

Remembering Ernst Mach (1838-1916) As A Scientist And An Educator

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ABSTRACT

On the occasion of the centenary of his death, we consider the figure and the work of the Austrian physicist, physiologist and philosopher Ernst Mach (1838-1916). We focus on Mach's role as an educator, at various levels, both as a professor and as a conference speaker. This becomes clear when considering his results in science education and, particularly, mathematics education. In Mach's vision, scientific education helps people to understand the world in which they live and so to act in an "economic" way (according to the very Machian meaning of the term). Therefore, with the aim of improving science and mathematics education, Mach suggested a set of operational guidelines to be implemented. We conclude by pointing out the modernity of Mach's ideas on education that still animate our present. **1. ERNST MACH AS A SCIENTIST**

The Austrian Ernst Mach, the centenary of whose death falls in 2016, is known worldwide as a physicist and philosopher who was also concerned with epistemology and the history of science, physiology of sensations and experimental psychology, especially perception problems (see Blackmore, 1972, 1992; Blackmore, Itagaki & Tanaka, 2001; Blüh, 1967; Cohen, 1968; Haller & Stadler, 1988; Heller, 1964; Janik & Toulmin, 1973; Matthews, 1990; Zudini & Zuccheri, 2016).

Mach was professor of Mathematics and of Physics at the University of Graz and then of Physics in Prague (at that time part of the Habsburg Empire) and thereafter held the chair in the History and Theory of Inductive Sciences in Vienna. In this context, he was the inspiration for the melting pot of ideas which would become famous as the "Vienna Circle" (called also "Verein Ernst Mach") and for the new generation of physicists that was growing up at that time (among them, Albert Einstein; see Heller, 1964).

Mach's scientific and philosophical programme is based, first of all, on the development of a theory of biologically based knowledge. Heavily influenced by evolutionism, his theory regarded knowledge construction as an essentially adaptive process, placed in continuity with biological processes by which living beings adapt to their natural environment (see, on the "genetic worldview" of Mach's ideas, Siemsen, 2014). The same principles that guide the adaptation of organisms to their environment also rule the biological and psychological dynamics of human beings, including the level of knowledge, both natural and scientific. Knowledge itself is a product of universal evolution, and the general imprint of evolution and transformation must also be noticeable in ideas (Mach, 1886, Engl. transl. 1996, pp. 71ff.).

In this vision, the conceptual and mathematical instruments used in that complex and sophisticated human activity called "science" constitute a strategy that people have put in place with the practical aim of achieving greater and more complete control on reality (see Mach, 1906). According to Darwin's conception, nature is structured in an organic and simple way, and each one of its elements finds its place without excess or waste, in an "economical" way. Therefore, if science aims to help guide human beings in the world around them, in order to be of real use, it must be able to provide a description of nature that is as "economical" as possible.

2. ERNST MACH AS AN EDUCATOR

One of Mach's hallmarks is that science, once constructed, should be taught and disseminated. Mach was among the most brilliant scientific propagators of his time, both as a professor and as a conference speaker, and contributed with his conceptions and theories to shaping modern culture and mores mirrored in the "great Vienna" of the early 1900's. Appointed member of the Upper House of the Parliament by the Emperor Franz Joseph, he played the role of advocate of a modern, scientific, interactive, anti-dogmatic and secular education;

he was aware of the necessity to propagate knowledge at all social levels (see Blüh, 1967; Hohenester, 1988; Matthews, 1990; Siemsen, 2010; Zudini & Zuccheri, 2016).

He promoted the “Volksbildungsverein” (“Association for popular culture”) and the “volkstümliche Vorlesungen” (“popular lectures”, i.e. the courses of lectures that were intended for the education of the working class and held by professors of the University of Vienna). He also edited the “Populär-wissenschaftliche Vorlesungen” (1896), in order to communicate his conceptions and theories even to a non-expert reading public (see, in particular, Zudini & Zuccheri, 2016).

3. MACH’S CONTRIBUTION TO MATHEMATICS AND SCIENCE EDUCATION

According to Mach’s ideas, studying science – in particular mathematics – proved to be fundamental in helping human beings to observe and understand the world around them and thus to act in an “economic” way (see, e.g., Mach, 1889, Engl. transl. 1989, pp. 577ff.); therefore, scientific education should be consistently pursued (Mach, 1896, Engl. transl. 1898, pp. 360ff.).

3.1 HUMANISTIC CULTURE AND SCIENTIFIC CULTURE

“Über den relativen Bildungswert der philologischen und der mathematisch-naturwissenschaftlichen Unterrichtsfächer der höheren Schulen” (“On the relative educational value of the classics and the mathematico-physical sciences in colleges and high schools”) is the title of a conference held by Mach in 1886, on the educational value of classical and scientific culture, and contained in his “Populär-wissenschaftliche Vorlesungen” (Mach, 1896, Engl. transl. 1898, pp. 338-374). In this conference, Mach showed himself to be a very modern scholar in his treatment of the relationship between humanities and sciences and their formative value (see Zuccheri & Zudini, 2016). He recognized that, within the cultural development of his time – by then focused on the technical-scientific aspect –, humanities could no longer be considered the only (or even the better) means to offer a higher education.

Mach countered the usual arguments in favour of the supremacy of humanistic culture with the greater value and effectiveness of teaching mathematics and science; he gave examples, claiming their superiority with regard to educational aims and for the development of ability in observation and logic (Mach, 1896, Engl. transl. 1898, pp. 344ff.).

