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Recycling waste and endogenous fluctuations in an OLG model

Mouez Fodha* and Francesco Magris†

We study an overlapping generations model composed of two productive sectors: the first produces a virgin good and the second recycles a good from a previous period. When the profit of the recycling sector is in its increasing region, endogenous fluctuations emerge through a flip bifurcation when leisure and consumption are complementary. Conversely, when profit is in its decreasing region, deterministic cycles emerge also under the gross substitutability assumption. The steady state becomes indeterminate through a flip or a Hopf bifurcation, provided leisure and consumption are complements. We also study the social optimum and the altruistic market economy.

Key words endogenous fluctuations, recycling activity, optimal policy, altruism

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1 Introduction

The recycling industry offers investors one of the few ways to make a profit during the current recession, according to a report released in 2008 by Progressive Investor *Investing in Recycling*.¹ The recycling industry now plays a central and growing role in the economy. In the USA alone, it pulled in \$236 billion in revenues and employed about 1.1 million people in 2007, accounting for about 2 percent of US gross domestic product. Waste Management, which is a leader in waste management in the United States, aimed to double its recycling activity, from 10 million tons per year to 20 million tons, by 2020. In Europe, the recycling sector should grow in a significant manner since the EU will impose a recycling rate of 50 percent for household waste and 70 percent for industrial waste in 2020, while the average rate was only 13 percent in France in 2008. Recycling is also beneficial for employment, since 100 tons of recycled waste per year would create 250 jobs, against 10 jobs when waste is sent to landfill. According to the OECD, recycling will be necessary in 2030 to alleviate the scarcity of natural resources exacerbated by a growing world population, as the latter could rise to 8.2 billion people, including 4.9 billion in urban areas. This demographic evolution will probably have serious consequences for the production of waste.

The economic literature has analyzed recycling activities from three theoretical points of view. First, recycling is integrated into the analysis of industrial organization; following the Alcoa case, several studies have analyzed the erosion of monopoly power by competitive firms of recycled

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¹ <http://bit.ly/1PiiQuw>

products (Grant 1999). Second, natural resource economics points out the usefulness of recycling for economies facing a decrease in the availability of resources; recycling postpones the exhaustion of mining resources and reduces the intensity of forest exploitation (Dasgupta and Heal 1979). Third, environmental economics analyzes recycling as an instrument to reduce environmental externalities (Lusky 1976).

However, this literature does not analyze the role of the life cycle of goods, in particular the impact of recovery and recycling of waste on growth and fluctuations. De Beir, Fodha, and Magris (2010) analyze an infinite-horizon model with waste recovery and show that deterministic cycles as well as sunspot fluctuations may arise when the economy is located in the region where profits are decreasing in the amount of the recycled good. The intuition rests upon the countercyclical behavior of the share of GDP due to waste recovery: when output increases, the amount of the recycled good declines and therefore the next period production will be lower. This in turn implies a larger amount of waste recovery activity and therefore an increase in the future GDP two periods later.

In the present paper, we generalize some standard results on endogenous fluctuations in overlapping generations (OLG) models. We show that, depending on the characteristics of the profit evolution of the recycling sector, the usual conditions entailing the emergence of fluctuations may be modified, namely deterministic cycles may emerge also under the gross substitutability assumption, a feature that is ruled out in the seminal works of Azariadis (1981), Grandmont (1985) and Azariadis and Guesnerie (1986).

The economy is kept as simple as possible. In each period two generations of agents coexist. There are two productive sectors. The first produces a virgin good, according to a constant returns to scale technology. A share of previous period consumption is recycled. Profits from the recycling activities are distributed to agents by means of lump-sum transfers. The dynamics describing intertemporal equilibrium is two-dimensional and can therefore be studied with the help of the geometrical approach presented in Grandmont, Pintus, and de Vilder (1998) and Cazzavillan, Lloyd-Braga, and Pintus (1998).

We show that endogenous fluctuations and sunspot equilibria always occur, whatever the slope of the profit function of the recycling sector.² The properties of such phenomena depend dramatically upon the behavior of the profits of the recycling activity. Specifically, if we are in the region in which profits are increasing in the amount of recovered waste, we find some standard results of the literature on fluctuations in OLG models as in Azariadis (1981), Geneakoplos and Polemarchakis (1984), Grandmont (1985) and Azariadis and Guesnerie (1986): there arise endogenous fluctuations and sunspot equilibria through a flip bifurcation only when consumption and leisure are complementary goods. On the other hand, when profits are decreasing in the amount of the recycled good, deterministic cycles may occur also under the gross substitutability assumption. These results mimic standard multisector models,³ but the mechanism does not rely upon dramatic reversals in capital intensity across sectors. Rather, cycles are due to the countercyclical behavior of the recycling sector's profits: when present consumption increases, tomorrow's waste recycling profits will be pushed down and the agents' total income will be lower. This, in turn, will entail an increase in the next period's total income and consumption. At the same time, we find a relevant result in terms of public policy. We show that the indeterminacy region shrinks as soon as the share of the profits obtained in the recycling sector and transferred to old agents increases. This is a direct consequence of the fact that

² By contrast, De Beir, Fodha, and Magris (2010) need a negative slope of the profit function in order to get endogenous fluctuations.

³ See, among others, Benhabib and Nishimura (1979), Benhabib and Farmer (1996), Druegeon and Venditti (2001), and Nishimura and Venditti (2002).

when profits are distributed to young agents, the latter invest them in money balances and therefore pay the inflationary tax. This in turn increases the scope for indeterminacy and deterministic cycles. Therefore, in order to stabilize the economy, the government should transfer the profits of the recycling sector entirely to old agents.

We also characterize the social optimum and show that equilibrium is always determinate and endogenous fluctuations are ruled out. Under the altruism hypothesis, we compute the optimal share of the profits to be granted to the old agents in order to mimic the social optimal outcome.

The rest of the paper is organized as follows. In Section 2 we study the decentralized economy and provide conditions for the occurrence of endogenous fluctuations and sunspot equilibria. We also derive explicitly the critical (bifurcation) points and appraise the evolution of the local dynamics as a function of the share of profits of the recycling activity granted to young agents. In addition, a discussion of the results is provided. Section 3 is devoted to the study of the social optimum. Section 4 concludes the paper.

2 The market economy

We consider an OLG model with two productive sectors producing a homogenous consumption good. One produces a virgin good, and the other employs the previous period consumption in order to produce a recycled good. We assume that there is a representative firm operating in each sector. The virgin good y^v is produced by means of a linear technology which employs only labor (l), i.e., the technological relationship is $y^v = l$ where v stands for virgin. We assume that firms are price-takers and that the profit is then $\pi_t^v = p_t y_t^v - w_t l_t$, i.e., $\pi_t^v = (p_t - w_t) l_t$, where w_t is the nominal wage and p_t the price of the consumption good. We depart from the industrial organization literature which generally assumes that the firm producing in the virgin sector is a monopolist which does not take the price p_t as given but rather considers the price as a function $p_t(l_t)$ in terms of labor demand: the optimal decision is therefore interior (as a solution of the equation $p'_t(l_t) = w_t$), i.e., the labor demand is defined as a function of the wage w_t . Assuming a monopolistic environment would display richer dynamics. However, the assumptions of a linear technology and price-taker firms make it possible to compare our results with those of Azariadis (1981), Grandmont (1985) and Azariadis and Guesnerie (1986). Actually, without a recycling sector, our economy boils down to the frameworks mentioned. Under our assumption concerning the technology and the competitive market configuration, at equilibrium one has $w_t = p_t$ and therefore $\pi_t^v = 0$ for each $t = 1, 2, \dots$. This point requires further clarification. To this end, one has to bear in mind that firms and agents are price-takers. Suppose, first, that $p_t < w_t$: in such a configuration, the profit corresponding to a positive amount of labor supply and hence of production would be negative. It follows that the demand for labor of the representative firm will be $l^d = 0$. But, the labor supply of the representative young agent is, as we shall see, positive and bounded away from zero. Therefore there will be an excess of labor supply and as a consequence the nominal wage will be pushed down, until it reaches exactly p_t . Suppose, on the other hand, that $p_t > w_t$. Now the profit increases linearly with the labor demand (and therefore with production) and the demand for labor will be $+\infty$. Since, again, the labor supply is finite, there will now arise an excess of labor demand: as a consequence, the nominal wage w_t will start to increase until it is equal to p_t . It follows that equilibrium in the labor market requires $p_t = w_t$ and the labor supply will be indeterminate, taking whatever value between zero and $+\infty$. In other words, firms are willing to absorb whatever amount of labor is supplied by young agents. This mechanism is the fruit of the perfect flexibility assumptions of the nominal wage and of the price level: the labor market will indeed always be at equilibrium, and the amount of labor is fixed by the

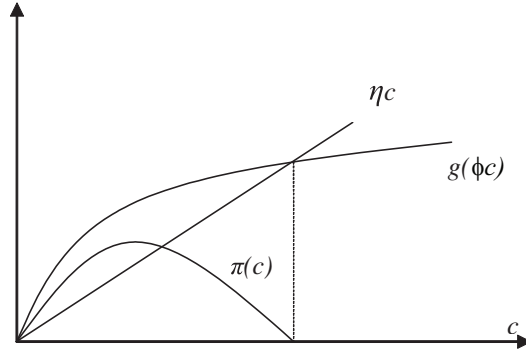


Figure 1 Profit of the recycling sector.

supply side of the market. The recycled good and the virgin good are perfect substitutes. A recycling sector competes directly with the virgin production sector. The representative firm includes recovery and recycling activities. Following Swan (1980) and Martin (1982), we suppose that the production factor is the recovered waste and that returns are decreasing. Diminishing returns are one of the main characteristics of the technology of recovery; they reflect rising marginal costs due to the increased difficulty of scrap recovery as larger quantities are recovered, and the fact that as the stock is depleted the brokers will have to pay a higher price to induce individuals to search for additional scrap. Hence, the recycled good is produced according to a concave production function g which recovers a share $\phi \in (0, 1)$ of the previous period consumption at the fixed cost⁴ $\eta > 0$. As the two goods are *in fine* perfect substitutes, we assume that it is possible to recycle the good infinitely many times. Assume that there is no profit maximization⁵ task. Hence, this sector is a public one or, as can be seen in European countries, private companies accredited by the public authorities (for example, DSD in Germany) which have to sort all the available household waste packaging. Therefore, as usual in recycling economic models (see Swan 1980; Martin 1982), we assume that the profit of the recycling sector first increases and then decreases. The production function g possesses the following properties.

