Vibration analysis of super-yachts: Validation of the Holden Method and estimation of the structural damping

Alessandro Zambon a, Lorenzo Moro a,*, Marco Biot b

a Department of Ocean and Naval Architectural Engineering, Faculty of Engineering and Applied Science, Memorial University of Newfoundland, St. John’s, Newfoundland and Labrador, A1B 3X5, Canada
b Department of Engineering and Architecture, University of Trieste, 34127, Trieste, Italy

ARTICLE INFO

Keywords:
Marine propeller
Propeller-induced hull pressure
Ship vibration
Ship structural damping
Super-yacht
On-board comfort

ABSTRACT

The hull excitation generated by marine propellers constitutes one of the most significant vibration sources affecting comfort on passenger ships. Consequently, the evaluation of the propeller-generated excitation through reliable numerical tools during the preliminary stage of ship design is fundamental. The Holden Method (HM) is an empirical tool utilized to calculate the propeller-induced pressure distribution on a ship hull. The present paper validates the HM, which is applied to a twin-screw, 54-m super-yacht. Finite Element Analysis (FEA) is used to benchmark the HM numerical predictions against a set of full-scale vibration measures. The outcomes show that the magnitude of the propeller-induced dynamic excitation predicted by the HM is over-estimated. Thereafter, the calibrated propeller-induced forces and the diesel engine excitation are applied to the FE model to perform a series of forced vibration analyses and estimate the global structural damping coefficient of the super-yacht. The study highlights the necessity of developing new empirical methodologies, analogous to the HM, to be applied to modern small luxury vessels.

1. Introduction

Over the last few decades, the International Maritime Organization (IMO) and the major Classification Societies have constantly improved the criteria concerning comfort on vessels. The boom of the cruise industry and of large pleasure yachts has increased the importance of comfort for passengers who are not accustomed to the marine environment. Guaranteeing a comfortable trip to these passengers has become a key factor in order to be competitive in the market [7]. Furthermore, comfort has also become of paramount importance on other merchant vessels, where safety is of major concern. Assuring crew members are properly rested improves their vigilance during work shifts, which decreases the number of injuries on the workplace, and thereby improves safety on board [15,23,42].

As noise and vibration mostly affect on-board comfort levels, international organizations such as the IMO and the International Organization for Standardization (ISO) have issued codes and standards which set noise and vibration limits on board of vessels. They also provide ship designers with procedures for the assessment of noise and vibration levels on board [19–21]. The major Classification Societies have also developed specific Class Notations for ships compliant with their strict rules on comfort, which include noise and vibration limits [5,9].

Regulations have been amended towards steadily-narrowing admissibility criteria; therefore, comfort assessment procedures that

* Corresponding author.
E-mail addresses: azambon@mun.ca (A. Zambon), lmoro@mun.ca (L. Moro), biot@units.it (M. Biot).

Accepted 9 June 2020
are employed during the design of ships should keep pace with those stringent standards. With regard to noise and vibration, researchers and ship designers have increasingly focused on the development of design procedures for the evaluation of on-board comfort. In order to be effective, these procedures should include an accurate simulation of the noise and vibration sources [8,25]; besides, a proper model of the vessel structural dynamics is necessary to simulate the means of transmission of the acoustic and vibration energy [11,24,30,36,46]. Additionally, standardized methods for the assessment of noise and vibration levels are needed to verify that new ships comply with relevant standards before it is delivered to the shipowner.

Marine propellers are one of the most prevalent on-board noise and vibration sources. Thus, over the last few years several studies simulated propeller-induced hull pressure fluctuations and developed effective methods for the characterization of the propeller as a source of noise and vibration. However, modeling the propeller-induced dynamic pressure on the stern hull is a challenging problem for several reasons. First, because of the continuous evolution of the shapes of ship sterns, it is necessary to update the methods used to evaluate propeller-induced excitation. Second, there are several parameters that define the hydrodynamic load in the propeller-hull system and decoupling them is challenging. Thorough investigations on the latter are found in Ref. [34,37], and [2], where the parameters affecting the propeller tip vortex flow and the consequent pressure field were examined and assessed. In particular, in Ref. [45] the investigation was focused on the relationship between the propeller dimensions and the hull geometrical region onto which the induced pressure was exerted.

Methods for predicting propeller-induced hull pressure fluctuations include empirical and theoretical or numerical techniques. Over the last few decades, as the efficiency of computational techniques has increased, the use of numerical methodologies has become more common. Voros [44] developed a diffraction Boundary Element Method (BEM) for the calculation of the hull-pressures on the area above a propeller. Using the Green Theorem, the author bypassed the pressure field integration to calculate the exciting load on the hull. These results also provided guidelines on the hull geometrical region that is affected by propeller-induced hull pressures. In particular, the pressure field on the hull should be calculated on a 2D-sided square area above each propeller, where D stands for the propeller diameter. More recently, Kinns and Bloor [22] and Spivack et al. [38] used an acoustic BEM to simulate twin screws of a cruise ship. The authors used a set of monopole and dipole sources and discussed how the different type of sources affected hull pressure fluctuations above the propeller, which cause hull vibration. Merz et al. [29] combined this technique with FEA to evaluate underwater pressures on a submarine structure and to simulate its structural and acoustic response. Analogously, the acoustic excitation generated by a submarine propeller-shaft system, which affects the structural response of the hull and structures, was studied in Ref. [33]. A fully coupled BEM-FE analysis was also used in Ref. [3], where the authors calculated propeller-induced hull pressure fluctuations on an ellipsoidal floating body, in order to validate the outcomes of numerical simulations against data from previous studies. Later, the authors applied this numerical procedure to the case of a twin-screw cruise ship, and their outcomes were benchmarked against experimental tests performed in a depressurized towing tank. Van Wijngaarden [43] also developed BE models to simulate propeller-induced pressure pulses and to determine the strength of sources of propeller noise. The validation of the numerical models showed that the results were within 15% of the measured values. Furthermore, Taskar et al. [39,40] studied the influence of operation in waves on propeller efficiency, focusing also on the estimation of hull pressure pulses simulated by means of BE analysis.

