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Computational hydroacoustic analysis of the BB2 submarine using the advective Ffowcs Williams and Hawkings equation with Wall-Modeled LES

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ABSTRACT

In the present work we perform the numerical characterization of the hydrodynamic and the hydroacoustic field of a notional submarine geometry (the BB2 submarine). Starting from a model-scale CFD simulation, the full scale acoustic signature of the submarine is derived. We use large eddy simulation with a wall-layer model and perform laboratory-scale numerical experiments at a value of Reynolds number equal to 1.2×10^6 . The conditions are those of a wind-tunnel test. The study is carried out focusing on the submarine in its own basic configuration without propeller to highlight the acoustic properties of the hull by itself and to replicate the silent advancing running (where all the non-essential source of noise are shut down and the speed is minimum to minimize the propeller noise). In this configuration, the wake is the main source of noise. The computational hydroacoustic analysis is based on the Ffowcs-Williams and Hawkings equation, here considered in advective form, which is the most suitable form for wind-tunnel-like problems. The results show that the acoustic spectrum is broadband in the far field and the SPL evaluated in the near field contains the signature of coherent structures associated to the presence of the sail and appendages.

Keywords:

Large-eddy simulation
Ffowcs Williams and Hawkings equation
BB2 submarine
Hydroacoustics

1. Introduction

The present research addresses the numerical simulation of the turbulent flow around a Notional submarine geometry (namely the BB2 Joubert, 2004, 2006) together with the characterization of its own hydroacoustic signature.

Because of their own intricate fluid-dynamic features, from an hydrodynamic perspective submarines can be considered among the most complex marine vessels. In fact, the interaction between the flow field and the submarine determines a strongly three-dimensional flow, with the development of typical vortical structures generated by the sail and appendages. Around the hull, a turbulent boundary layer develops which interacts with the sail wake, potentially yielding to three-dimensional flow separations. In addition, the propeller's wake and cavitation make the problem even more complex. This intricate dynamics generates a rich and broad-banded acoustic field, whose characterization is of fundamental importance for military purposes (Renilson, 2018); the noise pollution is also one of the major threat to the animal species populating the seas, and its reduction is required by recent regulations.

Also in consideration of the importance of minimizing the noise emission, the design of submarines is continuously improved; this

optimization is driven from the idea that noise is not an unavoidable side-effect of the motion (Ianniello, 2016). Thus, the development of reliable and accurate predictive tools assumes a central role in the analysis of hydrodynamics and hydroacoustics of underwater vehicles. In particular, the role of the Computational HydroAcoustic (CHA) is to investigate the noise generated as a consequence of the solid-flow interaction. This noise is the result of linear and non-linear effects, as for example the pressure acting on the body surface and the turbulence propagated in the near field, respectively.

CHA can be performed using either a *direct method* or an *hybrid-approach*. Wang et al. (2006) provide a comprehensive review of computational techniques for flow-noise prediction. The direct method is based on the solution of the *compressible* Navier-Stokes equations (NSEs) which delivers the acoustics together with the fluid dynamic solution over the computational grid; when used in conjunction with a direct solution of the NSEs (DNS) it is the most accurate technique but unfeasible for most flows in the turbulent regime, since the computational cost increases as $Re^{3.5}$ in wall-bounded turbulence (Re is the Reynolds number of the flow), as shown for example in Piomelli and Balaras (2002), Wang et al. (2006). Also, the direct method is unfeasible for additional reasons: it would require a computational

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domain much larger than that required for the solution of the fluid-dynamic field; in case of incompressible flow field (namely Mach number smaller than 0.3) the numerical solution of the compressible NSEs is unfeasible due to numerical issues.

As an alternative to the direct method, a very efficient way to face this problem is the hybrid-approach, also known as *acoustic-analogy* (AA). It consists in decoupling the solution process of the fluid-dynamics part from the hydroacoustics one. The idea behind AA is to use a reliable fluid dynamic solution, constituting the noise source, as an input for the acoustic solver. The noise is then radiated in the far field by solving a non-homogeneous wave equation. There exist different versions of AAs. They differ by the field variable used (typically it is a density perturbation) and also by the noise sources considered. The technical report Uosukainen (2011) presents a complete overview of the most popular AAs.

The present work is based on the Ffowcs-Williams and Hawkings equation (FWH-equation) (Williams and Hawkings, 1969), which is an extension of the early works of Lighthill (1952), Curle (1955) and it accounts for the sound generated by the relative motion between a fluid and rigid body. For this reason it is widely applied in estimating the sound generated by helicopter rotors, airplanes or marine propellers and aircraft engine fans.

When the AA is utilized to study problems characterized by the presence of an upstream velocity (Najafy-Yazdi et al., 2010), the FW-H equation has to be written in its advective form. In acoustics, the case of a stationary observer receives the sound in a moving stream is generally called a “wind-tunnel problem”. Worth to be mentioned that the name is related to the motion conditions between flow and body characterizing a wind tunnel test, rather than to the choice of the fluid. Starting from the works of Farassat (1981) and Farassat and Succi (1980) (formulations 1 and 1A of the FW-H equation, respectively), Najafy-Yazdi et al. (2010) derived the linear terms of the FW-H equation in advective form. This is known as *formulation 1C*. Recently Cianferra et al. (2019a) derived the advective form also for the non-linear terms of FW-H equation. This is the *formulation 1D* and represents the most complete advective form of the FW-H equation, allowing to account for the noise generated by turbulence.

To obtain the hydroacoustic solution using the acoustic analogy is necessary to get an accurate time-resolved hydrodynamic solution first, which successively has to be post-processed by the acoustic solver. Apart DNS, several techniques are available to calculate a reliable approximation of the unsteady fluid-dynamic flow, as, among others, the Unsteady Reynolds-Averaged Navier-Stokes equations (URANS), Large-Eddy Simulation (LES) and Detached Eddy-Simulation (DES). In most applications dealing with acoustics, the fluid-dynamic is generally computed solving URANS, providing the non-stationary mean turbulent field. In this case the effect of turbulent fluctuations on the acoustic signature is not detectable, since the turbulent field is not resolved at all and, as a consequence, most of the important information on the frequency content of the resulting noise is lost (Piomelli et al., 1997; Ianniello, 2016). This approach is still suitable in presence of tonal noise associated with deterministic and periodic flow structures (for example the tip vortex of propellers or the von Karman vortex street behind slender bluff bodies).

The growing availability of High-Performance Computing (HPC) resources allows investigation of the complex flow field using DES and even LES. They provide the unsteady, instantaneous and three dimensional turbulent field, at a reasonable computational cost, even in the high-Re number limit. In LES, the large and energy-carrying scales of turbulence are directly solved by an unsteady 3D simulation, while the smallest ones, more isotropic and dissipative, are modeled through a suitable sub-grid scale parametrization. A main drawback of LES of wall-bounded flows is the need to solve the thin viscous sublayer developing near the wall. The computational cost required to solve the boundary layer using LES increases with $Re^{2.5}$ (see Piomelli and Balaras, 2002 for a discussion), a bit less than that of a DNS, still

unaffordable for high Re-number flows. This problem can be mitigated by modeling the near wall region through the use of special functions (wall-layer models) allowing for a dramatic reduction of the computational cost up to acceptable levels. In this case the simulation is named Wall-Modeled LES (WLES). Nowadays LES, WLES and also DES are in use in the hydroacoustic research field (Wagner et al., 2007; Cianferra et al., 2019a,b; Keller et al., 2018).

To the best of our knowledge, there is not literature about the evaluation of the acoustic field generated by a submarine, and, specifically, for the BB2 model, neither numerical nor experimental; rather, recent research has been devoted to the characterization of the hydrodynamic field. WLES has been successfully applied to the analysis of the flow around a model of a submarine by Anderson et al. (2012), Fureby et al. (2016); they investigated the BB1 model (which is a geometry similar to the BB2, described here below, the main difference being the absence of the forward planes), at 0 and 10 degrees yaw and at Reynolds number $Re_L = 4.5 \times 10^6$. In their work, data from WLES and RANS were validated against experimental data obtained in low speed wind tunnel tests at $Re_L = 2.7 \times 10^6$. The focus of the work was on the analysis of the sail wake and of the fin-tip vortex displacement. The solutions were obtained using the OpenFOAM platform. The Pressure Implicit with Splitting of Operators (PISO) (Issa et al., 1986) procedure is used to handle the pressure velocity coupling in WLES. The unstructured grid is composed of $340 \cdot 10^6$ cells and wall-stress models are applied at the solid surfaces. The authors used the sub-grid scale Mixed Model (MM) and the Localized Dynamic Kinetic energy Model (LDKM). In successive papers Norrison et al. (2016) and Petterson et al. (2018), WLES was employed for the analysis of the self-propelled BB2 submarine advancing in straight ahead motion. Alin et al. (2010) and Bhushan et al. (2013) provide interesting comparisons between LES, DES and RANS predictions for the idealized submarine shape named DARPA Suboff (DSub), an archetypal of a nuclear submarine (Groves et al., 1989). DES and RANS approaches have been adopted by Chase et al. (2013), Chase and Carrica (2013) for the analysis of the flow field and the performances of the DSub in self-propulsion, in captive steady turn and horizontal overshoot maneuvers.

When dealing with submarines the wake assumes a crucial importance. It can be considered the footprint left by a submarine and propagates downstream for very long distances leaving an unavoidable trace and constituting an important source of noise. For this reason the characterization of the wakes is of fundamental importance in engineering applications and numerical simulations of turbulent wakes thus require a fine computational grid extending far away downstream the body generator. Posa and Balaras (2016) performed LES of the flow over the fully appended DSub at 0° yaw angle and Reynolds number equal to $Re_L = 1.2 \times 10^6$ using a 2.8 billion grid cells. The authors used a direct-forcing immersed boundary method (Posa and Balaras, 2014; Fadlun et al., 2000) solving the governing equations over a staggered grid in cylindrical coordinates. They adopted the Wall-Adapting Local Eddy-Viscosity (WALE) sub-grid scale model of Nicoud and Ducros (1999), which was proven to be as accurate as any dynamic model at a lower computational cost. The mean velocity and Reynolds stress fields were calculated and validated against experimental and DNS data. Mean velocity and Reynolds stress profiles in the intermediate wake, at 6 submarine’s diameters downstream, were compared with experimental data from Jiménez et al. (2010b). The authors also discussed the bimodal behavior of the wake and the evolution downstream towards self-similarity. Kumar and Mahesh (2018) investigated the axisymmetric body of revolution, i.e. the submarine DSub without the appendages and propeller, using a body fitted unstructured grid, at Reynolds number $Re_L = 1.1 \times 10^6$ based on the length of the hull and the free-stream velocity. They used the dynamic Smagorinsky sub-grid scale stress model (Germano et al. (1991), Lilly (1992)). The authors carried out a detailed analysis of the turbulent boundary layer and of the wake till a distance of 15 Suboff’s diameters downstream the

stern. In both LES studies, the authors found a good agreement with the experimental data of Jiménez et al. (2010a) and Jiménez et al. (2010b).

The acoustic analysis of BB2 model was performed by Fischer et al. (2017) at the DST Group's Low Speed Wind Tunnel using 64 microphones randomly located. The model was placed at different pitch and yaw angles and considering two flow speeds. Spectra obtained with and without the model in the tunnel were compared and it was observed that the noise from the model is the same as the background except at one single frequency and only for the highest flow speed where a peak was observed when the model is present.

To summarize, most studies were devoted to the analysis of the flow around archetypal submarines, mostly for the study of performance under manoeuvring and also focused on the characteristics of the wake. A considerable amount of studies used eddy resolving numerical techniques. Comparatively few studies focused on the analysis of the acoustic signature of submarines.

In the present work we focus on the characterization of the hydroacoustics signature of the fully-appended BB2 submarine model, without propeller. We choose the BB2 submarine since it has been and it is still considered for several experimental and numerical research campaigns; therefore, a large amount of data are available. Among others, Kumar et al. (2012) performed particle image velocimetry measurements, while Quick et al. (2012) and Quick and Woodyatt (2014) performed measurements of hydrodynamic forces on the hull. Anderson et al. (2012) reported accurate skin-friction coefficient measurements (using different tripping strategies), while (Anderson et al., 2012; Fureby et al., 2016; Lee et al., 2019) provides interesting flow visualizations. Moreover, the BB2 is also the object of various CFD studies, for example, carried out within the NATO AVT-301 collaboration group for flow field prediction for manoeuvring underwater vehicles (Toxopeus, 2017).

The propeller is not considered in the present study for a main reason. Since this is the first literature study where the acoustic signature of the BB2 submarine is investigated, it is more convenient to focus on the main body in order to obtain a full characterization. Another reason is that literature works here referenced for the wake do not consider the propeller too. To be noted that the interaction with the propeller may vary the acoustic signature also in view of the characteristics of the propeller itself. The hydrodynamic simulation is carried out at Reynolds number equal to $Re_L = 1.2 \times 10^6$ using WLES, paying special attention to the axisymmetric wake development. The flow conditions are the same as in Jiménez et al. (2010b) and Posa and Balaras (2016). Modeling the near-wall flow allows for a substantial reduction of the cell amount in the grid; we use about $40 \cdot 10^6$, making the simulation more affordable. The acoustic field is evaluated using the AA in its formulation 1D. To the best of our knowledge, the application of such methodology to submarines represents a novelty in the literature. For this reason, validation of the acoustic field cannot be assessed. However, our methodology has been extensively validated in a number of cases. Specifically, in Cianferra (2017) the authors considered the acoustic field generated by bluff bodies; in Cianferra et al. (2019a) the case of an elongated cylinder was considered, in Cianferra et al. (2019b), Cianferra and Armenio (2021), Posa et al. (2022b) the acoustic field generated by isolated marine propellers was deeply analyzed. Further, in Posa et al. (2022d), Posa et al. (2022c) and Posa et al. (2022a) the same methodology has been employed for the analysis of a system composed by a submarine propeller and a rudder.

The paper is organized as follows. The set up of the simulation including numerical method, computational grid and boundary conditions is discussed in Section 2. Results and validations against available data are described and discussed in Section 3. The pressure coefficient C_p and skin friction C_f over the hull are presented in Section 3.2. The boundary layer, first order velocity statistics and velocity spectra are in Section 3.4. The analysis of the intermediate self-similar wake is discussed in Section 3.3 showing local Reynolds number behavior, self-similar profiles, pressure and velocity spectra. Section 4 presents the results for the hydroacoustic analysis of the BB2 submarine, including the analysis of directivity patterns and the acoustic spectra of the wake. Finally, concluding remarks are in Section 5.

2. The mathematical framework

This Section contains the mathematical framework for both the fluid dynamic and the hydroacoustic part. Specifically, Section 2.1 describes the fluid dynamic conservation laws and the subgrid scale model. Section 2.2 discusses the AA herein employed, namely the integral form of the advective FW-H equation. The statement of the problem and the computational setup are described in Section 2.3.

