, the problem of beam scanning by aperiodic linear antenna arrays is introduced. Then, in Section 3, the Gaussian synthesis procedure is explained, and in Section 4, is extended to deal with beam scanning. A first numerical example is shown in Section 5 and an improvement of the solution is presented in Section 6, where comparison examples are also presented. Finally, Section 7 concludes the paper.

Notation. Throughout the paper the following notation is used: lower-case bold letters denote vectors, where a_n is the nth element of $\mathbf{a}$, j is the imaginary unit, and $\mathrm{erf}(x)$ denotes the error function.

2. Phase-only beam scanning with linear aperiodic arrays

With reference to a Cartesian system $O(x, y, z)$, a linear array consisting of N isotropic radiators lying at the positions $z_n, n = 1, \ldots N$ on the z-axis is considered. The array factor of such an array is given by:

$$F(\mathbf{a}, \mathbf{z}; u) = \sum_{n=1}^{N} a_n \exp(jz_n u) \tag{1}$$

where $\mathbf{a}$ and $\mathbf{z}$ are the complex excitations and positions of the array elements, respectively, $u{=}k\sin\theta$, with θ the angle from broadside (i.e., the elevation angle) and $k = 2\pi/\lambda$ is the wave number, being λ the wavelength. When a uniform distribution of excitation amplitudes is assumed with $|a_n| = 1, n = 1, \ldots, N,$ the array factor in (1) takes the form [34, Eq. (6–7)]:

$$F(\boldsymbol{\alpha}, \mathbf{z}; u) = \sum_{n=1}^{N} \exp(j\alpha_n) \exp(jz_n u) \tag{2}$$

where $\boldsymbol{\alpha}$ is the vector of excitation phases. So, in order to point the beam toward a generic direction θ_0, the excitation phases are chosen in such a way that the N exponential terms in (2) are in phase at the desired direction, that is

$$\alpha_n = -kz_n \sin\theta_0, n = 1, \ldots, N. \tag{3}$$

Here, it is to be noted that when the elements are equally spaced (i.e., $z_n = nd, n = 1, \ldots, N,$ having denoted by d the inter-element distance), this is exactly the well-known *progressive-phase method* [19]. In fact, the excitation phases can be written as $\alpha_n = n\alpha_0, n = 1, \ldots, N,$ with $\alpha_0 = -kd\sin\theta_0,$ so that each succeeding element of the array has an α_0 progressive phase lead current excitation with respect to the preceding element.

Now, with the excitation phases in (3), the array factor in (2) represents a pencil beam pointing at the generic direction θ_0, which is written as:

$$F(\boldsymbol{\alpha}, \mathbf{z}; u) = \sum_{n=1}^{N} \exp(jz_n u) \tag{4}$$

where the auxiliary variable u is defined here as:

$$u = k(\sin\theta - \sin\theta_0). \tag{5}$$

The problem addressed in this paper is that of finding the optimal positions of N elements of an aperiodic array having uniform amplitude distribution, in such a way as to realize continuous phase-only beam scanning in an angular region of interest.

3. Gaussian synthesis procedure

The synthesis procedure that we propose for phase-only scanning is based on the Gaussian approach recently presented in [32], which is summarized here for convenience to the reader.

First of all, a desired pattern $F_\mathrm{d}(u)$ is introduced, defined as a Gaussian function. This is a particularly convenient choice for two reasons. First, a pencil beam can be approximated by a Gaussian function whose width is controlled by the standard deviation, σ. Second, the Fourier transform of a Gaussian function is still a Gaussian function. And, as is well known, the Fourier transform of a radiation pattern produced by a continuous current distribution $a(z)$ is proportional to $a(z)$ [19]. So, $a(z)$ being a Gaussian function, the density tapering approach [21] can be easily applied, and the element positions can be evaluated in closed form. Details of this approach are given in [32], and are briefly summarized below.