These numerical simulations require a time-consuming modeling process and high computational costs. For those reasons, designers need reliable and practical design tools for the evaluation of propeller-induced hull pressure fluctuations, to be used in the preliminary design of ships. This is the case of the empirical methods. On this matter, Holden et al. [17] performed experimental tests on 72 ships with conventional hull shapes. The regression analysis they performed on the experimental data related the ship propulsion parameters, propeller load, wake field, and propellers geometrical properties to the propeller-induced hull pressure fluctuations on the hull plates above each propeller. In Ref. [16,46], the method developed by Holden et al. was applied to a patrol ship and to a chemical tanker respectively, and the equivalent dynamic forces that simulated the action of the propellers as an integrated pulsing force were calculated. Domínguez Ruiz et al. included the axial and transverse components of the propeller dynamic pressures, in addition to the external moment acting on the shaft bossings [16]. Besides that, Holtrop [18] and Choi et al. [12] conducted studies similar to the work presented by Holden et al.

The limits of applicability of the HM are related to the main parameters of the propellers which were taken into account in the statistical analysis. Moreover, the HM only evaluates the first two harmonics of the propeller-induced hull pressures. For this reason, De Bruijn et al. [13] and Raestad [35] developed empirical methods which are built on the HM, and take into consideration the broad band pressure due to cavitation volume variation. Furthermore, Buannic et al. [10] implemented a method where semi-empirical and acoustic FE analysis were applied simultaneously to investigate the broad-band cavitating pressure pulses.

Nowadays, the Holden Method is widely used as a practical design technique, especially when quick evaluations of propeller-induced hull vibrations are needed in the early design stages of ships. Nonetheless, the major Classification Societies mainly recommend the use of direct calculation of propeller-induced hull pressures via numerical analysis. The American Bureau of Shipping (ABS) proposes the use of either numerical or empirical methods [1]. The latter consists of a simplified version of the HM, applicable to ships of all lengths. In particular, the HM algorithm is reduced into direct formulas where the second harmonic component of the propeller-induced hull pressure is neglected. In the past, Det Norske Veritas (DNV) proposed a procedure that involves both numerical and empirical methods [14]: it included a simplified formulation of the HM, analogous to the one provided by ABS [1]. Lloyd’s Register (LR) has recommended different approaches to the problem. In addition to the direct evaluation of propeller-induced hull pressures via direct numerical analysis [27], in previous guidelines, LR recommended an empirical method that defines ranges of propeller-induced pressure peaks for different types of ships [26]. These guidelines do not specify the spatial distribution of propeller-generated pressure field to be applied to the stern of the hull. Other major Classification Societies, such as DNV GL, ClassNK, RINA, and RS-Class, do not provide explicit indications on the procedure to estimate the propeller-induced hull load, and specific evaluations may be performed.
on a case-by-case basis.

The present paper examines the outcomes of a study carried out to validate the use of the Holden Method for the vibration analysis of super-yacht structures. This study is part of a wider research project involving thorough experimental and numerical studies on super-yachts structural dynamics [28,32,47], on-board structure-borne noise [6], and simulation of the hull frequency dependent added mass [31].

In this research activity, the authors implement the HM to perform a series of forced vibration analyses of a super-yacht. The calculated pressure pulses are integrated over a selected area of the hull surface, in order to obtain an equivalent resultant dynamic load. The amplitude of the calculated dynamic load is then calibrated using on-board, full-scale vibration measures and linear FE dynamic analysis of the vessel structures. The obtained dynamic load is then used to test the validity of the empirical formulas provided by ABS and DNV, in the case of the considered super-yacht. Finally, the authors use the estimated value of the propeller-induced hull dynamic loads and full-scale engine vibration measures to evaluate the overall structural damping ratio of the super-yacht structures.

2. Methods

The flowchart in Fig. 1 represents the procedure followed in this work, which is briefly described here. Firstly, a fine-mesh 3D FE model of the super-yacht structures is created in MSC.Patran (step 1), as described in Subsection 2.1.

Then, the propeller-induced hull pressure pulses are calculated via a Matlab script which implements the Holden Method. Then, the pressure distributions are integrated on the hull surface to obtain their equivalent dynamic forces (step 2 in the flowchart). This step is described in Subsection 2.2.

The equivalent dynamic forces are then used as input forces in a series of FE dynamic linear analyses, which aim to evaluate the dynamic response of a set of nodes of the hull shell (step 3 A). After this first series of simulations, a second series of FE dynamic linear analyses is performed to evaluate the response on these same nodes (step 3 B) in the case when the input loads are simulated using acceleration data from full-scale measured data (step M Subsection 2.3).

The outcomes of the first run of simulations (step 3 A) are then benchmarked against the outcomes of the second simulation set (step 3 B) in order to validate the dynamic forces calculated according to the HM (step 3C). This validation leads to the estimation of the actual value of an equivalent dynamic force which simulates the propeller-induced hull loads in hull vibration analysis of the super-yacht. Step 3 is presented in Subsection 2.4.

Finally, the estimated propeller-induced hull forces are used to perform a series of FE dynamic linear analyses for the estimation of the vibration levels on different deck areas of the vessel. The outcomes of these simulations are then benchmarked against the vibration

![Fig. 1. Methods flowchart.](image-url)
levels measured in sea-trials. Following the procedure presented in Ref. [32], a damping ratio of the hull structures is estimated in a frequency range which includes the first harmonics of the diesel engines. Step 4 is described in Subsection 2.5.

This study was performed on a 54-m super-yacht, designed and built by an Italian shipyard and kindly made available for the present study. The case study ship is a twin-screw diesel powered vessel, whose main characteristics are reported in Table 1. The hull of the super-yacht is steel-made, whereas its superstructures are made of aluminum alloy. The ship is designed to accommodate 12 passengers and 14 crew members.

