

Supplementary Material

Although the idea underlying the derivation of the main result of this Letter is quite simple, the mathematics leading to it is delicate. The aim of this Supplementary Material is to guide the reader through the technical details of the calculations leading to the master equation (20). Furthermore, we provide the plots showing the behavior of the functions displayed by the master equation (20) and by the Bloch vector (23).

Equations of motion and contractions.

One can easily see that the Heisenberg equations of motion for the free Hamiltonian (1) are:

$$\begin{aligned}\dot{\hat{\sigma}}^x &= -\frac{\epsilon}{\hbar}\hat{\sigma}^y, \\ \dot{\hat{\sigma}}^y &= \frac{\epsilon}{\hbar}\hat{\sigma}^x + \Delta\hat{\sigma}^z, \\ \dot{\hat{\sigma}}^z &= -\Delta\hat{\sigma}^y.\end{aligned}\tag{S.1}$$

Since this is a linear system, one can always find a unique solution, provided three boundary conditions. Since these can be freely chosen, we set them at time t , because they will be convenient to switch from interaction to Schrödinger picture. The solution of the system at any time $s \leq t$ reads

$$\hat{\sigma}^i(s) = b_j^i(s-t)\hat{\sigma}^j(t),\tag{S.2}$$

where

$$b(t) = \begin{pmatrix} 1 + \frac{\epsilon^2}{\hbar^2\omega^2}(\cos\omega t - 1) & -\frac{\epsilon}{\hbar\omega}\sin\omega t & \frac{\Delta\epsilon}{\hbar\omega^2}(\cos\omega t - 1) \\ \frac{\epsilon}{\hbar\omega}\sin\omega t & \cos\omega t & \frac{\Delta}{\omega}\sin\omega t \\ \frac{\Delta\epsilon}{\hbar\omega^2}(\cos\omega t - 1) & -\frac{\Delta}{\omega}\sin\omega t & 1 + \frac{\Delta^2}{\omega^2}(\cos\omega t - 1) \end{pmatrix},\tag{S.3}$$

and $\omega^2 = \Delta^2 + \epsilon^2/\hbar^2$. We stress that if ϵ and Δ were time dependent functions, the system (S.1) would still be linear and it would still admit a solution of the type (S.2). Accordingly, our formalism can be applied also to time dependent detuning and dephasing.

The standard definition of Wick contraction for spin 1/2 particles is

$$\overline{\hat{\sigma}(s_1)\hat{\sigma}(s_2)} = -\{\hat{\sigma}(s_1), \hat{\sigma}(s_2)\}\theta_{s_2, s_1},\tag{S.4}$$

where the unit step function θ is needed because we are not dealing with normal ordered products. One should also keep in mind that we have dropped the superscript z . In order to obtain the explicit expression of the contraction one simply needs to replace Eq. (S.2) and exploit the anticommutation properties of the Pauli matrices. The result is

$$\overline{\hat{\sigma}(s_1)\hat{\sigma}(s_2)} = -2[b_x^z(s_1)b_x^z(s_2) + b_y^z(s_1)b_y^z(s_2) + b_z^z(s_1)b_z^z(s_2)]\theta_{s_2, s_1}.\tag{S.5}$$

One now understands how crucial is to have linear equations of motion: only in this case one can write a solution in the form (S.2) and obtain a contraction that is a c-function. If this is not the case, one cannot explicitly exploit the Wick's theorem and obtain the main result of this Letter. This explains why the formalism does not apply to spin chains. In fact, the equation of motion of e.g. the x component of the j -th spin of a Heisenberg chain reads

$$\dot{\hat{\sigma}}_j^x = -2J_y(\hat{\sigma}_j^z\hat{\sigma}_{j+1}^y + \hat{\sigma}_{j-1}^y\hat{\sigma}_j^z) + 2J_z(\hat{\sigma}_j^y\hat{\sigma}_{j+1}^z + \hat{\sigma}_{j-1}^z\hat{\sigma}_j^y),\tag{S.6}$$

where $J_{y,z}$ are coupling constants displayed by the Hamiltonian. An equation of this kind does not admit a solution of the type (S.2).

