

TRUTH PLURALISM AND MANY-VALUED LOGIC: LESSON FROM SUSZKO'S THESIS

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According to truth pluralism, sentences from different areas of discourse can be true in different ways. This view has been challenged to make sense of logical validity, understood as necessary truth preservation, when inferences involving different areas are considered. To solve this problem, a natural temptation is that of replicating the standard practice in many-valued logic by appealing to the notion of designated values. Such a simple approach, however, is usually considered a non-starter for strong versions of truth pluralism, since designation seems to embody nothing but a notion of generic truth. In this paper, I explore the analogy with many-valued logic by comparing the problem of mixed inferences with Suszko's thesis, and argue that the strong pluralist has room to resist the commitment to a generic property of truth by undermining the semantic significance of Suszko's reduction.

Keywords: truth pluralism, mixed inferences, logical validity; designation, Suszko; many-valued logic.

I. TRUTH PLURALISM, MIXED INFERENCE AND MANY-VALUED LOGIC

In its standard formulation,¹ truth pluralism has it that sentences² belonging to different areas of discourse can be, and in some cases are, true in different ways. Truth pluralism usually goes with a reasonable distinction between the truth predicate and the associated concept, on one hand, and the property or properties they denote, on the other. Truth pluralism can then be framed at the metaphysical level by claiming that, while the truth predicate and the truth concept are possibly unique, there are many (namely more than one) truth properties.³ For example, intuitively, truth about middle-size concrete

¹ Wright (1999) and Lynch (2009).

² Since I focus on logic here, I assume sentences as truth bearers for the sake of simplicity.

³ For ease of exposition, I conform to the usual practice and adopt a seemingly realist vocabulary with respect to properties.

objects might be correspondence to facts, while truth about morality might be super-assertability and truth in mathematics coherence with certain axioms.

Not long after its introduction in the contemporary debate,⁴ it became quickly clear that truth pluralism seems to conflict with the orthodox understanding of logical validity, conceived as necessary truth preservation.⁵ The reason is that such a traditional account seems to rely on a postulation of a single truth property. The problem distinctively emerges in the case of so-called mixed inferences, namely apparently valid inferences where different areas of discourse are mixed.⁶ In such cases, by pluralist assumption, there is not a single area-specific truth property to be preserved from premisses to conclusion, so that the validity of the inference can hardly be accounted for in terms of *necessary truth preservation*. What truth property is preserved in such cases? Truth pluralism is in trouble.

Despite being a straightforward difficulty, anyone familiar with many-valued logic⁷ might find it hard to see where the problem is. Indeed, the alleged obstacle looks easy to overcome once the customary many-valued definition of validity is adopted. As is well known, many-valued logic extends the traditional set of truth values (Truth and Falsehood), admitting also values that represent other semantic properties. At least *prima facie*, the analogy with truth pluralism should be apparent. Indeed, many-valued logic looks like a formal and more general counterpart of truth pluralism. For example, assume, for the sake of simplicity, a form of truth pluralism according to which there are only two areas of discourse, two corresponding truth properties and one property of falsehood.⁸ This simple case can be modelled by a many-valued logic treating the set of truth values V to be $\langle t_1, t_2, f \rangle$, where ‘ t_1 ’ and ‘ t_2 ’ stand for the properties of truth and ‘ f ’ for the property of falsehood. Clearly, to yield a corresponding many-valued logic, many details must be filled in, and several non-trivial choices done. For example, suitable semantic evaluations for the connectives must be defined.⁹ The relevant point for us, however, is that once many-valued logic is invoked, the problem of mixed inferences seems to fade away, since

⁴ Wright (1992).

⁵ Williamson (1994) and Tappolet (1997).

⁶ The issue of mixed inferences has received considerable attention. See, for example, Pedersen (2006), Cotnoir (2013) and Stollo (2018a). Recently, Gamester (2019) has cleverly argued that the problem might be only illusory. Here I do not enter this dispute since I am using mixed inferences mostly as an illustration case to introduce the issue of validity and the status of designation from a truth pluralist perspective. I also believe that even if mixed inferences do not pose an authentic problem, a serious one—namely explaining the unity of logical validity—is still there. This paper has an immediate bearing on this unity problem too. Since, however, the unity problem is still largely neglected, and the reader is likely more familiar with the problem of mixed inferences, I focus on it.

⁷ Malinowski (1994).