2.1. Finite Element model of the vessel structures

A 3D Finite Element model of the ship structures was developed with MSC.Patran to study the dynamic response of the vessel structures and is presented in Ref. [32]. That model is also used in this work to evaluate the dynamic response of the ship structures via linear frequency response analyses performed in MSC.Natran. Fig. 2 shows the 3D FE model, and Table 2 shows the model main properties.

The FE model consists of 2D isoparametric quadrilateral and triangular plane stress elements (Nastran Bulk Data items CQUAD4 and CTRIA3), 1D beam elements (CBEAM), and 0D elements (CONM2). The latter are used to distribute non-structural, lumped masses installed on board. For instance, the inertia of each main diesel engine is modeled via 24 0D CONM2 elements, distributed in correspondence of the 4 resilient mountings of the engine. Other types of machinery installed on board are modeled using the same approach. Distributed non-structural masses are also applied to 2D shell elements in order to simulate the inertia of on-board fittings and equipment. The average size of the plane strain and beam elements is 300 mm, which corresponds to the half-span of two consecutive transversal frames of the ship structures. This element size allows for a proper simulation of the structural dynamics up to 100 Hz [32], which is the frequency range of interest to evaluate the ship comfort [20].

The numerical model takes into account the strongly coupled fluid-structure interaction, within the frequency domain, of the wetted hull surface. The fluid-structure interaction is simulated by means of a fully coupled BEM-FE analysis. The FE elements of the wetted hull surface constitutes the boundary of the fluid domain onto which the frequency-dependent added mass is calculated and
applied. Overall, the wetted hull surface is simulated with 7204 shell elements, which represent the boundary of the BEM analysis.

2.2. Propeller-induced hull excitation according to the Holden Method

Propeller-induced hull pressure fluctuations are calculated using the algorithm presented by Holden et al. [17]. The analysis is limited to the first and second exciting harmonics. The fundamental frequency is called Blade Passing Frequency (BPF) and is defined as $BPF = Zn$, where $Z$ is the number of blades of each propeller and $n$ is the propeller shaft rotational speed in revolutions per second (rps). The algorithm employs three sets of input parameters:

1. Distance between the hull surface and the propeller position;
2. Geometry data of the propeller and its blades;
3. Main ship propulsion data;

Non-cavitating and cavitating components (respectively $\Delta P_0$ and $\Delta P_{1c}$) are determined, associated to a hull point located at a certain distance from the propeller center. In the following, the units of measure adopted by Holden et al. have been converted into S.I. units, for consistency with the results presented in Section 3. In detail, $\Delta P_0$ is determined directly as follows:

$$
\Delta P_0 = 12.45 g \cdot \rho \cdot n^2 \cdot D^2 \cdot Z^{-1.53} \left( \frac{b_0 \gamma}{D} \right)^{1.33} \frac{1}{(d(\bar{r})/R)^2} \text{ [Pa]}
$$

where $g = 9.81 \text{ m s}^{-2}$ is the gravity acceleration, $\rho$ is the water density in $10^3 \text{ kg m}^{-3}$, $D = 2R$ is the diameter of the propeller, and $b_0 \gamma$ is the maximum blade thickness at a radial distance of $0.7R$ from the rotation axis. Then, $\bar{r}$ is the distance from the propeller center to a selected hull point, whereas $d(\bar{r})$ is measured from the point on the propeller disk placed vertically above the hub center at a radial distance equal to $0.9R$, and finally $\mathcal{H} = (d(\bar{r})/R)$ is a dimensionless parameter defined as:

$$
\mathcal{H} = \begin{cases} 
1.8 + 0.4(d(\bar{r})/R) & \text{if } d(\bar{r})/R \leq 2 \\
2.8 & \text{if } d(\bar{r})/R > 2 
\end{cases}
$$

On the other hand, the cavitating component $\Delta P_{1c}$ is not determined by means of a direct equation, but it results from an algorithm where different intermediate parameters are calculated step by step. Following these steps, we can calculate $\Delta P_{1c}$ which is written as follows:

$$
\Delta P_{1c} = 0.098 g \cdot \rho \cdot n^2 \cdot D^2 \cdot (J_1 - J_M)^{0.01} \cdot f_2^{1.06} \sigma^{-0.5} \frac{1}{(d(\bar{r})/R)^2} \text{ [Pa]}
$$

where $J_1$ and $J_M$ are two advance coefficients corresponding to the mean and maximum blade loading respectively; in particular, $J_1$ is determined through different steps which involve both propulsive and geometrical variables of the ship and propeller. $\sigma$ is the cavitation number, the coefficient $f_2$ depends on the propeller blade pitch $P$ and camber length $f$ at $\bar{r}/R = 0.80$ and 0.95, and $\mathcal{H}_1$ is defined as:

$$
\mathcal{H}_1 = \begin{cases} 
1.7 - 0.7(d(\bar{r})/R) & \text{if } d(\bar{r})/R \leq 1 \\
1.0 & \text{if } d(\bar{r})/R > 1 
\end{cases}
$$

Both $\Delta P_0$ and $\Delta P_{1c}$ contribute for the magnitude of the 1st harmonic component, whereas the 2nd harmonic amplitude is proportional to the cavitating contribution only. The propeller-induced hull pressures at the first harmonic of the BPF is calculated as
follows:

\[\Delta P_z = \sqrt{\Delta P_0^2 + \Delta P_{1c}^2} - 2 \Delta P_0 \Delta P_{1c} \cos(\pi - \varphi)\]  

