Multi-fidelity Gaussian Process Regression for Propeller Optimisation Under Uncertainty

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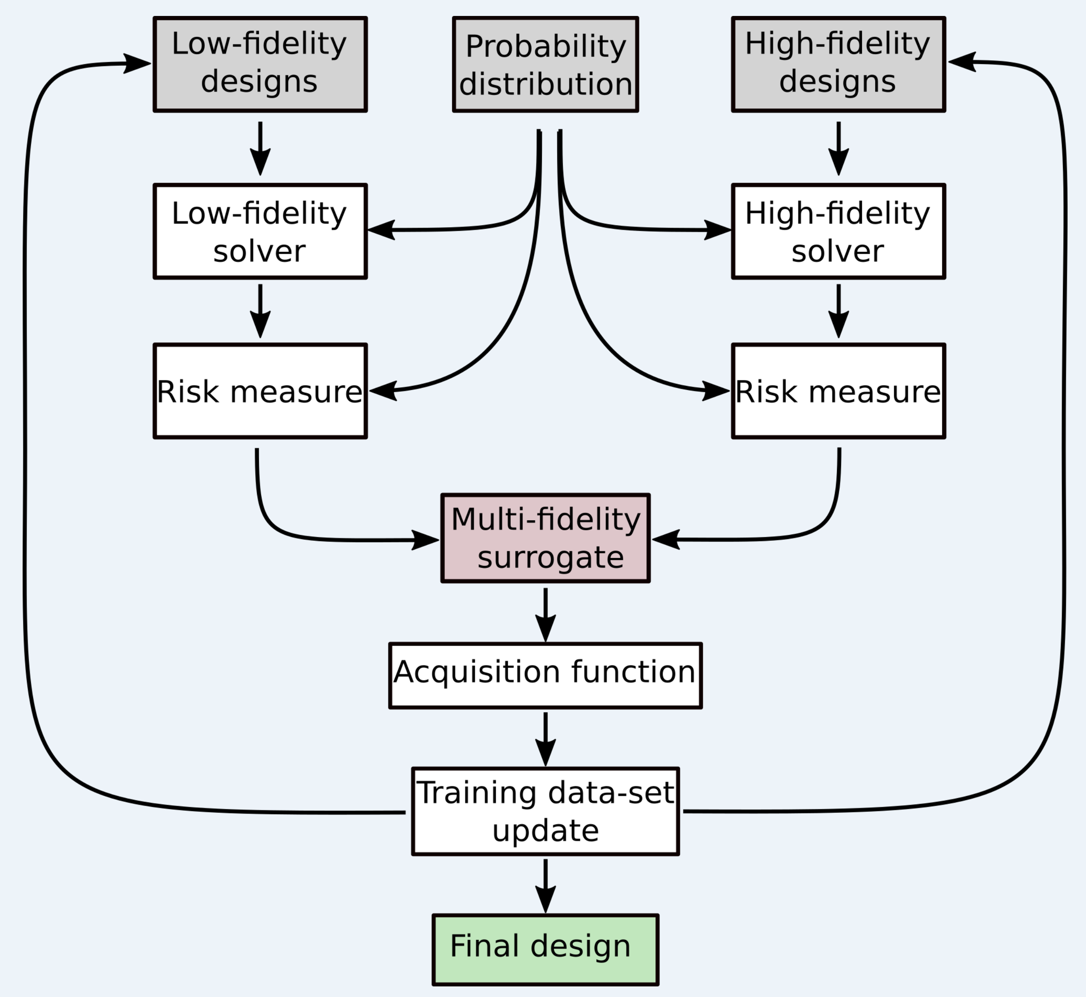
UTOPIÆ Uncertainty Treatment and Optimisation in Aerospace Engineering
Handling the unknown at the edge of tomorrow

Introduction

The motivation of this research is to optimise the propeller of a small-scale electrical aircraft under uncertainty. The expensive performance evaluation prohibits the application of standard optimisation techniques and the direct calculation of statistical measures. This motivates the use of cheap low-fidelity simulations to obtain more information about the unexplored locations of the input space. The information stemming from the low- and high-fidelity simulations are fused together with multi-fidelity Gaussian Process Regression to build an accurate surrogate model despite the low number of high-fidelity simulations. The proposed surrogate-based optimisation workflow allows us to efficiently carry out an optimisation problem which otherwise would be impracticable.