⁸ Posing two properties of falsehood would be irrelevant for present purposes.

⁹ The task is far from easy given the similar problem of mixed compounds: What is the truth property of a true compound with parts belonging to different areas?

equipping a many-valued logic with a definition of validity is routine. Given that not all semantic values must be on a par, we just need to decide what values are to be preserved in valid inferences. In other words, a subset of the set of semantic values (the designated values) is to be displayed and validity defined accordingly.¹⁰ Since we have two values for truth and one value for falsehood, which values should play a logical role is an easy choice. The natural set of designated values is $D = \{t_1, t_2\}$. It is then enough that we keep in mind that there is more than one value in our subset of designated values and formulate the definition accordingly: an inference is valid iff necessarily if each premiss has a *designated value*, also the conclusion has a *designated value*. (See the Appendix for a formal version.)

The analogies between truth pluralism and many-valued logic are so striking, and the above strategy so customary, that this potential solution had been pointed out very soon.¹¹ Equally soon, however, it was rejected.¹² The reason is that the notion of designation, unproblematic as it might look, is troublesome in the context of truth pluralism. It is not hard to see why. The definition of validity is made possible by having at disposal a set of designated values that allows to speak of a sentence having an unspecified designated value. In particular, D only includes values that represent truth properties (t_1 and t_2). It thus follows that *having a designated value* is equivalent to *having value t_1 or t_2* , namely to *being true in some way*. Since this seems to amount to a generic notion of truth, designation arguably carries with it a commitment to a generic property of truth. Consequently, if truth pluralists reject generic truth, the notion of designation is anathema to them.

At this point, an important specification is in order. In general, a commitment to a generic property of truth is not fatal to any pluralist conception. According to weak versions of pluralism, beside a plurality of area-specific truth properties, also a generic property of truth, exemplified by every true sentence, is to be admitted. Clearly, if such a weak form of pluralism is endorsed, a resort to designation is unproblematic. However, once a generic property is introduced, the detour through many-valued logic becomes unnecessary. In the hands of the weak pluralist, validity can be immediately accounted for in terms of necessary preservation of generic truth. By contrast, the problem persists for those strong pluralists who are reluctant to introduce a generic property of truth. Given that designation carries a commitment to generic truth, strong pluralists must look for alternative solutions and cannot simply subscribe to many-valued logic. This shows why, although for

¹⁰ Although Dummett (1978) is often cited, the notion of designated values dates at least to Łukasiewicz (1930), where Łukasiewicz uses the term ‘ausgezeichnet’, which Lewis & Langford (1932) translate as ‘designated’. After that, ‘designated’ has been regularly used by those working in many-valued logic. Thanks to Diego Marconi for pointing this out.

¹¹ Beall (2000).

¹² Tappolet (2000).

opposite reasons, standard many-valued logic and designation lost its prima facie attractiveness to both weak and strong pluralism.¹³

The main purpose of this paper is showing that truth pluralism and many-valued logic can strongly benefit from an inquiry keeping an eye on both. In particular, I demonstrate that the analogy between truth pluralism and many-valued logic naturally extends well beyond the above point. First, the objection against designation raised in the context of truth pluralism parallels a similar objection moved against the legitimacy of many-valued logic. This is remarkable on its own, especially because it passed unnoticed, as most truth pluralists seem to ignore that many-valued logicians faced a similar challenge. Secondly, some strategy implemented by many-valued logicians to counteract the objection can be exploited by truth pluralists too, shedding light on interesting neglected options. Once the formal details are adequately developed, it can be shown that a solution based on designation is available to truth pluralism, even strong. Finally, while I mainly focus on truth pluralism, also many-valued logic has valuable insights to earn, since the discussion on the status of designation and its interpretation contributes to the understanding of the foundation of many-valued logic itself.