(5)

where \(\varphi\) depends on \(\bar{r}\) and \(Z\) and represents an equivalent phase angle between the two pressure components. The amplitude of the propeller-induced hull pressures at the second harmonic of the \(BPF\) (named \(\Delta P_{2c}\)) is proportional to \(\Delta P_{1c}\):

\[\Delta P_{2c} = 0.245 (J_1 - J_M)^{-0.18} f_1^{1.06} \Delta P_{1c}\]  

(6)

Following the procedure just described, the propeller-induced hull pressure fluctuations are calculated on two areas of the hull shell above the propellers, as recommended in Ref. [17,44]. Fig. 3 shows the pressure domain associated to each propeller on the FE model. The centers of these areas correspond to the vertical projections of each propeller center on the hull surface. Each area measures 3.0 m and 2.6 m, along the longitudinal and transverse directions respectively, in accordance to the criteria described in Ref. [44]. The domains are symmetrical to the centerplane, and each of them contained 100 shell elements. \(\Delta P_0\) and \(\Delta P_{1c}\) are calculated at the centroids of every element in the selected areas and considered constant over the element.

Once the \(\Delta P_z\) values are calculated for each element, they are integrated over each area to obtain the overall vertical force \(F_1\), acting at \(f_1 = BPF\). In the case when \(\Delta P_{1c} \gg \Delta P_0\) and \(\Delta P_{2c} \propto \Delta P_{1c}\) (Eq. (6)), the 2nd harmonic pressure values are approximated as proportional to \(\Delta P_z\). Thereby, we calculate \(F_2 = hF_1\), where \(F_2\) is the vertical force at the frequency \(f_2 = 2BPF\) and \(h\) is the constant of proportionality.

Even if the simulation of the propeller-induced excitation by means of concentrated forces is an approximation of the physical phenomenon, it has been shown that the dynamic response of the ship structures is not affected by this simplification [16,46]. In our FE model, the propeller-induced hull excitation is then modeled by means of frequency-dependent forces applied to the stern.

2.3. Full-scale measurements of hull vibrations

Full-scale measurement surveys were performed to evaluate on-board vibration levels. The measurements were performed according to the ISO Standard 6954:1984 [21]. It was deliberately chosen to use the first version of this Standard, rather than the most recent version issued in 2016 [20], as it is more suitable to evaluate the overall structural damping, and to identify the response to each harmonic of the vibration sources, as explained in Ref. [27,32].

The data were acquired in time domain using Brüel & Kjær IEPE accelerometers, which were calibrated before and after the measurement survey. The analog data from the sensors were converted into digital signals using an 8-channel 01 dB data acquisition system, and stored on a hard drive. The sample rate was 12.8 kHz. During the measurements, the acceleration data were also analyzed in real-time in order to evaluate the quality of the measured vibration levels and to identify possible unwanted input vibration. At each measurement point, three 1-min records were acquired and resulting velocity spectra were averaged to minimize transient disturbances. The surveys were carried out at the vessel cruising speed, \(V = 12\) kn. At this speed the main diesel engine velocity was 1600 rpm, and the propeller velocity was 400 rpm. In order to avoid the influence of the ship motions on the measured data, the measurements were performed while the sea state was 1–2. Once the measurement surveys were completed, the data were then analyzed to
obtain narrow-band velocity spectra in the range from 3 Hz to 100 Hz. The spectra were obtained using a flat-top window function in order to better estimate the amplitudes of the harmonics. The spectra frequency resolution was 0.5 Hz.

Fig. 4 shows the general arrangement of the vessel and the positions of the measurement points. The location of these points on the hull structures is shown in Table 3.

Another set of measures were performed to evaluate the dynamic response of the stern structures excited by the propeller-induced hull pressure pulses. In particular, for each propeller, a grid of 9 points was identified on an area of the hull shell immediately above the propellers (Fig. 5). These data were later used to validate the dynamic forces generated by the propellers, and to evaluate the vibration levels generated by the main diesel engines. The latter set of vibration measures were acquired at the diesel engine foundations. During navigation at cruising speed, the acceleration data were measured in time domain, and were post-processed in frequency domain using the same analysis parameters that were considered in the comfort assessment and previously described. With regard to the vibration measures at the diesel engine foundations, a set of 8 measurement points was selected, with one point in correspondence of each resilient mount, which were used to decouple the diesel engines from the ship structures (Fig. 6).

Table 3
Locations of the deck points.

<table>
<thead>
<tr>
<th>Point</th>
<th>Deck</th>
<th>Height from B.L. [m]</th>
<th>Frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>Sun deck</td>
<td>10.250</td>
<td>10</td>
</tr>
<tr>
<td>101</td>
<td>Upper deck</td>
<td>7.700</td>
<td>7-8</td>
</tr>
<tr>
<td>104</td>
<td>Upper deck</td>
<td>7.700</td>
<td>15-16</td>
</tr>
<tr>
<td>35</td>
<td>Main deck</td>
<td>5.050</td>
<td>5</td>
</tr>
<tr>
<td>40</td>
<td>Main deck</td>
<td>5.050</td>
<td>14-15</td>
</tr>
</tbody>
</table>

Fig. 5. Grid of measurement points. The acceleration values measured in sea-trials on these points were used to estimate the propeller-induced hull load.