First, the desired Gaussian pencil beam is:

$$F_\mathrm{d}(u) = \exp\left(-\frac{u^2}{2\sigma^2}\right). \tag{6}$$

The mean-square deviation σ is used to control the beamwidth:

$$\sigma = k\sqrt{\frac{10}{b\ln 10}}\sin\left(\frac{\pi \mathrm{BW}_\mathrm{deg}}{360}\right) \tag{7}$$

for a desired b-dB beamwidth of BW_deg (in degrees). Then, using the Fourier transform relation [19], the excitation density $a(z)$ of the continuous source that (exactly) produces $F_\mathrm{d}(u)$ is:

$$a(z) = \frac{\sigma}{\sqrt{2\pi}}\exp\left(-\frac{\sigma^2 z^2}{2}\right), \quad -\infty{<}z{<}{+}\infty. \tag{8}$$

Finally, the optimal positions of a finite number of elements N belonging to an array of assigned length L_a and having uniform excitation amplitude are selected as[1]:

$$z_n = \frac{s_{n-1} + s_n}{2}, \tag{9}$$

with

$$s_n = \frac{\sqrt{2}}{\sigma} \operatorname{erf}^{-1}\left[\left(\frac{2n}{N} - 1 \quad \operatorname{erf}\left(\frac{\sigma L}{2\sqrt{2}}\right)\right)\right], \tag{10}$$

for $n = 1, \ldots, N,$ where the length L is obtained by solving the equation[2]:

$$L = \frac{2\sqrt{2}}{\sigma} \operatorname{erf}^{-1}\left\{\frac{N}{2-N} \operatorname{erf}\left[\frac{\sigma}{\sqrt{2}}\left(-L_a + \frac{L}{2}\right]\right]\right\} \tag{11}$$

as described in [32, Appendix B]. The above closed form solution in (10) is obtained by the so-called density tapering approach. Precisely, given the infinite continuous source distribution $a(z)$, the positions of a desired number of N elements of an aperiodic array of length L_a are found by imposing that the area between the amplitude distribution $a(z)$ and the z-axis be subdivided into N regions of equal area, where $z_N - z_1 = L_a$. An interesting graphical representation of this approach is shown in [35, Fig. 4].

4. Beam scanning

Once the element positions in (9) are determined, in order to point the beam at the angle θ_0, the excitation phases of the array elements are evaluated by (3) and the array pattern is provided by (4).

However, before showing a numerical example, it is worth emphasizing here that position synthesis by the Gaussian approach allows one to realize phase-only beam scanning in terms of the new definition of the auxiliary variable u (5), which implies a different visible region for the synthesized pattern. In fact, u is an auxiliary variable that takes all real values, unlike the angles θ and θ_0. Nevertheless, for a given pointing direction θ_0 in (4), since $-\pi/2 \le \theta \le \pi/2$, only values of u in the interval

$$I_{VP} = [k(-1 - \sin\theta_0),\ k(1 - \sin\theta_0)] \tag{12}$$

specify a visible pattern, while the remaining values specify invisible patterns, which are not of interest. Furthermore, when beam scanning in an angular region $\Theta = [\theta_{0\,\min}, \theta_{0\,\max}]$ is of concern, a wider interval has to be considered, namely:

$$I_\Theta = [k(-1 - \sin\theta_{0\,\max}), k(1 - \sin\theta_{0\,\min})]. \tag{13}$$

Then, for each arbitrary pointing direction $\theta_0 \in \Theta$, the visible region is the interval $I_{VP} \subset I_\Theta$, as expressed in (12). The following sections propose some interesting numerical results, endowed with many general comments on the synthesis procedure.

5. Numerical example

The method described in the previous section is used here to synthesize the positions of an aperiodic array of assigned length $L_a = 10\lambda$ and is composed of $N = 21$ isotropic elements. The limits of the desired scanning interval Θ are $\theta_{0\,\min} = -50^\circ$ and $\theta_{0\,\max} = 0^\circ$. The root-mean-square deviation σ of the desired Gaussian pattern in (6) is chosen in such a way as to have the same beamwidth as that of a periodic array of equal length and the same number of isotropic elements with unitary excitation, resulting in $\sigma = 0.1763$ rad/m. Using (9), in conjunction with (10) and (11), provides the optimal element positions, $z_n, n = 1, \ldots, N,$ in approximately[3] 10 ms, which are listed in Table 1. The array pattern is provided by (4) with these optimized positions, and Fig. 1 shows the desired pattern $F_d(u)$, the array factor produced by the periodic array, and that obtained with the synthesized positions. In Fig. 1(a) the array factors are plotted versus the auxiliary variable u in the interval $[-4k, 4k]$. Here, it can be noticed that the pattern radiated by the periodic array is a periodic function. On the contrary, the radiation pattern of the optimized aperiodic array is not periodic and does not have grating lobes. However, some high sidelobes still appear. This is the typical result provided by the Gaussian approach. Importantly, in general, this kind of plot can also be used to determine the maximum scanning range. Precisely, in this example, one obtains a maximum scanning angle of $\pm 34^\circ$ for a maximum sidelobe level of -15 dB, and of $\pm 44^\circ$ for a maximum sidelobe level of -10 dB. Fig. 1(b) shows the same patterns versus u, but is limited to the scanning interval, I_Θ, where the maximum sidelobe level is -9.95 dB. Finally, Figs. 1(c), (d), and (e) show the same radiation patterns, but versus the angle θ for three selected values of the beam pointing direction $\theta_0 \in \Theta$ (respectively, $\theta_0 = 0^\circ, \theta_0 = -10^\circ$, and $\theta_0 = -50^\circ$).