II. A MAGNIFICENT CONCEPTUAL DECEIT

An account of validity based on designation seems at odds with (strong) truth pluralism because it apparently imports a commitment to a generic property of truth. Remarkably, a resort to designation looks problematic, and for analogous reasons, also in the context of many-valued logic. Consider the standard account according to which an inference is valid if in any case in which the premisses have a designated value, also the conclusion has a designated value. It is easy to see that once designation is exploited, the plurality of values hosted in a many-valued model becomes logically irrelevant: what counts is only that a value is designated, not what specific value is. This simple observation provided the inspiration for a deep criticism of many-valued logic put forward by the Polish logician Roman Suszko. Suszko claimed that many-valued logic is, literally, *a magnificent conceptual deceit*,¹⁴ since every logic is two-valued.¹⁵ Suszko

¹³ For example, Pedersen (2006), Stollo (2018a) and Gamester (2019). Other works, like Yu (2017a), focus more on the right semantic and logic and less on the problem of validity as such. For instance, Yu implicitly assumes a notion similar to designation and accepts its apparent commitment to generic truth. Cotnoir (2013) is an interesting attempt at developing a many-valued approach avoiding the pitfalls of designation. See, however, Stollo (2018b) and Keefe (2018) for criticisms.

¹⁴ Suszko (1977: 377–8).

¹⁵ Similar remarks are also found in Dummett: ‘What point there may be in distinguishing between different ways in which a statement may be true or between different ways in which it may be false?’ (Dummett, 1959: 153).

grounded this philosophical thesis in a formal result known as *Suszko's theorem*. The gist of the theorem is that any logic (and many-valued logic in particular) meeting some minimal formal constraints,¹⁶ can be given a bivalent semantics that is provably adequate (namely correct and complete). Based on such a neat mathematical result, Suszko argued that every logic is two-valued, so that the very idea of a many-valued logic is illusory. Let me illustrate both the mathematical result and the philosophical argument in more detail.

The idea behind Suszko's mathematical reduction is extremely simple and, at this point, quite unsurprising. As just mentioned, the bipartition of the set of values in designated and non-designated follows the intuitive distinction between true and untrue values, with the designated values being those to be preserved in valid inferences. Indeed, given that validity is entirely defined in terms of designation, such a bipartition of values is the only logically significant classification of sentences. If so, validity seems equally supported by a semantics where only the designated/non-designated distinction is drawn and logically irrelevant values neglected. Suszko's reduction implements this simple idea: an equivalent two-valued model is constructed by evaluating a formula as 1 if it receives a designated value in the initial many-valued model, and as 0 otherwise. In this new model, the original plurality of values vanishes in favour of a mere duality (0,1). Clearly, this strategy is independent of the initial number of values and can be pursued as long as the definition of validity is based, as customarily done, on a bipartition between designated and non-designated values. In order to keep the philosophical discussion smooth, I postpone the formal description for the interested reader to the Appendix.

From a merely mathematical point of view, the result is hardly questionable, and, as such, it can just be acknowledged. What is controversial, however, is how it should be interpreted, and how significant it should be taken to be. Suszko himself proposed an interpretation (known as *Suszko's thesis*), which is a philosophical thesis and should not be confused with the mathematical result (*Suszko's theorem*) on which it is grounded. It is the philosophical thesis that concerns us here, given that it is this philosophical thesis that provides the criticism of many-valued logic paralleling the objection raised against truth pluralism. Suszko's thesis is the philosophical view according to which every logic is two-valued. While sounding as an extreme idea, under the light of Suszko's theorem, its justification seems quite natural. After all, if any logic has an adequate two-valued semantics, it is tempting to conclude that every logic is two-valued. To block this conclusion, however, it could be quickly objected that the formal result only shows that every logic has *also* a two-valued semantics. Why should the bivalent semantics be privileged, given that often,

¹⁶ The logic must respect some basic formal constraints identifying a structural Tarkian logical consequence relation: reflexivity, transitivity and monotonicity. Actually, the requirements can be even weakened (see Font 2009; Shramko and Wansing 2008, 2018).