Fig. 6. Resilient mounts used to decouple the diesel engine from the ship structures, with an accelerometer used to measure the vibration levels at the diesel engine foundation.
2.4. Validation of the Holden Method

The implementation of the HM to the case study is performed in three parts, as outlined in the flowchart shown in box 3 of Fig. 1. First, the propeller-induced hull load, calculated according to the procedure presented in Subsection 2.2, is simulated in the FE analysis by means of a frequency-dependent force $F(f)$, whose dimensionless spectrum $S(f)$ has two harmonic peaks: $S(BPF) = 1$ and $S(2BPF) = h$. Hence, $F(f)$ is obtained by multiplying the spectrum $S(f)$ by the force magnitude $F_1$, which is determined by integrating the propeller-induced hull pressures:

$$
\begin{align*}
F(BPF) &= ||F_1|| \cdot S(BPF) = F_1 \\
F(2BPF) &= ||F_1|| \cdot S(2BPF) = ||F_1|| \cdot h = F_2
\end{align*}
$$

(7)

The use of an equivalent force $F(f)$ in place of the HM pressure fields is justified if the dynamic response of the ship structures is not evaluated in the neighborhood of the structures surrounding the propellers. As an example, Fig. 7 shows the results of a frequency response analysis performed to evaluate the dynamic response of the ship structures at the measurement point 40, when the propeller-induced hull pressures (blue dots) or the equivalent forces (red dots) are applied. The plot in Fig. 7 also shows the velocity levels...
measured in the same point—black solid line. The discrepancy in the velocity peak levels, calculated and measured, is due to the structural damping coefficient that will be discussed in detail in Sections 2.5, 3.3, and 4.2. In particular, the responses shown in Fig. 7 are obtained by applying a global viscous damping ratio of \( \zeta = 10\% \) to the ship structures. The plot of the calculated vertical velocities \( v_{c,sl} \) shows that the discrepancy of the numerical results is negligible and confirms what was already found by Dominguez Ruiz et al. [16] and Yucel and Arpaci [46].

The propeller-induced dynamic forces \( F(f) \) are applied to two nodes located at the centers of the two enforced acceleration areas. In Fig. 8, the nodes where those forces are applied are labeled with \( F \). These are the closest FE nodes to the geometrical centers of the propeller-induced pressure domains.

Once the propeller-induced forces have been applied to the FE model, a frequency response analysis is performed in order to evaluate the structural dynamic response on a set of nodes, which are highlighted with red-yellow labels in Fig. 8. These 18 nodes are selected on a perimeter surrounding the area of application of the propeller-induced pressures. As those nodes are not part of the domain of the propeller-induced hull pressures, their responses only depend on the transfer mobility functions, which are calculated between the nodes of application of the dynamic forces or accelerations and the reference nodes. Hence, their responses do not depend on the direct mobility functions. This implies the equivalence of the effect of a propeller-induced nodal excitation and the actual pressure distribution. In this series of simulations, the damping ratio of the structures in the stern of the ship is taken equal to \( \zeta = 2\% \), since the local structures on that area of the hull stern are made of steel and no damping treatments are applied.

In the second place, the acceleration levels measured during the sea trials on the hull plating above the propellers (Fig. 5) are applied as enforced motion to the corresponding nodes of the FE model. The acceleration levels include the peaks of the first and second harmonics of the propeller excitation. The blue points in Figs. 5 and 8 represent the 18 nodes of the FE model where the acceleration levels are enforced. In this way, the response of the hull structures immediately above the propeller, excited by the propeller-induced hull pressure fluctuations, is accurately simulated. In particular, the structural response using the enforced motion is evaluated on the same eighteen nodes highlighted with red-yellow labels in Fig. 8.

Therefore, two sets of responses are obtained, in terms of acceleration peaks, for each one of the 18 reference nodes—one resulting from the application of the nodal propeller forces, and the other induced by the enforced accelerations. For both excitation harmonics, the ratio between the force-induced peak response and the enforced acceleration-induced peak response—\( r_1 \) and \( r_2 \) for the first and second BPF harmonic respectively—is calculated. These values are then used to evaluate the ratios \( r_1 \) and \( r_2 \), which are then utilized to update the propeller-induced force calculated according to the HM.

### 2.5. Estimation of the overall damping coefficient evaluation

Once the magnitudes of the two components of the propeller-induced hull load were evaluated, we performed a series of simulations to estimate the overall damping ratio \( \zeta \) of the super-yacht structures. This analysis is based on the procedure presented in Ref. [32]. While Pais et al. present the structural damping values only for the first BPF harmonic, in the present study we estimate the structural damping at higher frequencies, which take into account the second harmonic of the blade passing frequency, as well as the first 7 harmonics of the main four-stroke medium speed diesel engines. In this way, the estimated value of the overall damping coefficient is validated up to 90 Hz, which covers the entire frequency range for the vibration comfort assessment of ships. In particular, the FE model is used to perform a series of linear frequency response analyses of the vessel structures. The propeller excitation is simulated by applying the estimated propeller-induced hull dynamic forces. The dynamic loads generated by the diesel engines are simulated by enforcing the acceleration levels, measured during the full-scale trials at the base of the resilient mountings, to the correspondent nodes of the FE model. Each one of the series of dynamic FE linear simulations differs from the other only for the value of the structural damping coefficient \( G \), which is defined as follows:

$$
\vec{F}_{\text{damp}}(\omega) = iG \mathbf{K} \vec{X}(\omega)
$$

where \( i \) is the imaginary unit, \( [\mathbf{K}] \) is the stiffness matrix of the FE model, and \( \vec{X}(\omega) \) is the vector of the displacement in frequency domain. The above definition entails an equivalence with the viscous damping ratio \( \zeta \) on a modal basis, and precisely \( \zeta = G/2 \) for every frequency value. Therefore, defining a constant value of the structural damping coefficient \( G \) means defining a constant viscous damping ratio in the frequency domain of the analysis.

It is worth pointing out that this damping ratio takes into account all the damping characteristics of the ship structures and fittings. This includes floating floors, visco-elastic materials, resilient mounts, and the damping properties of any superficial treatment. Therefore, the damping ratio or the structural damping coefficient evaluated in this study is not a characteristic of the material itself, such as the steel of the hull structures or the aluminum of the superstructures. Rather, it should be considered as an overall damping coefficient that can be used to perform linear dynamic FE analyses for the simulation of the vibration levels on this type of vessel. This means that by tuning the structural damping coefficient through the procedure presented in Ref. [32], we took into account the dissipative effects of several local details, which are neglected when modeling the ship structures in FE analysis, and thereby they are included in an overall damping coefficient.