Assumption 1 $g(0) = 0$, $g'(c) > 0$, $g''(c) < 0$ for $c > 0$.

In Figure 1 we have depicted the shape of the function of the real profit $\pi_t(c_{t-1})$ given by

$$\pi_t(c_{t-1}) = g(\phi c_{t-1}) - \eta \phi c_{t-1}. \quad (1)$$

Of course the nominal profit is obtained by multiplying $\pi_t(c_{t-1})$ by p_t . According to Assumption 1, one has $\pi(0) = 0$, $\pi'(c) > 0$ for $c < \arg \max_c g(\phi c) - \eta \phi c$, $\pi'(c) < 0$ for $c > \arg \max_c g(\phi c) - \eta \phi c$, and $\pi''(c) < 0$ for all $c > 0$.

In each period t , the economy is populated by two types of agents, whose size, for the sake of simplicity, is normalized to unit: young and old. There is no population growth. Young agents supply

⁴ η is the shrinkage cost. It represents the costs linked to the recycling activity: a share is lost during the waste recovery stage or during the recycling process.

⁵ More generally, this assumption ensures that the unique objective of the recycling sector is to recover all the available waste so long as its profit is non-negative. We thus exclude inefficiencies related to optimal waste demand, and we do not need to determine a market price for waste.

labor and receive a share $q \in [0, 1]$ of the profit obtained in the waste activity. q is a policy instrument chosen by the government that agents take as given. Young agents invest all their income in money balances. Therefore the nominal budget constraint of a young agent born in period t is

$$M_{t+1} = w_t l_t + q p_t \pi_t (c_{t-1}), \quad (2)$$

where M denotes nominal balances. When old, the individual born in period t consumes and receives a share $1 - q$ of current profits of the waste activity: he or she will face therefore the following budget constraint:

$$p_{t+1} c_{t+1} = M_{t+1} + (1 - q) p_{t+1} \pi_{t+1} (c_t). \quad (3)$$

From (2) and (3) it is possible to obtain the intertemporal budget constraint of a individual belonging to generation t :

$$p_{t+1} c_{t+1} = w_t l_t + q p_t \pi_t (c_{t-1}) + (1 - q) p_{t+1} \pi_{t+1} (c_t). \quad (4)$$

The preferences of an agent born in period t are represented by a separable utility function defined over labor l_t and consumption c_{t+1} :

$$u(c_{t+1}) - v(l_t). \quad (5)$$

The utility functions u and v satisfy standard assumptions.

Assumption 2 u is C^r for r large enough with $u'(c) > 0$, $u''(c) < 0$, $\lim_{c \rightarrow 0^+} u'(c) = +\infty$, and $\lim_{c \rightarrow +\infty} u'(c) = 0$. v is C^r for r large enough with $v(0) = 0$, $v'(l) > 0$, $v''(l) < 0$, and $\lim_{l \rightarrow +\infty} v'(l) = 0$.

By maximizing (5) under constraint (4) we obtain the first-order condition

$$\frac{u'(c_{t+1})}{v'(l_t)} = \frac{p_{t+1}}{w_t}. \quad (6)$$

Condition (6) claims that the effort of supplying one additional unit of labor must be equal to the correspondent increase in utility multiplied by the number of additional units of consumption an individual can buy. Using the intertemporal budget constraint and the market clearing condition $p_t = w_t$ in the market of the virgin good, we obtain the following expression for p_{t+1}/w_t :

$$\frac{p_{t+1}}{w_t} = \frac{l_t + q \pi(c_{t-1})}{c_{t+1} - (1 - q) \pi(c_t)}. \quad (7)$$

Notice that from condition (7) one can derive implicitly the labor supply as a function of the price level and of the nominal wage, together with the consumption level in periods $t - 1$, t and $t + 1$. As a matter of fact, (7) is a standard arbitrage (or Euler) condition. Combining (6) with (7) yields

$$\frac{u'(c_{t+1})}{v'(l_t)} = \frac{l_t + q \pi(c_{t-1})}{c_{t+1} - (1 - q) \pi(c_t)}. \quad (8)$$

Notice that if there were no waste recycling activity ($\pi(c_{t-1}) \equiv 0$ for every $t = 0, 1, 2, \dots$), condition (8) would boil down to $u'(c_{t+1})c_{t+1} = v'(l_t)l_t$ as in a standard OLG model, as studied by, among others, Azariadis (1981), Grandmont (1985), Azariadis and Guesnerie (1986), and Benhabib and Laroque (1988).

2.1 Intertemporal equilibrium and steady-state analysis

There are three markets in the economy: the goods market, the money market, and the labor market. In period t , the goods market equilibrium is expressed as

$$y_t^v + \pi(c_{t-1}) = c_t. \quad (9)$$

Notice that in our economy, the combination of the equilibrium conditions in the labor market (i.e., $p_t = w_t$ and the amount of labor is therefore fixed by the supply side since firms are willing to employ as much labor as households wish to supply) and the technology we assume to be used in the virgin sector ($y_t^v = l_t$) yields the following condition in the goods market:

$$l_t = c_t - \pi(c_{t-1}). \quad (10)$$

Setting the constant supply of nominal balances $\bar{M} > 0$, by Walras's law, the money market is at equilibrium too. By combining the arbitrage equation (8) with the equilibrium condition (9) and (10), we find that the intertemporal equilibrium is described by a sequence $\{c_t\}_{t=-1}^{+\infty}$ satisfying the following second-order difference equation:

$$u'(c_{t+1})[c_{t+1} - (1-q)\pi(c_t)] = v'(c_t - \pi(c_{t-1}))[c_t - (1-q)\pi(c_{t-1})] \quad (11)$$

subject to the initial consumption level c_{-1} . System (11) will be then indeterminate if and only if the dimension of the stable manifold is two, i.e., if and only if both eigenvalues of the Jacobian matrix evaluated at the steady state belong to the unit circle. In the opposite case, the steady state will be locally determinate. The steady state of system (11) is a level $\bar{c}$ of consumption solving

$$u'(c) = v'(c - \pi(c)). \quad (12)$$

Notice that the policy instrument q does not influence the stationary level of the economy. We now have the following proposition which ensures the existence of exactly one interior stationary solution (from now on we will omit any suffix denoting the steady-state value of a variable).

Proposition 1 *There exists a unique stationary consumption level $c > 0$.*

PROOF: The proof follows from the fact that $\lim_{c \rightarrow +\infty} u(c) = 0$, $\lim_{c \rightarrow 0^+} u(c) = +\infty$, and from the fact that, for $c > c_{\min}$, where $c_{\min} = \arg \min c - \pi(c)$ (i.e., $\pi'(c_{\min}) = 1$), $v'(c - \pi(c))$ is increasing and diverging to $+\infty$. $\square$

The next lemma will be very useful.