2.1. Fluid dynamic model

Large-eddy simulation resolves the filtered unsteady and three-dimensional Navier-Stokes equations, which in their incompressible form read as:

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial \bar{U}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{U}_i \bar{U}_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_i} + \nu \nabla^2 \bar{U}_i, \quad (2)$$

where the symbol $\bar{\cdot}$ denotes a filtering operation. In the equations above, U_i denotes the velocity component along the i -direction, ρ and ν are respectively the fluid density and kinematic viscosity, p is the hydrodynamic pressure and $\tau_{ij} = \rho(\bar{U}_i \bar{U}_j - \bar{U}_i U_j)$ is the subgrid-scale stress (SGS) tensor; it comes out from the filtering operation of the non linear (advective) term of the NSEs. It represents the effects of the unresolved scales of motion on the larger, resolved ones. In the present paper, it has been modeled following the eddy-viscosity assumption:

$$\tau_{ij}^R = 2\rho\nu_{sgs}\bar{S}_{ij} + \frac{1}{3}\delta_{ij}\tau_{kk}^R = 2\rho\nu_{sgs}\bar{S}_{ij} + \frac{2}{3}k_r\delta_{ij} \quad (3)$$

where ν_{sgs} is the Sub-Grid Scale (SGS) eddy viscosity; k_r is the residual kinetic energy, which is included into thermodynamic pressure. For simplicity the new variable is denoted again with p ; $\bar{S}_{ij}$ is the resolved rate-of-strain tensor; δ_{ij} is the Kronecker symbol. In this study, the SGS eddy viscosity is calculated using the Wall-Adapting Local Eddy-viscosity (WALE) model, developed by Nicoud and Ducros (1999) has been used:

$$\nu_{sgs} = (C_w \Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}}, \quad (4)$$

where S_{ij}^d is the traceless symmetric part of the square of the filtered-velocity gradient tensor; C_w is a model constant and Δ is a length scale related to the local size of the computational grid. This model is suited for flows around complex geometries. Also, from computational point of view it is not expensive and takes into account the effect of the local strain and of rotation. The quantity ν_{sgs} tends naturally to 0 at the wall as $\sim x_n^2$ and for this reason it is particularly suited for LES (Posa and Balaras, 2016). Also, the fact that the model does not need an *ad-hoc* damping functions near the solid boundaries makes it suited for WLES. Posa and Balaras (2020) used the WALE model coupled with the standard equilibrium wall-layer model for the simulation of the DARPA Suboff at very high Reynolds number, obtaining rather a good agreement with the experimental data.

2.2. The formulation 1D of the Ffowcs Williams and Hawking equation

The Ffowcs Williams and Hawking equation (Williams and Hawking, 1969) provides the acoustic pressure generated by the relative motion between a fluid flow and a rigid body. Due to its better computational performances, the integral formulation of the FW-H equation generally is adopted. In the FW-H equation, the acoustic pressure is expressed as a sum of three terms:

$$p'(\mathbf{x}, t) = p_T'(\mathbf{x}, t) + p_L'(\mathbf{x}, t) + p_Q'(\mathbf{x}, t). \quad (5)$$

The first term p_T' is called Thickness component. It represents the noise generated by the fluid displaced by the body in motion. The Loading

component p_L' represents the noise coming from the fluid dynamic pressure acting on the body surface. Both p_T' and p_L' are surface integrals, calculated on the surface of the solid body, and representing the linear part of FW-H equation. The term indicated with p_Q' is known as *Quadrupole* term (Farassat, 2007). It is composed of the sum of volume integrals and represents the contribution from non linear sources, namely the pressure and velocity disturbances developing around the generator body and in its wake; Ianniello et al. (2013) and successive literature, showed that differently from the aeroacoustic problem, where the quadrupole term is relevant just for transonic or supersonic flows, in hydroacoustics it affects significantly the pressure field. The evaluation of quadrupole term is very expensive in terms of computational time and data storage because of the *time-delays*. In fact, these integrals contain terms to be evaluated at specific times τ , whose contribution determines the pressure signal at the present time t .

When CHA is utilized for wind-tunnels problems, the FW-H equation has to be written in advective form. Najafy-Yazdi et al. (2010) derived the advective loading and thickness terms (formulation 1C), while only recently Cianferra et al. (2019a) provided the advective form of the quadrupole term a general criterion for an accurate and efficient evaluation of these contributions to the acoustic signal. To distinguish the formulation developed by Najafy-Yazdi et al. (2010) from that one provided by Cianferra et al. (2019a), the latter will be named **formulation 1D**, meaning that in the FW-H equation appear both linear and non-linear terms in advective form.

In the present case, we consider a fixed-in-space non-permeable body (i.e. $U_n = V_n$, with $\mathbf{V}$ the body velocity), so the thickness term is null and loading term assumes the simple form of the Curle equation (Curle, 1955) in advective formulation:

$$4\pi\hat{p}_L(\mathbf{x}, t) = \frac{1}{c_0} \frac{\partial}{\partial t} \int_S \left[\frac{\hat{p}\hat{n}_i\hat{r}_i}{r^*} \right]_\tau dS + \int_S \left[\frac{\hat{p}\hat{n}_i\hat{r}_i}{r^{*2}} \right]_\tau dS. \quad (6)$$

The advective quadrupole term is given by:

$$4\pi\hat{p}_Q(\mathbf{x}, t) = \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \int_V \left[T_{ij} \left(\frac{\hat{r}_i\hat{r}_j}{r^*} \right) \right]_\tau dV + \frac{1}{c_0} \frac{\partial}{\partial t} \int_V \left[T_{ij} \left(\frac{2\hat{r}_i\hat{r}_j}{r^{*2}} + \frac{\hat{r}_i\hat{r}_j^* - R_{ij}^*}{\beta^2 r^{*2}} \right) \right]_\tau dV + \int_V \left[T_{ij} \left(\frac{3\hat{r}_i\hat{r}_j^* - R_{ij}^*}{r^{*3}} \right) \right]_\tau dV. \quad (7)$$

The kernels of the integrals are evaluated at the time τ , which is the time of noise emission from the source, that is located at $\mathbf{y}$. The acoustic signal reaches the receiver in $\mathbf{x}$ at the present time t . Emission time and receiving time are related by the equation:

$$\tau = t - r/c_0 = t - \frac{|\mathbf{x}(t) - \mathbf{y}(\tau)|}{c_0}. \quad (8)$$

In Eq. (6), the domain of integration is denoted as S ; the surface S may be coincident with the solid body or may be larger and include it when the porous formulation is employed (see Cianferra et al., 2019a and literature therein reported). In this case S coincides with the solid wall of the immersed body. To be noticed that Curle's analogy does not provide for the computation of the quadrupole terms, and it turns out to be a special case of the more general FWH equation. The integrals of Eq. (7) are computed in the fluid volume V surrounding the surface S . $\hat{n}$ indicates the unit normal to S ; while $\hat{r}_i$ and $\hat{r}_i^*$ are the components of the unit radiation vectors, given by:

$$\hat{r}_1 = -\frac{M_0}{\beta^2} + \frac{1}{\beta^2} \frac{(x_1 - y_1)}{r^*}; \quad \hat{r}_2 = \frac{(x_2 - y_2)}{r^*}; \quad \hat{r}_3 = \frac{(x_3 - y_3)}{r^*}, \quad (9)$$

and

$$\hat{r}_1^* = \frac{(x_1 - y_1)}{r^*}; \quad \hat{r}_2^* = \beta^2 \frac{(x_2 - y_2)}{r^*}; \quad \hat{r}_3^* = \beta^2 \frac{(x_3 - y_3)}{r^*}, \quad (10)$$

where

$$r^* = \sqrt{(x_1 - y_1)^2 + \beta^2 [(x_2 - y_2)^2 + (x_3 - y_3)^2]}; \quad \beta = \sqrt{1 - M_0^2}. \quad (11)$$



Fig. 1. Lateral view of the BB2 submarine and frame of reference.

r and r^* indicate the modules of the radiation vectors $\mathbf{r}$ and $\mathbf{r}^*$, respectively; $M_0 = U_\infty/c_0$ is the free-stream Mach number, where c_0 is the speed of sound.

In Eq. (7) T_{ij} is the Lighthill second order stress tensor (Lighthill, 1952) and is given by:

$$T_{ij} = \rho_0 U_i U_j + (\bar{p} - c_0^2 \bar{\rho}) \delta_{ij}. \quad (12)$$

The first term composing T_{ij} is $\rho_0 U_i U_j$ and is associated to the velocity perturbations in the flow; the second term $\bar{p} - c_0^2 \bar{\rho}$ describes possible non-isentropic acoustic generation processes, where $\bar{\rho} = \rho - \rho_0$ is the density perturbation. R_{ij}^* is the diagonal matrix $R_{ij}^* = (1, \beta^2, \beta^2)$.

As it was already mentioned, the time delays plays a crucial role in the propagation of sound waves. Indeed, since the sound wave travels at finite speed, the observer $\mathbf{x}$ will receive the signal emitted at τ only after the wave covered the source-observer distance r . It is clear that the computation of time-delays would require conspicuous data storage; however, if the *Maximum Frequency Parameter* (Cianferra et al., 2019a) satisfies the condition:

$$\frac{1}{MFP} = \Delta_{del} f_{max} < 1, \quad (13)$$

the effect of compressibility can be ignored and the sound pressure wave can be considered traveling at infinite speed. In Eq. (13), Δ_{del} is given by:

$$\Delta_{del} = \frac{\max_{\mathbf{y} \in S} |\mathbf{y} - \mathbf{x}_{mic}| - \min_{\mathbf{y} \in S} |\mathbf{y} - \mathbf{x}_{mic}|}{c_0}, \quad (14)$$

where f_{max} is the highest frequency at which the fluid dynamic phenomena is observed. f_{max} is chosen to be representative of the flow phenomena. From a practical point of view, once f_{max} is estimated from fluid dynamic data, to satisfy the condition $MFP > 1$ corresponds to limit the value of Δ_{del} .

2.3. Computational setup

We perform LES of the turbulent flow around the BB2 submarine, which is a modification of the BB1 submarine (Joubert, 2004, 2006). It is representative of a standard Diesel submarine, with a large casing, two fore fins placed on the side of the sail and an X-form rudder (Overpelt et al., 2015). The Reynolds number considered is $Re_L = LU_\infty \nu^{-1} = 1.2 \times 10^6$, with $L = 3.826$ m the submerged length at model scale, U_∞ the far field velocity equal to 3 m s^{-1} and ν the kinematic viscosity of the fluid. The submarine is advancing in straight ahead conditions. Fig. 1 shows the BB2 in its fully appended configuration without propeller and the frame of reference adopted. The origin of right-handed coordinate system is located at the stern. With x directed forward, y to port side and z vertically upward.

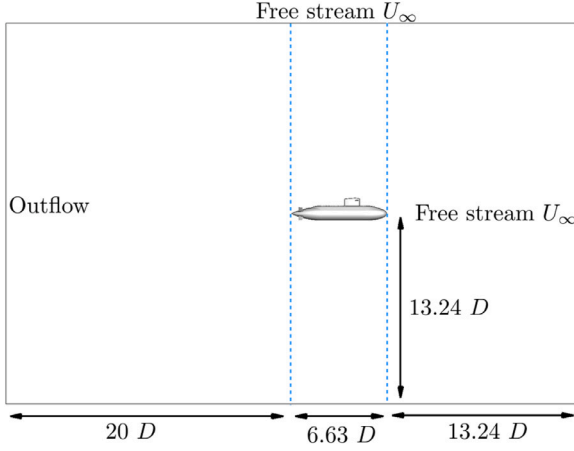
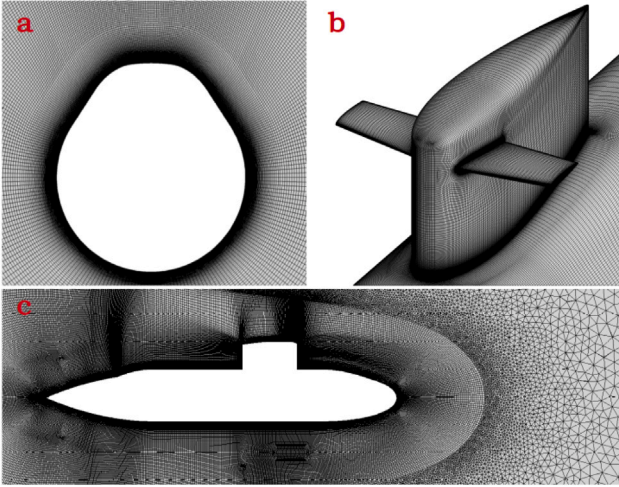
2.3.1. Computational fluid domain and grid

The computational domain is a rectangular box, whose its longitudinal mid-plane section is depicted in Fig. 2. Along the y - and z -directions the domain bounds are located at $\pm 13D$, with D the diameter of the hull at $x = 0.5L$. The distance of the stern from the outlet is $20D$, while the distance between bow and inlet is $13D$. The grid employed for the WLES is composed of 40,000,000 cells. Blocks composed by hexahedral cells are used in the near body and the wake regions, while tetrahedral cells are used to fill the rest of the domain. The present grid is a modification of that of Bettie (2020), for Wall-Resolved RANS.

Table 1

Evaluation of LES grid quality parameters.

Quality criteria	Value
Max cell-openness	10^{-15}
Max aspect-ratio	52
Min volume	10^{-12}
Max non-orthogonality	74.57
Average non-orthogonality	15
Max-skewness	2.22

**Fig. 2.** Longitudinal mid-plane section of the computational domain.**Fig. 3.** Computational grid: (a) plane W ($x/L = 0.5$); (b) sail and sail planes; (c) transverse plane Y ($y = 0$).

Modifications were directed to adapt it to WLES and to maintain high resolution also in the wake. Focusing on the main-body ($0.25 < x/L < 0.75$), where a fully developed turbulent boundary layer is expected, the heights of the first layer of cells in wall normal direction is of 50 wall units; the resolution in streamwise and circumferential directions is $\Delta x^+ = 100$ and $\Delta \theta^+ = 100$ respectively. [Table 1](#) contains the values of the grid quality parameters obtained using the OpenFOAM utility *checkMesh*. [Fig. 3](#) shows three views of the grid: a section at $x/L = 0.5$, the sail and sail planes and a section at plane $y = 0$.

2.3.2. Solver and numerical schemes

The solution algorithm for pressure-velocity coupling is a customized PISO scheme, with 2 corrections loop and one extra loop for non-orthogonality. The solver used for the pressure is the iterative Preconditioned Conjugate Gradient (PCG) coupled with a Faster version

of the Diagonal-based Incomplete Cholesky preconditioner (FDIC). The solver used for the advective term is the Preconditioned Bi-Conjugate Gradient (PBiCG), with Diagonal-based Incomplete LU preconditioner (DILU).

[Table 2](#) collects the relevant information about spatial discretization schemes used in LES. In order to improve the stability, especially at the interface between tetrahedral and hexahedral grid elements, the LUST scheme has been adopted. It is a blend between linear (75%) and second order upwind (25%). Time derivative was discretized with an explicit Euler scheme, and the time step size is $1.57 \times 10^{-4} t_c$ (with $t_c = \frac{U_\infty}{L}$) which guarantees $CFL = 0.5 < 1$.

2.3.3. Boundary and initial conditions

At the inlet the velocity field is considered uniform and equal to U_∞ and a zero gradient (in the direction normal to the inlet boundary) condition is imposed for pressure. At the outlet, the streamwise velocity gradient is set to zero while the gauge pressure is zero. A zero-gradient condition was applied to the physic variables at top, bottom, and the two lateral patches of the computational domain. No-slip boundary condition was enforced on the solid walls together with a wall-model, based on the Spalding's continuous formulation of the law of the wall ([Spalding, 1961](#)).