Table 1
First example. Positions obtained by the Gaussian approach, and phases obtained by the alternating projection approach. (Due to symmetry, the remaining values are $z_n = -z_{N-n+1}, n = 12, \ldots, 21$ and $\alpha_n = -\alpha_{N-n+1}, n = 12, \ldots, 21$.).

n	z_n/λ	$\alpha_n(^\circ)$	n	z_n/λ	$\alpha_n(^\circ)$
1	−5.00	−17.88	7	−1.80	4.37
2	−4.38	59.44	8	−1.34	11.85
3	−3.80	−2.71	9	−0.89	5.44
4	−3.27	−12.68	10	−0.44	−1.37
5	−2.76	−18.60	11	0.00	0.00
6	−2.27	−14.36			

Now, it is useful to focus attention on the following characteristics of this numerical example, which are common to other tested examples. Firstly, when passing from the auxiliary variable u to the angular variable θ, a broadening and asymmetry of the beam has to be expected when the beam is scanned away from broadside. This phenomenon is also experienced for beam scanning with periodic arrays employing the progressive-phase method [19]. Thus, the considered method is only suitable in cases where these effects can be tolerated. Then, for pointing directions θ_0 near broadside, the performance of the synthesized aperiodic array is satisfactory. On the other hand, when the beam is scanned away, the pattern degradation may not be acceptable, as is the case for $\theta_0 = -50^\circ$, where high sidelobes appear near the endfire direction [see Fig. 1(e)]. However, there are many applications where the scanning interval is limited, as for example in communications between geostationary satellites and the Earth [36]. In these cases, the results provided by the proposed approach are very satisfactory. Finally, note that the only degrees of freedom with the adopted formulation of the problem are the positions of the array elements (which, appear in the exponential terms), whereas the excitation phases are evaluated by (3). Thus, it is to be expected that an improvement of the solution may be obtained by also optimizing the element excitations, which is shown in the next section.

6. Improved solution

Here, a method is proposed for lowering high sidelobes that might arise near the endfire direction, i.e., for pointing angles far from broadside. Precisely, the scanning performance of the

[1] Alternatively, [32, Eq. (13)] can be used, which is not reported here.

[2] If [32, Eq. (13)] is used for the element positions, [32, Eq. (16)] is to be used to evaluate L.

[3] All the provided numerical examples are obtained using Matlab on a laptop with 8GB RAM.