Calculation details leading to the master equation of Eq.(20). We start from the second line of Eq.(10) of the main text and we apply the Wick's theorem. For simplicity we focus only on the contribution from $\hat{\sigma}_L$ (the calculations for $\hat{\sigma}_R$ are similar).

$$T\left[\left(\int_0^t ds_1 D(t, s_1)\hat{\sigma}_L(s_1)\right)\prod_{i=2}^n \diamond_i\right] = \left(\int_0^t ds_1 D(t, s_1)\hat{\sigma}_L(s_1)\right)T\left[\prod_{i=2}^n \diamond_i\right] + \int_0^t ds_1 D(t, s_1)\sum_i \overline{\hat{\sigma}_L(s_1)T\left[\prod_{i=2}^n \diamond_i\right]},\tag{S.7}$$

where \diamond_i is given by Eq.(9). Since all contractions contribute in the same way, we can rewrite the last term of Eq. (S.7) as follows:

$$(n-1) \int_0^t ds_1 D(t, s_1) \left(\overbrace{\hat{\sigma}_L(s_1) T \left[\int_0^t dt_2 \int_0^{t_2} ds_2 D(t_2, s_2) [\hat{\sigma}_L(t_2) \hat{\sigma}_L(s_2) - \hat{\sigma}_R(t_2) \hat{\sigma}_L(s_2)] \prod_{i=3}^n \diamond_i \right]}^{\text{long overbracket}} \right. \\ \left. - \hat{\sigma}_L(s_1) T \left[\int_0^t dt_2 \int_0^{t_2} ds_2 \left(D(s_2, t_2) [\hat{\sigma}_L(t_2) \hat{\sigma}_R(s_2) - \hat{\sigma}_R(t_2) \hat{\sigma}_R(s_2)] \prod_{i=3}^n \diamond_i \right) \right] \right), \quad (\text{S.8})$$

where we simply extracted \diamond_2 from $\prod_{i=2}^n \diamond_i$, and the long overbracket denotes the contraction of $\hat{\sigma}_L(s_1)$ with this term. We now exploit the rules of Eqs.(12)-(13) to express Eq. (S.8) in terms of single contractions as follows:

$$(n-1) \int_0^t ds_1 D(t, s_1) \int_0^t dt_2 \int_0^{t_2} ds_2 \left[\overbrace{\hat{\sigma}(s_1) \hat{\sigma}(t_2)}^{\text{bracket}} (D(t_2, s_2) \hat{\sigma}_L(s_2) - D(s_2, t_2) \hat{\sigma}_R(s_2)) \right. \\ \left. - \overbrace{\hat{\sigma}(s_1) \hat{\sigma}(s_2)}^{\text{bracket}} D(t_2, s_2) (\hat{\sigma}_L(t_2) + \hat{\sigma}_R(t_2)) \right] T \left[\prod_{i=3}^n \diamond_i \right], \quad (\text{S.9})$$

where we also exploited the fact that contractions are c-functions and commute with T-ordering. By manipulating the integral limit and rearranging the terms, one can rewrite this equation as follows:

$$(n-1) \left(\int_0^t ds_1 D(t, s_1) \int_0^t dt_2 \int_0^t ds_2 \overbrace{\hat{\sigma}(s_1) \hat{\sigma}(t_2)}^{\text{bracket}} [\bar{D}(t_2, s_2) \hat{\sigma}_L(s_2) - D^*(t_2, s_2) \hat{\sigma}_R(s_2)] \right) T \left[\prod_{i=3}^n \diamond_i \right], \quad (\text{S.10})$$

where we have exploited the relation $D(s_2, t_2) = D^*(t_2, s_2)$ and we have defined

$$\bar{D}(t_2, s_2) = D^{Re}(t_2, s_2)(2\theta_{t_2 s_2} - 1) + iD^{Im}(t_2, s_2). \quad (\text{S.11})$$