As initial condition we use a steady-state RANS solution obtained using the same grid at $Re_L = 1.2 \times 10^6$. In this case the turbulence model adopted was a standard $k - \epsilon$ with the Spalding's continuous wall-function.

2.3.4. Numerical stimulator of turbulence

The velocity at the inlet patch is uniform and equal to U_∞ . Starting from the bow of the submarine, where a stagnation point is present, a laminar boundary layer develops up to a transition to turbulence. Likewise laboratory experiments, in order to stimulate the transition to turbulence, a numerical stimulator is placed in the bow region. It consists of a random volume forces, with zero mean value, applied over a small portion of the submarine surface. [Fig. 4](#) shows the trace of the tripping loads on the submarine's surface ($0.9 \leq x/L \leq 0.925$). In this case the transition occurs at $x/L \approx 0.84$. The body forces are generated at every step of the PISO loop, based on the resolved LES velocity, on the mean velocity and on the amount of turbulent kinetic energy (both computed in the precursor RANS, which becomes part of the overall model).

2.4. RANS simulations of the BB2

The present section presents the numerical settings of the RANS simulations. We show them because RANS results have been utilized both as initial condition of LES and for comparison with LES. RANS has been performed with and without wall models. The grid utilized in these two cases is the same with the exception of the first wall-adjacent layer of cell, whose the cell height has been modified to obtain a $y^+ = 1$ in the case of wall-resolved simulation, or $y^+ = 50$ in the case of wall-function. The grid has been selected after a mesh refinement study. The Large-Eddy Simulation has been conducted at the same Re number of the RANS, using the WM-RANS solution as initial condition.

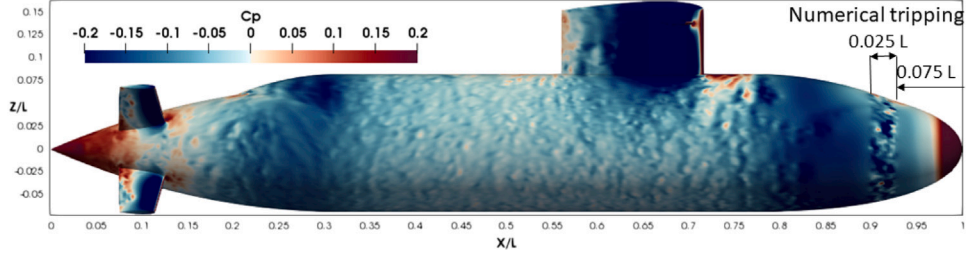
The RANS results (computed with OpenFOAM) were validated with those obtained by the other participants in the AVT-301 collaborative work-group.

The use of wall-functions at these relatively low Reynolds number is not straightforward. We experienced many issues about setting up suitable numerical parameters for this case and choosing the best performing turbulence model. The $k - \omega$ SST equipped with wall-functions responded badly in estimating forces and the wall-shear stress close to the bow. For this reason we adopted the classic $k - \epsilon$ turbulence model and hereby we propose a comparison between the results from the wall-resolving simulation obtained using $k - \omega$ SST and the wall-modeled simulation obtained using the $k - \epsilon$ (see [Fig. 7](#)).

Table 2

LES discretization schemes.

Item	Discretization	Type
$\nabla \bar{U}$	Gauss cellMDlimited Gauss Linear 1	limited central differencing
$\nabla \bar{p}$	Gauss linear	central differencing
$\nabla \cdot (\phi, \bar{U})$	Gauss LUST	75% linear and 25% 2nd order upwind
$\nabla \cdot \bullet$	Gauss linear	central differencing
$\nabla \cdot v(\nabla \bar{U})^T$	Gauss linear	unbounded central differencing
∇^2	Gauss linear corrected	unbounded central differencing

**Fig. 4.** Body forces of the numerical stimulator of turbulence acting on the submarine.**Table 3**

Discretization schemes wall-resolving and wall-modeled RANS simulation.

Item	WR-RANS	WM-RANS
∇U	cellMDlimited Gauss linear 0.5	cellLimited Gauss linear 0.5
$\nabla \{p, k, \omega\}$	Gauss linear	Gauss linear
$\nabla \cdot (\phi, U)$	bounded Gauss linearUpwind Gauss linear	bounded Gauss linearUpwind cellLimited Gauss linear 0.5
$\nabla \cdot (\phi, k)$	bounded Gauss linearUpwind Gauss linear	bounded Gauss linearUpwind cellLimited Gauss linear 0.5
$\nabla \cdot (\phi, \omega)$	bounded Gauss upwind	bounded Gauss upwind
$\nabla \cdot v(\nabla U)^T$	Gauss linear	Gauss linear
∇^2	Gauss linear limited corrected 0.5	Gauss linear limited corrected 0.5

Table 4

RANS initial conditions.

Item	Units	Value	Description
U	[m/s]	(-3 00)	mean velocity field
p	[m ² /s ²]	0	mean kinematic pressure
k	[m ² /s ²]	0.00135	turbulent kinetic energy
ω	[s ⁻¹]	150	specific dissipation rate
ϵ	[m ² /s ³]	0.015	rate of dissipation of turbulent kinetic energy

2.4.1. Solver and numerical schemes (RANS)

The solution algorithm for pressure-velocity coupling used is SIM-PLC scheme (Semi-Implicit Method for Pressure Linked Equations Consistent) (Patankar, 1980). The solver used for the pressure is the standard OpenFOAM multigrid GAMG with Gauss-Seidel smoother. The solver used for the momentum and the other transport equations is a stabilized preconditioned bi-conjugate gradient, PBICGStab (for asymmetric matrices), with the diagonal incomplete-LU preconditioner.

Table 3 collects all the relevant information about spatial discretization schemes used in RANS computations. Obviously this is not the only possible choice. Based on our tests, the combination of schemes reported gives superior behavior for the flow conditions analyzed in the present research. Specifically an intensive campaign of numerical tests was carried out, simulating the turbulent flows at different Reynolds numbers and using a variety of strategies for the treatment of the near-wall region.

Table 4 shows the initial conditions of the RANS, the boundary conditions are listed in the following Table 5, grouped by patches. With **sub-walls** we intend the solid boundaries of the submarine: hull, sail, sail plane, rudders.

As a final remark, the convergence history, for both WR-RANS and WM-RANS, shown that pressure residuals drop about three orders of magnitude. The residuals of the three momentum equations and of the

two turbulence model equations dropped by more than five orders of magnitude.

3. WLES results

3.1. Overview of the flow field

We consider here the submarine in straight ahead advancement, as shown in Fig. 5. Initially we focus on the bottom side of the hull ($z < 0$). Starting the flow description from the nose of the submarine, a first stagnation point occurs at the bow; here the velocity approaches to zero and the pressure coefficient increases to a maximum. The boundary layer is initially laminar and the flow develops under an adverse pressure gradient. At about $0.1L$ (L is the submarine length) from the bow, the boundary layer undergoes a transition to turbulence and the pressure gradient along the streamwise direction goes rapidly to zero. Despite the circumferential pressure gradient is not uniform, the friction coefficient resembles that of a canonical boundary layer over a Zero-Pressure-Gradient Turbulent Flat Plate (ZPGTFP). Moving downstream, close to the stern, the flow separates and a turbulent axisymmetric two-dimensional wake develops. As suggested by Townsend (1956), sufficiently far from the generator, the wake reaches a state of moving-equilibrium and becomes self-similar. Under these conditions, the mean velocity profile can be described using a velocity scale u_0 and a length scale l_0 only. The same does not hold for the Reynolds stresses that in the wake never attain self-similarity, as discussed in Jiménez et al. (2010a), Posa and Balaras (2016) and Kumar and Mahesh (2018).

The presence of the appendages makes the flow substantially more complex. They modify the turbulent field, in fact, the shear layer from the trailing edge of the fins affects the wake. This was shown experimentally by Jiménez et al. (2010b) and numerically by Posa and Balaras (2016). Further, the sail on the upper side of the hull generates an adverse pressure gradient that determines the formation of a horse-shoe vortex. This vortex structure surrounds the sail and it is advected

Table 5
Boundary conditions RANS simulations.

Item	Units	Inlet	Outlet	Sub-walls WR	Sub-walls WF
U	[m/s]	(-3 0 0)	zero gradient	noSlip	noSlip
p	[m ² /s ²]	zero gradient	0	zero gradient	zero gradient
k	[m ² /s ²]	0.00135	zero gradient	0	kqRWallFunction
ω	[s ⁻¹]	150	zero gradient	omegaWallFunction	-
ϵ	[m ² /s ³]	0.15	zero gradient	-	epsilonWallFunction
ν_t	[m ² /s]	calculated	calculated	0	nutUSpaldingWallFunction

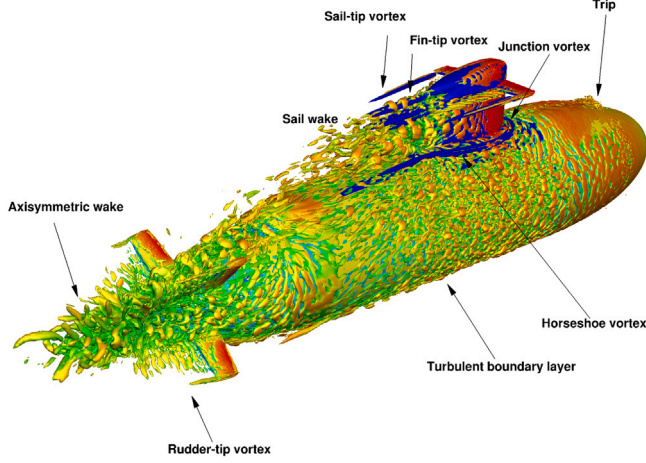


Fig. 5. Visualization of the turbulent structures around the BB2 submarine at Reynolds number $Re_L = 1.2 \times 10^6$. Q-criterion iso-surfaces ($Q \frac{L}{U_\infty} = 127$) colored by pressure coefficient.

downstream by the free stream current, interacting with the boundary layer over the hull. From the top side of the sail, additional vortices develop from the trailing edge and from the widest section of the sail (Fureby et al., 2016). The wake, on the side of the sail, cannot be considered axisymmetric due to the sail-wake effect on the velocity deficit at the center line of the wake. Further, three dimensional vortical structures arise from the sail, from the horizontal hydroplanes and from the rudders, whereas a couple of counter rotating vortices develop from the bottom of the hull.

Among these complex processes, turbulent wakes and boundary layers can be considered the fundamental fluid dynamic canonical flows developing around streamlined bodies as submarine vehicle. As suggested by Kumar and Mahesh (2018), far away downstream from the stern the wake attains self-similarity and Reynolds number independence.

3.2. Pressure and skin friction coefficients

Fig. 6 shows the mean resolved pressure coefficient over the submarine. Statistics have been computed with data sampled over the solid surfaces for $9t_c$, with $t_c = tU/L$. The pressure coefficient is defined as $C_p = (p - p_\infty)/(\frac{1}{2}\rho U_\infty^2)$. Fig. 7 panel (a) shows the time-averaged pressure coefficient over the intersection curve between the submarine and the semi-plane $z \leq 0$. Comparison is carried out with RANS results, both for the wall-resolved mode and for the wall-modeled one, and with experimental data by Huang et al. (1992) for DARPA submarine. Experiments were conducted at $Re=12 \times 10^6$. On the central region, $0.3 < x/L < 0.7$, all the numerical and experimental data are in a rather good agreement. DARPA's data move away at the bow and at the stern, probably due to the geometry differences between DARPA and BB2.

Fig. 7 panel (b) shows the skin-friction coefficient C_f on the same location as for the C_p . C_f is defined as $C_f = \tau_x/(\frac{1}{2}\rho U_\infty^2)$. The comparison is made again between WLES, scaled experimental data by Huang

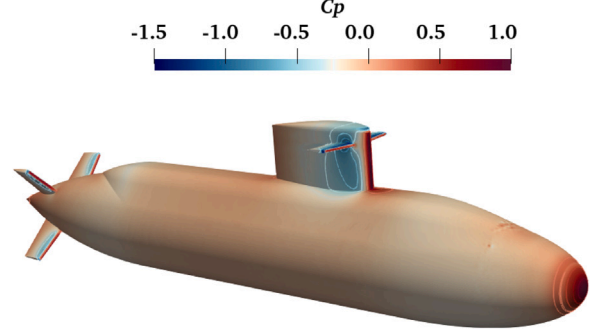


Fig. 6. Time-averaged pressure coefficient.

et al. (1992) and Anderson et al. (2012) and the two RANS simulations (WR and WM) carried out together with the present LES. WLES underestimates C_f , whereas wall-resolved RANS shows rather good agreement with experiments. RANS with wall-model overestimates C_f in the central region of the hull. Possible causes of disagreement are the different value of the Reynolds number among the cases compared in Fig. 7 and the stimulation of turbulence. Specifically, for the cases compared in the figure Reynolds similarity is not expected and the friction coefficient may be sensitive to the values of Re ; Also, Fig. 7 shows that the main difference arises in the region of the bow and this difference is translated back over the region of the ZPGTFP. Probably a better mechanism to trigger turbulence is required. This is an issue worth of future investigation. However, it has to be highlighted that, since we are focused on the hydro-acoustics response of the submarine, our interest is not on the flow field close to the wall by itself, rather on the flow field outside the boundary layer and in the wake. With this focus, the results of our WLES is fully satisfactory; In fact, as it will be shown later on (Section 3.4), the WLES estimation of the flow field in the wake is in rather good agreement with available experimental data, theory and WLES predictions by other authors.

3.3. Turbulent boundary layer

Figs. 8 (a) and (b) show a cross-sectional picture of instantaneous velocity and vorticity contours over a plane located just behind the sail, at 50% of the submarine length (W-plane), while Figs. 8 (c) and (d) show the mean axial velocity u and the resolved non dimensional averaged turbulent kinetic energy k , made non dimensional with U_∞^2 . Looking at the instantaneous velocity and vorticity contours we observe the footprint of the sail and sailplane wake. The Turbulent Boundary Layer (TBL) surrounds the hull, depicting the characteristic eddies. It is well known that TBL is characterized by a complex dynamics (Pope, 2000). In the present case, since we parametrize the part of the boundary layer, up to $y^+ \approx 50$, we cannot see structures up the early part of the log-layer, because the grid does not resolve them (Piomelli and Balaras, 2002). The wall-model is able to model the effect of these structures in a statistical sense; with reference to Fig. 9(a), where the outermost part of the boundary layer is depicted, we can clearly see ejections of low-speed fluid from the wall (indicated

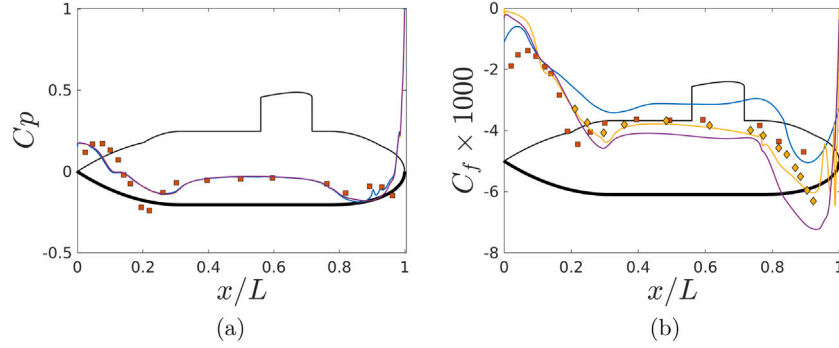


Fig. 7. Time-averaged pressure and skin friction coefficients over the bottom side of the hull. Present LES computation (—); Experimental data over DARPA-SUBOFF at $Re_L = 12 \times 10^6$ (■) (Huang et al., 1992); Experimental data over DSTO generic submarine (BB1) at $Re_L = 5.4 \times 10^6$ (◆) (Anderson et al., 2012); Wall-resolved RANS $k-\omega$ SST (—) at $Re_L = 1.2 \times 10^6$; Wall-modeled RANS $k-\epsilon$ (—) at $Re_L = 1.2 \times 10^6$; Experimental data were rescaled, based on the Reynolds numbers ratio (Posa and Balaras, 2016).

with a blue arrow pointing upward) and sweeps of high-speed fluid towards the wall (indicated with a black arrow pointing downward). In Fig. 9(b) streamwise vorticity contours suggest the presence of large eddies or bulges; They are δ scale motions capped in the outer turbulent/potential interface (Robinson, 1991). Considering that we have transition at about $0.2L$ from the bow (see Fig. 4), at $x/L = 0.4$ an estimation of the non-dimensional boundary layer thickness based on present LES simulation is $\delta/L = 0.013$ (non-dimensional boundary layer thickness from theoretical power-law is $\delta/L = 0.012$). Vorticity is distributed along areas inclined by about 20–25 degrees, in the direction of the free stream. These areas are separated by valleys of non-turbulent flow. Looking for tiny white areas in the picture, we can recognize valleys.