and typically in the case of many-valued logic, adequate many-valued semantics are available too? To neutralise such a natural reply, Suszko introduces a crucial distinction. If the theorem is carefully scrutinised, it is easy to see that the reduction relies on something that is already present, albeit implicitly, in how logical validity is defined in the initial many-valued model itself. Also in the many-valued model, the logical contribution of each value is negligible with respect to the set of designated ones. Indeed, it is exactly on such a logical irrelevance that the construction of the reduced bivalent model relies. Since, to define validity, what only matters is whether a value is designated or not, the initial plurality of values is logically inert already in the many-valued case. Accordingly, the original many-valued model is itself logically bivalent, and the seeming plurality of logical values chimerical. In other words, the reduction to bivalence provided in the two-valued semantics just brings to surface the logical irrelevance of the initial plurality of values. To articulate this insight and mark the difference between values that are semantic referents of sentences without tracking logically relevant distinctions, and values that instead do play an authentic logical role, Suszko distinguishes between *algebraic* and *logical* values, respectively. The values hosted in a many-valued semantics are algebraic values, whereas the values embodied in the designated/non-designated distinction and exploited in the definition of validity are the logical ones. With this contrast in mind, Suszko's thesis can be formulated in a more careful and effective way. According to Suszko, although not every logic is *algebraically* two-valued, the theorem shows that every logic is *logically* two-valued. It follows that, although a many-*algebraically*-valued semantics can have many virtues and even be necessary for some purposes, a definition of logical validity only requires a bivalent distinction between designated and non-designated values. Therefore, being designated or not are the only *logical* values. In this sense, every logic is ultimately logically bivalent, no matter how many algebraic values other semantics host. Reframed in these terms, it is a little step to conclude that the truth/falsehood distinction is the only logical distinction that matters. To reach this conclusion, it is enough to identify designation and truth, as it is natural to do if validity is understood as preservation of truth.

Once carefully crafted in this way, Suszko's thesis seems harder to reject and the justification grounded on the mathematical result quite solid. If such an analysis is correct, it follows that many-valued logic is, as a *logic*, unreachable. We can have, at most, many-valued algebraic semantics, where the supported notion of logical validity inescapably relies on just two logical values, Truth and Falsehood. Suszko's thesis shakes the very foundation of many-valued logic.

III. SUSZKO'S THESIS AND SUSZKO'S THEOREM

The analogies between Suszko's objection and the problems of truth pluralism should be evident, since the inspiring idea is the same. In the definition

of logical validity standardly provided by many-valued logicians, the crucial appeal to designation obscures the initial plurality of values, making them logically irrelevant. Given the natural identification of designation and generic truth, every (many-valued) logic embodies a generic notion of truth in terms of which logical validity is defined. The analogies are so clear that Suszko's thesis may appear as a mere regimentation and generalisation of the objection against pluralism. In both cases, the multiplication of semantic values (or truth properties) contrasts with the bivalent nature of logical validity, grounded in the preservation of a single logical value (generic truth).

Despite these *prima facie* easy analogies, the issue is subtler and its significance deeper. In particular, understanding where and how the commitment to generic truth is incurred in Suszko's theorem is a critical point. The key point is far from obvious and, while crucial, it is hardly explicitly discussed even in logical contexts.¹⁷ To introduce the issue, consider the argument above. I stressed that a many-valued model is implicitly logically bivalent because of the dual distinction between designated and non-designated values. Since finer-grained distinctions among designated values are logically irrelevant, the resulting logic is not logically many-valued (although it can have an algebraically many-valued semantics). At this point, from a philosophical point of view, one might wonder what the general need for the mathematical machinery of Suszko's reduction is, even with respect to the foundation of many-valued logic. If many-valued logic is implicitly two-valued because the definition of logical validity essentially relies on designation, then Suszko's thesis can be raised and defended independently of Suszko's theorem. The mathematical result certainly strengthens the thesis, makes it formally precise and allows further mathematical investigations, but its indispensability for the philosophical criticism fades away. Many-valued logic could have been attacked without relying on a formal theorem, exactly like the appeal to designation was criticised in the context of truth pluralism in informal terms. It is important to consider such a dismissive reaction because it neglects a crucial aspect of Suszko's attack. Indeed, far from being a philosophically unnecessary step, Suszko's theorem provides an essential premiss for the philosophical conclusion. This point should be well understood also because it is precisely what bears on truth pluralism, making the analogies between the two issues not just a matter of intellectual curiosity.

As mentioned above, Suszko put forward a distinction between algebraic and logical values. The opposition between 'algebraic' and 'logical', however, may obscure an important point: that both are *values*. Given that a value is a referent of an expression in a certain semantics, and that this is what the notion of 'algebraic value' stands for, it follows that also logical values are algebraic values, as they are the algebraic values hosted in the two-valued semantics. Indeed, what Suszko's reduction shows is that *designation is a logical*

¹⁷ To be clear, what follows has no exegetical purpose. It is just my reading of Suszko.

notion that can be turned into an algebraic value of a suitable semantics. In the two-valued model, designation is an algebraic value that also plays a logical role. By contrast, in the initial many-valued model, the algebraic values have a semantic but not a logical role, and they cannot be turned into logical values in an authentically many-valued account of logical validity.¹⁸ Unlike designation, the semantic values of the many-valued semantics are *merely* algebraic values. The real opposition, strictly speaking, is between the *merely* algebraic values of the many-valued model—which implicitly also involves two extra (logical) values (designation and not designation)—and the algebraic *and* logical values of the two-valued semantics.