The results of the simulations consist in the acceleration levels, which are calculated on the nodes of the FE model that correspond to the measurement points shown in Fig. 4 and in Table 3. Later, the outcomes of the numerical simulations are post-processed in MATLAB® and compared with the measured data. This allows us to evaluate the overall structural coefficient \( G \) that better simulates the overall damping of the ship structures.
3. Results

3.1. Propeller-induced hull pressures according to the Holden Method

The propeller-induced hull pressure pulses on the stern of the super-yacht are calculated by implementing the Holden Method in a MATLAB script. Fig. 9 shows the pressure distribution of the port side propeller while Fig. 10 shows the pressure distribution for the case of the starboard propeller. In both Figures, the x axis corresponds to the centerline of the super-yacht; in particular, the origin of the reference frame is placed on the astern extremity of the ship baseline. The red points represent the calculated pressure values at the centroids of the FE elements, and the interpolating surfaces are shown to display the pressure spatial distribution. The surfaces are obtained through triangulation-based linear interpolations of the red-point scatterings, along x and y. The two pressure fields are symmetrical with respect to the vessel centerline.
Fig. 11 shows the pressure values $\Delta P_z$ [Pa] calculated at the centroid of each shell element of the area above the port side propeller. In Fig. 11, the yellow-red labeled nodes are the nodes used to calibrate the pressure amplitudes as described in Validation and in Fig. 8. Fig. 12 shows the pressure field on the propeller disks.

The maximum values of the propeller-induced hull pressure pulses at the 1st and 2nd harmonics are equal $\Delta P_{z,\text{max}} = 1.640$ Pa and $\Delta P_{z,2\text{c,\text{max}}} = 575$ Pa respectively, and their ratio, as introduced in Subsection 2.2, equals $h = 0.35$. The integration of each pressure field gives two equivalent forces $F_1 = 4.470$ kN and $F_2 = hF_1 = 1.564$ kN for the 1st and 2nd harmonics, respectively.

3.2. Validation against experimental data

The equivalent forces at the 1st and 2nd harmonics of the BPF are applied to the FE model and then calibrated according to the procedure introduced in Validation. In detail, the FE linear dynamic simulations are performed with MSC.Nastran. The analysis aims to calculate the velocity amplitudes at the reference points, whose positions are described in Figs. 8 and 11. The acceleration levels
Fig. 13. Full-scale vibration measures on the stern structures above each propeller.

Fig. 14. Ratios $r_1$ calculated in the examined reference nodes.

Fig. 15. Ratios $r_2$ calculated in the examined reference nodes.
measured during the sea trials, acquired on the grid points highlighted with blue dots in Figs. 5, 8 and 11, are averaged to obtain the spectrum shown in Fig. 13. This acceleration spectrum is applied to the FE model as an enforced acceleration on the nodes corresponding to the measurement points, as shown in Fig. 11.

Fig. 14 shows the values of the ratio $r_1$ between the structural response at the reference nodes, calculated when the model is excited by the dynamic force determined according to the HM, and the response evaluated when the model is excited by the enforced accelerations from the measured spectra, as described in Subsection 2.4. The responses are evaluated at the first harmonics of the BPF, whereas the values of the ratio $r_2$ evaluated at the second harmonics of the BPF on the reference points are shown in Fig. 15.

The $r_1$ values range from 4.11 to 6.87, and its average value is $r_1,\text{average} = 5.77$. On the other hand, $r_2$ ranges between 0.89 and 3.29, and its average value is $r_2,\text{average} = 2.05$.

The results presented in the following Subsections are obtained from analyses performed using the ratios $r_1 = 5$ and $r_2 = 1$. Therefore, new ratio of the tuned dynamic force between the first and the second harmonic is now $h = 1.75$. Table 4 compares the calculated values of the updated propeller-induced dynamic forces (Tuned values) with the outcomes obtained by applying the HM.

### Table 4
Calibration of the propellers dynamic loading.

<table>
<thead>
<tr>
<th></th>
<th>1st harmonic [N]</th>
<th>2nd harmonic [N]</th>
<th>Ratio 2nd/1st [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HM</td>
<td>4470</td>
<td>1564</td>
<td>0.35</td>
</tr>
<tr>
<td>Tuned</td>
<td>894</td>
<td>1564</td>
<td>1.75</td>
</tr>
</tbody>
</table>

### Table 5
Vibration source harmonics, in Hertz.

| | Propeller | Engine |
|-----------------|-----------------|
| 1st harmonic    | 33.4            | 66.8           |
| 2nd harmonic    | 13.4            | 13.4           |
| Ratio 2nd/1st   | 2.50            | 4.80           |
| 3rd harmonic    | 26.8            | 53.5           |
| 4th harmonic    | 40.1            | 53.5           |
| 5th harmonic    | 53.5            | 66.8           |
| 6th harmonic    | 66.8            | 80.1           |
| 7th harmonic    | 80.1            | 93.5           |

**Fig. 16.** Point 201, sun deck, $f = BPF$.

### 3.3. Determination of the overall structural damping coefficient

The calibrated propeller-induced hull force is applied to the super-yacht FE model to perform a series of frequency response analyses of the structures, and hence to evaluate an overall structural damping coefficient, as described in Subsection 2.5. The Figures presented in this section show the spectra of the vibration measures obtained during sea-trials as described in Subsection 2.3. The plots also show the results of the numerical simulations for different values of the damping coefficients $G$, ranging from 0.10 to 0.40, with steps of 0.02. In order to investigate the response of the structures to possible shifts of the exciting frequencies in navigation, the plots show the vibration levels at a set of frequencies centered at the nominal excitation frequency. Table 5 shows the harmonic frequencies for the two dynamic excitation sources: the propeller and engine forcing terms include two and seven harmonic peaks, respectively. In particular, the propeller 2nd harmonic coincides with the engine 5th level.
Figs. 16–20 show the results of the vibration measures at the measurement points presented in Fig. 4, together with the results of the simulations calculated at the BPF. Figs. 21–25 show the velocity spectra at the same measurement points and the results of the FE frequency response analysis at the second harmonic of the BPF, which also corresponds to the fifth harmonic of the engine excitation. These results are presented in different plots from the previous ones, in order to separate responses that are produced from different vibration sources. Figs. 26–30 present the vibration spectra measured at the different measurement points, together with the results of the simulations performed at the 1st, 2nd, 3rd, 4th, 6th, 7th harmonics of the engine firing frequency. In these graphs a logarithmic scale for the response axis is adopted, to make the plots easier to read. The legend in the following graphs is the same employed in the previous plots.