Repeating similar calculations for $\hat{\sigma}_R(s_1)$ and recollecting the results, one eventually obtains:

$$T \left[\left(\int_0^t ds_1 D(t, s_1) \hat{\sigma}_L(s_1) - D^*(t, s_1) \hat{\sigma}_R(s_1) \right) \prod_{i=2}^n \diamond_i \right] = \left(\int_0^t ds_1 D(t, s_1) \hat{\sigma}_L(s_1) - D^*(t, s_1) \hat{\sigma}_R(s_1) \right) T \left[\prod_{i=2}^n \diamond_i \right] \\ + (n-1) T \left[\left(\int_0^t ds_1 D_{(2)}(t, s_1) \hat{\sigma}_L(s_1) - D_{(2)}^*(t, s_1) \hat{\sigma}_R(s_1) \right) \prod_{i=3}^n \diamond_i \right], \quad (\text{S.12})$$

with

$$D_{(2)}(t, s_1) = \int_0^t dt_2 \int_0^t ds_2 \overbrace{\hat{\sigma}(s_2) \hat{\sigma}(t_2)}^{\text{bracket}} [\bar{D}(t_2, s_1) D(t, s_2) + D(t_2, s_1) D^*(t, s_2)]. \quad (\text{S.13})$$

The important lesson we learn from Eq. (S.12) is that the odd T-product of the left hand side, can be decomposed in an even T-product (that can be linked to M_t^n) plus another odd T-product of lower order with the same structure as the left hand side. This implies that one just needs to perform the substitution $D \rightarrow D_{(2)}$ and repeat these calculations to obtain $D_{(3)}$, and so on. This iteration leads to the following expression:

$$D_{(n)}(t, s_1) = \int_0^t dt_n \int_0^t ds_n \overbrace{\hat{\sigma}(s_n) \hat{\sigma}(t_n)}^{\text{bracket}} [\bar{D}(t_n, s_1) D_{(n-1)}(t, s_n) + D(t_n, s_1) D_{(n-1)}^*(t, s_n)]. \quad (\text{S.14})$$

The result of this procedure is that we have decomposed the initial odd T-product of Eq. (S.7) in a sum of even T-products, that can be linked to M_t^k :

$$T \left[\left(\int_0^t ds_1 D(t, s_1) \hat{\sigma}_L(s_1) - D^*(t, s_1) \hat{\sigma}_R(s_1) \right) \prod_{i=2}^n \diamond_i \right] = \\ \sum_{k=0}^{n-1} \frac{(n-1)!}{k!} \left(\int_0^t ds_1 D_{(n-k-1)}(t, s_1) \hat{\sigma}_L(s_1) - D_{(n-k-1)}^*(t, s_1) \hat{\sigma}_R(s_1) \right) M_t^k. \quad (\text{S.15})$$

By substituting this equation in Eq. (10), and by exploiting the definition of Cauchy product of two series one obtains

$$\dot{\mathcal{M}}_t = - \sum_{n=1}^{\infty} (-1)^{n-1} [\hat{\sigma}_L(t) - \hat{\sigma}_R(t)] \left(\int_0^t ds_1 D_{(n)}(t, s_1) \hat{\sigma}_L(s_1) - D_{(n)}^*(t, s_1) \hat{\sigma}_R(s_1) \right) \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} M_t^k. \quad (\text{S.16})$$

By applying this equation to $\hat{\rho}_0$, one easily finds Eqs.(18),(19), where by definition $D_{(1)} \equiv D$, and we have added the subscript zz for coherence with the notation of Eq.(5).

It is interesting to observe that Eq. (S.13) can be interpreted as the action of an operator \mathfrak{D} on $D(t, s_2)$, i.e. $D_{(2)}(t, s_1) = \mathfrak{D}[D(t, s_2)]$, which for a general $D_{(n)}$ leads to $D_{(n)}(t, s_1) = \mathfrak{D}^{n-1}[D(t, s_2)]$. According to this notation, one can rewrite Eq.(18) in a more elegant way, by formally summing the series:

$$\mathbb{D}(t, s_1) = \frac{1}{1 + \mathfrak{D}} [D(t, s_2)]. \quad (\text{S.17})$$

The last step in the derivation of Eq.(20) requires the solution of the Heisenberg equations of motion for Eq.(1), which are provided in the first section of this Supplementary Material.

Plots of the functions in the master equation (20).

In the main Letter, we provided a plot showing the time evolution of $B_{zz}^{Re}(t)$ for an increasing number of terms in the series (19) defining it. We provide here the plots for the functions $B_{zx}(t)$ and $B_{zy}(t)$, which rule the evolution of a density matrix according to Eq.(20).