Lemma 1 *At the steady state one has $\pi'(c) < 1$.*

PROOF: The proof follows from the fact that $\pi'(c_{\min}) = 1$, $c > c_{\min}$, and the profit is a concave function. $\square$

2.2 The linearized system

By linearizing system (11) around its (unique) stationary solution and setting the elasticity of marginal utility $-u''(c)c/u'(c) \equiv \varepsilon_u \in (0, +\infty)$, the elasticity of labor marginal disutility $v''(l)l/v'(l) \equiv \varepsilon_v \in (0, +\infty)$, the elasticity of the profit function $\pi'(c)c/\pi(c) \equiv \varepsilon_\pi \in (-\infty, c/\pi(c))$,⁶ and $x_t \equiv c_{t-1}$, we obtain the linearized dynamics in terms of the deviations of the variables with respect to their stationary values:

$$\begin{bmatrix} dc_{t+1} \\ dx_{t+1} \end{bmatrix} = J \begin{bmatrix} dc_t \\ dx_t \end{bmatrix}, \quad (13)$$

where

$$J = \begin{bmatrix} \frac{(1-q)\pi(c)}{[l+q\pi(c)]}\varepsilon_\pi + \frac{c}{l}\varepsilon_v + \frac{c}{[l+q\pi(c)]} & -\frac{\pi(c)}{l}\varepsilon_v\varepsilon_\pi + \frac{(1-q)\pi(c)}{[l+q\pi(c)]}\varepsilon_\pi \\ -\varepsilon_u + \frac{c}{[l+q\pi(c)]} & -\varepsilon_u + \frac{c}{[l+q\pi(c)]} \\ 1 & 0 \end{bmatrix} \quad (14)$$

is the Jacobian matrix of system (11) evaluated at its fixed point. We therefore find that the trace T of the Jacobian is given by

$$T = \frac{\frac{(1-q)\pi(c)}{[l+q\pi(c)]}\varepsilon_\pi + \frac{c}{l}\varepsilon_v + \frac{c}{[l+q\pi(c)]}}{-\varepsilon_u + \frac{c}{[l+q\pi(c)]}} \quad (15)$$

and the determinant D is

$$D = \frac{\frac{\pi(c)}{l}\varepsilon_v\varepsilon_\pi + \frac{(1-q)\pi(c)}{[l+q\pi(c)]}\varepsilon_\pi}{-\varepsilon_u + \frac{c}{[l+q\pi(c)]}}. \quad (16)$$

In view of the complicated form of the above expressions, it may seem that the study of the local dynamics of system (11) requires long and tedious computations. However, by applying the geometrical method adopted in Grandmont, Pintus, and de Vilder (1998) and Cazzavillan, Lloyd-Braga, and Pintus (1998), it is possible to analyze qualitatively the (in)stability of the characteristic roots of J and their bifurcations (changes in stability) by locating the point (T, D) in the plane and studying how (T, D) varies when the value of some parameter changes continuously. If T and D lie in the interior of the triangle ABC depicted in Figure 2, the stationary solution is a sink, hence locally indeterminate. In the opposite case, it is locally determinate: it is either a saddle when $|T| > |1 + D|$,

⁶ Notice that, since $c = l + \pi(c)$, we have $c/\pi(c) > 1$.

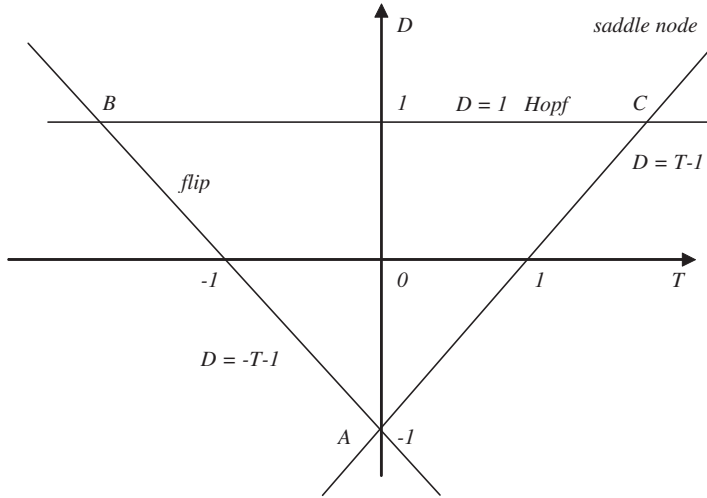


Figure 2 General case.

or a source in the opposite case. If we fix all the parameters of the model with the exception of ε_u (which we let vary from zero to $+\infty$) we obtain a parameterized curve $\{T(\varepsilon_u), D(\varepsilon_u)\}$ that describes a line Δ starting from the point (T_0, D_0) when ε_u is close to zero. The linearity of such a locus can be verified by direct inspection of the expressions for T and D and from the fact that they share the same denominator. This geometric method makes it possible also to characterize the different bifurcations that may arise when ε_u moves from zero to $+\infty$. In particular, as shown in Figure 2, when the half-line Δ intersects the line $D = T - 1$ (at $\varepsilon_u = \varepsilon_u^s$), one eigenvalue goes through unity and a saddle node bifurcation generically occurs; accordingly, we should expect a change in the number and in the stability of the steady states. When Δ goes through the line $D = -T - 1$ (at $\varepsilon_u = \varepsilon_u^f$), one eigenvalue is equal to -1 and we expect a flip bifurcation: it follows that there will arise nearby two-period cycles, stable or unstable, according to the direction of the bifurcation. Eventually, when Δ intersects the interior of the segment BC (at $\varepsilon_u = \varepsilon_u^H$), the modulus of the complex conjugate eigenvalues is unity and the system undergoes, generically, a Hopf bifurcation. Therefore, around the stationary solution, there will emerge a family of closed orbits, stable or unstable, depending on the nature of the bifurcation (supercritical or subcritical).

Following Grandmont, Pintus, and de Vilder (1998) and Cazzavillan, Lloyd-Braga, and Pintus (1998), this analysis is also powerful enough to characterize the occurrence of sunspot equilibria around an indeterminate stationary solution of system (11) as well as along flip and Hopf bifurcations.⁷ Actually, as is the case in Grandmont, Pintus, and de Vilder (1998) and Cazzavillan, Lloyd-Braga, and Pintus (1998), system (11) has at each period t one predetermined variable, the inherited consumption c_{t-1} , and one which is not, namely current consumption c_t . Accordingly, multiple equilibria and sunspot fluctuations occur when the modulus of both eigenvalues is lower than unity, i.e., the steady state is located in the interior of the triangle ABC or along a supercritical flip bifurcation or a supercritical Hopf bifurcation.

⁷ In the case of a supercritical flip bifurcation and a supercritical Hopf bifurcation, sunspot equilibria remain in a compact set containing in its interior, respectively, the stable two-period cycle and the stable closed orbit.

We will now fix the structural parameters ε_π , ε_v as well as $(1 - q) \pi(c) / [l + q\pi(c)]$, c/l , and $c / [l + q\pi(c)]$, which are all positive, and let ε_u vary within its domain of definition, namely $(0, +\infty)$. It is easy verifiable that when ε_u increases from zero to infinity, it describes a part of the line $D = ST$ which goes through the origin and whose slope is

$$S = \frac{\frac{\pi(c)}{l} \varepsilon_v \varepsilon_\pi + \frac{(1-q)\pi(c)}{[l+q\pi(c)]} \varepsilon_\pi}{\frac{(1-q)\pi(c)}{[l+q\pi(c)]} \varepsilon_\pi + \frac{c}{l} \varepsilon_v + \frac{c}{[l+q\pi(c)]}}. \quad (17)$$

It is worth noticing that since $T(+\infty) = D(+\infty) = 0$, by continuity, indeterminacy always arises for values of ε_u sufficiently large, i.e., for strong enough income effects. Before going through the stability analysis, we appraise a feature which will turn to be quite useful.

Lemma 2 *The origin (T_0, D_0) corresponding to $\varepsilon_u = 0$ of the curve described by ε_u lies below the line $D = T - 1$.*

PROOF: We first compute the ordinate D' of the half line $D = T - 1$ when $\varepsilon_u = 0$. This ordinate is easily obtained by simply setting $D'(\varepsilon_u = 0) = T(\varepsilon_u = 0) - 1$, which gives

$$D'(\varepsilon_u = 0) = \frac{\frac{(1-q)\pi(c)}{[l+q\pi(c)]} \varepsilon_\pi + \frac{c}{l} \varepsilon_v}{\frac{c}{[l+q\pi(c)]}}.$$

In order to prove the thesis of the proposition, inequality $T(\varepsilon_u = 0) - 1 > ST(\varepsilon_u = 0)$ must hold, i.e.,

$$\frac{\frac{(1-q)\pi(c)}{[l+q\pi(c)]} \varepsilon_\pi + \frac{c}{l} \varepsilon_v}{\frac{c}{[l+q\pi(c)]}} > \frac{\frac{\pi(c)}{l} \varepsilon_v \varepsilon_\pi + \frac{(1-q)\pi(c)}{[l+q\pi(c)]} \varepsilon_\pi}{\frac{c}{[l+q\pi(c)]}},$$

which, after some straightforward computation, becomes $\frac{c}{l} \varepsilon_v > \frac{\pi(c)}{l} \varepsilon_v \varepsilon_\pi$ or, equivalently, $c > \pi(c) \pi'(c) c / \pi(c)$, which leads, eventually, to the condition $1 > \pi'(c)$ which, in view of Lemma 1, is satisfied. $\square$

In view of the above expressions, one immediately verifies that two main regimes may occur, namely $\varepsilon_\pi \in (0, c/\pi(c))$ and $\varepsilon_\pi < 0$.

2.3 Case $\varepsilon_\pi \in (0, c/\pi(c))$

Under such a configuration, the agents' consumption falls within such a range of values that profits in the recycling sector are increasing. Before going through the analysis of the local dynamics generated by system (11), we first introduce a useful lemma.

Lemma 3 *Let $\varepsilon_\pi > 0$. Then the slope S of the line described by ε_u lies in $(0, 1)$.*

PROOF: From the expression for S provided in (17), since $\varepsilon_\pi > 0$, it is easily verifiable that $S > 0$. In addition, in order for S to be less than unity, we must have $\frac{\pi(c)}{l} \varepsilon_v \varepsilon_\pi < \frac{c}{l} \varepsilon_v + \frac{c}{[l+q\pi(c)]}$.