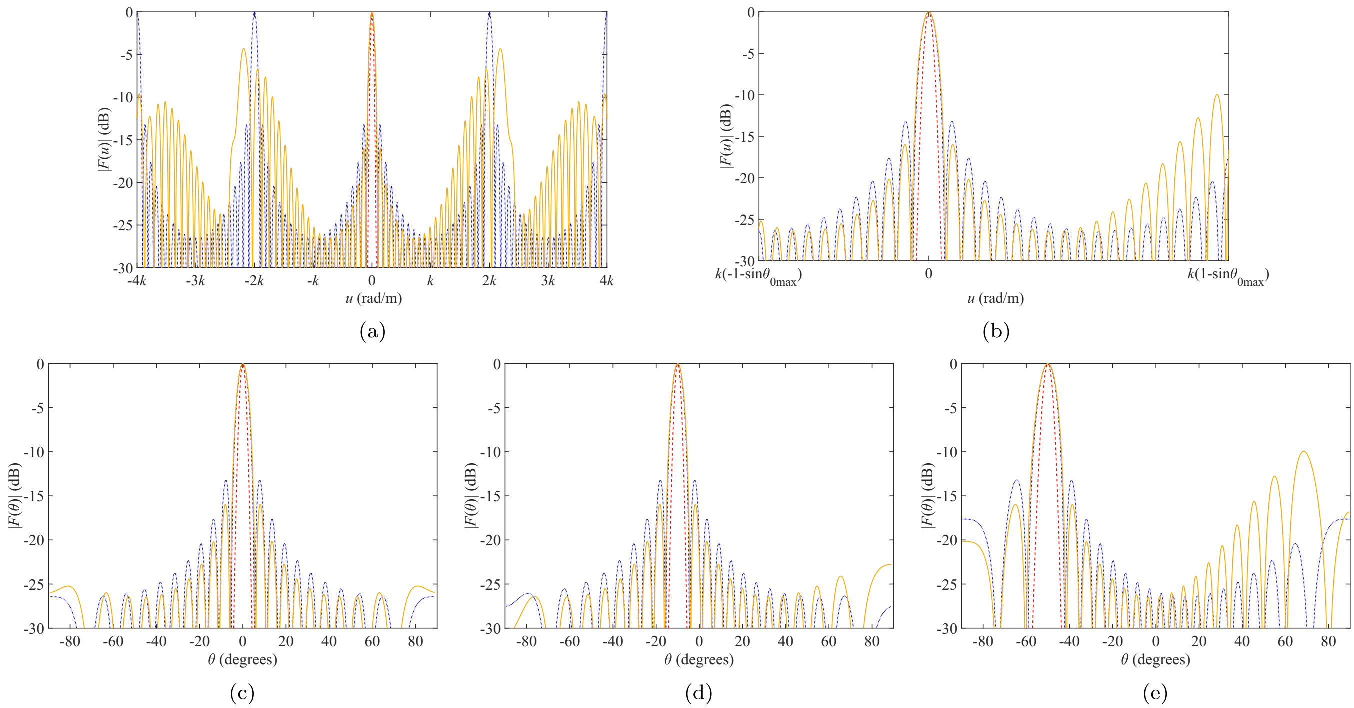

Fig. 1. First example: $N = 21$ and $L_a = 10\lambda$, showing desired (red) Gaussian pattern (outside of the mainbeam, the amplitude is below -30 dB), array factor (blue dotted) of the periodic array, and synthesized array factor (yellow). (a) Versus u in the interval $[-4k, 4k]$. (b) Versus u in the scanning interval I_Θ. (c) Versus θ with $\theta_0 = \theta_{0\,\max} = 0^\circ$. (d) Versus θ with $\theta_0 = -10^\circ$. (e) Versus θ with $\theta_0 = \theta_{0\,\min} = -50^\circ$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

aperiodic array, whose positions are synthesized in the previous section, is enhanced by optimizing the element excitations. The algorithm in [33] is chosen, which uses an alternating projection approach for finding the optimal element excitations of an array of arbitrary (but fixed) geometry, in such a way that the radiation pattern lies within a given mask, while the DRR of the excitations is constrained to not exceed a given threshold. This algorithm also allows control of the cross-polar component of the pattern and is suitable for reconfigurable arrays. However, when the problem is formulated in terms of the auxiliary variable u in the interval I_Θ, continuous beam scanning can be realized with a unique mask, so reconfigurability is not required and the algorithm is applied as briefly described below.

- At first, a (unique) suitable mask is introduced, whose lower bound is the desired pattern $F_d(u)$ and whose upper bound $F^{\text{up}}(u)$ is used to control the maximum sidelobe level, $M = \{f(u) : F_d(u) \le |f(u)| \le F^{\text{up}}(u)\}$. In order to maintain the uniform amplitude distribution of the array elements, the maximum DRR of the excitation is constrained to $D = 1$. Therefore, the constraints of the synthesis problem are:

$$F(\mathbf{a}, \mathbf{z}; u) \in M \tag{14}$$

$$\text{DRR}(\mathbf{a}) \le D. \tag{15}$$

- Then, in order to formulate this problem as an intersection-finding problem, a set W is introduced, composed of inhomogeneouos elements as follows:

$$W = \{\tilde{\mathbf{w}} : \tilde{\mathbf{w}} = [g(u), \mathbf{w}]\}, \tag{16}$$

where $g(u)$ is an arbitrary scalar complex function defined in I_Θ, and $\mathbf{w}$ is an arbitrary vector with N complex components. In W, two subsets are introduced:

$$U = \{\tilde{\mathbf{u}} : \tilde{\mathbf{u}} = [f(u), \mathbf{u}]\}, \tag{17}$$

and

$$V = \{\tilde{\mathbf{v}} : \tilde{\mathbf{v}} = [F(\mathbf{v}, \mathbf{z}; u), \mathbf{v}]\}, \tag{18}$$

such that $f(u) \in M$ and $\text{DRR}(\mathbf{u}) \le D$. In other words, the elements of U satisfy the constraints (14) and (15), whereas in (18), $\mathbf{v}$ is an arbitrary complex vector and $F(\mathbf{v}, \mathbf{z}; u)$ is the array pattern evaluated by replacing $\mathbf{a}$ with $\mathbf{v}$ in (1).