Table 6 shows the values of the damping coefficient $G$ that best match the results of the FE analysis with the velocity peaks of the measured spectra, for each deck zone and harmonic. In the Table, the values of the damping coefficient that are higher than the highest value considered in the present study ($G = 0.40$) are labeled as $G > 0.40$. We considered $G = 0.40$ as the upper value of damping coefficient in this analysis, as higher values can underestimate the vibration levels on several areas of the vessel.
4. Discussion

In the following Subsections the results concerning the HM validation and the evaluation of the global structural damping coefficient of the ship will be discussed.

4.1. Validation of the Holden Method to the case of a super-yacht

The results presented in Subsection 3.1 and in Subsection 3.2 show that applying the Holden Method to the case study entails an overestimation of the propeller-induced hull excitation. This is evident for the first harmonic of the excitation, where the Holden Method overestimates the propeller-induced hull excitation of about 5 times, as presented in Subsection 3.2.

The results described in Section 3.1 are here compared with the outcomes of the recommendations provided by ABS [1] and the old recommendations of DNV [14], which determine the propeller-induced pressure distribution on the hull by means of two simplified versions of the Holden Method. The DNV recommendations are taken into account in this Section because they have been widely used.
in the recent decades, before their withdrawal. Differently from HM, these two guidelines consider only the first harmonic component for the pressure excitation. The propeller-induced pressure fields were calculated for the case study considered in this research following ABS and DNV, and the same spatial integration carried out in Section 2.2 was performed to obtain the propeller-induced equivalent forces. Table 4, compares the resulting forces at the first harmonic of the BPF $F_1$.

The propeller-induced hull forces of Table 7 were used to evaluate the velocity peak response at $f = \text{BPF}$ in the measurement zone 201 (Sun deck). The numerical simulations were performed as described in Subsections 2.5 and 3.3 with a constant damping coefficient $G = 0.20$. At that frequency and with that damping coefficient, the results of the numerical simulations performed using the tuned value of the Holden Method coincided with the measured data. Fig. 31 shows the results of the simulations performed with the four different values of propeller forces from Table 7, together with the measured velocity spectrum.

The velocity peaks obtained from the application of the ABS and DNV propeller-induced forces resulted 4.1 and 4.5 times higher than the velocity peak obtained with the original HM-integrated force respectively. Therefore, those results were found to be respectively 20.5 and 22.5 times higher than the actual velocity peaks.

The simulations presented in this paper underline the extent by which the Holden Method predictions largely overestimates the

![Fig. 21. Point 201, Sun Deck, $f = 2\text{BPF}$ and engine 5th harmonic.](image1)

![Fig. 22. Point 101, Upper Deck astern, $f = 2\text{BPF}$ and engine 5th harmonic.](image2)
actual vibration levels measured on board. The unfitness of HM for its application on this type of ships, in particular super-yachts or small cruise ships with flat-shaped sterns, is particularly evident for the 1st harmonic component of the propeller-induced load. This is mainly due to the structural and hydrodynamic characteristics of the super-yacht, which are different from the characteristics of the 72-ship sample that was considered as the statistical basis for the regression models included in the HM. In particular, the 72 ships, studied by Holden et al. differed from the super-yacht analyzed in the present paper by:

1. Larger sizes;
2. Different shapes of the stern vault above propellers;
3. Different flow and wake-hull interaction;
4. Different propeller blade geometry;
5. Slower propeller rotation rate;
6. Larger propeller tip-hull clearance;
7. Larger cavitation state.
Furthermore, the HM-simplified formulas provided by ABS in Ref. [1] and by DNV in Ref. [14] produce even larger pressure magnitudes for the 1st harmonic component. This implies that an updated regression-statistical analysis, focused on super-yachts and small cruise ships, should be developed in order to provide shipbuilders an accurate and trustworthy numerical method to be utilized during the early stage design of super-yachts. New full-scale vibration measurements need to be carried out in order to characterize the hull pressure pulses generated by propellers on modern passenger ships and luxury super-yachts. In particular, throughout new measurements on a larger sample of super-yachts and small cruise ships, the tuned ratios elaborated in Subsection 3.2 should be validated to confirm their general applicability.

4.2. Evaluation of the overall damping coefficient

The data plotted in Figs. 16–30, and resumed in Table 6, show that several velocity peaks of the measured spectra do not match the response values, as they are lower than the corresponding FE responses. The threshold value $G = 0.40$ in Table 6 was chosen as it corresponds to a viscous damping ratio equal to $\zeta = 20\%$, and a damping that exceeded that limit is considered to be not suitable to be used in the vibration design of a ship. Indeed, a global structural damping coefficient should prevent the calculated vibration responses
on the ship’s decks to be underestimated. Higher values of $G$ should be taken into consideration when FE analyses are used to simulate the dynamic response of local structures and areas as deck floors and cabins, whereas a global vibration analysis, which involves the entire vessel structures, needs to be conservative. To estimate the structural damping coefficient $G$, we calculated its average from the values that fall below $G = 0.40$, and it equals 0.24. After that, the outlier values of the $G$-value distribution, which lie above $G = 0.30$ as shown in Table 6, are discarded in order to consider the main cluster of the value scattering. In the latter case, the mean value of the damping coefficient results to be 0.20. Hence, the overall damping coefficient $\bar{G}$ for the whole ship structures can be reasonably estimated as $\bar{G} = 0.22$, which is the average value between the two average $G$ values found above. Hence, its equivalent viscous damping ratio, which remains constant for all frequencies, equals $\zeta = G/2 = 0.11 = 11\%$. This result is in accordance with the recommendations provided by Ref. [4,41], which suggest to consider the global viscous damping ratio of ship structures as $\zeta = 10\%$, for all frequencies. Pais et al. in Ref. [32] found a similar result, i.e. $\zeta = 10\%$, when they studied the structural damping of the same super-yacht presented in this study considering the propellers as only excitation source.

