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Models of spontaneous wave function collapse: what they are, and how they can be tested

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Abstract. There are few proposals, which explicitly allow for (experimentally testable) deviations from standard quantum theory. Collapse models are among the most-widely studied proposals of this kind. The Schrödinger equation is modified by including nonlinear and stochastic terms, which describe the collapse of the wave function in space. These spontaneous collapses are rare for microscopic systems, hence their quantum properties are left almost unaltered. On the other hand, collapses become more and more frequent, the larger the object, to the point that macroscopic superpositions are rapidly suppressed. The main features of collapse models will be reviewed. An update of the most promising experimental tests will be presented.

1. Introduction

Collapse models [1, 2] are nonlinear and stochastic (phenomenological) modifications of the Schrödinger equation, which add the collapse of the wave function to the standard quantum evolution. The fundamental dynamics, stripped from all details, which are important to have a physically consistent model, but which are not necessary to understand the fundamental mechanism, is given by the following stochastic differential equation for the wave function:

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}Hdt + \sqrt{\lambda}(A - \langle A \rangle_t)dW_t - \frac{\lambda}{2}(A - \langle A \rangle_t)^2dt \right] |\psi_t\rangle, \quad (1)$$

where H is the usual quantum Hamiltonian of the system, $\langle A \rangle_t = \langle \psi_t | A | \psi_t \rangle$ is the quantum expectation of the operator A , here assumed to be self-adjoint, λ is a positive constant, which sets the strength of the collapse mechanism, and W_t is a standard Wiener process.

Eq. (1) is rather easy to understand [3]. It describes a diffusion process on the unit sphere of the Hilbert space, since the norm is conserved. H as usual induces a unitary “rotation” on the sphere, while the remaining terms tend to collapse the wave function towards one of the eigenstates of the operator A , in a stochastic fashion. The composite effect depends on the strength of one term with respect to the other. If the dynamics induced by H is dominant, then the evolution is deterministic and only slightly blurred by the collapse terms, for very long times. If on the other hand the collapse terms are dominant, then very rapidly the wave function randomly collapses, with a probability distribution, which is almost equal to the Born probability rule.



It is interesting to note that all known collapse models, in spite of the differences among them, sometimes also significant, have the same structure, which is captured by Eq. (1). This is not an accident. It can be shown [4] that the requirement of no-faster-than-light signalling, together with norm conservation, almost uniquely fixes the form of the possible modifications of the Schrödinger equation. Therefore, in this sense, one can claim that collapse models are not just a possible way of modifying quantum theory, but are the only way.

2. Collapse models in space

The choice of the operator A in Eq. (1) determines the basis upon which the wave function is localised. If $A = H$, then the collapse will occur in the energy basis, if $A = p$, it will occur on the momentum basis, and so on. Since the main reason for introducing collapse models is to justify the classical properties of macroscopic objects, the most obvious one being that they always occupy a well defined position in space, then the simplest and natural choice is to take A equal to the position operator q , or some function of it. All collapse models considered in the literature are of this kind.

The first model of this type was the GRW (Ghirardi–Rimini–Weber) model [5]. It is defined in terms of a discrete jump process instead of a continuous one, but otherwise the dynamics is equivalent to that of Eq. (1). Subsequently it was generalised to the CSL (Continuous Spontaneous Localisation) model [6], to include identical particles in the description. Parallel to that the QMUPL (Quantum Mechanics with Universal Position Localisation) model [7] was proposed, which can be seen as a short distance approximation of both the GRW and CSL models [8]. To mention also the DP (Diosi–Penrose) model [9], as a first attempt to connect the collapse of the wave function to gravity.

All these models are equivalent from the qualitative point of view: they induce the collapse of the wave function in *space*, and the larger the object, the faster the collapse, as we will see in Section 4. They differ, sometimes significantly, from the quantitative point of view. They share another property: the noise inducing the collapse of the wave function also induces a Brownian type of motion on the system. In momentum space, the wave function picks higher and higher components, meaning that its kinetic energy steadily increases. Although the increase is small and hardly detectable with present-day technology, nevertheless it is rather unpleasant feature of these models.