All these chaotic eddies give rise to a behavior which can be represented in a statistical way. In this central zone of the hull, the boundary layer can be considered similar to that developing over a ZPGTFP, so that it is possible to make comparison with well-established theoretical results (Pope, 2000; Davidson, 2004). Fig. 10(a) shows the mean axial velocity profile, made dimensionless using the friction velocity u_τ , as a function of the non-dimensional distance from the wall (expressed in wall units). The data are sampled along a line at $(x/L = 0.5, 0, z < 0)$. The sampling frequency is 100 Hz and the sampling period is equal to $9 t_c$. The velocity is plotted using two different values of the friction velocity: in the green dashed line it is scaled using the wall-shear stress coming from the present simulation, while in the dash-dotted blue line, it is calculated using the nominal wall-shear stress of the wall-resolved RANS. In this way it is possible to give a measure of the log-layer mismatch (Mukha, 2018; Mukha and Liefvendahl, 2015; Larsson et al., 2016; Yang et al., 2017). Following the definition given by Yang et al. (2017), the log-layer mismatch is a chronic problem affecting WLES, where the modeled wall-shear stress deviates from the true one by approximately 15%. Even more evident (and may be expected) is the mismatch for the second order statistics in comparison with available data for a ZPGTFP estimated with LES (Eitel-Amor et al., 2014), as shown in Fig. 10(b).

3.4. Turbulent axisymmetric wake

The validation of the LES wake is made against the theoretic axisymmetric wake model, and against numerical and experimental data available. Because of the submarine appendages, the analysis is limited to the semi-space $z < 0$.

Fig. 11 shows the instantaneous turbulent wake in terms of non-dimensional velocity contours on two planes: the horizontal plane H ($z = 0$) (figure (a)) and the transverse plane Y ($y = 0$), (figure (b)). In Fig. 11 panel (a), it is observed that the wake is that typically generated by a streamlined body, whereas, on plane Y (Fig. 11 panel (b)) the trace of the sail along the entire wake is evident. To develop

the analysis of the wake it is common to use a local frame of reference in cylindrical coordinates as the one shown in Fig. 12 (George, 1989). The origin of the frame of reference is located at the stern of the submarine and the streamwise direction is oriented as positive x . The study of the axisymmetric turbulent wake requires the definition of a set of new variables (Tennekes and Lumley, 1972; Pope, 2000). The velocity along the symmetry axis (Center Line) will be denoted with U_{CL} . The difference between the free-stream velocity U_∞ and U_{CL} is the velocity deficit, U_0 . The radial distance from the center-line to the radius where U_0 is zero is indicated by δ . The radial distance from the center line where the velocity deficit is $U_0/2$ is the half-wake width, l_0 . The turbulent axisymmetric wake can be characterized by these two scales U_0 and l_0 . From a practical point of view it is more convenient to calculate δ_* rather than l_0 , which is defined as:

$$\delta_*^2 = \lim_{R \rightarrow \infty} \frac{1}{U_0} \int_0^R (U_\infty - U) r dr. \quad (15)$$

Once δ_* is calculated, the derivation of l_0 is straightforward:

$$l_0 = \delta_* \sqrt{2 \ln 2}. \quad (16)$$

Taking the mean axial momentum equation and integrating its leading terms over a cross section (Tennekes and Lumley, 1972), yields:

$$U_0 \delta_*^2 = U_\infty \theta^2, \quad (17)$$

where θ is the momentum thickness, defined as:

$$\theta^2 = \lim_{R \rightarrow \infty} \frac{1}{U_\infty^2} \int_0^R U (U_\infty - U) r dr. \quad (18)$$

Moving downstream from the body, the wake decays and spreads, the mean velocity profiles changes, maintaining its own shape (self-similarity), as shown in Fig. 13.

The region of the wake close to the body is called “near wake”. Here the average axial velocity profiles $U(x, r)$ scaled by the velocity deficit U_0 and δ_* collapse over a single curve, as shown in Figs. 14. To allow an easier visualization of the data they have been plotted into two different figures: Fig. 14(a) shows the mean non-dimensional axial velocity profiles for locations in the region $10 < x/\theta < 59$; Fig. 14(b) shows the results for $65 < x/\theta < 107$ (up to $x/D = 18$).

We will follow the equilibrium similarity analysis formulated by George (1989) and extended by Johansson and George (2001), Johansson et al. (2003). Differently from the classical theory, here the local Reynolds number plays a fundamental role. Two different equilibrium similarity solutions exist, and they depend on the local Reynolds numbers. So it is possible to make a distinction between “high local Reynolds number” and “low local Reynolds number” solutions. The mathematics yielding these two formulations is accurately formulated and developed in Johansson and George (2001), Johansson et al. (2003); When reaching the state of equilibrium self-similarity, the

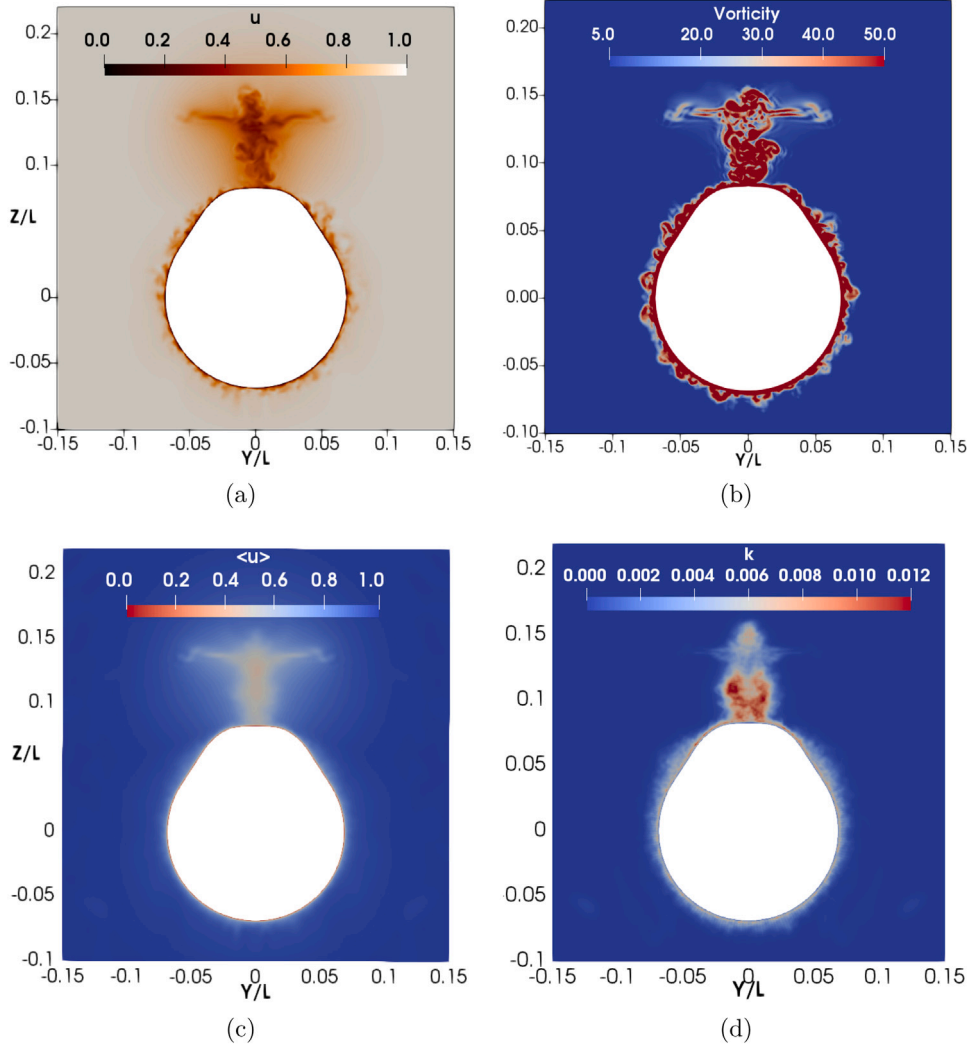


Fig. 8. Instantaneous and time-averaged flow quantities at the plane W: (a) axial velocity field $u = U/U_\infty$; (b) Vorticity magnitude $|\omega| * U_\infty/L$; (c) Mean axial velocity field $\langle u \rangle = \langle U \rangle/U_\infty$; (d) mean turbulent kinetic energy $k = \frac{u^2 + v^2 + w^2}{2}$, made non dimensional with U_∞^2 .

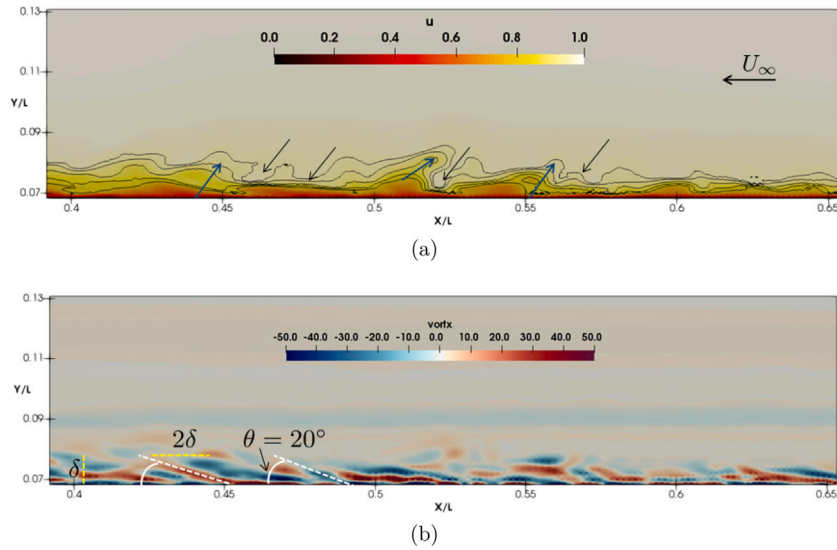


Fig. 9. Instantaneous physical quantities at the plane H: (a) axial velocity field $u = U/U_\infty$; (b) non dimensional x-vorticity $\omega_x * U_\infty/L$.

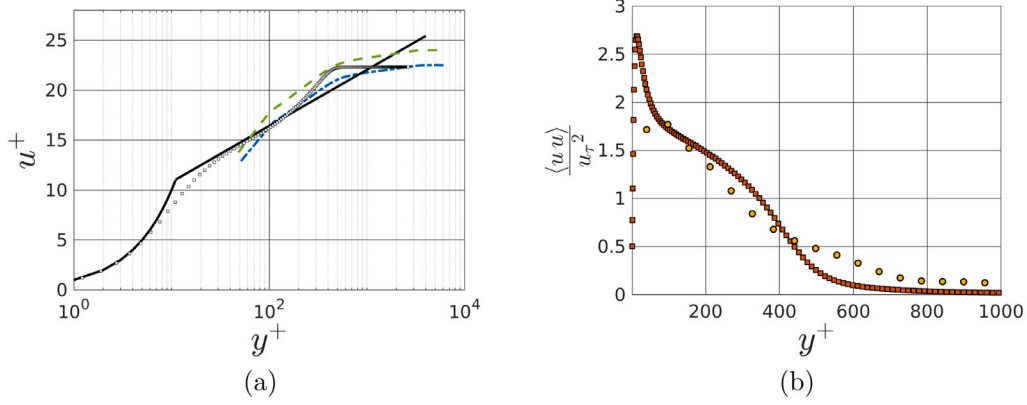


Fig. 10. (a) First order statistics. The law-of-the-wall (—); Present computation (—); Present computation scaled using u_τ from Wall-resolved RANS (—); Wall-resolved LES data of turbulent flat plate at $Re_\theta = 1410$ from Eitel-Amor et al. (2014) (□); (b) Second order statistics. Present computation (○) Wall-resolved LES data of turbulent flat plate at $Re_\theta = 1410$ from Eitel-Amor et al. (2014) (■);.

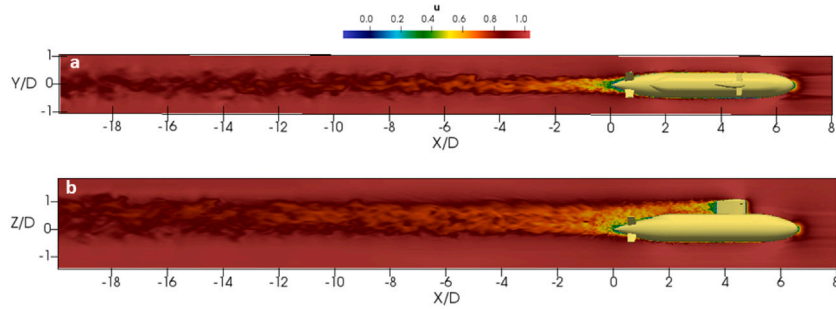


Fig. 11. The turbulent wake: contours of instantaneous axial velocity $u = \bar{U}/U_\infty$. (a) Horizontal plane H ($z = 0$). (b) Transverse plane Y ($y = 0$).

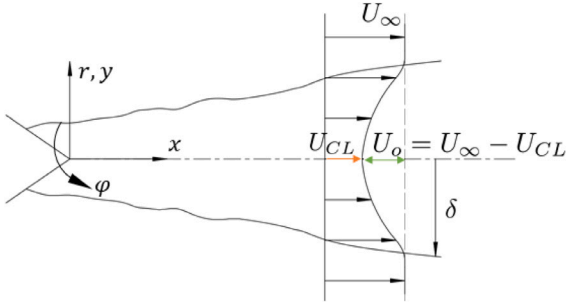


Fig. 12. The turbulent wake: wake's coordinate system and schematic (George, 1989).