If the point of the mathematical reduction is showing that designation can be turned into an algebraic value of a reduced semantics, we must ask why this is so important for the philosophical thesis. The answer is quite simple, yet subtle. According to Suszko's thesis, every logic is logically two-valued, because in every valid inference only one and the same value is necessarily preserved. No matter what logic is considered and how many elements there are in its algebraic semantics, validity always consists in preservation of one value: truth. To arrive at this claim, given that validity is, *de facto*, defined in terms of designation, it is necessary that designation is turned into a *semantic value for truth*. Suszko's theorem covers one fundamental prerequisite of this claim by showing that designation can be turned into a semantic value. In this way, designation can be considered a value even if it is not explicitly included in the initial many-valued semantics. This is crucial for two reasons. One, quite clear, is that it makes designation a semantic value transmittable in inferences. The second, less obvious, is to allow the eventual identification of designation with truth. Let me explain this latter point better.

The general argument for Suszko's thesis rests on the implicit assumption that to identify a property with truth, at the bare minimum, it must be possible to treat such a property as a semantic value of sentences. This is a reasonable assumption, since truth and falsehood are not just semantic properties, but the semantic properties *par excellence* of sentences. If a property could not be treated as a value of sentences in a suitable semantics, then an identification with truth would be prevented.¹⁹ Suszko's theorem fills this possible gap between designation and truth by showing that designation can actually be turned into a semantic (namely algebraic) value. Clearly, the mere possibility of treating a property as a semantic referent of a suitable semantic is not sufficient to conclude that it represents truth. After all, a semantic value could also stand for other semantic properties, like, for instance, falsehood. Suszko's theorem

¹⁸ Some strategies to undermine the reduction question exactly this. See Malinowski (1994).

¹⁹ One might resist it either by claiming that semantic properties need not be modelled in terms of formal values or by rejecting the idea that truth is a semantic property at all (for example, by rejecting truth-theoretical semantics). Both options are theoretically costly or plainly unattractive and would lead us astray here.

alone only shows that designation can be identified with a semantic value, but it does not show that it is a semantic value for truth. This, however, is a little step easily covered once it is accepted that validity consists in necessary *truth* preservation. If valid inferences are those necessarily preserving truth, and designation is the value preserved in valid inferences, then designation and truth must be identical. That validity consists in truth preservation is a reasonable philosophical assumption whose legitimacy Suszko accepts without further inquiry, and that it is also assumed in the truth pluralist debate.²⁰ The crucial and critical step, thus, is exactly the one handled by Suszko's theorem, which shows that designation can be turned into a semantic (algebraic) value of an adequate semantics.

To make the point more vivid, and clearly see why the theorem is crucial to defend the philosophical thesis, suppose that Suszko's theorem did not go through, and designation could not be turned into an algebraic value of a suitable semantics. In this case, two ingredients would be disentangled in the formulation of a many-valued logic: a level of semantic properties (like truth, falsehood, and possibly others) represented by algebraic values and a level of logical notions employed to define validity (like designation and non-designation). Algebraic values would correspond to properties of sentences that characterise their semantic status. Logical notions, by contrast, would not characterise any semantic feature of sentences, but just display their logical relations. While certainly connected, logic and semantics would be taken apart. In particular, logical notions like designation would not be, even implicitly, semantic values. Designation, far from being a value standing for a semantic property on its own, would just be a meta-logical notion employed to characterise what relations among semantic values take place in valid inferences.²¹ In such a case, the philosophical claim that every logic is logically *two-valued* would vanish. First of all, with the reduction neutralised, the semantic values transmitted in valid inferences could only be those in the many-valued semantics, because there would not be other values at all. Strictly speaking, designation could not be a preserved value, because it could not even be a value. Secondly, given that designation could not be a value, let alone the preserved value, it could not be identified with truth. Thirdly, given that designation would not stand for a value, *logical bivalence* (reflected in the bivalent distinction between designated and non-designated) would not collapse into semantic *two-valuedness*. Many-valued logic would be authentically semantically many-valued and truth one of the semantic values—and indeed the preserved value—despite logical validity being defined in bivalent terms by means of

²⁰ Through the entire debate on mixed inferences, the traditional account of logical validity as necessary truth preservation is assumed. If the assumption were rejected, the very issue would evaporate, since there will be no single truth property to be transmitted, as validity would consist in something else.