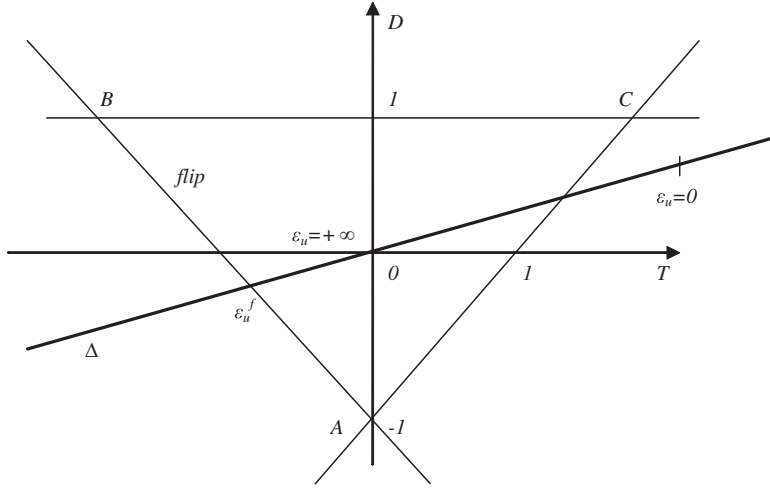


Figure 3 The case $\varepsilon_\pi \in (0, c/\pi(c))$.

But $\varepsilon_v \pi(c) \pi'(c) c / l \pi(c) < \varepsilon_v c / l$ since $\pi'(c) < 1$ as stated in Lemma 1, from which the thesis follows. $\square$

One now immediately verifies that the abscissa T is equal to $(\frac{(1-q)\pi(c)}{[l+q\pi(c)]} \varepsilon_\pi + \frac{c}{l} \varepsilon_v + \frac{c}{[l+q\pi(c)]}) / \frac{c}{[l+q\pi(c)]}$ for $\varepsilon_u = 0$; it becomes infinite when ε_u converges to $\frac{c}{[l+q\pi(c)]}$ from the left, then it is $-\infty$ for ε_u converging to $\frac{c}{[l+q\pi(c)]}$ from the right, and is zero for $\varepsilon_u \rightarrow +\infty$ (and $D(+\infty)$ is equal to zero too). We have depicted these features in Figure 3. Looking at this, and taking into account Lemma 2, it is possible to appreciate that the steady state is first a saddle (locally determinate), for $0 < \varepsilon_u < \varepsilon_u^f$, then a sink, hence locally indeterminate, for $\varepsilon_u > \varepsilon_u^f$. In addition, when $\varepsilon_u = \varepsilon_u^f$, the steady state undergoes a flip bifurcation. This result is quite similar to the findings of the two-dimensional model studied by Geanakoplos and Polemarchakis (1984) which proves that in an OLG model with capital accumulation, elastic labor supply and Cobb–Douglas technology, indeterminacy and cycles require strong enough income effects in intertemporal consumption in order to emerge. In our model, we are able to mimic the same features, without accounting for capital accumulation, but relying only on waste activity: here past consumption plays the role of state variable and the induced local dynamics is strongly affected by individual smoothing behavior.

Our findings can be summarized by the following proposition.

Proposition 2 *Let $\varepsilon_\pi \in (0, c/\pi(c))$. Then there exists $\varepsilon_u^f > 0$ such that:*

- (i) *when $\varepsilon_u < \varepsilon_u^f$ the steady state is a saddle (locally determinate);*
- (ii) *when $\varepsilon_u > \varepsilon_u^f$ the steady state is a sink (locally indeterminate).*

In addition, when $\varepsilon_u = \varepsilon_u^f$ the steady state undergoes a flip bifurcation.

2.4 Case $\varepsilon_\pi < 0$

Now consumption lies in the region where profits in the waste activity are decreasing. As we have seen, this is fully compatible with firms' rational behavior: there is no profit maximization task, but

production in the recycling sector is the fruit of past period consumption, subject to the unique constraint that profits must be non-negative. In such a case, the sign of the slope S is positive if and only if

$$\varepsilon_\pi < - \left(\frac{c}{l} \varepsilon_v + \frac{c}{[l + q\pi(c)]} \right) \bigg/ \frac{(1-q)\pi(c)}{[l + q\pi(c)]} \equiv \varepsilon_\pi^+ < -1. \quad (18)$$

In addition, under condition (18), the slope S of the half-line Δ lies in the interval $(1, +\infty)$; i.e., the slope S is larger than one (but finite) for ε_π converging to $-\infty$ and converges to $+\infty$ when ε_π approaches ε_π^+ from the left. When (18) is not satisfied, one easily verifies that the slope S lies in $(-\infty, 0)$. More precisely, it is $-\infty$ for ε_π converging to ε_π^+ from the right and 0 for $\varepsilon_\pi \rightarrow 0$. By inspecting (15), one immediately verifies that, when inequality (18) is not verified, the trace T is increasing in ε_u : it goes to $+\infty$ for $\varepsilon_u \rightarrow \frac{c}{[l+q\pi(c)]}$ from the left, then to $-\infty$ for $\varepsilon_u \rightarrow \frac{c}{[l+q\pi(c)]}$ from the right, and, for $\varepsilon_u = +\infty$, it converges to zero. In addition, in view of Lemma 3, $D(\varepsilon_u = 0) < T(\varepsilon_u = 0) - 1$. Under this configuration, two regimes for the local dynamics may arise. The first regime corresponds to the case in which line Δ crosses line $D = 1$ on the right of point B . This is the case if the slope S is less than $-1/2$, i.e.,

$$\varepsilon_\pi < \varepsilon_\pi^H \equiv \frac{-\frac{1}{2} \left[\frac{c}{l} \varepsilon_v + \frac{c}{[l + q\pi(c)]} \right]}{\frac{\pi(c)}{l} \varepsilon_v + \frac{3(1-q)\pi(c)}{2[l + q\pi(c)]}} < 0. \quad (19)$$

Notice that we must have $\varepsilon_\pi^+ < \varepsilon_\pi^H$, a condition requiring

$$\frac{\frac{1}{2} \left[\frac{c}{l} \varepsilon_v + \frac{c}{[l + q\pi(c)]} \right]}{\frac{\pi(c)}{l} \varepsilon_v + \frac{3(1-q)\pi(c)}{2[l + q\pi(c)]}} < \left(\frac{c}{l} \varepsilon_v + \frac{c}{[l + q\pi(c)]} \right) \bigg/ \frac{(1-q)\pi(c)}{[l + q\pi(c)]}$$

which is obviously satisfied. Under inequality (19) two regimes are possible according to the location of $(T_0, D_0) = (T(\varepsilon_u = 0), D(\varepsilon_u = 0))$. Actually, (T_0, D_0) may or may not satisfy the inequality $D_0 > -T_0 - 1$. If the answer is positive, there will exist ε_u^f such that for $\varepsilon_u = \varepsilon_u^f$ there arises a flip bifurcation. This is the case when

$$0 > \varepsilon_\pi > \varepsilon_\pi^f \equiv - \frac{\frac{2c}{[l + q\pi(c)]} + \frac{c}{l} \varepsilon_v}{2 \frac{(1-q)\pi(c)}{[l + q\pi(c)]} + \frac{\pi(c)}{l} \varepsilon_v} \quad (20)$$

as depicted in Figure 4. Notice that there is a whole range for the parameter values so that inequalities (19) and (20) are simultaneously satisfied. As a matter of fact, this requires $\varepsilon_\pi^H > \varepsilon_\pi^f$, i.e.,

$$\frac{\frac{1}{2} \left[\frac{c}{l} \varepsilon_v + \frac{c}{[l + q\pi(c)]} \right]}{\frac{\pi(c)}{l} \varepsilon_v + \frac{3(1-q)\pi(c)}{2[l + q\pi(c)]}} < \frac{\frac{2c}{[l + q\pi(c)]} + \frac{c}{l} \varepsilon_v}{2 \frac{(1-q)\pi(c)}{[l + q\pi(c)]} + \frac{\pi(c)}{l} \varepsilon_v} \quad (21)$$

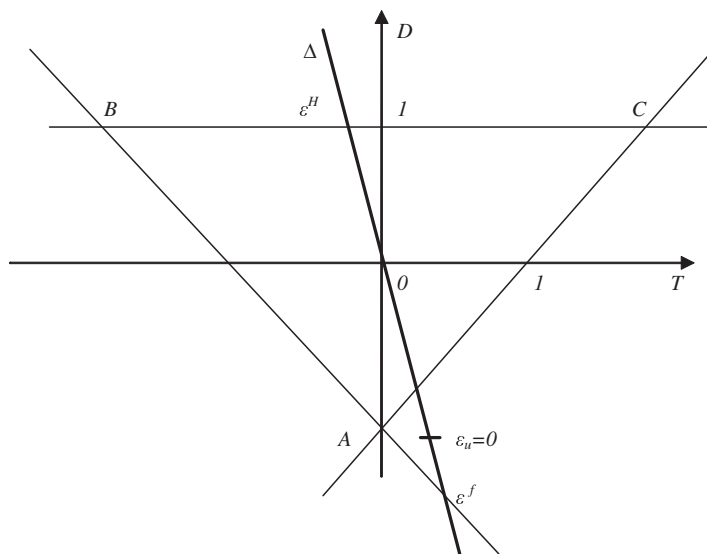


Figure 4 The case $\varepsilon_\pi < 0$, $D_0 > -T_0 - 1$ and negative slope.

which, by setting for example $\varepsilon_v = 0$, is trivially satisfied. Provided (19) and (20) are simultaneously satisfied, the steady state will first be a saddle for $\varepsilon_u < \varepsilon_u^f$, then it will become a source for $\varepsilon_u^f < \varepsilon_u < \varepsilon_u^H$ and eventually a sink, hence locally indeterminate, for $\varepsilon_u > \varepsilon_u^H$. The alternative case where $D_0 < -T_0 - 1$ is straightforward, as depicted in Figure 5.