Optimisation Workflow



Conclusion

Multi-fidelity optimisation techniques require good cross-correlation between fidelity levels. Multi-fidelity surrogate techniques can help construct accurate surrogates when only sparse high-fidelity samples are available. An acquisition function calculating the expected variance reduction can efficiently choose where to sample next. The separate design and probability space modelling approach facilitates the use of appropriate surrogate techniques in each space. Multi-fidelity techniques in aerospace applications can increase efficiency as well-calibrated low-fidelity formulas are available.

Publications

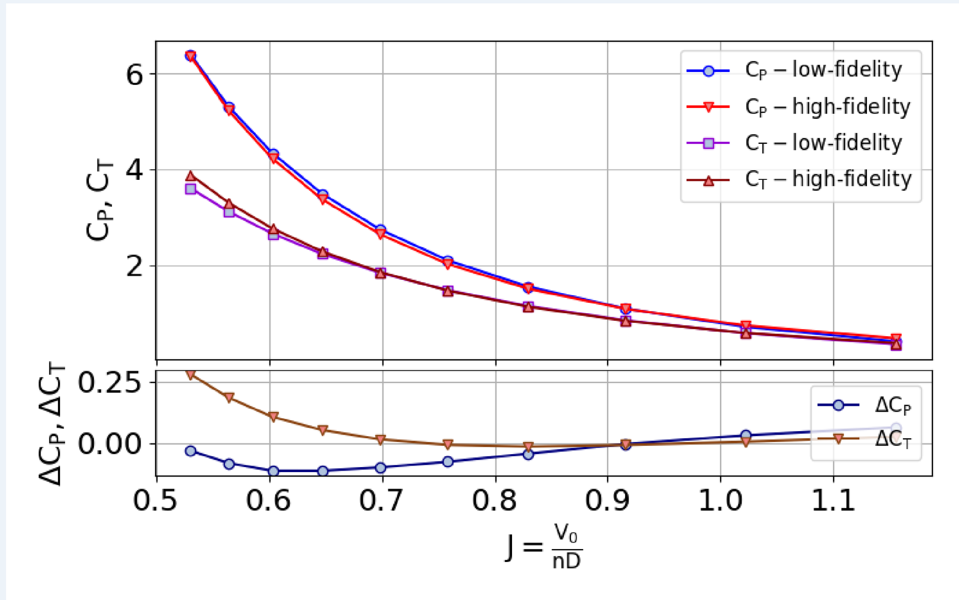
[1] P.Z. Korondi, M. Marchi, C. Poloni, L. Parussini. **Recursive Polynomial Chaos Co-Kriging for Reliability-based Design Optimisation.** *UQOP Conference, 18-20 March, 2019, Paris, France*

[2] P.Z. Korondi, L. Parussini, M. Marchi, C. Poloni. **Reliability-based design optimisation of a ducted propeller through multi-fidelity learning**, *UNCECOMP, June 24-26, 2019, Creta, Greece*

[3] P.Z. Korondi, L. Parussini, M. Marchi, C. Poloni. **Multi-fidelity Gaussian Process Regression for Propeller Optimisation Under Uncertainty,** *EUROGEN Conference, September 12-14, 2019, Guimarães, Portugal*

[4] M. Marchi, P.Z. Korondi, L. Parussini, C. Poloni. **New Approach for the Optimisation of a Ducted Propeller Under Uncertainty,** *NAFEMS: Simulation-Based Optimisation, October 16, 2019, London, UK*

Propeller Performance Solvers

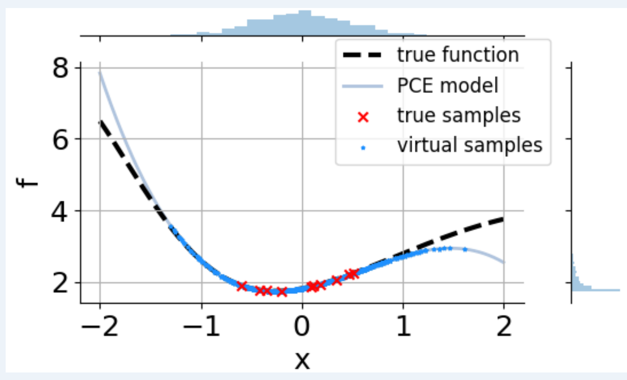


Solver	Speed	Samples	Accuracy
CFD[1]	~3.5 h	few	highest
XROTOR[2]	~30 s	some	intermediate
BEMT[3]	~0.1 s	many	lowest

[1] Navier-Stokes Solver (SU2)
[2] Lifting-line Theory
[3] Blade Element Momentum Theory

Multi-fidelity techniques require high correlation between the fidelities. Fortunately, in aerospace engineering even low-fidelity models (Lifting-line Theory, Blade Element Momentum Theory) are well calibrated and the correlation with high-fidelity Navier-Stokes Solvers is good. This allows us to obtain valuable information on the performance of a design at a cheaper cost.