Recently a partial resolution of the problem was found. It was shown that it is possible to include dissipative effects in collapse models, without altering the collapse properties. This has been successfully accomplished for the QMPUL model [10], the GRW model [11], the CSL model [12], and in this case one speaks of dissipative models (dQMUPL, dGRW, dCSL). Interestingly enough, the same procedure turned out to be more problematic for the DP model [13]. In all these cases, the energy does not increase anymore, but thermalises to a finite temperature, which can be associated to the noise inducing the collapse. This means not only that the energy increase is not an unavoidable feature of collapse models (a naive argument was often told, that by collapsing the wave function in space, one increases the uncertainty in momentum—because of Heisenberg’s uncertainty principle—thus increasing the kinetic energy) but also that, actually, the energy can decrease (if initially higher than that of the noise), while the system’s wave function collapses in space.

Dissipative collapse models suggest the following picture: the noise causing the collapse is a *real* field filling space, at thermal equilibrium at some temperature T . It couples nonlinearly to quantum systems, causing the localisation of the wave function in space. At the same time, it thermalises them to its own temperature. This second effect is similar to what happens to a classical particle in a thermal bath. Energy is not conserved of course, but now the path towards restoring energy conservation is clear: besides the action of the noise on a quantum system, one should consider the reaction of the system on the noise. The combination of both

effects might lead to energy conservation, as classically the fundamental equations of motion (Newton's laws), out of which the phenomenological description of a particle in a gas emerges, are energy conserving. In the case of collapse models, this requires the definition of an underlying theory, out of which they emerge as phenomenological models. This is not available yet (see [4] for a first attempt); our argument here shows that its existence is not unreasonable.

All models so far considered are defined in terms of a noise field, which is white in time. This assumption is very convenient from the mathematical point of view, because the associated dynamics is given in terms of a standard stochastic differential equation, which is relatively easy to analyse. But white noises are only a mathematical idealisation; real noises necessarily have a frequency cut off. Collapse models defined in terms of coloured noises have been introduced [14, 15, 16, 17, 18]. It has been shown that they share the same properties as the original models. Details could be different, especially for cut offs at low frequencies.

3. The mass proportional CSL model

The most widely analysed model in the literature is the mass-proportional version of the CSL model [19]. Its dynamics is given by the following stochastic differential equation:

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}Hdt + \frac{\sqrt{\gamma}}{m_0} \int d^3x (M(\mathbf{x}) - \langle M(\mathbf{x}) \rangle_t) dW_t(\mathbf{x}) - \frac{\gamma}{2m_0^2} \int \int d^3x d^3y G(\mathbf{x} - \mathbf{y}) (M(\mathbf{x}) - \langle M(\mathbf{x}) \rangle_t) (M(\mathbf{y}) - \langle M(\mathbf{y}) \rangle_t) \right] |\psi_t\rangle, \quad (2)$$

where γ (having the dimensions of $[L^3T^{-1}]$) is the collapse strength of the model, m_0 is a reference mass, which is taken equal to that of a nucleon, and $M(\mathbf{x}) = ma^\dagger(\mathbf{x})a(\mathbf{x})$ is the *mass density* operator ($a^\dagger(\mathbf{x})$ and $a(\mathbf{x})$ are, respectively, the creation and annihilation operator of a particle at point $\mathbf{x}$ of space—here we are ignoring spin), which is a suitable replacement for the position operator in the second-quantized language. $W_t(\mathbf{x})$ is a family of Wiener processes, one for each point of space, with average equal to zero. They are “white” in time, while they have a Gaussian correlation function in space:

$$\mathbb{E}[W_t(\mathbf{x})W_s(\mathbf{y})] = \delta(t-s)G(\mathbf{x}-\mathbf{y}), \quad G(\mathbf{x}-\mathbf{y}) = \frac{1}{(4\pi r_C^2)^{3/2}} \exp[-(\mathbf{x})^2/4r_C^2]. \quad (3)$$

As we can see, the CSL model is defined in terms of two phenomenological parameters: the collapse strength γ and the correlation length of the noise r_C . Usually, one replaces γ with $\lambda = \gamma/(4\pi r_C^2)^{3/2}$, which has the dimensions of a rate $[T^{-1}]$. Experimental analysis is placing stronger and stronger constraints on these parameters.