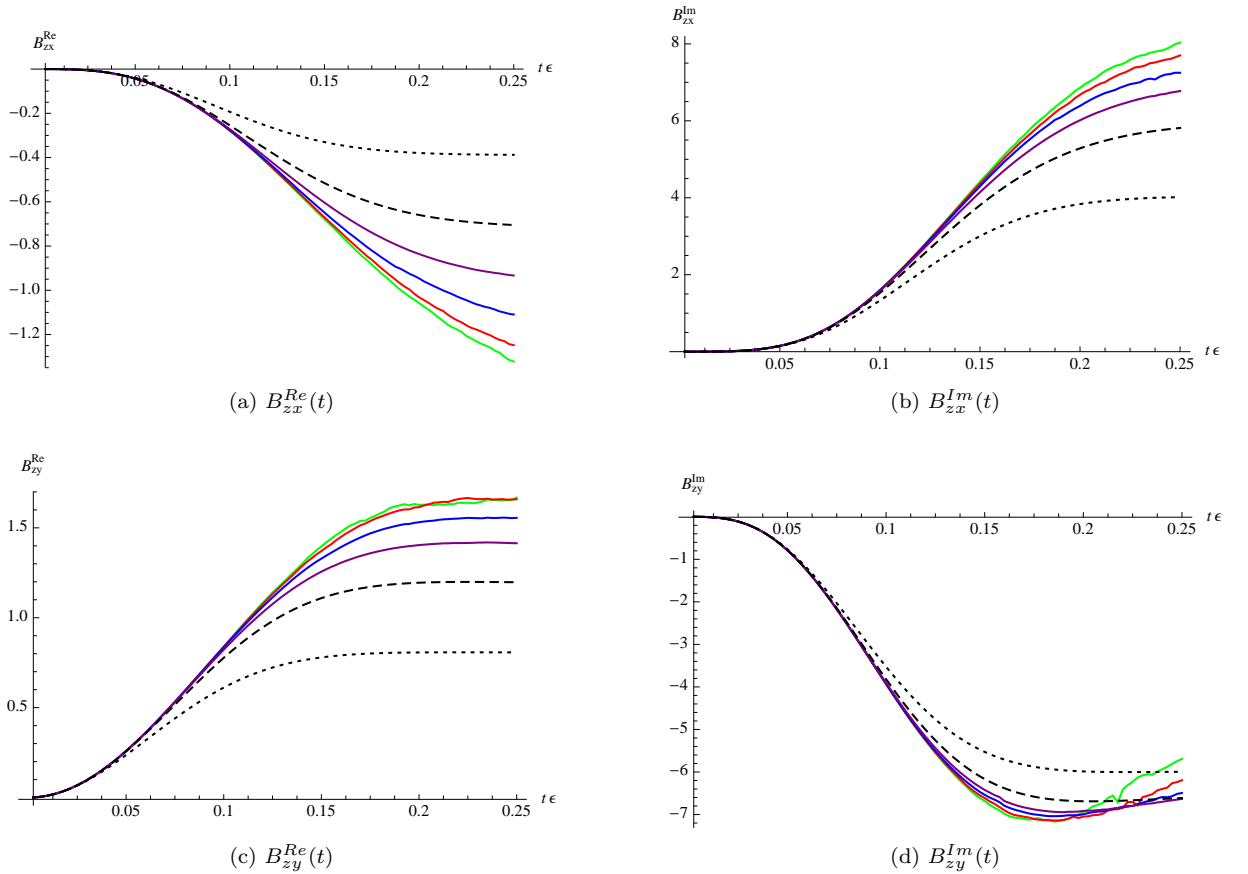


FIG. S.1: Dotted line is $n = 1$, dashed $n = 2$. Solid lines are respectively [bottom to top in insets (b) and (c), top to bottom in insets (a) and (d)]: $n = 3$ (purple), $n = 4$ (blue), $n = 5$ (red), $n = 6$ (green). Bath with ohmic spectral density and Gaussian cutoff: $J(\omega) = 2\pi\omega \exp[-\omega^2\Lambda^{-2}]$. Other parameters are: $\epsilon = 10$, $\Delta = \epsilon$, $k_0^2 = 0.04\epsilon$, $k_B T = 0.1\epsilon$, $\Lambda = 2\epsilon$.

These plots clearly show how previous results (black lines) are improved, and that the series converges quite fast.

We stress that these functions and the evolution of $\hat{\rho}$ strongly depend on the bath structure and the other parameters of the model.