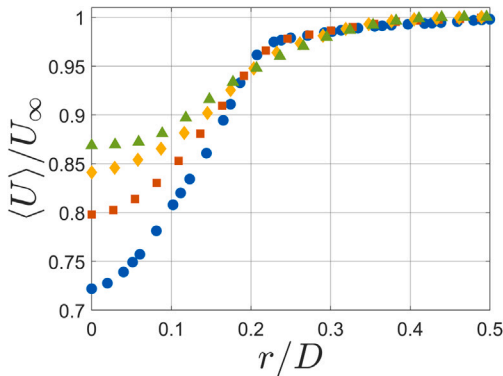


Fig. 13. The turbulent wake: mean velocity profiles scales by far field velocity: $x/D = 3$ (circles); $x/D = 6$ (squares); $x/D = 9$ (diamonds); $x/D = 12$ (triangles).

wake continues to evolve and decays, but the terms in the governing equations (momentum and Reynolds stress components) maintain the same relative importance. During this equilibrium decaying process, the local Reynolds number monotonically decreases. For high local Reynolds number, viscous diffusion is negligible compared to inertial effects, and an infinite-Reynolds-number solution does exist. Momentum is conserved and the ratio between the maximum (radial direction) of the mean axial velocity fluctuation over the mean velocity deficit, $\max(u)/U_0$, is constant. Moreover, in this region the velocity spectrum exhibits a developed inertial subrange region. In accordance with this theory, when the local Reynolds number drops below a certain level, the wake goes into unsteadiness, in the sense that the relative importance in the terms in the flow equations changes, evolving into a new equilibrium state; In this zone, $\max(u)/U_0$ assumes a constant value again, which is different from the value in the high-Re regime. This new equilibrium regime is called *low-Reynolds*. According to theoretical arguments of Gourlay et al. (2001) (supported by DNS data analysis), in this region $\delta_* \sim x^{1/2}$ and $U_0 \sim x^{-1}$. Johansson et al. (2003) and Gourlay et al. (2001), analyzed the wake generated by a disk and a sphere and found that the transition from high-Re to a low-Re regime occurs at downstream distances of $x/\theta \approx 500-1000$ (and Re_{l_0} about 500). Following Johansson et al. (2003), Kumar and Mahesh (2018) found that for streamlined bodies, i.e. the axisymmetric not-appended DARPA submarine, the transition from high-Re to low-Re solution occurs at $x/D \approx 2$ and local Reynolds number $Re_{l_0} \approx 7000$. The velocity spectra evolution did not exhibit any change in the inertial region, apparently contradicting the analysis of George (1989). On the other hand, Posa and Balaras (2016) in the analysis of the same submarine with appendages, did not observe the transition to low-Re solution (they studied the wake up to a distance of $12 D$ from the stern).

Johansson et al. (2003) found that, in the high-Reynolds number regime, similarity scales U_0 and l_0 follow power-law relationships of

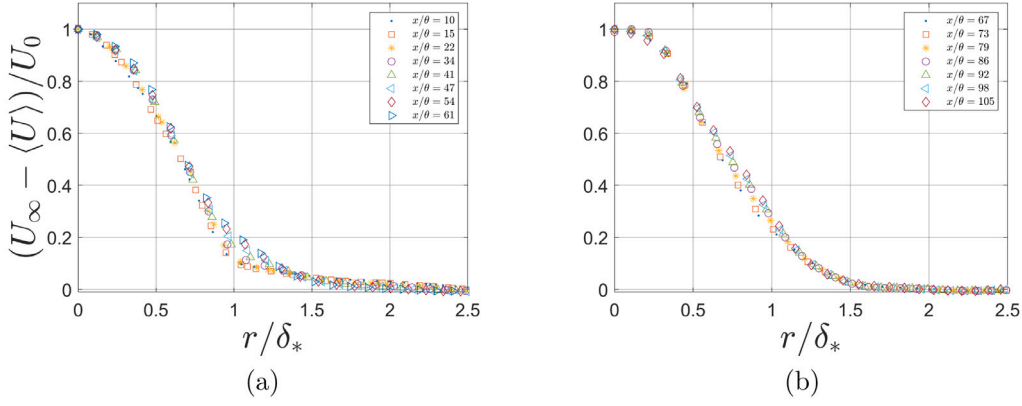


Fig. 14. The turbulent wake: mean axial velocity against radial distance scaled by velocity deficit and the wake's width. Data are averaged between the semi-planes at $\varphi = 0$ and $\varphi = \pi$.

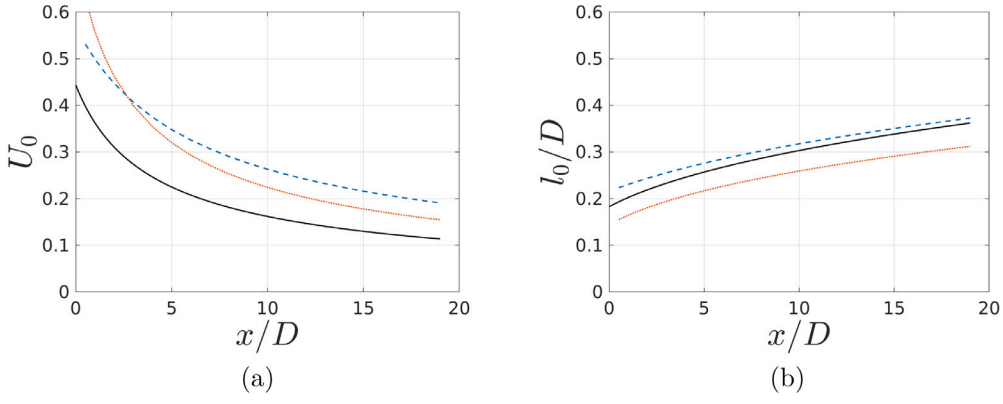


Fig. 15. The turbulent wake: data fitting for velocity and length scales. Present LES (—); Posa and Balaras (2016) (dashed line); Jiménez et al. (2010b) (dotted line).

Table 6
The turbulent wake: power laws relationships coefficients for u_0 and l_0 .

Item	Re_L	A	B	x_0/D
Present LES	1.2×10^6	0.89	0.13	2.83
Jiménez et al. (2010b)	1.1×10^6	1.18	0.113	2.08
Posa and Balaras (2016)	1.2×10^6	1.57	0.13	4.58

Table 7
The turbulent wake: power laws relationships coefficients for δ_*/θ .

Item	Re_L	A	x_0/D
Present LES	1.2×10^6	1.16	1.89
Kumar and Mahesh (2018) High-Re	1.1×10^6	1.17	2.08
Kumar and Mahesh (2018) Low-Re	1.1×10^6	0.78	2.08

the form:

$$U_0 = A \left(\frac{x + x_0}{D} \right)^{-2/3} \quad \text{and} \quad (19)$$

$$l_0 = B \left(\frac{x + x_0}{D} \right)^{1/3}. \quad (20)$$

This is clearly the same result as in the physical law of classic theory (Townsend, 1956). Applying the similarity scales of Eq. (20) to the present LES of the BB2 submarine, we can obtain the value of A , B and x_0 using a non-linear fit. In particular, the best fit is obtained by minimization of the squared residuals σ^2 and keeping x_0/D to be equal in the two relationships:

$$U_0 = 0.887 \left(x/D + 2.827 \right)^{-2/3} \quad \text{and} \quad (21)$$

$$l_0 = 0.129 \left(x/D + 2.827 \right)^{1/3}. \quad (22)$$

For comparison purposes, Table 6 lists the present power laws, together with the power laws proposed by Jiménez et al. (2010b) and Posa and Balaras (2016) based on experimental and numerical data respectively.

Fig. 15 shows the behavior of $u_0 = U_0/U_\infty$ and l_0 during the spatial evolution of the wake. The correlations for the DARPA-Suboff

proposed by numerical work of Posa and Balaras (2016) (dashed lines) and experimental data from Jiménez et al. (2010b) (dotted) are shown for comparison.

Fig. 16 shows the local Reynolds number evolution. At 18 diameters downstream from the stern its value is still rather high. As shown in Fig. 17, the present solution is well fitted using a power of 1/3, and it is close to the high-Re solution of Kumar and Mahesh (2018). Table 7 shows comparison between the curve fit coefficients for δ_*/θ .

In addition, looking at the velocity spectra of Fig. 18 we do not see changes in the extension of the inertial subrange region which, as suggested by Johansson et al. (2003), would predict the transition of the wake from high- to low-Re regime. Further, looking at the ratio between turbulence velocity fluctuations and velocity deficit, Fig. 19, we can assert that turbulence governing laws are still evolving to a state of equilibrium, as intended by George (1989). For all these reasons we may argue that the present wake is still in a high-Re number solution state.

Fig. 20 shows the turbulent kinetic energy profiles at 3, 6, 9 and 12 diameters from the stern. Solution at 9D is in rather good agreement with numerical data from Posa and Balaras (2016) at the same distance from the stern. The two peaks characterizing k in the horizontal plane derive from the turbulent boundary layer advected downstream (Posa

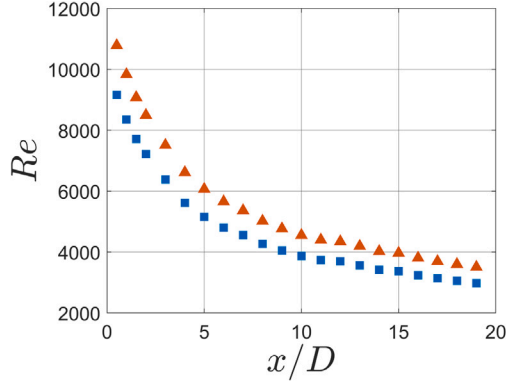


Fig. 16. The turbulent wake: the local Reynolds number $Re_{l_0} = U_0 l_0 / \nu$ (orange triangles) and $Re_{\delta_*} = U_0 \delta_* / \nu$ (blue squares).

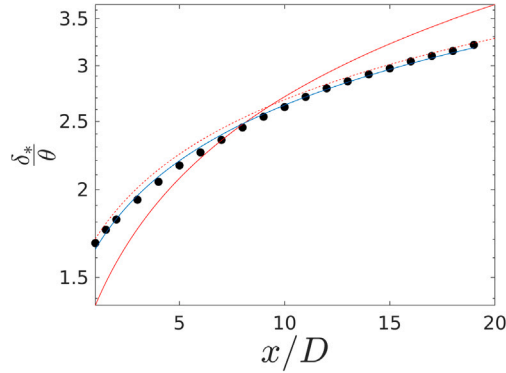


Fig. 17. The turbulent wake: evolution of the wake width made non-dimensional by momentum thickness. Present simulation ($\bullet$). Power law for the present simulation $\delta_* / \theta = 1.16(x/D + 1.89)^{1/3}$ (—); High-Re solution $\delta_* / \theta = 1.17(x/D + 2.08)^{1/3}$ (---) (Kumar and Mahesh, 2018). Low-Re solution $\delta_* / \theta = 0.78(x/D + 2.08)^{1/2}$ (—) (Kumar and Mahesh, 2018).

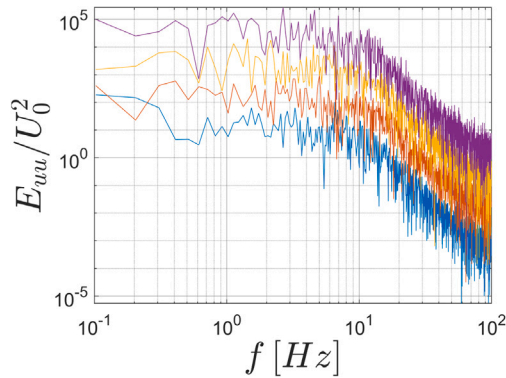


Fig. 18. The turbulent wake: evolution of streamwise velocity spectra. 6D (—); 9D (—); 12D (—); 15D (—); (Data are moved upward as: at 6D, $E_{uu} \times 10^0$; at 9D, $E_{uu} \times 10^1$; at 12D, $E_{uu} \times 10^2$; at 15D, $E_{uu} \times 10^3$).

and Balaras, 2016). Away from the stern the velocity deficit rapidly decreases, and turbulent fluctuations maintain their intensity also in the far wake (Gourlay et al., 2001; Johansson et al., 2003). The two peaks of turbulent kinetic energy are also clearly visible looking at the color plot of k/U_∞^2 in the near-wake region, Fig. 21.

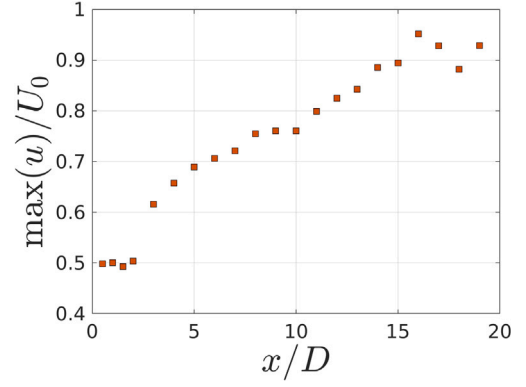


Fig. 19. The turbulent wake: evolution of maximum velocity fluctuations scaled by the velocity deficit.

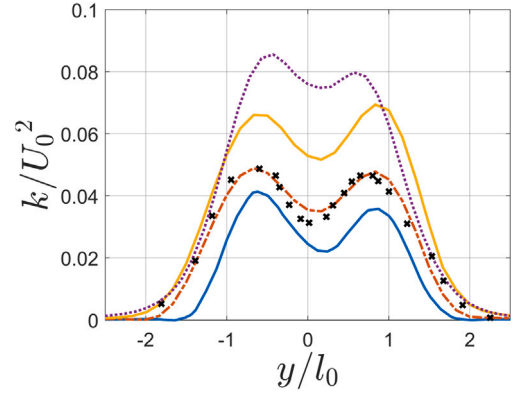


Fig. 20. The turbulent wake: evolution of the non-dimensional turbulent kinetic energy. 6D (—); 9D (—); 12D (—); 15D (—); numerical data from Posa and Balaras (2016) 9D($\times$).

4. CHA of the fully appended BB2 submarine

This section presents the results of the hydroacoustic analysis conducted on the BB2 submarine.

The large-eddy simulation conditions are those of a standing observer receiving the sound in a moving water stream. Thus, the formulation of the FW-H equation adopted here is the formulation 1D, specific for wind tunnel problems, and given by the sum of Eq. (6) (Najafy-Yazdi et al., 2010) and Eq. (7) (Cianferra et al., 2019a). The hydroacoustic solver has been extensively validated in Cianferra (2017) and Cianferra et al. (2019a,b). Both the loading and the non-linear volume terms contribute to the noise, while the thickness term is null everywhere, as already discussed in Section 2.2.

The computation is carried out under the common hypothesis that the noise generated by viscous forces on the hull is negligible compared to pressure and turbulence contribution. As a consequence, the compression tensor P_{ij} can be approximated as:

$$P_{ij} = (p - p_0) \delta_{ij} - \sigma_{ij} \approx (p - p_0) \delta_{ij}, \quad (23)$$

where σ_{ij} is the viscous stress tensor. It has to be highlighted that, this approximation is consistent with the use of wall-functions.