“I shall meet with no contradiction when I say that without at least an elementary mathematical and scientific education a man remains a total stranger in the world in which he lives, a stranger in the civilisation of the time that bears him. Whatever he meets in nature, or in the industrial world, either does not appeal to him at all, from his having neither eye nor ear for it, or it speaks to him in a totally unintelligible language.

A real understanding of the world and its civilisation, however, is not the only result of the study of mathematics and the physical sciences. Much more essential for the preparatory school is the formal cultivation which comes from these studies, the strengthening of the reason and the judgment, the exercise of the imagination.” (Mach, 1896, Engl. transl. 1898, pp. 359f.)

Mach indicated a set of operational guidelines for the improvement of mathematics and science education (Mach, 1896, Engl. transl. 1898, pp. 364ff.), which can be outlined as follows (see Zudini & Zuccheri, 2016):

1. avoiding premature abstraction and reducing the amount of the subject matter, inasmuch it was impossible to accumulate ideas beyond a certain level in a brain all at once;
2. using, in the presentation of a subject, examples (taken also from reading selected passages of the great classics of mathematics and science) and practical (manual and, generally, bodily) experiences to cause students to take an active part in their learning;
3. presenting, in the final classes of the higher schools, a division of the subjects into compulsory and optional, so as to enhance students’ interests and inclinations;
4. preventing the most talented students, who, for some reason, had not been able to follow a regular curriculum, from being excluded a priori from entering university and academic professions; generally, there should be no restriction of access to education and the professions.

The ideal proposed by Mach (Mach, 1896) was therefore, in general, that of a non-dogmatic teaching of mathematics and science, in which the concepts should be introduced accompanied by an experience of the events that led to their formation. This should be done in order to take into account how they actually developed in the course of mathematics and science, following a historical and natural approach, with the goal of an “economical” adaptation of thoughts to facts.

In this vision, and with psychological attention to the development of the intellectual abilities, rigour should be reached in a gradual way, without an excessive use of formulas and chains of reasoning, so as not to burden the

students and prevent them from forming unnecessary or incorrect concepts. As far as possible, everyday language and concepts in common use should be applied, at least in the initial phase; new concepts, hypotheses and theories should be introduced only when actually needed for the handling of a subject.

3.2 THE “JACOB METHOD”: AN EXAMPLE OF APPLICATION OF MACH’S IDEAS IN MATHEMATICS EDUCATION OF HIS TIME

On the basis of Mach’s ideas, a didactical method was developed, called the “Jacob method” – after Josef Jacob, who proposed it – and applied at the beginning of the twentieth century in the teaching of mathematics in the Austrian “Gymnasium” (pupils aged 11-18) (see Zuccheri & Zudini, 2007a, 2007b, 2008, 2010; Zudini & Zuccheri, 2016). This “practical method”, supported by Jacob’s teaching experience, was illustrated in a textbook to be used in training “Gymnasium” mathematics teachers and published in 1913 with a preface by Mach himself (Jacob, 1913).

Jacob’s treatise proceeded step by step giving practical suggestions and including very precise didactical examples, explaining to the teachers, in a clear and practical manner, how to introduce any subject, ranging from simple arithmetical operations to calculus. This was put into effect with the target of a mathematical teaching method for secondary school level, which, in line with Mach’s ideas, had the following aims:

1. to make the mathematical “facts” (for instance, the sum of the angles of a triangle) and their interrelations comprehensible to young people as concisely as possible, with first simple, schematic representations;
2. to educate pupils on conceptual thought, encouraging intuition and avoiding premature abstraction;
3. to increase the value of mathematics by applying it in practical life, in technology and in science.

Therefore, Jacob proposed already at the beginning of the twentieth century a didactical method with conscious scientific foundation (i.e. Mach’s theory), supported by his own teaching practice. A case study of application of the “Jacob method” is demonstrated through an examination of documents (school year-books and archival documents) concerning the teaching of mathematics in the city of Trieste, which at that time and until the end of the First World War belonged to the Habsburg Empire, and now is part of Italy (see Zuccheri & Zudini, 2007a, 2010; Zudini & Zuccheri, 2016).

4. CONCLUSION: THE MODERNITY OF MACH’S IDEAS

The modernity of Mach emerges in the light of his ideas on education, which were destined to have rich developments in the twentieth century and which still animate current research in the field.

The hallmark of modernity is particularly evident when considering the two following perspectives that are closely related: the cultural point of view and the educational point of view.

Mach’s focus on science and on scientific culture can without doubt be acknowledged as modern from the first point of view. This aspect is, moreover, strictly connected, both in Mach’s vision and in the present day, with the focus on the relationship between scientific culture and humanistic culture and on their formative value (see, among the modern treatises on the subject, Snow, 1993).

Other elements of Mach’s conceptions are to be stressed from the educational point of view, such as the importance given, respectively, to teacher training and to students’ intellectual development, which resound in modern studies on teaching and learning processes.

As explained in Zudini & Zuccheri (2016), an important role with educational aims is also played in Mach’s theory of knowledge by manual and, generally, physical activities within the process of creation and development of concepts. As described above, on the basis of this theory, a didactical method (the so-called “Jacob method”) was conceived, supplying a historical example of mind-body grounded methodology that could be compared to some aspects of modern theory of embodied cognition applied to mathematics teaching (see, e.g., Arzarello and Robutti, 2008, as well as Edwards et al., 2009, and, for more recent developments, Edwards et al., 2014).

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