- Now, it is evident that a point $\tilde{\mathbf{u}} \in U \cap V$ is a solution to the problem. Since $U \cap V$ might be empty (unfeasible problem), the solution is considered as a point of U "sufficiently" close to V. Such a solution is found following the iterative scheme:

$$\tilde{\mathbf{u}}_{n+1} = P_U P_V \tilde{\mathbf{u}}_n, \qquad n = 0, 1, 2, \ldots \tag{19}$$

where P_U and P_V are the projection operators onto the sets U and V, respectively, and $\tilde{\mathbf{u}}_0 = P_U(\tilde{\mathbf{w}}_0)$ is a suitable starting point in U. Due to the definition and properties of the projection operators, the points $\{\tilde{\mathbf{u}}_n\}$ of the sequence generated by (19) get closer and closer to the set V, as the sequence $\{d_n\}$ of the distances d_n between $\tilde{\mathbf{u}}_n$ and V is non-increasing, and therefore is convergent. The iteration is stopped when

$$d_n \le \varepsilon_1 \text{ or } (d_{n-1} - d_n)/d_n \le \varepsilon_2 \tag{20}$$

where ε_1 and ε_2 are suitably small thresholds. (Further details on the alternating projection approach and the formulas for implementation can be found in [33, Appendix].)

Now, the above described alternating projection approach is applied to improve the scanning performance of the array synthesized in the previous section. A starting point $\tilde{\mathbf{u}}_0 = P_U(\tilde{\mathbf{w}}_0)$ is chosen, with the function $g(u) = \sum_n \exp(jz_n u)$ and with the vector $\mathbf{w}$ having all the components equal to unity, $w_n = 1, n = 1, \ldots, N$. The upper bound of the mask $F^{\text{up}}(u)$ is chosen so as to obtain a maximum sidelobe level of -15.32 dB. The scheme in (19) is applied with the thresholds $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-6}$, and a sampling

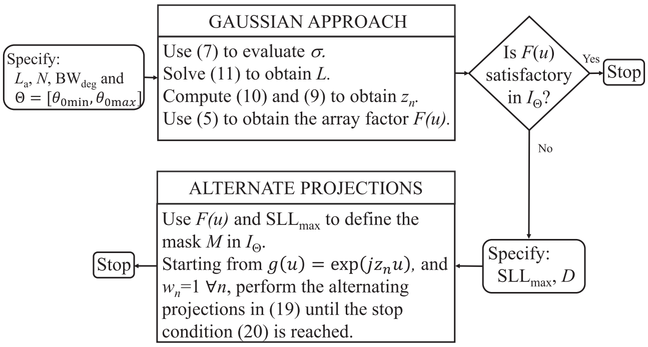

Fig. 2. Graphical representation of the proposed design strategy.

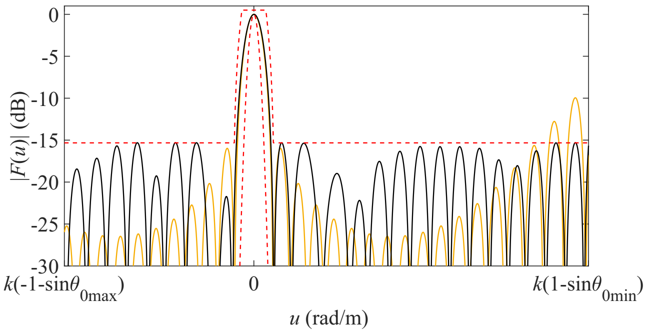

Fig. 3. Array factor (black) of the linear aperiodic array synthesized using the alternating projection approach versus u in the interval I_Θ within the mask (dotted red), compared to the array factor of the Gaussian approach (yellow). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