4. The amplification mechanism and the numerical choice of the parameters

The CSL equation (2) does what all collapse equations do: it tries to localize the wave function in space, depending on the relative strength with respect to the Hamiltonian H . More specifically, the collapse terms work as follow [20].

Consider a system (in particular, a rigid object) whose center of mass is in a state, which is delocalized over a distance $\Delta x \ll r_C$. One can show that in this case the collapse is ineffective, almost absent. In a way, the delocalization is too small and the noise, which can resolve only distances greater than r_C , sees it already as well-localized.

If on the other hand the delocalization is larger than r_C , then the noise forces the wave function to collapse with a rate, which is roughly given by:

$$\Lambda = \lambda n^2 N, \quad (4)$$

where λ , introduced before, is the collapse rate for a single nucleon, n is the number of nucleons within a volume r_C^3 (a measure of the density of matter) and N counts how many such volumes can be accommodated in the space occupied by the system (a measure of the total size of the system).

Eq. (4) is the mathematical expression of the *amplification mechanism*: the larger the system, the faster the collapse. This means that it is possible to choose a unique value for λ and r_C such that, at the microscopic level, CSL differs little from ordinary quantum mechanics, while at the macroscopic level it forces wave functions to be well localized in space. In this way, the model is capable of providing a unique description of the quantum properties of macroscopic systems, and the classical properties of macroscopic objects—in particular, a resolution of the quantum measurement problem—within a unique dynamical framework.

In the literature, two values for the CSL parameters have been proposed. The first proposal was [6]: $\lambda \simeq 10^{-17} \text{ s}^{-1}$ and $r_C = 10^{-7} \text{ m}$. More recently, a strongly enhanced value for λ has been suggested by Adler [20], motivated by the requirement of making the wave-function collapse effective at the level of latent image formation in photographic process. The values suggested by Adler are $\sim 10^{9\pm 2}$ times larger than standard values at $r_C = 10^{-7} \text{ m}$, and $\sim 10^{11\pm 2}$ times larger at $r_C = 10^{-6} \text{ m}$.

5. Experimental tests

Collapse models explicitly modify the Schrödinger equation, therefore they make predictions, which differ—in most cases, only slightly—from those of ordinary quantum mechanics, and which can be tested experimentally. There are two type of tests: interferometric and non-interferometric ones.

Interferometric experiments are the most direct way of testing collapse models. Since their purpose is to forbid the existence of macroscopic superpositions, then the obvious way to test them is to take an object, which is as big as possible, create a large enough superposition and keep it there for as long a time as possible, and then detect the presence of quantum interference. Collapse models predict a negative outcome, contrary to ordinary quantum mechanics. Of course, performing such an experiment is not easy at all, for several technical reasons.

The group of Markus Arndt in Vienna succeeded in superimposing and detecting quantum interference of the center of mass of a macro-molecule of 20,000 a.m.u. [21]. Currently this is the world record. Fig. 1 shows the bound such an experiment places on the CSL parameters. As we can see, neither Adler's values for λ and r_C , nor GRW's, are disproved by the experiment, meaning that the mass here involved, the superposition times and distances are large enough. Optomechanics promises to significantly go beyond the state of the art [22], though that requires a improvement of the technology.

Non-interferometric tests [23, 24, 25, 26] aim at measuring the Brownian motion induced by the noise, independently of the collapse. There are two effects, which have been investigated. The first one is the increase of the kinetic energy, discussed in Sec. 2. A system confined in a (harmonic) trap should tremble around the equilibrium position more than what is predicted by quantum mechanics. A recent experiment with a cantilever [27] was able to place a bound, which is strong enough to partly exclude Adler's value for λ . The second effect is the spontaneous emission of radiation from charged particles, due to the acceleration induced by the noise. Experimental data [28] have been taken in the X-ray region, which also exclude Adler's value for λ . Future experiment promise to set bounds, which are at least two orders of magnitude stronger.