Uncertainty Quantification



The probability space is modelled by Polynomial Chaos Expansion (PCE). The PCE is a very efficient technique and depending on its polynomial family it can model the probability space of various probability distributions.

$$f(x) \cong \sum_{i=1}^{k} \alpha_i p_i(x), \quad (1)$$

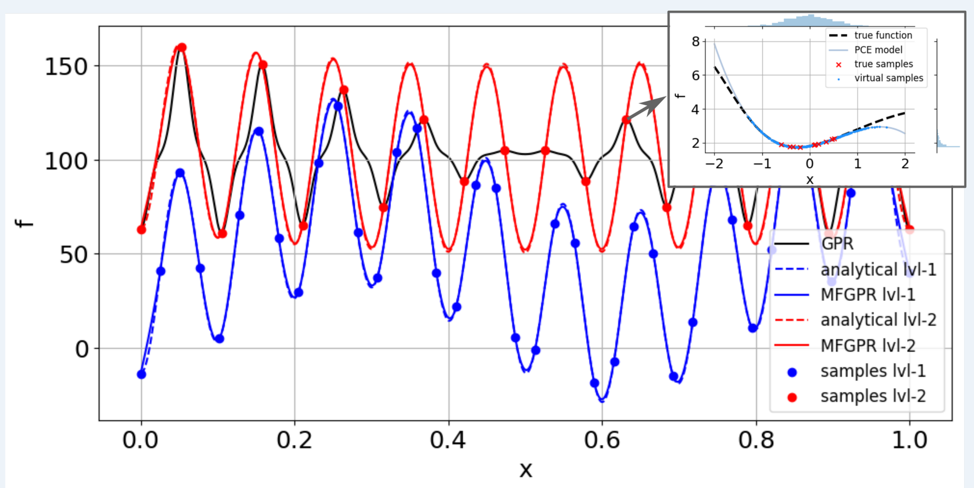
Multi-fidelity Gaussian Process Regression

A complex process can be approximated by the sum of two processes. The low-fidelity process can be modelled accurately as many data can be generated. The difference term is typically a less complex process.

$$Z_t(\mathbf{x}) = \rho_{t-1}(\mathbf{x}) Z_{t-1}(\mathbf{x}) + Z_{\delta_t}(\mathbf{x}) \quad (2)$$

$$\hat{m}_{Z_t}(\mathbf{x}) = \rho_{t-1} \hat{m}_{Z_{t-1}}(\mathbf{x}) + \hat{m}_{\delta_t}(\mathbf{x}) \quad (3)$$

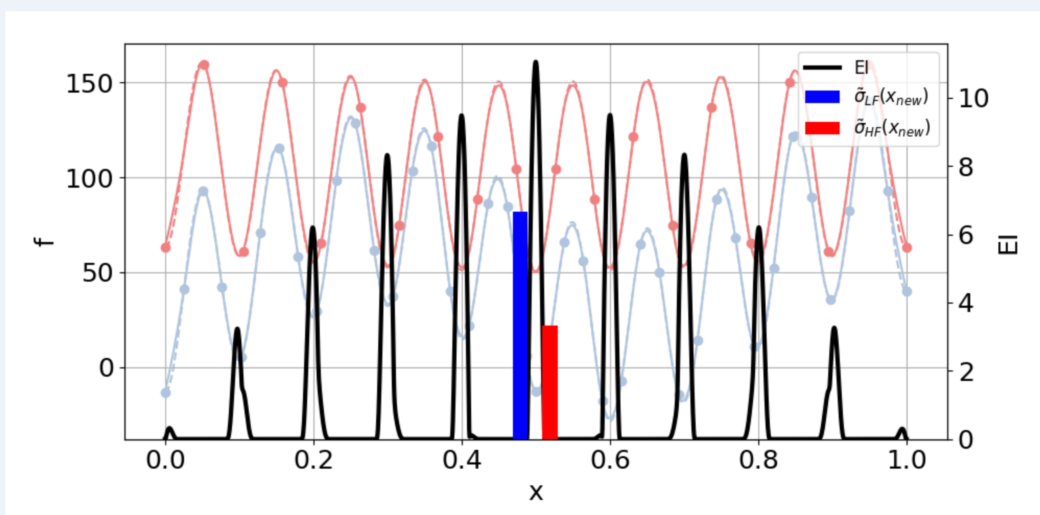
$$\hat{s}^2_{Z_t}(\mathbf{x}) = \rho_{t-1} \hat{s}^2_{Z_{t-1}}(\mathbf{x}) + \hat{s}^2_{\delta_t}(\mathbf{x}) \quad (4)$$