During the LES, the pressure signal has been sampled at specific locations near the submarine to determine the maximum relevant frequency at which collect data. For instance, Fig. 22 shows the spectrum of the pressure signal sampled at a probe located in the plane behind the sail, with coordinates $x_p = (0.5, 0, 0.12) L$. The sampling period is about $1.5 \tau_c$. The maximum relevant fluid-dynamic frequency is equal to $f'_{max} = 100$ Hz. Where f' indicates the frequency at model scale. The

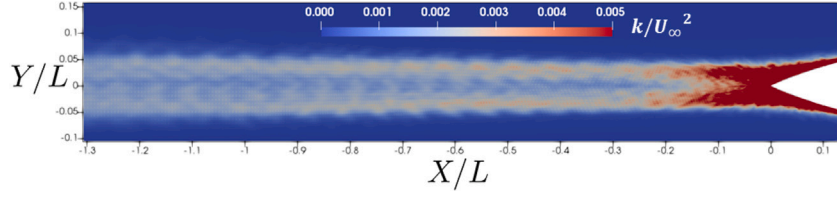


Fig. 21. The turbulent wake: turbulent kinetic energy in the near-wake region.

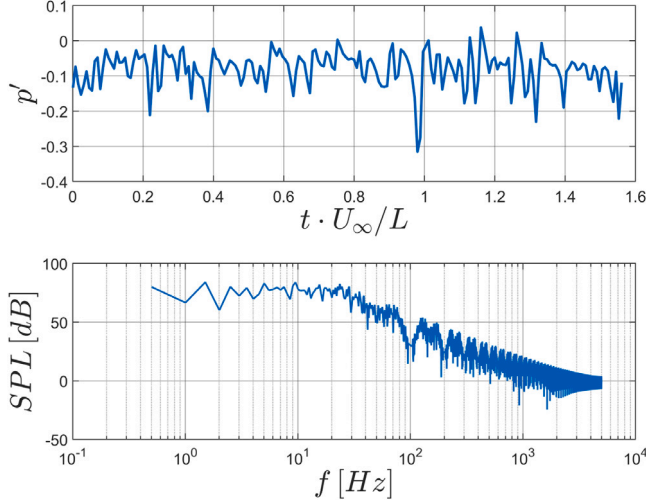


Fig. 22. Pressure signal and relative spectrum sampled at $x_p = (0.5, 0, 0.12) L$.

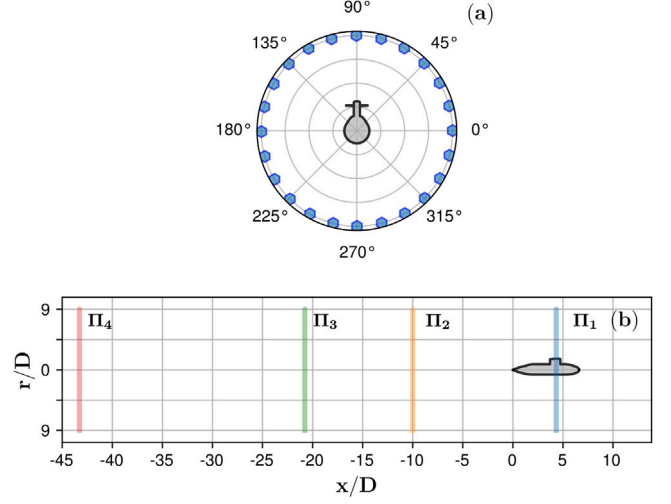


Fig. 23. (a) Front view of the receivers on a generic transversal plane. (b) Lateral view of the planes Π_1 , Π_2 , Π_3 and Π_4 .

spectra of the pressure signals sampled at other characteristic positions, as for example close to the hull, in the plane behind the sail or in the wake, decays even faster.

Since $f'_{max} = 100$ Hz the calculation of time delays can be avoided, because the condition on the MFP(Eq. (13)) is satisfied.

Following [Cianferra and Armenio \(2021\)](#), a relationship of perfect similarity between the acoustics of the submarine at model scale and its full scale representation can be established. Thus, the full scale acoustic field derives directly from the model scale fluid-dynamic computation reducing the speed of sound. The scaling is obtained through the geometrical scaling-factor of the problem λ , that in the present case is equal to:

$$\lambda = \frac{L_f}{L_m} = \frac{70.200 \text{ m}}{3.826 \text{ m}} = 18.348, \quad (24)$$

where L is the length of the submarine. The subscripts m and f indicate the “model-scale” and “full-scale” quantities, respectively. It follows that perfect similarity regarding the acoustic field is obtained using a model-scale speed of sound given by

$$c_{0m} = \frac{c_{0f}}{\sqrt{\lambda}} = 326.84 \text{ ms}^{-1}. \quad (25)$$

Obviously, scale effects associated to the different values of Re at the two scales cannot be removed.

The first analysis concerns the directivity of the acoustic signature at four distinct transversal planes: Π_1 , Π_2 , Π_3 and Π_4 . Each plane contains 24 hydrophones distributed along a circumference with radius $r/D = 9$ centered at $C = (x, 0, 0)$, where x denotes the position of the plane with respect to streamwise direction, as shown in [Fig. 23\(a\)](#). [Fig. 23\(b\)](#) shows a side view of the four planes and their axial position, i.e. $x/D = \{4.5; -10; -21; -43\}$ respectively. Planes Π_1 , Π_2 and Π_3 intersect the computational domain. The fourth plane is located in the very far field.

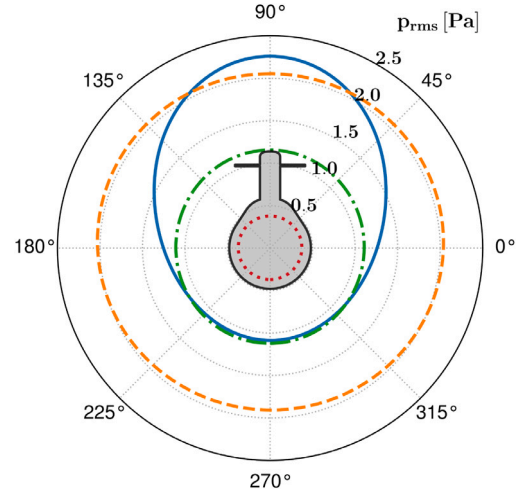


Fig. 24. Directivity patterns at planes Π_1 (—), Π_2 (---), Π_3 (-.-) and Π_4 (....).

[Fig. 24](#) shows the directivity patterns at transversal planes. Data are plotted in a polar-coordinate frame of reference (θ, p_{rms}) , where the radial coordinate indicates the root-mean-squared pressure at the observer location. At Π_1 the sound pressure draws an egg-like shape. To be noticed that the region of larger values of p_{rms} is found in correspondence of the sail. From Π_2 and beyond the profile of the directivity attains axial symmetry and the shape is almost circular. This indicates that the contribution of the sail has decayed and the axial-symmetric wake of the submarine dominates the acoustic response. At Π_3 the profile of the directivity is similar to Π_2 , almost circular although slightly stretched upwards; here the intensity of the signal has

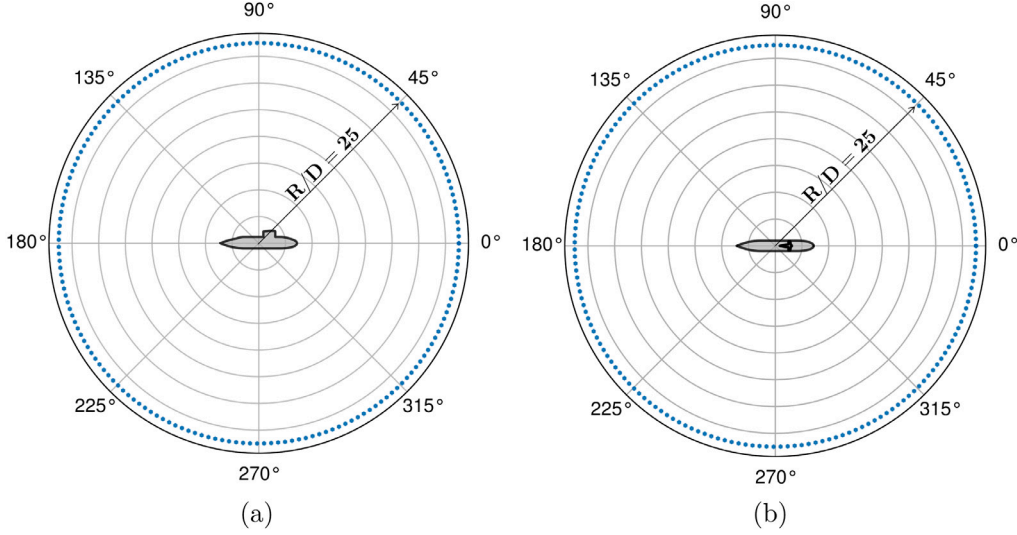


Fig. 25. Position of the hydrophones on: (a) plane XZ; (b) plane XY.

decayed more than 50%. The signal related to the plane Π_4 , located in the farfield, shows radial symmetry and p_{rms} is one order of magnitude smaller than those measured at Π_1 .

The second analysis concerns the directivity at the XZ and YZ planes. Fig. 25 shows a schematic of the source-observer system. In particular, panel (a) shows the hydrophones laying on the vertical plane, whereas panel (b) those laying on the horizontal plane. Each plane contains 180 receivers. Fig. 26, panel (a), shows the directivity pattern in the vertical plane. The energetic turbulent wake gives rise to a quadrupole-like acoustic signal. The lobe in direction of the wake (180°) is remarkably stretched and, as can be noticed, it is not perfectly symmetric with respect to the submarine longitudinal axis. This can be explained by the presence of the sail, which slightly enhances the acoustic signal in the range of angles ($135^\circ \leq \theta \leq 180^\circ$) with respect to the signal in the range ($180^\circ \leq \theta \leq 225^\circ$).

The lobe at 0° is characterized by p_{rms} more than four times smaller compared to that one in the wake direction. Recalling that, in this case, the solid body is transparent to acoustic waves, it can be concluded that the reduction of p_{rms} in this direction is a consequence of the flow advection and the non-homogeneous concentration of noise sources, which is foremost in the wake.

The two vertical lobes are even less intense because the pressure signal related to hydrophones orthogonal to the flow is dominated by sources depending mainly on transversal velocity components. Moreover, they are slightly inclined towards the direction of motion. Panel (b) shows the directivity on the horizontal plane. Similar arguments can be applied, with difference that in this case the directivity patterns are symmetric with respect to the longitudinal axis of the submarine.

As noticed, the flow field is streamwise dominated and the two lobes in y -direction are not perpendicular to the mean flow direction but slightly distorted downstream. The particular shape of the directivity, shown in Fig. 26, is strictly related to the nature of the acoustic source rather than the shape of the body. This can be demonstrated considering equation (7). In the present case we have that $M_0 = U_0/c_0 \ll 1$ and $\beta = \sqrt{1 - M_0^2} \approx 1$ (see Eq. (11)), and Eq. (7) can be recast in the more familiar form:

$$4\pi\hat{p}_Q(\mathbf{x}, t) = \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \int_V \left[\frac{T_{rr}}{r|1 - M_r|} \right]_r dV + \frac{1}{c_0} \frac{\partial}{\partial t} \int_V \left[\frac{3T_{rr} - T_{ii}}{r^2|1 - M_r|} \right]_r dV + \int_V \left[\frac{3T_{rr} - T_{ii}}{r^3|1 - M_r|} \right]_r dV \quad (26)$$

Looking at the kernel of the last two integrals, which dominate at intermediate distances, and using the Einstein's summation rule, yields

$$3T_{rr} - T_{ii} = 3(U_1r_1 + U_2r_2 + U_3r_3)^2 - (U_1^2 + U_2^2 + U_3^2), \quad (27)$$

where U_i and r_i are the instantaneous velocity and the component of the source-observer unit vector along the i -direction ($i = 1, 2, 3$), respectively.

Since we consider microphones at constant distance r from the source, we may assume that the shape of the directivity depends only on $3T_{rr} - T_{ii}$. In particular, the canonical quadrupole shape can be easily achieved by drawing the term $3T_{rr} - T_{ii}$ and considering $(U_1, U_2, U_3) = (a, a, a)$, with a an arbitrary value. Thus, the quadrupole with four identical lobes is obtained when the velocity components U_1, U_2 and U_3 have the same magnitude.

On the other hand, when the flow field has a dominant streamwise velocity, as in the present case, the quadrupole results deformed and elongated along the direction of the dominant velocity.

As an example, we may consider the two-dimensional case of the two point sources S_1 and S_2 , shown schematically in Fig. 27(a). The two sources are located at $P_1 = (0, D/2)$ and $P_2 = (0, -D/2)$; P_1 and P_2 are symmetric with respect to the streamwise direction. The two sources move with distinct velocities $U_1 = (-3a, a)$ and $U_2 = (-3a, -a)$. This configuration would be a representation of the flow field to the sides of the submarine, on the horizontal plane ($z = 0$). The receivers are located on a circumference with center $C = (0, 0)$ and radius $R = 1$.

Considering again only the last kernel of the non-linear part, which is predominant, it follows that the acoustic pressure $\hat{p}$ is proportional to

$$\hat{p}_1 \propto 3(-3ad_{x1} + ad_{y1})^2 - 10 \quad (28a)$$

$$\hat{p}_2 \propto 3(-3ad_{x2} - ad_{y2})^2 - 10 \quad (28b)$$

where d_x and d_y represent the projection of the source-receiver distance in x and y direction, respectively. The directivity pattern is the one depicted in Fig. 27(b) with blue solid curve. In this case the lobe at 0° and the lobe at 180° have the same maximum intensity but the rear lobe (180°) has higher aspect ratio. The two small vertical lobes are distorted and this is the effect of the y -velocity rather than the advection. In fact, suppressing the y -velocity components of the previous analysis, the directivity pattern is that one shown in Fig. 27(b) with the orange dashed curve, and in this case the two vertical lobes are perfectly aligned with the 90° and 270° directions.

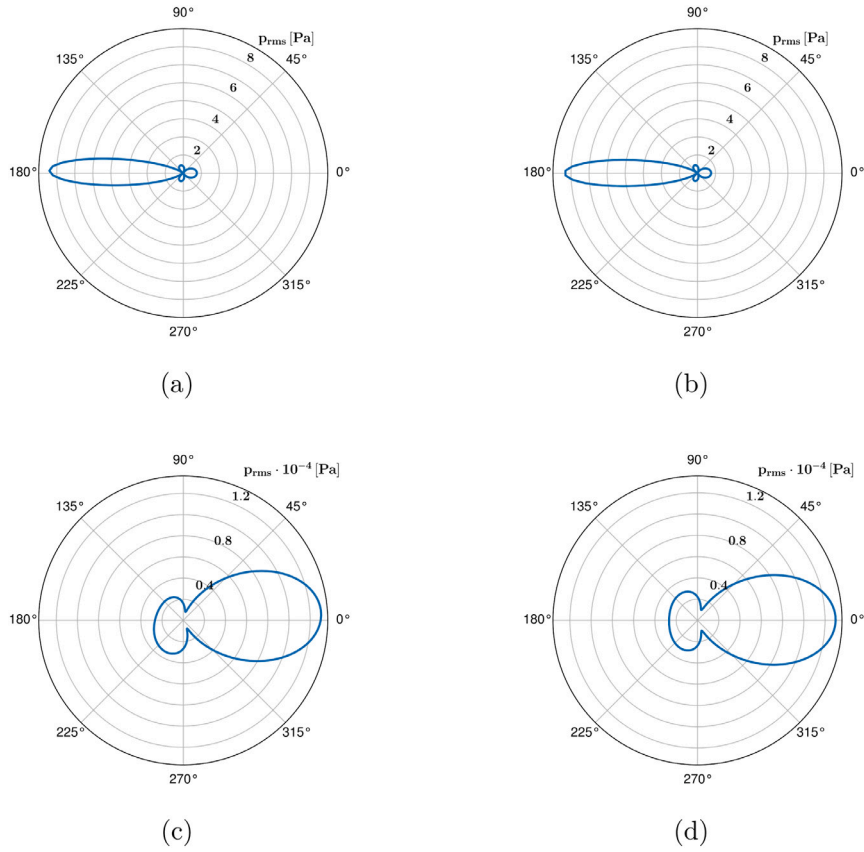


Fig. 26. Directivity patterns on: (a) XZ plane; (b) XY plane. Directivity pattern of the only loading term on: (c) XZ plane; (d) XY plane.