²¹ See Caret & Cotnoir (2008).

bivalent meta-theoretical notions. In this way, the threat to many-valued logic would be thwarted.

IV. TAMING THE REDUCTION

By shedding light on Suszko's thesis and the role of Suszko's theorem in supporting it, the above reflections point to a possible way to avoid the commitment to a generic truth property seemingly carried by designation. If the significance of Suszko's theorem could be undermined, a strong pluralist could counteract by replicating the considerations put forward in the considered scenario in which Suszko's reduction is blocked. In that case, designation could not be turned into an algebraic value of an acceptable semantics, and since truth must be liable to such a treatment, designation and truth could not be identified. Notwithstanding its logical significance, the notion of designation would not represent a semantic value, and, a fortiori, a truth property. In a nutshell, if the significance of Suszko's theorem can be undermined, then the strong pluralist can rely on designation. The crucial point thus becomes: Can the significance of Suszko's reduction be undermined? To answer this question, following the debate in many-valued logic is useful again.

Although Suszko's thesis casts some shadows on the general foundation of many-valued logic, it is clear that logicians have not stopped working in the field and scepticism about its legitimacy is far from widespread. This general reaction is not due to negligence or insensitivity to the issue. Rather, it is the consequence of reasonable conclusions that can be drawn from a serious scrutiny of Suszko's argument.²² Indeed, from the point of view of many-valued logic, Suszko's attack can be counteracted in several ways.²³ Since most of these moves can hardly be transferred to the context of pluralism, however, for space reasons, I put them aside²⁴ and focus on the transferable options.

²² See Malinowski (1994), Shramko & Wansing (2008, 2018), Font (2009) and Chemla & Egré (2019).

²³ Given how the two-valued semantics is defined, one might deflate designation arguing that it is not a fundamental value. After all, a sentence has value 1 only because it has a certain value in the many-valued semantics. In a similar way, some pluralists (see, for instance, Ferrari, Pedersen and Moruzzi 2020) argued that domain-specific truth properties have priority over generic truth. However, it is not so clear that designation is entirely parasitic of the pluralist values. As Suszko's reduction indicates, designation plays an explanatory role in logical validity that the pluralist values by themselves cannot play. Such a possibly irreducible role does not consist in providing a preserved value—as argued in this paper—but in something else, such as accounting for the unity of logical validity. For the matter of space, however, a full exploration of this issue must be left to another paper.

²⁴ I just give some hints. To block the reduction, one option could be that of giving logical relevance also to the plurality of algebraic values, thus defining an authentically many-valued validity (Malinowski 1994; Shramko and Wansing 2008). This strategy, however, can be hardly adopted by a truth pluralist, since if values other than truth were preserved, then, against the

In order to defend the idea that every logic is two-valued, a suitable two-valued semantics in which designation is turned into a semantic value must exist. The considered strategies to undermine Suszko's argument hinge exactly on this key point, stressing that not every mathematical construction counts as a legitimate semantics. Indeed, although Suszko's reduction yields an algebraic structure from a many-valued logic whatsoever, one could still wonder why the obtained algebraic structure should be considered semantically relevant. In general, when a logic is designed, the role of semantics is twofold. In one sense, it provides means to prove mathematical properties of the logic under study. In this merely mathematical sense, the semantics is a formal instrument chosen and evaluated for its mathematical services. Its possible connections with the general motivation of the logic are thus relaxed. In a second sense, the semantics is intended to give an interpretation doing justice to what the logic is meant to model. In order to undermine Suszko's reduction, two kinds of remarks can then be offered. One is about the *formal constraints* that an algebraic structure must meet to count as a semantics rather than as a mere algebra. The other concerns whether the obtained semantics is an *intended* semantics.