The large amount of $G$ values that fall above the threshold value $G = 0.40$, as reported in Table 6, are mainly due to over-damping phenomena that take place in local areas of the ship decks, which directly depend on the effects of the deck damping treatments and coverings. A trustful representation of the complex damping behavior of super-yacht decks requires a detailed FE analysis of the local structures. Therefore, the obtained value for $\bar{G}$ is sufficiently reliable and meaningful when it is meant to be used in a global vibration analysis during the preliminary stage of the design of ships, in which the vibration levels should not be underestimated.
Fig. 29. Point 35, Main Deck astern, engine harmonics.

Fig. 30. Point 40, Main Deck amidship, engine harmonics.

Table 6
$G$ – values distribution along the frequency and the deck zones.

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>$f$ [Hz]</th>
<th>201</th>
<th>101</th>
<th>104</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prop. I</td>
<td>33.4</td>
<td>0.20</td>
<td>&gt; 0.40</td>
<td>0.10</td>
<td>&gt; 0.40</td>
<td>0.16</td>
</tr>
<tr>
<td>Prop. II, Eng. V</td>
<td>66.8</td>
<td>0.10</td>
<td>0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
</tr>
<tr>
<td>Eng. I</td>
<td>13.4</td>
<td>&gt; 0.40</td>
<td>0.14</td>
<td>0.26</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
</tr>
<tr>
<td>Eng. II</td>
<td>26.8</td>
<td>0.38</td>
<td>&gt; 0.40</td>
<td>0.18</td>
<td>0.18</td>
<td>&gt; 0.40</td>
</tr>
<tr>
<td>Eng. III</td>
<td>40.1</td>
<td>0.20</td>
<td>&gt; 0.40</td>
<td>0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
</tr>
<tr>
<td>Eng. IV</td>
<td>53.5</td>
<td>0.16</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
</tr>
<tr>
<td>Eng. VI</td>
<td>80.1</td>
<td>0.26</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
</tr>
<tr>
<td>Eng. VII</td>
<td>93.5</td>
<td>0.36</td>
<td>&gt; 0.40</td>
<td>&gt; 0.40</td>
<td>0.28</td>
<td>&gt; 0.40</td>
</tr>
</tbody>
</table>
4.3. Strengths and limitations of this study

The results presented in this paper pointed out the necessity either to develop new empirical methods for the calculation of the propeller-induced hull excitation on super-yachts and small cruise vessels, or to expand the HM algorithm when it has to be applied to those ships. A limitation of the results obtained in this paper is that the HM has been implemented to a single case study, therefore further similar studies should be carried out in order to validate these results further. So far, few studies have been developed on this matter, and the results shown in this paper can ignite a broad research in terms of full-scale measurement campaigns and statistical analyses, which should be undertaken on several super-yachts and small cruise ships. Therefore, the applicability and reliability of an updated regression analysis from these tests would have to be concentrated on this particular vessels and their propeller-hull geometries. In addition to that, the results concerning the overall structural damping coefficient agree with the available recommendations on this matter, although they are limited to be used in a global vibration analysis. The estimated value of $G$ is conservative, as the uncertainty in the determination of a global damping coefficient is high. In more advanced design stages, where simulations are applied to local ship structures, higher values for the structural damping coefficient should be used to take into account local damping characteristics.

5. Conclusions

In this paper, we presented the results of the implementation of the Holden Method for the analytical evaluation of the propeller-induced hull dynamic pressures to the case of a twin-screw, 54-m super-yacht. The hull pressure distributions generated by the two propellers and their equivalent concentrated loads were calculated by implementing the HM on a FE model of the ship. Therefore, a set of full-scale vibration measures on the ship stern structure was used to validate the FE simulations, and obtain scale factors to be applied to the dynamic excitation calculated with the HM. Thereafter, by means of the calibrated propeller-induced forces and the engine excitation, a series of forced vibration simulations were performed, and their outcomes validated against vibration measurements taken during sea trials. This validation allowed us to estimate the structural damping coefficient of the studied vessel.

The results of this study highlighted how the actual propeller-induced pressures on the hull are overestimated by the HM. This means that the regression algorithm of the HM could not be reliably applied to ships similar to the super-yacht considered in this paper.

Table 7
Comparison between HM, ABS, and DNV predictions at $f = \text{BPF}$.

<table>
<thead>
<tr>
<th>Method</th>
<th>$F_1 [N]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HM, tuned</td>
<td>894</td>
</tr>
<tr>
<td>HM, original</td>
<td>4470</td>
</tr>
<tr>
<td>ABS</td>
<td>18,400</td>
</tr>
<tr>
<td>DNV</td>
<td>19,900</td>
</tr>
</tbody>
</table>

Fig. 31. Point 201, Sun Deck, $f = \text{BPF}$, $G = 0.20$, comparison of different prediction methods.
and highlighted the necessity of new or updated simplified formulations to predict propeller-induced hull pressures on this type of ships. The analysis of the damping coefficient of the ship structures confirms findings from other relevant studies and showed the need for different values of damping coefficients for local and global structural dynamic analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[17] Holden KO, Fagerjord O, Frostad R. Early design-stage approach to reducing hull surface forces. SNAME Trans 1980;88:403