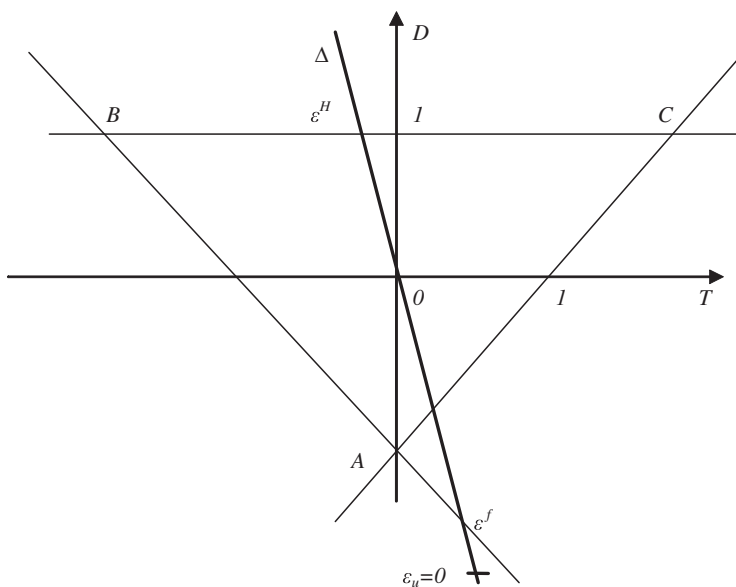


Figure 5 The case $\varepsilon_\pi < 0$, $D_0 < -T_0 - 1$, and negative slope.

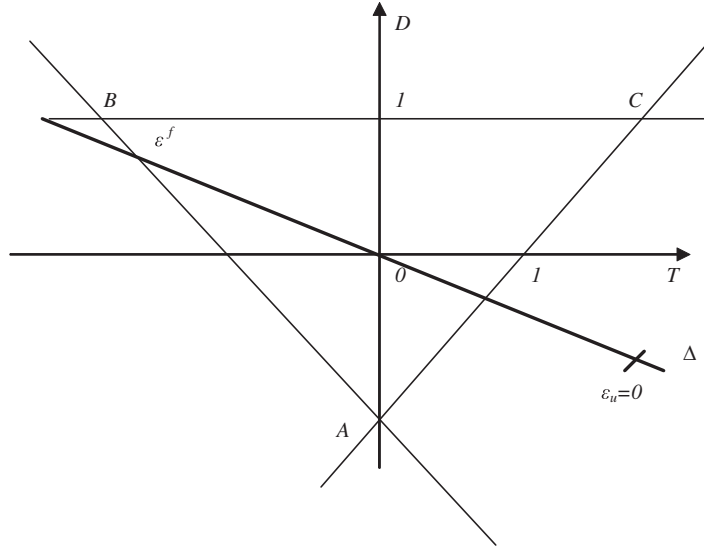


Figure 6 The case $\varepsilon_\pi^H < \varepsilon_\pi < 0$, and negative slope.

The steady state is now a source for $\varepsilon_u < \varepsilon_u^H$ and then a sink, hence locally indeterminate, for $\varepsilon_u > \varepsilon_u^H$. In addition, for $\varepsilon_u = \varepsilon_u^H$, the steady state undergoes a Hopf bifurcation. The case $0 > \varepsilon_\pi > \varepsilon_\pi^H$, where ε_π^H is defined in (19), is depicted in Figure 6.

When $0 < \varepsilon_u < \varepsilon_u^f$ the steady state is a saddle. When ε_u goes through ε_u^f the steady state undergoes a flip bifurcation. When $\varepsilon_u^f < \varepsilon_u$ the stationary solution is a sink, hence locally indeterminate. When $\varepsilon_u = \varepsilon_u^f$ the steady state undergoes a flip bifurcation.

We now analyze the case in which the slope of the line Δ is positive, i.e., where the elasticity of the profit function is low enough and satisfies inequality (18). We have seen that the slope S is now positive and larger than unity. As depicted in Figure 7, the trace T_0 obtained by setting $\varepsilon_u = 0$ is negative. It converges to $-\infty$ for ε_u converging to $\frac{c}{[l+q\pi(c)]}$ from the left, and to $+\infty$ for ε_u converging to $\frac{c}{[l+q\pi(c)]}$ from the right. In addition, $\{T(0), D(0)\}$ lies below the line $D = T - 1$. Therefore the steady state is a source (locally determinate) for $0 < \varepsilon_u < \varepsilon_u^H$ and a sink (locally indeterminate) for $\varepsilon_u > \varepsilon_u^H$. When ε_u goes through ε_u^H the steady state undergoes a Hopf bifurcation. Notice that in this configuration the steady state does not undergo any flip bifurcation.

The results in the case $\varepsilon_\pi < 0$ are summarized in the following proposition:

Proposition 3 Assume $\varepsilon_\pi < 0$.

- (i) When ε_π does not satisfy inequality (18) but satisfies inequalities (19) and (20), the steady state is a saddle (locally determinate) for $\varepsilon_u < \varepsilon_u^f$, a source (locally determinate) for $\varepsilon_u^f < \varepsilon_u < \varepsilon_u^H$, and a sink (locally indeterminate) for $\varepsilon_u > \varepsilon_u^H$. The steady state undergoes a flip bifurcation at $\varepsilon_u = \varepsilon_u^f$ and a Hopf bifurcation at $\varepsilon_u = \varepsilon_u^H$.
- (ii) When ε_π does not satisfy inequality (18), and satisfies (19) but not (20), the steady state is a source (locally determinate) for $\varepsilon_u < \varepsilon_u^H$ and a sink (locally indeterminate) for $\varepsilon_u > \varepsilon_u^H$. The steady state undergoes a Hopf bifurcation at $\varepsilon_u = \varepsilon_u^H$.

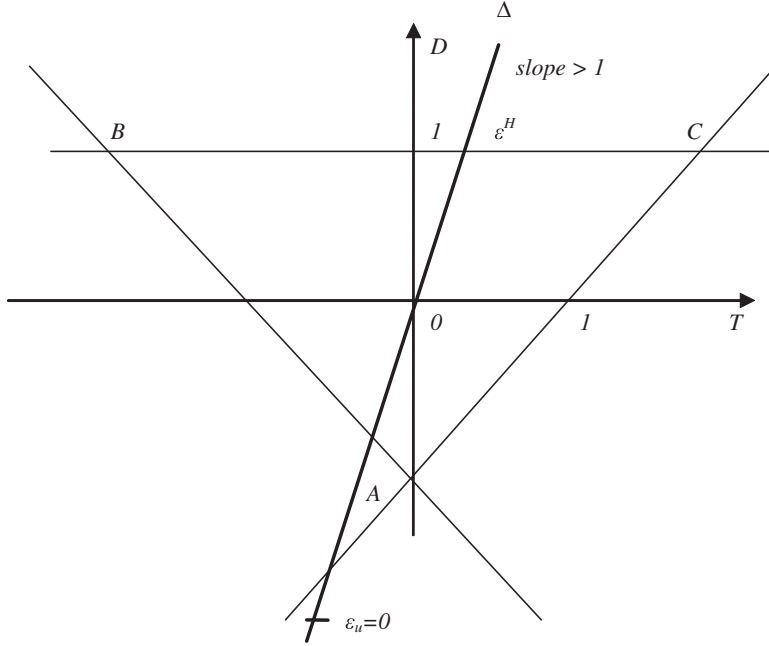


Figure 7 The case $\varepsilon_\pi < 0$ and positive slope.

- (iii) When ε_π does not satisfy inequality (18) or inequality (19), then the steady state is a saddle (locally determinate) for $\varepsilon_u < \varepsilon_u^f$ and a sink (locally indeterminate) for $\varepsilon_u > \varepsilon_u^f$. The steady state undergoes a flip bifurcation at $\varepsilon_u = \varepsilon_u^f$.
- (iv) When ε_π satisfies inequality (18) the steady state is a saddle (locally determinate) for $0 < \varepsilon_u < \varepsilon_u^H$ and a sink (locally indeterminate) for $\varepsilon_u > \varepsilon_u^H$. The steady state undergoes a Hopf bifurcation at $\varepsilon_u = \varepsilon_u^H$.