Surrogate	Samples	$\text{Error}_{\text{avg}}$[1]	$\text{Error}_{\text{opt}}$[2]
GPR	20	21.628	0.3949
MF-GPR	20(HF) 40(LF)	0.881	0.0002

[1] Mean squared error
[2] Absolute error of the location of the optimum

Acquisition Function



The fidelity level (l) is chosen which provides the highest scaled variance reduction:

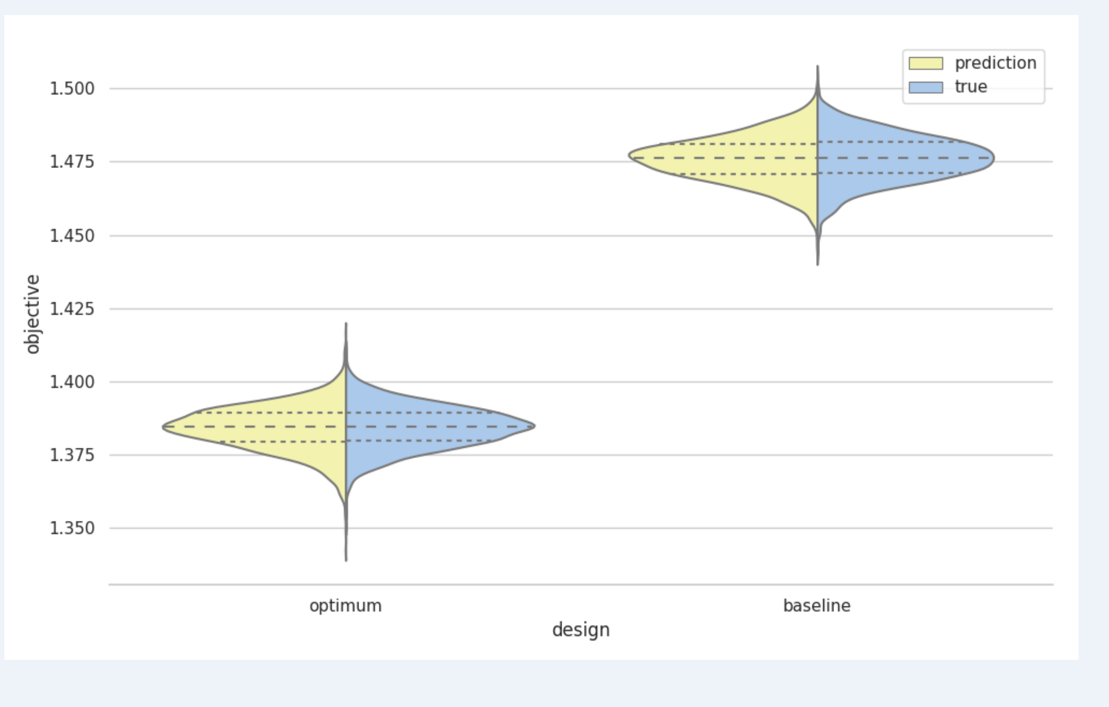
$$l = \arg\max_{LF,HF} \tilde{\sigma}_l, \quad (5)$$

where the $\tilde{\sigma}_l$ is defined as:

$$\tilde{\sigma}_{\text{HF}} = \hat{s}^2_{Z_t}(x_{new}) / cost_{HF} \quad (6)$$

$$\tilde{\sigma}_{\text{LF}} = \rho_{t-1} \hat{s}^2_{Z_{t-1}}(x_{new}) / cost_{LF} \quad (7)$$

Results



The proposed surrogate-based optimisation algorithm achieved 6% improvement of the objective. The uncertain nature of the problem was captured with a nested probability modelling.

Design	C_T[1]	C_P[2]	Objective[3]
baseline	2.281	3.367	1.476
optimum	1.173	1.624	1.384

[1] Mean of the thrust coefficient
[2] Mean of the power coefficient
[3] Inverse of mean efficiency

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