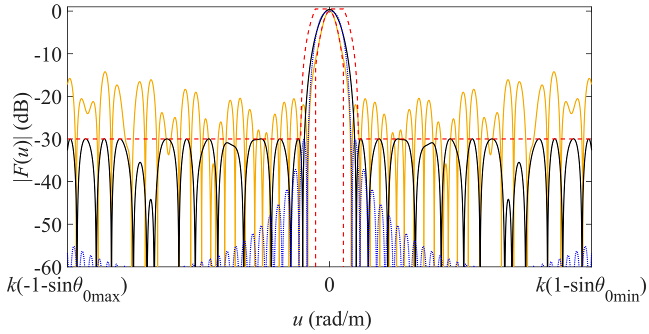

Fig. 5. First comparison example, showing a Taylor pattern (blue dotted) of the reference periodic array in [26], array factor obtained by the Gaussian approach (yellow), and array factor synthesized by the proposed approach (black), all within the mask (red dashed) in the scanning interval I_Θ. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
First comparison example, where positions are obtained by the Gaussian approach, and excitations are obtained by the alternating projection approach. (Due to symmetry, the remaining values are $z_n = -z_{N-n+1}, n = 17, \ldots, 32$ and $a_n = a_{N-n+1}, n = 17, \ldots, 32$.).

n	z_n/λ	a_n	n	z_n/λ	a_n
1	−9.75	0.0392	9	−2.17	0.6706
2	−5.87	0.0373	10	−1.85	0.8863
3	−4.93	0.2569	11	−1.54	0.7784
4	−4.27	0.3667	12	−1.25	0.9431
5	−3.73	0.7392	13	−0.96	0.7294
6	−3.28	0.7333	14	−0.68	1.0000
7	−2.88	0.5980	15	−0.41	0.7961
8	−2.51	0.7706	16	−0.14	0.8863

step of $5 \cdot 10^{-4}$ rad/m, where the process stops after 7947 iterations (approximately 162 s). Fig. 2 offers a graphical representation of the complete design procedure. Fig. 3 shows the obtained array factor versus the u variable, which exhibits a maximum sidelobe level of −15.30 dB in the scanning interval I_Θ. Table 1 lists the optimized excitation phases (with unity DRR of the excitations). Finally, in Fig. 4, the obtained array factor is plotted versus the variable θ for different pointing directions and is compared to that of the Gaussian approach. As can be seen, an overall increase of the sidelobes is payed back by the desired lowering of the high sidelobes, which arises near the endfire direction when the beam is scanned away from broadside. It is worth noting that the reduction of the maximum sidelobe level is obtained thanks to the increased degrees-of-freedom of the problem, due to the excitation phases.

Now, in order to compare the proposed approach with a state-of-the-art algorithm, the beam scanning example of [26] is considered, where a periodic array with 40 isotropic elements and $d = \lambda/2$ is taken as a reference, which radiates a Taylor pattern with PSL=−30 dB, shown in Fig. 5. The $M = 7$ scanning angles $\theta_0 = 0^\circ, \pm 10^\circ, \pm 20^\circ, \pm 30^\circ$ are considered in [26], and an enhanced matrix pencil method is used to synthesize the positions and excitations of an aperiodic array with a reduced number of elements, $N = 32$. The algorithm requires 0.38 s and achieves good scanning performance for the desired pointing directions [26, Fig. 11(c)]. Here, the Gaussian approach proposed in Section 4 is used to synthesize the positions of an equal number of elements $N = 32$ on an aperture with the same assigned length, $L_a = 19.5\lambda$, with a desired pattern having the same beamwidth as the reference pattern ($\sigma = 0.2892$ rad/m) and a scanning interval with $\theta_{0\,min} = -30^\circ$ and $\theta_{0\,max} = 30^\circ$. The synthesized element positions are listed in Table 2. The radiation pattern exhibits high sidelobes, as can be seen in Fig. 5. But, this is to be expected since the Gaus-