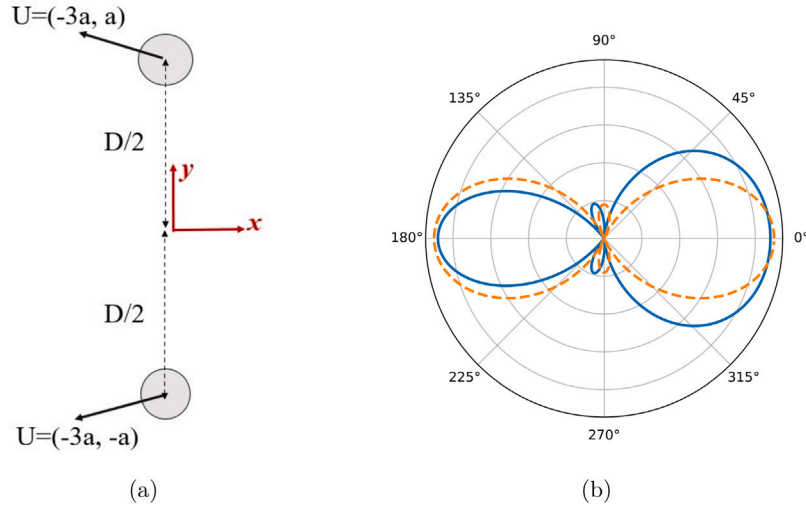


Fig. 27. Schematic of sources distribution (a); Directivity pattern of two sources located symmetrically with respect to the x direction and moving with velocities: $U = (-3a, \pm a)$, solid blue (—); $U = (-3a, 0)$, orange dashed (---).

Notice that in this analysis we do not consider the first volume term since it has little influence on medium distances. Panels (c) and (d) of Fig. 26 show the directivity of linear loading terms on the planes XZ and YZ (see Fig. 25), respectively. To be noticed that at the distance considered in this discussion, their magnitude is one order of magnitude smaller than that of the total acoustic signal, which appears dominated by the non-linear terms.

As expected, their directivity pattern resembles the one of a canonical dipole. The rear lobe is not symmetric because the pressure contribution coming from the bottom side of the hull is foremost. As

expected, the frontal lobe is the major one, because the static pressure is in general higher on the frontal side of the submarine due to the stagnation points at the bow, at the sail and the sail planes, as shown in Fig. 6.

Fig. 28 shows the Sound Pressure Level (SPL) at some receivers selected among those of Fig. 25(a), against frequency $f = f'/\sqrt{\lambda}$, relative to the prototype scale; here f' indicates the frequency at model scale. Panel (a) considers four locations, corresponding to 0° , 90° , 180° and 270° . Panel (b) shows only the spectra relative to the receivers at 90° and 270° , because those at 0° and 180° are the same as reported in

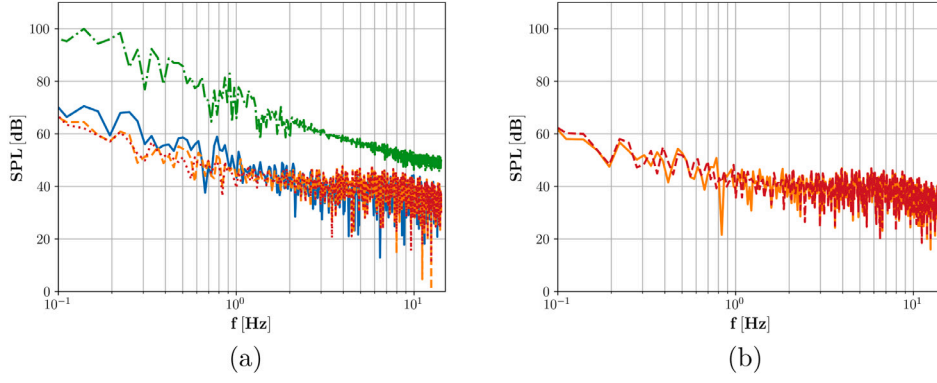


Fig. 28. Sound pressure level of the full-scale submarine on: (a) XZ plane at: 0° (—); 90° (---); 180° (-.-.); 270° (....); (b) XY plane at 90° (---); 270° (....).

Table 8

Coordinates of the receivers in the near submarine field.

Receivers	x/L	y/D	z/D
1–5	{1, 0.75, 0.5, 0.25, 0}	0	3
6–10	{1, 0.75, 0.5, 0.25, 0}	2	3

(a). The radial distance from the origin of the coordinate system of the receivers is $R/D = 25$.

The SPL is reported in decibels (dB) and calculated as

$$SPL = 20 \log_{10}(FFT(\hat{p}\sqrt{\lambda})/p_{ref}), \quad (29)$$

where FFT stands for Fast Fourier Transform (normalized by the number of sampling points) and $p_{ref} = 1 \mu Pa$ is the reference pressure adopted for water sound pressure level measurements.

The spectra at 90° and 270° are broadband at all locations. Conversely, the spectrum relative to the source at 180° presents a characteristic peak at about 0.90 Hz. The spectrum relative to the source at 0° presents also a characteristic peak at a frequency a bit less than 0.80 Hz. We can argue that these two peaks are related to turbulent structures released from the submarine, and the difference in frequency between them is due to two factors: the asymmetry of the source with respect to the two receivers and the mean flow direction.

To relate the spectra to specific fluid dynamic structures, ten hydrophones has been placed closer to the submarine, as shown in Fig. 29. Table 8 shows the coordinate of the receivers arranged in the near submarine fields.

The lift-coefficient C_L is employed as a measure of the pressure signal on the hull and its spectrum, expressed in decibels for better visualization, is compared with the sound pressure level at specific positions. Coherence measures the normalized correlation between the two signals and is expressed as (Welch, 1967):

$$C_{xy} = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}, \quad (30)$$

where $G_{xy}(f)$, $G_{xx}(f)$ and $G_{yy}(f)$ are, respectively, the cross spectral density of the signals x and y , the power spectral density of the signal x and the power spectral density of the signal y .

Fig. 30 shows a comparison of the spectra and the coherence between four representative receivers labeled as 3, 5, 8 and 10. In the low-frequency range, sound pressure level and C_L are not related. Panel (b) suggests that there are two characteristic frequencies where pressure fluctuations on the hull surface are strictly correlated to sound pressure level, namely 0.7 Hz and 1.7 Hz. Since the receiver 5 is located just above the stern, it is probably catching the noise associated with massive separations at the stern.

Fig. 31 shows the comparisons of the spectra at different downstream locations: $10D$, $15D$ and $20D$. Hydrophones are located at the bottom side of the hull, with $z/D = -1$, and distributed along

the $y/D = 0$ axis. As we can see, the noise associated with high frequencies grows going farther from the submarine. The reason of this behavior is that, close to the submarine, the characteristic length of the largest turbulent structures is of the same order of magnitude of the diameter of the main body. These vortices are characterized by large characteristic times and so they have low characteristics frequencies. The breaking up process of these vortices starts just downstream the propeller plane and, as a consequence, the concentration of small structures increases downstream. Conversely, small eddies have short characteristic time and thus high characteristic frequency. All these three spectra are characterized by a peak corresponding to a full-scale frequency of 0.6 Hz.

5. Conclusions

This paper presents original results concerning the hydroacoustic analysis of the BB2 submarine. Consistently with most literature, we did not consider the presence of the propeller. The methodology herein proposed has been shown to be adequate for both the flow field computation and the acoustic evaluation. Wall-modeled LES allows to compute turbulence at rather accessible cost keeping accuracy satisfactory, especially in the wake. The acoustic analogy, based on the Ffowcs Williams and Hawkings equation and here formulated in advective form for both linear and non linear terms (formulation 1D), allows to calculate the acoustic field by post-processing the fluid-dynamic data. Perfect similarity conditions between the acoustics of the submarine at model scale and at full scale allow to derive the full scale noise of the submarine starting from model-scale simulation data.

First we analyzed the hydrodynamic field and we found that in the near field, the wake of the superior half-plane is affected by the sail and the appendages whereas in the farther field it develops according to the equilibrium similarity theory for an axial-symmetric body. Regarding the acoustic field, we have processed the results to obtain information at the full scale.

The directivity on the horizontal plane ($z = 0$) and on the transverse plane ($y = 0$) is dominated by non-linear sources of noise and the pattern resembles that of a canonical quadrupole, distorted in the direction of the wake. It has been demonstrated that a qualitatively similar directivity pattern can be obtained by properly feeding the kernel of the integrals constituting the non-linear counterpart of the FW-H equation. An elementary system of two sources, characterized by a velocity analogous to that one observable close to the hull, contains all the features necessary to represent a schematic of the directivity.

The analysis of the sound pressure level shows rather broad-band spectra although at some particular frequencies they present characteristic peaks. Coherence analysis allowed to link these spectra with specific flow phenomena, namely the massive separations at the stern. The noise associated with high frequencies grows far from the submarine as a consequence of the breaking up process of the eddies.

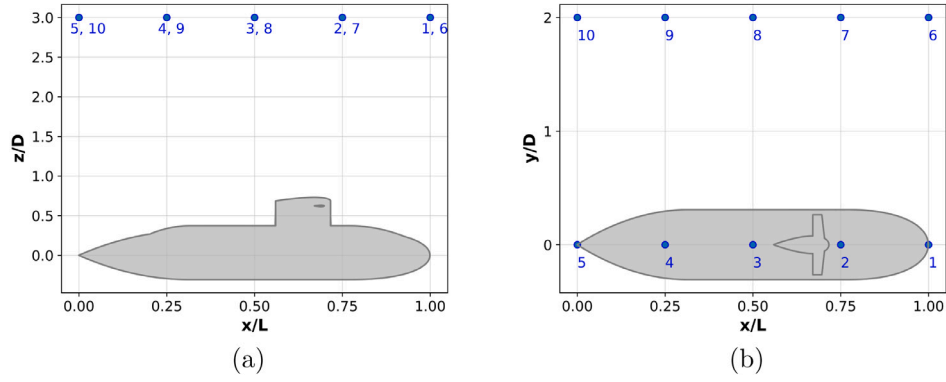


Fig. 29. Position of the receivers for comparative analysis between SPL and C_L .

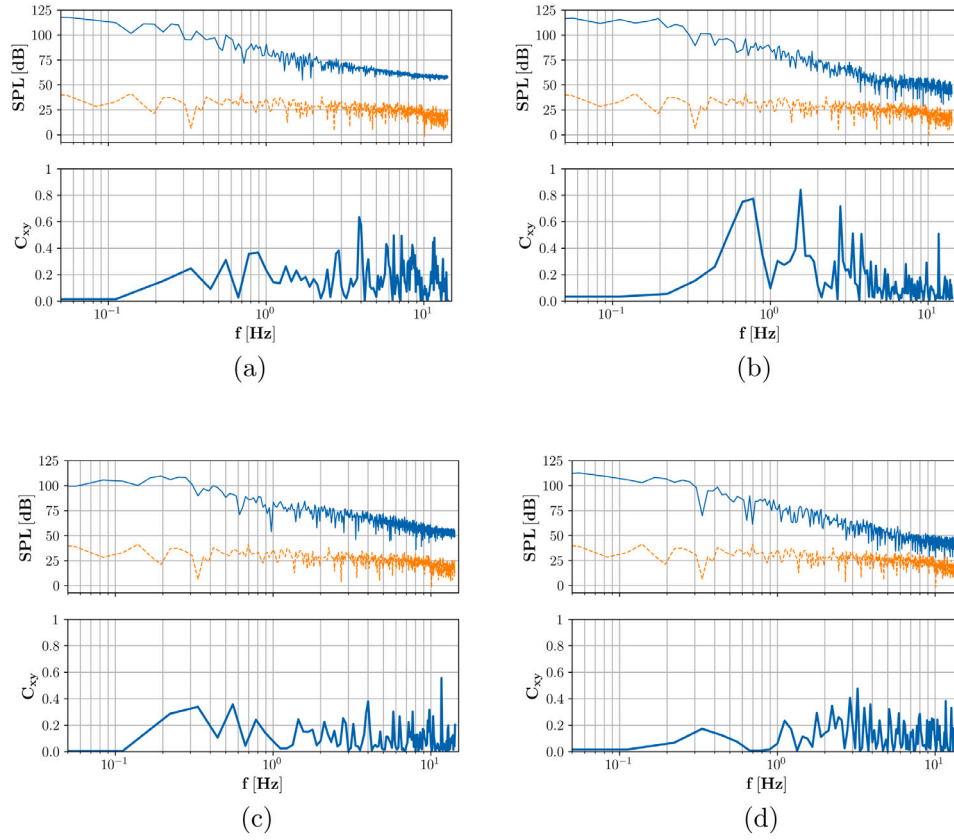


Fig. 30. (top panel) Comparison between SPL (—) and C_L (- -). (bottom panel) Coherence analysis between the two signals. (a) receiver 3; (b) receiver 5; (c) receiver 8; (d) receiver 10 (see Fig. 29).

CRedit authorship contribution statement

Andrea Rocca: Methodology, Software, Validation, Formal analysis, Investigation, Writing – original draft. **Marta Cianferra:** Methodology, Software, Validation, Formal analysis, Investigation, Writing – review & editing. **Riccardo Broglia:** Conceptualization, Writing – review & editing. **Vincenzo Armenio:** Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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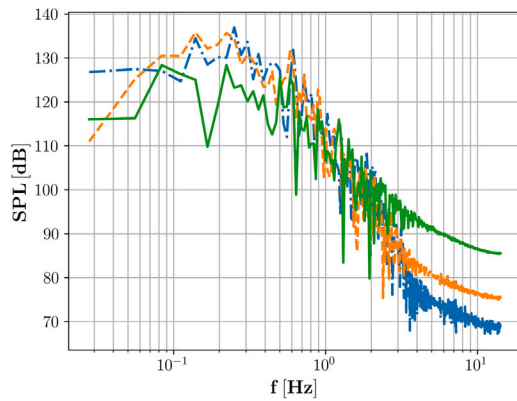


Fig. 31. Wake spectra analysis. SPL against scaled frequency for receivers at $y/D = 0$, $z/D = -1$ and $x/D = -10$ ($\square$), -15 ($\circ$) and -20 ($\bullet$).