2.5 Bifurcation values

We have seen that under the condition $\varepsilon_\pi > 0$ the steady state undergoes only a flip bifurcation, while for $\varepsilon_\pi < 0$ there is room for both flip and Hopf bifurcations. At the same time, we have found the result that the economy never displays a saddle node bifurcation. In particular, when the elasticity ε_π of the profit of the waste activity is positive, indeterminacy arises for all the elasticities ε_u of the intertemporal substitution in consumption larger than its (flip) bifurcation point ε_u^f . On the other hand, when the elasticity ε_π is negative, indeterminacy arises for the elasticities ε_u larger than $\max\{\varepsilon^f, \varepsilon^{H^\circ}\}$, where ε^{H° is such that $D(\varepsilon^{H^\circ}) = 1$. Therefore, in order to analyze how the region of indeterminacy evolves when the parameter q is relaxed from zero to one, we must consider separately the two cases $\varepsilon_\pi > 0$ and $\varepsilon_\pi < 0$. In the first case the indeterminacy region will be $(\varepsilon_u^f, +\infty)$, while in the second it will be $(\max\{\varepsilon^f, \varepsilon^{H^\circ}\}, +\infty)$. We now derive the expressions for such critical points for the economy and study how they vary when the parameter q is relaxed from zero to one.

2.5.1 Flip bifurcation

In order to obtain the flip bifurcation value, we must keep in mind that the latter occurs when the line $D = ST$ crosses the line $D = -T - 1$. This implies that $ST = -T - 1$ or, equivalently, $T = -1/(1 + S)$. From the expression for S in (17), we obtain the value ε_u^f for the elasticity of the marginal utility such that the system undergoes a flip bifurcation:

$$\varepsilon_u^f = \frac{2c}{[l + q\pi(c)]} + \frac{c}{l}\varepsilon_v + \left(2\frac{(1-q)\pi(c)}{[l + q\pi(c)]} + \frac{\pi(c)}{l}\varepsilon_v\right)\varepsilon_\pi. \quad (22)$$

One immediately verifies that for $\varepsilon_\pi > 0$, a flip bifurcation always arises and, since $\frac{2c}{[l + q\pi(c)]} > 1$, one has $\varepsilon_u^f > 1$. Conversely, when $\varepsilon_\pi < 0$, the flip bifurcation occurs only for not too low values for ε_π , namely for values satisfying inequality (20). From (22) one immediately verifies that a flip bifurcation may occur also when consumption is intertemporally substitutable, i.e., when $0 < \varepsilon_u < 1$. Setting $A \equiv \frac{2c}{[l + q\pi(c)]} + \frac{c}{l}\varepsilon_v > 1$ and $B \equiv 2\frac{(1-q)\pi(c)}{[l + q\pi(c)]} + \frac{\pi(c)}{l}\varepsilon_v > 0$, this is actually the case when

$$-\frac{A}{B} < \varepsilon_\pi < \frac{1 - A}{B}. \quad (23)$$

It would be easy to show that when inequalities (23) are satisfied, inequality (21) also holds. It follows that the flip bifurcation is compatible with the gross substitutability in intertemporal consumption–leisure arbitrage only when the indeterminacy region is reached through a Hopf bifurcation. This means that under condition (23), the steady state undergoes a flip bifurcation and from a saddle becomes a source (this corresponds to Figure 4). This feature represents an important departure from classical OLG models, where cycles, in order to arise, require strong enough income effects in intertemporal consumption–leisure substitutability (see, among others, Azariadis 1981; Geanakoplos and Polemarchakis 1984; Grandmont 1985; Azariadis and Guesnerie 1986). It is also easily verifiable that as soon as q increases from zero to one, provided $\varepsilon_\pi > 0$, ε_u^f decreases. On the other hand, when ε_π is negative, the sign of $\partial\varepsilon_u^f/\partial q$ is ambiguous; i.e., it is given by the expression

$$\partial\varepsilon_u^f/\partial q = -\frac{2c\pi(c)}{[l + q\pi(c)]^2} - 2\varepsilon_\pi \left[\frac{\pi(c)[l + q\pi(c)] + (1-q)\pi(c)^2}{[l + q\pi(c)]^2} \right]. \quad (24)$$

From (24) one easily verifies that when $\varepsilon_\pi > -1$, the response of ε_u^f to q is negative, while from $\varepsilon_\pi < -1$ it is positive. It follows that for $\varepsilon_\pi \in (\varepsilon_\pi^+, -1)$, $\partial\varepsilon_u^f/\partial q > 0$, otherwise it is negative. Notice that for $\varepsilon_\pi < \varepsilon_\pi^+$ there is no room for the flip bifurcation. For $-A/B > \varepsilon_\pi$ the flip bifurcation also disappears (which corresponds to Figure 7). Therefore, since the indeterminacy region in terms of ε_u under the regimes described by part (iii) of Proposition 3 is the interval $(\varepsilon_u^f, +\infty)$, the best stabilization policies are those which set the value of q equal to zero, namely policies that ensure that the entire profits of the recycling sector are granted to old agents.

2.5.2 Hopf bifurcation

In order to get a Hopf bifurcation, the determinant must be equal to unity, since both complex conjugate eigenvalues have unit modulus. From the expression (16) for the determinant D we have

$$\varepsilon_u^H = \frac{c}{[l + q\pi(c)]} - \frac{(1-q)\pi(c)}{[l + q\pi(c)]}\varepsilon_\pi - \frac{\pi(c)}{l}\varepsilon_v\varepsilon_\pi. \quad (25)$$

This value, in order to undergo a Hopf bifurcation, must be coupled with the value for ε_π satisfying inequality (19). Since, as we have seen, the Hopf bifurcation occurs only when $\varepsilon_\pi < 0$, one has $\varepsilon_u^H > 1$, in view of the fact that $\frac{c}{[l + q\pi(c)]} \geq 1$. In order to get multiple equilibria, in such a configuration the income effect in intertemporal consumption must be larger than the substitution one, namely larger than the unit elasticity in intertemporal consumption corresponding to the logarithmic case. It is easy to verify that

$$\frac{\partial \varepsilon_u^H}{\partial q} = -\frac{c\pi(c)}{[l + q\pi(c)]^2} + \varepsilon_\pi \left[\frac{\pi(c)[l + q\pi(c)] + (1-q)\pi(c)^2}{[l + q\pi(c)]^2} \right],$$

which, under the inequality $\varepsilon_\pi < 0$, is always negative. As in the case of the flip bifurcation, we then see that in order to obtain to get a shrinking indeterminacy region, one must set $q = 0$, i.e., all the profits of the recycling sector must be transferred to the old agents.

2.5.3 Ruling out the saddle node bifurcation

We now prove that there is no room for a saddle node bifurcation. To obtain such a bifurcation, the curves $D = ST$ and $D = T - 1$ must intersect each other. This requires $T = 1/(1 - S)$. From the expressions (17) for the slope S and (15) for the trace T , this implies

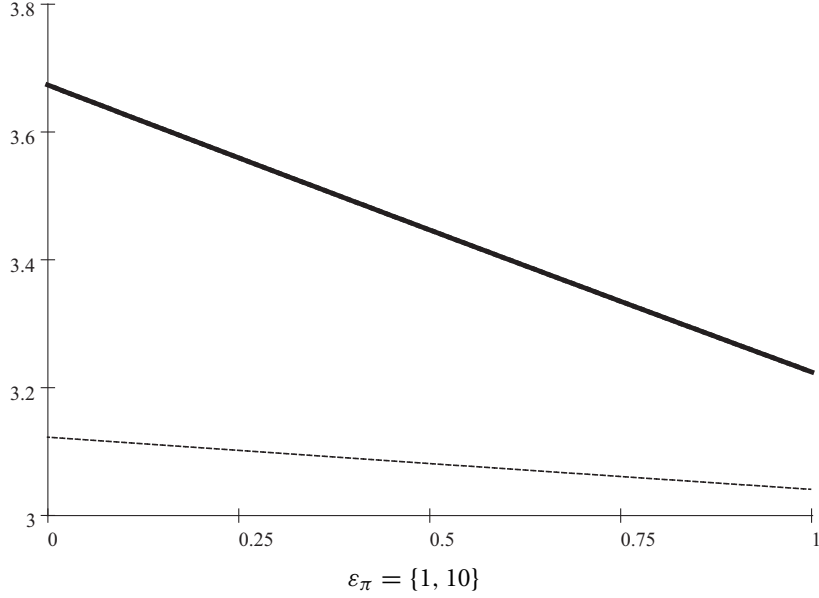
$$\varepsilon_u^S = \frac{\pi(c)}{l}\varepsilon_v\varepsilon_\pi - \frac{c}{l}\varepsilon_v = \frac{c\varepsilon_v}{l}(\pi'(c) - 1) < 0,$$

which cannot be satisfied in view of Lemma 1.