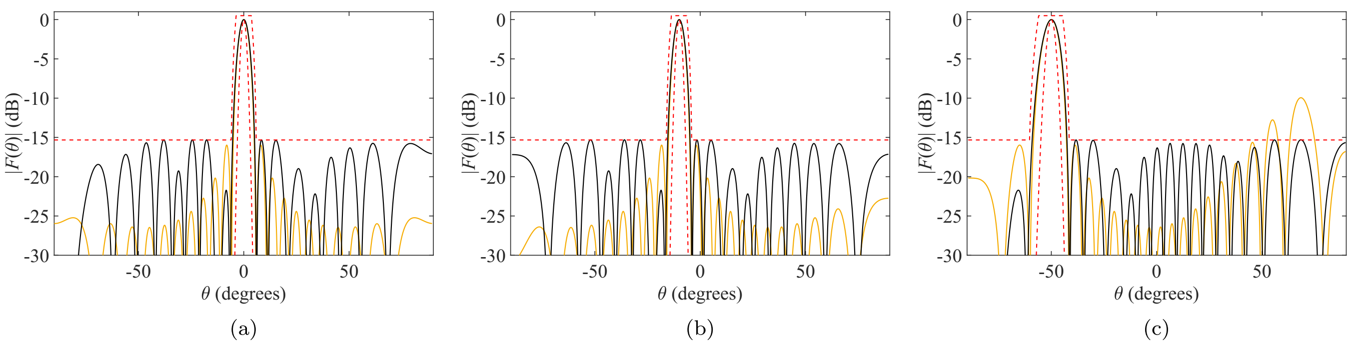

Fig. 4. Array factor (black) synthesized by the alternating projection approach, compared to the array factor of the Gaussian approach (yellow) versus the angle θ. (a), $\theta_0 = \theta_{0\,max} = 0^\circ$. (b), $\theta_0 = -10^\circ$. (c), $\theta_0 = \theta_{0\,min} = -50^\circ$.

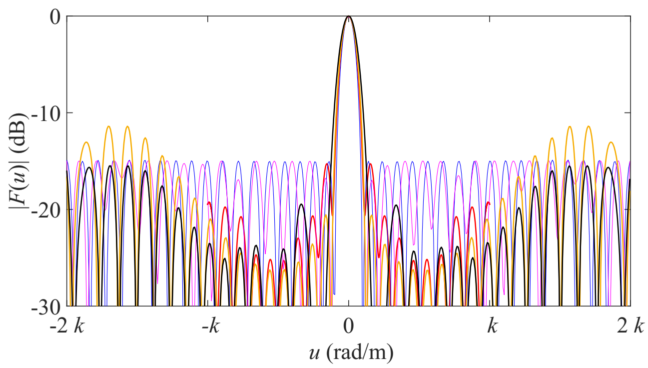

Fig. 6. Second comparison example, showing the array factors obtained: by the method in [38] (thin magenta), the method in [37] (thin blue), the Gaussian approach (thick yellow), the proposed design procedure (thick black) and radiation pattern taking into account mutual coupling (thick red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

sian approach assumes a uniform amplitude distribution. So, the alternating projection approach described in Section 6 is used to synthesize the element excitations, imposing a maximum sidelobe level of -30 dB and without considering the DRR constraint (15). The optimized element excitations are reported in Table 2 and have DRR = 26.79. Fig. 5 shows the obtained pattern. As can be seen, the high sidelobes are lowered below the desired level of -30 dB. The computational time to obtain the final result is approximately 1.5 s (99 iterations).

Finally, the performance of the proposed algorithm is compared with that obtained by the matrix pencil method (MPM) of [37], and by the compressive sensing (CS) approach developed in [38]. In both of these methods, the amplitude distribution of the array element is not uniform, and the number of elements is not fixed a priori, but is optimized during the synthesis procedure. An array with $L_a = 9\lambda$ is considered. Firstly, the MPM [37] is applied, taking as reference a Chebyshev pattern with PSL=-15 dB. The algorithm requires 53 ms to obtain the optimized positions and excitations of 21 elements. Secondly, the CS approach [38] is used. The algorithm requires approximately 1550 s to obtain the optimal positions and excitations of a 20-element array. Thirdly, the Gaussian approach described in Section 4 is applied to obtain the positions of $N = 20$ elements. Fig. 6 shows the array factors obtained by the MPM, CS and Gaussian approach. Then, in order to lower the sidelobes exceeding the desired -15 dB, the alternating projection approach described in Section 6 is applied, without considering the DRR constraint (15). The alternating projection requires 11 ms (76 iterations). As can be seen in Fig. 6 the sidelobes are below the desired value. Table 3 summarizes the most relevant results.

Table 3
Most relevant results of the second comparison example: number of elements, aperture length, minimum inter-element distance, DRR, CPU time.