References

- Alin, N., Bensow, R., Fureby, C., Huuva, T., Svennberg, U., 2010. Current capabilities of DES and LES for submarines at straight course. *J. Ship Res.* 54, 184–196.
- Anderson, B., Chapuis, M., Erm, L., Fureby, C., Giacobello, M., Henbest, S., Jones, D., Jones, M., Kumar, C., Liefvendahl, M., Manovski, P., Norrison, D., Quick, H., Snowden, A., Valiyff, A., Widjaja, R., Woodyatt, B., 2012. Experimental and computational investigation of a generic conventional submarine hull form. In: 29th Symposium on Naval Hydrodynamics. Gothenburg, Sweden.
- Bettle, M., 2020. Development of CFD meshes for the generic BB2 submarine: evaluation of discretization uncertainty in model-scale hydrodynamic forces for straight-ahead flight. Technical Report, (DRDC-RDDC-2020-R066), Defence Research and Development Canada.
- Bhushan, S., Alam, M.F., Walters, D.K., 2013. Evaluation of hybrid RANS/LES models for prediction of flow around surface combatant and suboff geometries. *Comput. & Fluids* 88, 834–849. <http://dx.doi.org/10.1016/j.compfluid.2013.07.020>.
- Chase, N., Carrica, P.M., 2013. Submarine propeller computations and application to self-propulsion of DARPA Suboff. *Ocean Eng.* 60, 68–80. <http://dx.doi.org/10.1016/j.oceaneng.2012.12.029>.
- Chase, N., Michael, T., Carrica, P., 2013. Overset simulation of a submarine and propeller in towed, self-propelled and maneuvering conditions. *Int. Shipbuild. Prog.* 60, 171–205.
- Cianferra, M., 2017. Use of the Ffowcs Williams and Hawkins equation for the prediction of hydroacoustic noise for single-phase and cavitating flows (Ph.D. thesis). School of Earth Science and Fluid Mechanics, University of Trieste.
- Cianferra, M., Armenio, V., 2021. Scaling properties of the Ffowcs-williams and hawkins equation for complex acoustic source close to a free surface. *J. Fluid Mech.* 927, A2. <http://dx.doi.org/10.1017/jfm.2021.723>.
- Cianferra, M., Ianniello, S., Armenio, V., 2019a. Assessment of methodologies for the solution of the Ffowcs Williams-Hawkins equation using LES of incompressible single-phase flow around a finite-size square cylinder. *J. Sound Vib.* 453, 1–24. <http://dx.doi.org/10.1016/j.jsv.2019.04.001>.
- Cianferra, M., Petronio, A., Armenio, V., 2019b. Non-linear noise from a ship propeller in open sea condition. *Ocean Eng.* 191, 1064–1074.
- Curle, N., 1955. The influence of solid boundaries upon aerodynamic sound. *Proc. R. Soc. A* 231 (1187), 505–514.
- Davidson, P.A., 2004. Turbulence. An introduction for scientists and engineers. Oxford University Press.
- Eitel-Amor, G., Örlü, R., Schlatter, P., 2014. Simulation and validation of a spatially evolving turbulent boundary layer up to $Re_\theta = 8300$. *Int. J. Heat Fluid Flow* 47, 57–69.
- Fadlun, E., Verzicco, R., Orlandi, P., Mohd-Yusof, J., 2000. Combined immersed-boundary finite-difference methods for three-dimensional complex flow simulations. *J. Comput. Phys.* 161 (1), 35–60. <http://dx.doi.org/10.1006/jcph.2000.6484>.
- Farassat, F., 1981. Linear acoustic formulas for calculation of rotating blade noise. *AIAA J.* 19 (9), 1122–1130.
- Farassat, F., 2007. Derivation of Formulation 1 and 1A of Farassat. Technical Report, NASA, TM-2007-214853.
- Farassat, F., Succi, G.P., 1980. A review of propeller discrete frequency noise prediction technology with emphasis on two current methods for time domain calculations. *J. Sound Vib.* 71 (3), 399–419.
- Fischer, J., Rowan, M., Jacquemin, P., Lamos, D., Vargas, O., Malcolm, J., Skvortsov, A., Doolan, C., 2017. Wind tunnel flow noise measurements using a random design acoustic array. In: Proceedings of Acoustics 2017. Perth, Australia.
- Fureby, C., Anderson, B., Clarke, D., Erm, L., Henbest, S., Giacobello, M., Jones, D., Nguyen, M., Johansson, M., Jones, M., Kumar, C., Lee, S.-K., Manovski, P., Norrison, D., Petterson, K., Seil, G., Woodyatt, B., Zhu, S., 2016. Experimental and numerical study of a generic conventional submarine at 10° yaw. *Ocean Eng.* 116, 1–20.
- George, W., 1989. The self-preservation of turbulent flows and its relation to initial conditions and coherent structures. In: Advances in Turbulence. Hemisphere Press, pp. 39–73.
- Germano, M., Piomelli, U., Moin, P., Cabot, W.H., 1991. A dynamic subgrid-scale eddy viscosity model. *Phys. Fluids A* 3, <http://dx.doi.org/10.1063/1.857955>.
- Gourlay, M.J., Arendt, S.C., Fritts, D.C., Werne, J., 2001. Numerical modeling of initially turbulent wakes with net momentum. *Phys. Fluids* 13 (12), 3783–3802. <http://dx.doi.org/10.1063/1.1412246>.
- Groves, N.C., Huang, T.T., Chang, M.S., 1989. Geometric characteristics of the DARPA SUBOFF models. Technical Report, (DTRC/SHD-1298-01), David Taylor Research Center, Bethesda, MD.
- Huang, T., Liu, H.L., Groves, N., Forlini, T., Blanton, J., Gowing, S., 1992. Measurements of flows over an axisymmetric body with various appendages in a wind tunnel: the DARPA SUBOFF experimental program. In: Proceedings of the 19th Symposium on Naval Hydrodynamics. National Academy Press.
- Ianniello, S., 2016. The Ffowcs Williams-Hawkins equation for hydroacoustic analysis of rotating blades. The rotapole. *J. Fluid Mech.* 797, 345–388.
- Ianniello, S., Muscarelli, R., Di Mascio, A., 2013. Ship underwater noise assessment by the acoustic analogy. Part I: Nonlinear analysis of a marine propeller in a uniform flow. *J. Mar. Sci. Technol.* 18, <http://dx.doi.org/10.1007/s00773-013-0227-0>.
- Issa, R., Gosman, A., Watkins, A., 1986. The computation of compressible and incompressible recirculating flows by a non-iterative implicit scheme. *J. Comput. Phys.* 62 (1), 66–82. [http://dx.doi.org/10.1016/0021-9991\(86\)90100-2](http://dx.doi.org/10.1016/0021-9991(86)90100-2).
- Jiménez, J.M., Hultmark, M., Smits, A.J., 2010a. The intermediate wake of a body of revolution at high Reynolds numbers. *J. Fluid Mech.* 659, 516–539. <http://dx.doi.org/10.1017/S0022112010002715>.
- Jiménez, J.M., Reynolds, R.T., Smits, A.J., 2010b. The effect of the fins on the intermediate wake of a submarine model. *ASME J. Fluids Eng.* 132 (3).
- Johansson, P.B.V., George, W.K., 2001. On the effect of finite Reynolds number and initial conditions on the axisymmetric wake. In: Proceedings of the Second International Symposium on Turbulence and Shear Flow Phenomena. pp. 323–328.
- Johansson, P.B.V., George, W.K., Gourlay, M.J., 2003. Equilibrium similarity, effects of initial conditions and local Reynolds number on the axisymmetric wake. *Phys. Fluids* 15 (3), 603–617. <http://dx.doi.org/10.1063/1.1536976>, [arXiv:http://dx.doi.org/10.1063/1.1536976](http://arxiv.org/abs/http://dx.doi.org/10.1063/1.1536976).
- Joubert, P.N., 2004. Some Aspects of Submarine Design Part 1 - Hydrodynamics, DSTO Report DSTO-TR-1622. Technical Report, DSTO Melbourne, Australia.
- Joubert, P.N., 2006. Some Aspects of Submarine Design Part 2 - Shape of a Submarine 2026, DSTO Report DSTO-TR-1920. Technical Report, DSTO Melbourne.
- Keller, J., Kumar, P., Mahesh, K., 2018. Examination of propeller sound production using large eddy simulation. *Phys. Rev. Fluids* 3, 064601. <http://dx.doi.org/10.1103/PhysRevFluids.3.064601>.
- Kumar, P., Mahesh, K., 2018. Large-eddy simulation of flow over an axisymmetric body of revolution. *J. Fluid Mech.* 853, 537–563. <http://dx.doi.org/10.1017/jfm.2018.585>.
- Kumar, C., Manovski, P., Giacobello, M., 2012. Particle image velocimetry measurements on a generic submarine hull form. In: 18th Australasian Fluid Mechanics Conference.
- Larsson, J., Kawai, S., Bodart, J., Bermejo-Moreno, I., 2016. Large eddy simulation with modeled wall-stress: recent progress and future directions. *Bull. JSME. Mech. Eng. Rev.* 3 (1), <http://dx.doi.org/10.1299/merc.15-00418>.
- Lee, S.K., Manovski, P., Kumar, C., 2019. Wake of a cruciform appendage on a generic submarine at 10° yaw. *J. Mar. Sci. Technol.* 25, 787–799.
- Lighthill, M.J., 1952. On sound generated aerodynamically. I general theory. *Proc. R. Soc. A* 211 (11079), 564–587.
- Lilly, D.K., 1992. A proposed modification of the Germano subgrid-scale closure method. *Phys. Fluids A* <http://dx.doi.org/10.1063/1.858280>.
- Mukha, T., 2018. Modelling Techniques for Large-Eddy Simulation of Wall-Bounded Turbulent Flows (Ph.D. thesis). Uppsala University.
- Mukha, T., Liefvendahl, M., 2015. Large-Eddy Simulation of turbulent channel flow. Technical Report, Uppsala University.
- Najaf-Yazdi, A., Bres, G.A., Mongeau, L., 2010. An acoustic analogy formulation for moving sources in uniformly moving media. *Proc. R. Soc.* 467, 144–165.
- Nicoud, F., Ducros, F., 1999. Subgrid-scale stress modelling based on the square of the velocity gradient tensor. *Flow Turbul. Combust.* 62 (3), 183–200.
- Norrison, D., Sidebottom, W., Anderson, B., Petterson, K., Fureby, C., 2016. Numerical study of a self-propelled conventional submarine. In: 31st Symposium on Naval Hydrodynamics. Monterey, CA, USA.
- Overpelt, B., Nienhuis, B., Anderson, B., 2015. Free running manoeuvring model tests on a modern generic SSK class submarine (BB2). In: Pacific International Maritime Conference. Sydney, Australia.
- Patankar, S.V., 1980. Numerical Heat Transfer and Fluid Flow. Taylor & Francis.
- Petterson, K., Fureby, C., Liefvendahl, M., Norrison, D., Sidebottom, W., 2018. LES of a full-scale self-propelled conventional submarine at $\pm 10^\circ$ yaw. In: 32nd Symposium on Naval Hydrodynamics. Hamburg, Germany.

- Piomelli, U., Balaras, E., 2002. Wall-layer models for large-eddy simulations. *Annu. Rev. Fluid Mech.* 34 (1), 349–374.
- Piomelli, U., Streett, C.L., Sarkar, S., 1997. On the computation of sound by large-eddy simulations. *J. Eng. Math.* 32, 217–236. <http://dx.doi.org/10.1023/A:1004236206327>.
- Pope, S.B., 2000. *Turbulent Flows*. Cambridge University Press.
- Posa, A., Balaras, E., 2014. Model-based near-wall reconstructions for immersed-boundary methods. *Theor. Comput. Fluid Dyn.* 28, 473–483. <http://dx.doi.org/10.1007/s00162-014-0326-5>.
- Posa, A., Balaras, E., 2016. A numerical investigation of the wake of an axisymmetric body with appendages. *J. Fluid Mech.* 792, 470–498. <http://dx.doi.org/10.1017/jfm.2016.47>.
- Posa, A., Balaras, E., 2020. A numerical investigation about the effects of Reynolds number on the flow around an appended axisymmetric body of revolution. *J. Fluid Mech.* 884 (A41).
- Posa, A., Broglia, R., Felli, M., 2022a. Acoustic signature of a propeller operating upstream of a hydrofoil. *Phys. Fluids* 34 (6), 065132. <http://dx.doi.org/10.1063/5.0096030>.
- Posa, A., Broglia, R., Felli, M., Cianferra, M., Armenio, V., 2022b. Hydroacoustic analysis of a marine propeller using large-eddy simulation and acoustic analogy. *J. Fluid Mech.* 947, A46. <http://dx.doi.org/10.1017/jfm.2022.692>.
- Posa, A., Felli, M., Broglia, R., 2022c. Influence of an upstream hydrofoil on the acoustic signature of a propeller. *Phys. Fluids* 34 (4), 045112. <http://dx.doi.org/10.1063/5.0086633>.
- Posa, A., Felli, M., Broglia, R., 2022d. The signature of a propeller–rudder system: Acoustic analogy based on LES data. *Ocean Eng.* 259, <http://dx.doi.org/10.1016/j.oceaneng.2022.112059>.
- Quick, H., Widjaja, R., Anderson, B., Woodyatt, B., Snowden, A., Lam, S., 2012. Phase I experimental testing of a generic submarine model in the DSTO low speed wind tunnel. In: Technical Note TN-1101. Technical Report, Defence Science and Technology, Australia.
- Quick, H., Woodyatt, B., 2014. Phase II experimental testing of a generic submarine model in the DSTO low speed wind tunnel. In: Technical Note TN-1274. Technical Report, Defence Science and Technology, Australia.
- Renilson, M., 2018. *Submarine Hydrodynamics*, second ed. Springer, Berlin, <http://dx.doi.org/10.1007/978-3-319-79057-2>.
- Robinson, S.K., 1991. Coherent motions in the turbulent boundary layer. *Annu. Rev. Fluid Mech.* 601–639.
- Spalding, D.B., 1961. A Single formula for the “law of the wall”. *J. Appl. Mech.* 28 (3), 455–458. <http://dx.doi.org/10.1115/1.3641728>.
- Tennekes, H., Lumley, J., 1972. *A First Course in Turbulence*. MIT Press, Cambridge, Mass.
- Townsend, A.A., 1956. *The Structure of Turbulent Shear Flow*. vol. 1, (5), Cambridge University Press, pp. 554–560. <http://dx.doi.org/10.1017/S0022112056210366>.
- Toxopeus, S., 2017. Report No. 29347-8-RD. Technical Report, Marin.
- Uosukainen, S., 2011. Foundations of acoustic analogies. Technical Report, (757), VTT Publications.
- Wagner, C., Huttli, T., Sagaut, P., 2007. *Large-Eddy Simulation for Acoustics*. Cambridge University Press, <http://dx.doi.org/10.1017/CB09780511546143>.
- Wang, M., Freund, J., Sanjiva, L.K., 2006. Computational prediction of flow-generated sound. *Annu. Rev. Fluid Mech.* 38 (1), 483–512. <http://dx.doi.org/10.1146/annurev.fluid.38.050304.092036>.
- Welch, S., 1967. The use of the fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms. *IEEE Trans. Audio Electroacoust.* 15, 70–73.
- Williams, J.F., Hawkins, D., 1969. Sound generation by turbulence and surfaces in arbitrary motion. *Phil. Trans. R. Soc. A264 (A1151)*, 321–342.
- Yang, X., Park, G., Moin, P., 2017. Log-layer mismatch and modeling of the fluctuating wall stress in wall-modeled large-eddy simulations. *Phys. Rev. Fluids*.