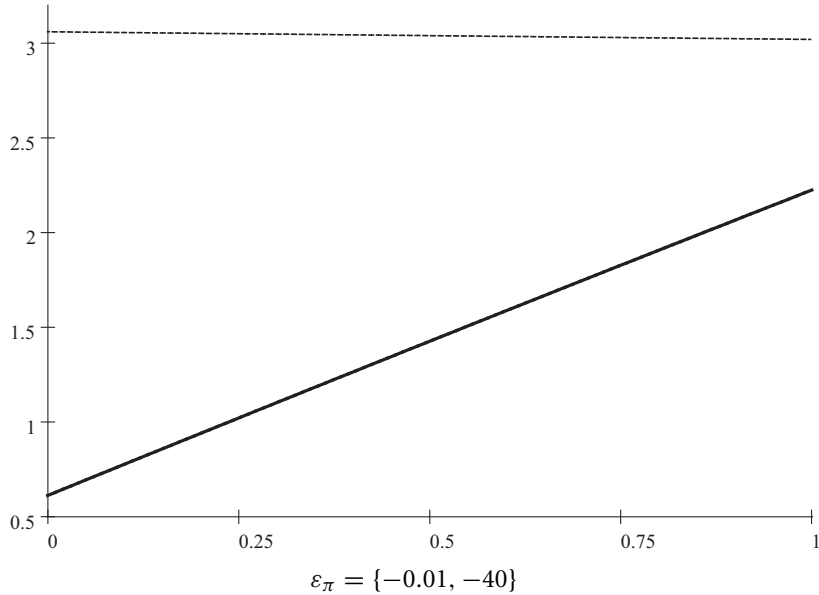
2.5.4 The indeterminacy region: some numerical examples

In order to appreciate the analytical results previously obtained, we provide some numerical examples. That is to say, we test how the indeterminacy region evolves as soon as the policy parameter q is continuously relaxed from one to zero. We have already seen that in such a case the indeterminacy region shrinks, but we want to provide a quantitative picture of the phenomenon. Remember that the indeterminacy region corresponds to $(\varepsilon_u^f, +\infty)$ when $\varepsilon_\pi > 0$ and to $(\max\{\varepsilon_u^f, \varepsilon_u^{H\circ}\}, +\infty)$ when $\varepsilon_\pi < 0$. To this end we report in the abscissas the value of q and in the ordinates the threshold of ε_u giving raise to indeterminacy. We specify our economy in the simplest way assuming, as in the case of the US economy, that the recycling sector accounts for 2 percent of the total GDP. Therefore, we can fix the following values for the structural variables: $c = 100$; $\pi = 2$, $l = 98$, $\varepsilon_v = 1$. In addition, we must keep in mind the bifurcation values ε_u^f and ε_u^H . Notice that under this parameterization the flip bifurcation always occurs, since inequality (20) holds.

Flip bifurcation As we have seen, in this case the flip bifurcation may exist whatever the sign of the elasticity ε_π . We then have, for $\varepsilon_\pi > 0$,



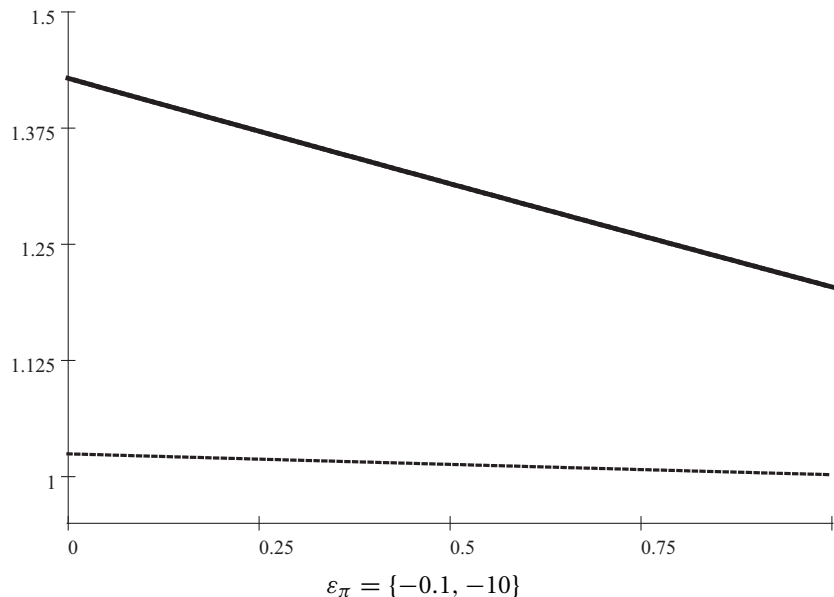
In such a picture, as one should expect, ε_f is decreasing in q and therefore the indeterminacy region shrinks as soon as the profits of the recycling sector granted to old agents become larger. Conversely, for $\varepsilon_\pi < 0$, we have the following picture:



Notice that the decreasing dashed line represents the values for the flip bifurcation summarized in part (iii) of Proposition 3. Actually, these are the values for ε_u , so that for superior values the steady state becomes indeterminate. In other words, they represent the frontier of the indeterminacy region. On the other hand, the increasing solid line corresponds to case (i) of Proposition 3: here the flip bifurcation implies a change in stability of the steady state from a saddle path configuration to a source one. In other words, this bifurcation value is not the lower bound of the indeterminacy region. It is

worth observing that such a flip bifurcation may occur even when labor and consumption are gross substitutes, i.e., $\varepsilon_u < 1$, a feature that usually rules out the occurrence of endogenous fluctuations (see Azariadis 1981; Grandmont 1985; Azariadis and Guesnerie 1986).

Hopf bifurcation We know that, in order to get a Hopf bifurcation, ε_π should be negative. So we have



This case corresponds to (iii) of Proposition 3. Here ε_c^H represents the lower bound of the indeterminacy region and, as we have already shown, it is decreasing in the amount of profits of the recycling sector granted to young agents.

All this calibration confirms the analytical result obtained: as soon as the share of profits of the recycling sector granted to old agents increases, the indeterminacy region shrinks.

2.6 Discussion of the results

As we have seen, in our economy the occurrence of indeterminacy and sunspot fluctuations mainly depends on the properties of the profit function in the recycling sector and on the amplitude of the share q of the profits granted to old agents. Our results are similar to those found in De Beir, Fodha, and Magris (2010) where indeterminacy and endogenous fluctuations emerge only when the economy is located in the region where profits in the recycling activity are decreasing. The economy studied by De Beir, Fodha, and Magris (2010) differs in many aspects from that considered in the present paper. First, De Beir, Fodha, and Magris (2010) analyze an infinite-horizon model. Second, they account for capital accumulation. Third, they assume that the waste recovery activity includes all production, that is to say, the production of the consumption good as well as that of the investment. The emergence of endogenous fluctuations is the fruit of the countercyclical behavior with respect to the GDP growth rate of the waste recovery activity: when the output in a given period becomes larger, the real profit in the sector of the recycling activity declines and therefore the next period's production will be lower. But this decrease in GDP will translate into a larger amount of waste

recovery activity and therefore into an increase in the future GDP two periods later. In addition, the dynamics in De Beir, Fodha, and Magris (2010) is three-dimensional, and therefore the stability analysis of the steady state and of the emergence of endogenous fluctuations is not suitable for a geometrical characterization.

In contrast, our study accounts for an OLG model without capital accumulation, where the constant returns to scale technology of the virgin good sector employs, as productive inputs, only labor. This enables us to reduce the dynamics of the economy to a simpler two-dimensional difference system. In view of the OLG structure of the model and the presence of a recycling activity, the occurrence of indeterminacy is the fruit of a twofold mechanism. On the one hand, endogenous fluctuations are made possible by the very same OLG structure of the model, provided that leisure and consumption are not gross substitutes, as is the case in the seminal works of Azariadis (1981), Geneakoplos and Polemarchakis (1984), Grandmont (1985), and Azariadis and Guesnerie (1986). On the other hand, in our study, deterministic cycles emerge also under the gross substitutability assumption, in so far as the profit of the recycling activity is decreasing in the amount of its inputs (namely, previous periods' consumption). Here the mechanism mimics that described in De Beir, Fodha, and Magris (2010) and rests upon the countercyclical behavior of the waste activity: when GDP increases, the profit in the recycling sector declines, provoking a decrease in next period GDP. As a consequence, the new profit in the recycling sector will increase and yield a larger GDP two periods later. We have also seen that the indeterminacy region shrinks with the share q of the profit of waste activity granted to old agents. This is because old agents can spend the profit of the recycling activity at the time they get it, and do not need a previous conversion into money balances as is the case when profits are granted to young agents who must reinvest into money, and therefore pay the inflationary tax, thereby increasing the scope for endogenous fluctuations.

3 The social optimum

We analyze the social optimum by assuming that the social planner discounts the utility of future generations at the rate $\theta > 0$. This implies a social discount factor $\beta < 1$. The difference in respect of the decentralized economy consists of the fact that the “externality” of current consumption on the profit function of the recycling activity is now taken into account. The social welfare function is simply

$$\Omega = u(c_0) + \sum_{t=0}^{\infty} \beta^{t+1} [u(c_{t+1}) - v(l_t)], \quad (26)$$

where β^{t+1} is the weight assigned to the generation born in period t and consuming in period $t + 1$. The resource constraint is the same as in the decentralized economy; see Equation (10). Maximizing (26) under constraint (10) is equivalent to choosing a sequence $\{c_t\}_{t=0}^{\infty}$ which maximizes the expression

$$u(c_0) + \sum_{t=0}^{\infty} \beta^{t+1} [u(c_{t+1}) - v(c_t - \pi(c_{t-1}))]$$

subject to the initial condition c_{-1} . The first-order condition for this problem in respect of c_t is

$$u'(c_t) - \beta v'(c_t - \pi(c_{t-1})) + \beta^2 v'(c_{t+1} - \pi(c_t)) \pi'(c_t) = 0, \quad (27)$$

The steady state is given by the equation

$$u'(c) = \beta v'(c - \pi(c)) (1 - \beta \pi'(c)). \quad (28)$$

Notice that the social optimum regime described by equation (27), is consistent only when the economy lies in the region in which profits of the recycled sector are increasing, i.e., $\pi' > 0$. In fact, if this was not the case, the social planner would have to decrease the exploitation of the recycled good until its profit were maximized, i.e., until the level of recycled good c became $\pi'(c) = 0$. Let us first consider the case where the social planner is able to choose the level of waste activity that maximizes exactly the profit in the recycling sector. In such a case the first-order condition reduces to

$$u'(c_t) = \beta v'(c_t - \pi_{\max}), \quad (29)$$

where $\pi_{\max} = \max_c \pi(c)$. Equation (29) implies that the economy does not display any dynamics but sticks to its fixed point. In view of Assumptions 1 and 2, it is easy to prove the existence of a unique solution for Equation (29). In fact, the left-hand side of (29) is smooth and decreasing and converging to 0 when c tends to $+\infty$, while its right-hand side, for $c > \pi_{\max}$, is positive, increasing and eventually diverges to infinity when $c \rightarrow +\infty$. It follows that it will cross $u'(c)$ exactly once, for some $\hat{c} > \pi_{\max}$.