	CS in [38]	MPM in [37]	Proposed
N	20	21	20
$z_N - z_1$	7.74λ	8.98λ	9λ
$d_{\min}$	0.13λ	0.42λ	0.37λ
DRR	-15 dB	-15 dB	-15.5 dB
CPU time	1550 s	53 ms	76 ms

Now, it is worth noting that the developed approach, as well as the considered references, do not take into account the mutual coupling effects. In [32], for the case of a broadside beam, it was shown that coupling causes an increase of the sidelobes nearer to the main beam. This is confirmed here by the red line of Fig. 6, which shows the radiation pattern obtained taking into account the coupling effects for an array of half-wave dipoles at the positions synthesized by the Gaussian approach. When the beam is scanned away from broadside, the pattern degradation becomes more severe. In order to mitigate this, one can use the alternating projection approach of [33], which is suitable for the phase-only synthesis of reconfigurable arrays. Here, it is applied to the linear array optimized by the Gaussian approach, without considering the DRR constraint and with a number of $S = 10$ masks, from $\theta_0 = 0^\circ$ to $\theta_0 = 45^\circ$, with an angular step of 5°. Fig. 7 shows the radiation patterns obtained for $\theta_0 = 0^\circ, 20^\circ, 45^\circ$. Satisfactory results are obtained, but for discrete beam scanning and with a longer computational time (approximately 1680 s).

7. Conclusion

In this paper, the very simple and fast Gaussian approach introduced in [32] was extended to the problem of position synthesis of phase-only beam scanning arrays. This is an important novelty with respect to the original Gaussian approach, which was suited for the position synthesis of broadside arrays. With the proposed method, the positions of the array elements are found in closed form, and the obtained array factors are quite acceptable, especially for scanning directions not too far from broadside. In many applications, as for example in satellite communications [36], this is sufficient. On the other hand, when the beam is pointed near the endfire direction, high sidelobes arise. This is a physical limitation, that cannot be avoided with the proposed solving procedure. So, when it is necessary to mitigate this issue, a suitable synthesis method may be applied that is able to deal with the excitation synthesis of linear arrays with arbitrary, but fixed, geometry. Also, in this paper, an alternating projection approach was chosen since it allows one to optimize the element excitations, simultaneously imposing a threshold on their maximum DRR, which can eventually be set equal to unity so as to maintain a uniform amplitude

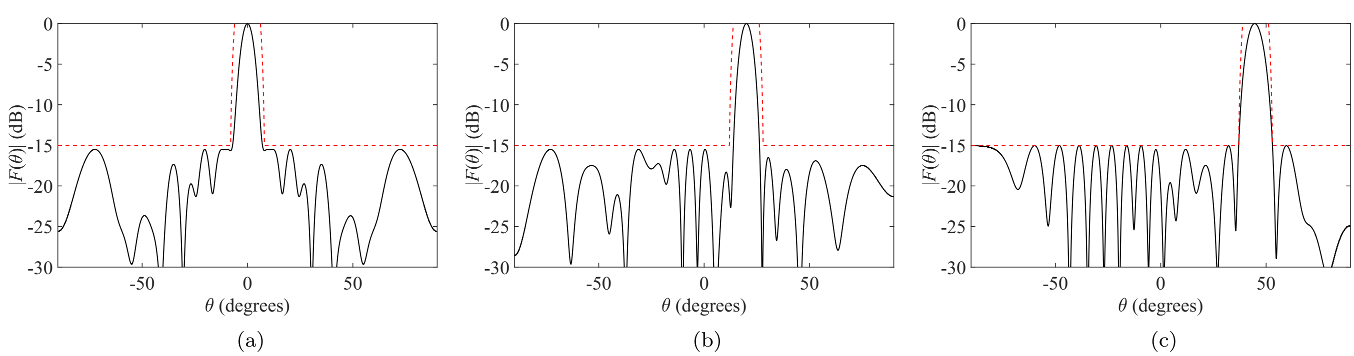

Fig. 7. Radiation patterns synthesized by the alternating projection approach [33] within the mask (red) versus the angle θ. (a), $\theta_0 = 0^\circ$. (b), $\theta_0 = 20^\circ$. (c), $\theta_0 = 45^\circ$.

distribution. If this is not necessary, higher values can be imposed (and eventually no thresholds), so as to allow further reduction of the sidelobe levels, thanks to the increased number of degrees of freedom available to the designer. This is an important novelty with respect to the approach in [32], which was conceived for arrays with a uniform amplitude distribution. Besides, thanks to the two-step procedure, the mutual coupling effects can be considered by applying the alternating projection approach as originally proposed in [33]. So, the position synthesis is performed by using the Gaussian approach described in Section 4 and the phase-only synthesis of the excitation is performed by the method in [33]. However, in this case, although the provided results are very satisfactory, some advantages of the proposed synthesis strategy are lost, i.e., the possibility of performing continuous beam scanning, and achieving very short computational time. These are important aspects, which will be the subject of future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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