One comment is in order. It has been shown that interferometric bounds coming from matter-wave interferometry are robust against changes in the collapse equation, e.g. by introducing dissipative effects and/or by changing the type of noise [29]. Therefore they serve the purpose of testing the whole class of collapse models (in space), not just a single

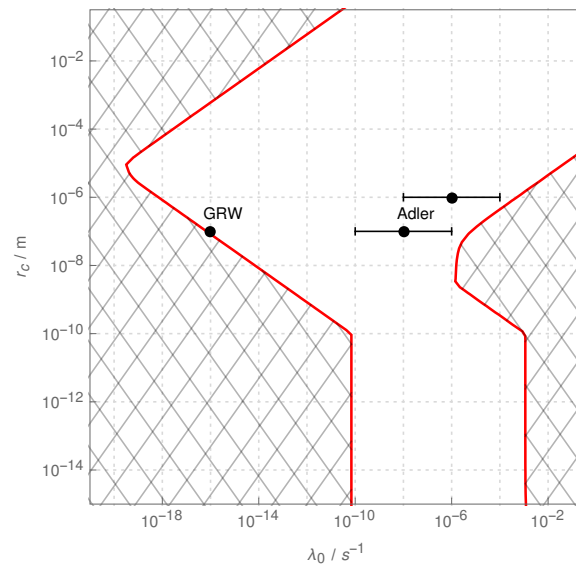


Figure 1. Left exclusion region: lower bound on the CSL parameters, coming from the requirement that macroscopic objects should be localized in space [29]. This is the fundamental motivation for introducing collapse models. Right exclusion region: upper bounds set by matter-wave interferometry [29]. The black dots/lines denote the numerical values for λ and r_C proposed by GRW and Adler.

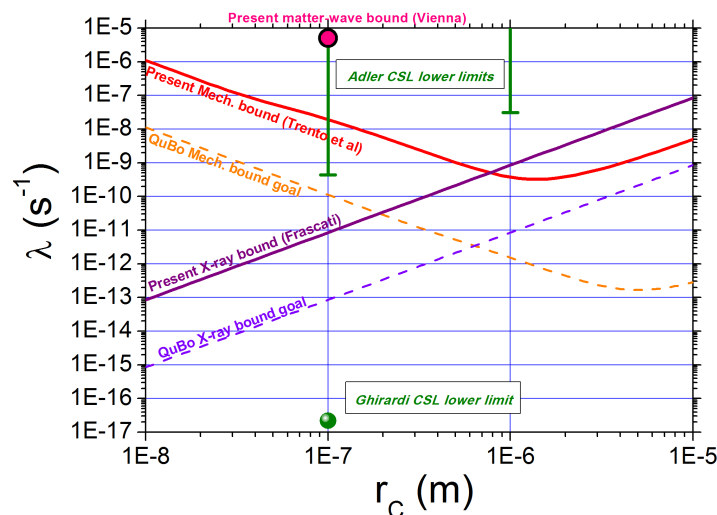


Figure 2. Upper bounds on the CSL parameters from non-interferometric experiments. The red solid line refers to the measurement of a violation of energy conservation with cantilevers [27], while the purple solid line refers to measurements of induced Brownian via spontaneous X-ray emission [28]. Adler's value for λ and r_C are excluded (but possibly rescued by suitable modifications of the CSL dynamics). Future upgrades of the experiments (dashed lines) promise to strengthen the bounds by two order of magnitude (QuBo project submitted to MIUR, Italy).

model—CSL, in our case. On the other hand, non-interferometric tests are more sensitive to the type of model. For example, predictions about spontaneous photon emission highly depend on the correlation function of the noise [20]. These bounds are strong, but probably are not able to span the whole class of collapse models. A combination of interferometric and non-interferometric experiments will probably be needed to test collapse models—therefore the quantum superposition principle—in a meaningful way.

6. Conclusion

Collapse models provide a consistent phenomenology for describing the collapse of the wave function as a dynamical effect, described by a suitable modification of the Schrödinger equation. One expects them to derive from an underlying theory, out of which both the quantum evolution and the collapse of the wave function emerge in a suitable coarse-grained description. This is a difficult problem, which has not been solved yet.

Meanwhile, it is interesting and relevant to test them experimentally, not least because any such test is a test of the superposition principle, the most important feature of quantum theory. Till recently, collapse models were confined to theoretical speculation. Now instead the experimental effort in testing them is increasing, and technological improvement allows (and will allow even more, in the near future) to explore significant regions of the parameter space. It will be interesting to see at the end, whether nature forbids nonlinear modification of quantum theory or, more exciting, whether the superposition principle will eventually break down.

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