Suppose now that $\pi'(c) > 0$. Again, one sees that the left-hand side of (28) is decreasing and then converges to zero when c tends to $+\infty$. Conversely, the derivative with respect to c of its right-hand side yields

$$\beta [v''(c - \pi(c)) (1 - \pi'(c)) (1 - \beta \pi'(c)) - v'(c - \pi(c)) \beta \pi''(c)]. \quad (30)$$

It is easy verifiable, since $\pi''(c) < 0$, that (30) is equal to zero if and only if $c = \pi(c)$. Then, for larger values of c , the derivative (30) is positive; since the right-hand side of (28) diverges to infinity when $c \rightarrow +\infty$, it follows that it will cross $u'(c)$ exactly once, for some $c^* > \bar{c}$, where $\bar{c}$ solves $c = \pi(c)$, with $\pi'(c^*) < 1$. Therefore, the steady-state level of labor will be $l^* = c^* - \pi(c^*) > 0$.

The dynamics of system (27) can be studied, in a neighborhood of the stationary solution, by analyzing the linear tangent. Setting $x_t \equiv c_{t-1}$, the linearized dynamics is described by (we again omit any suffix denoting the steady-state value of a variable)

$$\begin{bmatrix} dc_{t+1} \\ dx_{t+1} \end{bmatrix} = J \begin{bmatrix} dc_t \\ dx_t \end{bmatrix},$$

where

$$J = \begin{bmatrix} -\frac{u''(c) - \beta v''(l) - \beta^2 v''(l) \pi'(c)^2 + \beta^2 v'(l) \pi''(c)}{\beta^2 v''(l) \pi'(c)} & -\frac{1}{\beta} \\ 1 & 0 \end{bmatrix}$$

is the Jacobian matrix evaluated at the steady state. From the form of J one can easily derive the expressions for the trace T and the determinant D :

$$T = -\frac{u''(c)}{v''(l) \pi'(c)} + \frac{1}{\beta \pi'(c)} + \pi'(c) - \frac{v'(l) \pi''(c)}{v''(l) \pi'(c)} > 0,$$

$$D = \beta^{-1} > 1.$$

It follows that the system possesses two positive eigenvalues and as a consequence its equilibrium is unique.

In Appendix (see below) we also characterize the behavior of the market economy with altruism and compare the first-order conditions with those obtained by the social planner. We show that the market economy with altruism mimics the optimum only for a very precise value of q .

4 Conclusion

In this paper we have studied an OLG economy with two sectors producing, respectively, a virgin good and a recycled good. We find that when profits in the recycling sector are increasing, endogenous fluctuations and sunspot equilibria occur through a flip bifurcation and only when leisure and consumption are complements. Conversely, when profits in the waste recovery technology are decreasing, multiple equilibria may occur also through a Hopf bifurcation. In addition, deterministic cycles arise even when leisure and consumption are gross substitutes, in contrast to standard results in OLG models (Azariadis 1981; Geneakoplos and Polemarchakis 1984; Grandmont 1985; Azariadis and Guesnerie 1986; Benhabib and Laroque 1988; Drugeon, Nourry, and Venditti 2010). This is due mostly to the countercyclical behavior of the profit function: if in a given period consumption increases, profits decrease and thus next period investment and consumption will be lower. Thus, the system moves back toward its initial position. We show that as soon as the share of the profits in the waste recovery activity transferred to old agents increases, the indeterminacy region shrinks: this is due to the fact that transfers to young agents must be previously invested in money balances and so they are affected by the inflationary tax. This in turn improves the scope for indeterminacy. We have also studied the equilibrium path chosen by a social planner, and considered the market economy with altruism. We have computed the optimal share of the profit to be granted to young agents so that the first-order condition reduces to that found at the social optimum. A suitable extension of the paper should take into account, on the one hand, capital accumulation and, on the other hand, a monopolistic firm operating in the virgin sector.

Appendix

The market economy with altruism

We study the equilibrium outcome of the economy when agents are supposed to be altruistic, i.e., when they care about the welfare of their offspring. We have seen that an agent born in period t faces the intertemporal budget constraint (4). This means, in particular, that his own utility also depends on the consumption c_{t-1} of the old born in period $t - 2$ and on the consumption c_t of the old born in period $t - 1$. Notice that the agent born in period t has no control over c_{t-1} and c_t : from his point of view c_{t-1} and c_t represent “state” variables. In fact he has to choose his old-age consumption c_{t+1} as well as his young-age labor supply l_t . If we assume that agents care about the next generations’ utility, it means that he decides how much they want to consume and how long they have to work. In order to do so, old agents will have to take into account the effects of their choices on their offspring. From the budget constraint (4) it is possible to express l_t as a function of c_{t-1} , c_t and c_{t+1} :

$$l_t = \frac{p_{t+1}c_{t+1} - qp_t\pi(c_{t-1}) - (1-q)p_{t+1}\pi(c_t)}{w_t}. \quad (\text{A1})$$

If we denote by $V(c_{t-1}, c_t)$ the value function of an agent born in period t , we obtain

$$V(c_{t-1}, c_t) = \max_{c_{t+1}} \left\{ u(c_{t+1}) - v(l_t) + \beta V(c_t, c_{t+1}) \right\},$$

where l_t is defined by (A1). Assuming we are in the region in which profit in the recycling sector is increasing in consumption,⁸ the first-order condition with respect to c_{t+1} is

$$u'(c_{t+1}) - v'(l_t) \frac{p_{t+1}}{w_t} + \beta V(c_t, c_{t+1}) = 0.$$

Let V_i , $i = 1, 2$, be the derivative of the value function V with respect to its i th argument, it is possible to get the expression for $V_2(c_{t-1}, c_t)$:

$$V_2(c_{t-1}, c_t) = \frac{v'(l_t)(1-q)p_{t+1}\pi'(c_t)}{w_t} + \beta V_1(c_t, c_{t+1}).$$

Furthermore, the expression for $V_1(c_{t-1}, c_t)$ is

$$V_1(c_{t-1}, c_t) = \frac{v'(l_t)qp_t\pi'(c_{t-1})}{w_t}.$$

By standard computations, we obtain

$$V_2(c_t, c_{t+1}) = \frac{v'(l_{t+1})(1-q)p_{t+2}\pi'(c_{t+1})}{w_{t+1}} + \beta \frac{v'(l_{t+2})qp_{t+2}\pi'(c_{t+1})}{w_{t+2}}. \quad (\text{A2})$$

Therefore, by combining (A2) with the zero-profit condition in the virgin sector and the intertemporal budget constraint, we obtain

$$\begin{aligned} u'(c_{t+1}) - v'(l_t) \frac{l_t + q\pi(c_{t-1})}{c_{t+1} - (1-q)\pi(c_t)} + \beta v'(l_{t+1})(1-q)\pi'(c_{t+1}) \frac{l_{t+1} + q\pi(c_t)}{c_{t+2} - (1-q)\pi(c_{t+1})} \\ + \beta^2 v'(l_{t+2})q\pi'(c_{t+1}) = 0, \end{aligned} \quad (\text{A3})$$

which obviously must be coupled with the good market equilibrium condition (10). We stick to the study of its stationary solution, which is a value for c solving

$$\begin{aligned} u'(c) - v'(l) \frac{l + q\pi(c)}{c - (1-q)\pi(c)} + \beta v'(l)(1-q)\pi'(c) \frac{l + q\pi(c)}{c - (1-q)\pi(c)} \\ + \beta^2 v'(l)q\pi'(c) = 0 \end{aligned} \quad (\text{A4})$$

under equilibrium equality $l = c - \pi(c)$. Comparing (A4) with the first-order condition (28) chosen by the social planner, it is easily verifiable that the two conditions merge together when q is set equal to $1 - (1/\beta\pi'(c))$. This implies that the market economy with altruism does not mimic the optimal path chosen by the social planner. This is due to the presence of transfers granted to young and old agents; such transfers are distortionary since they entail the presence of two state variables, c_{t-1} and c_t , instead of only one, which in the planned economy is represented by c_{t-1} . This optimal value of q confirms all our previous results: actually, it tends to zero when $\pi'(c) \rightarrow 0$.

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⁸ If we were in the region in which profit in the recycling sector is decreasing, agents would face a trade-off between reducing their consumption to the advantage of the next generation and maximizing their own welfare. In such a configuration, the corner solution would arise and the standard method of dynamic programming would no longer be available.

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