Consider the second kind of remarks first. Granted the mathematical legitimacy of Suszko's two-valued semantics, a many-valued logician could still question its legitimacy as an *intended* one. Given that the basic rationale for a many-valued logic is modelling cases where more than two (algebraic) values are admitted, providing two-valued models is a betrayal of such a basic motivation. Accordingly, the many-valued logician has a simple reason to reject the significance of Suszko's reduction: the provided semantics is not the intended one, so that it can only be used, at most, for mathematical purposes. Since the intended semantics is the original, many-valued, the two-valued one lacks semantical relevance and the reduction does not longer lend support to Suszko's thesis. In other words, the only semantic values are those explicitly included in the intended semantics, and not those possibly hosted in others.

Such a reply has a certain appeal, given that Suszko's manoeuvre of replacing an originally motivated model with another looks suspicious. However, the reduced semantics is not intended to replace the original intended many-valued semantics. Its aim is not that of showing that the many algebraic values can be dispensed under any respect. On the contrary, their semantical significance as referents of sentences is not in question, as the many-valued semantics

assumption, validity would no longer consist in truth preservation. Thus, the originally assumed standard truth-theoretic account would just be replaced with a different one. Moreover, the suspicious notion of designation would still be involved in the articulation of this proposal. Another approach could be that of avoiding any set of values, defining validity in terms of preservation of an order among values (Font 2009). This strategy cannot be exploited by the pluralist because, apparently, the only possible ordering is a trivial one (but see Cotnoir for a clever move, and Stollo 2018b and Keefe 2018 for criticisms). Moreover, if validity is defined as order (rather than truth) preservation, the initial truth-theoretic approach would again be replaced by a different one.

can be necessary and irreplaceable to model certain semantic phenomena. The role of the two-valued semantics is different. The new semantics is intended to show something about the formal nature of designation, namely that it can be treated as a semantic value, even though it is not hosted in the intended semantics. Designation is shown to be liable to a semantic reduction, even if, in the intended many-valued model, it plays a merely logical and not a semantic role. Designation is not needed as referent of sentences in the intended semantics, but as the value of generic truth that is implicitly preserved in valid inferences according to the standard account. The reduction shows that designation itself can be treated as a semantic value and thus eventually identified with truth, even though it is not an element of the intended semantics.

Let me rephrase this from another angle. Take a many-valued semantics equipped with designation to define validity. In order to identify designation and truth, designation must be treatable as a semantic value. Since Suszko's reduction, arguably, covers this step, a many-valued logic is shown to be implicitly committed to an extra value, whose role is being preserved in valid inferences. In particular, this value is semantic because it can be hosted as an algebraic element in a semantics, but it is an extra value because it is not an element of the intended semantics.²⁵ Thus, with respect to the commitment to generic truth, the fact that the two-valued semantics is not intended is irrelevant. What counts is just that it is a semantics. The only way out would be to insist that a non-intended semantics is not a semantics at all. This, however, is a radical move that would make the dispute eventually hard to adjudicate. Given the problems afflicting this option, to resist Suszko's thesis, it is more convenient if the obtained two-valued structure could be argued not to be a semantics for formal reasons. I now turn to this strategy—the first mentioned above—eventually showing that it can be used on behalf of strong pluralism.

As repeatedly stressed, the crucial point of Suszko's theorem is that designation can be turned into a value of a semantics. However, not every mathematical structure counts, in general, as a semantics. If some minimal formal constraints are not met, the obtained structure arguably results in a mathematical artefact, instrumental to prove mathematical theorems, but lacking semantic significance. In this case, the structure is not just an *unintended* semantics, but *not* a semantics at all. The formal constraints a structure needs to satisfy to count as a semantics can be general or logic specific. An example of the latter is the notion of *acceptable* semantics that logicians use to limit the range of mathematical structures in a particular context. Not all constraints are logic specific, however. There are also general features that a mathematical construction whatsoever should arguably meet to count as a semantics. A notably and fairly traditional one emerges in relation with language itself,

²⁵ Font speaks of designation as a truth value of the meta-theory, but I have qualms about this reformulation. I prefer not to rely on it and leave that option to the sympathetic reader.

namely compositionality. At least from Frege, compositionality is considered as a basic mark of an adequate semantics. Compositionality is often assumed in the study of language, from linguistics to philosophy of language, for several (and even logic-independent) reasons. Regardless of any other feature, if an algebraic structure is assigned to a language in a non-compositional way, it can be reasonably accused of failing to provide an acceptable interpretation to that language.²⁶ If it is not compositional, arguably it is not acceptable as a semantics. Of course, one might challenge a general requirement of compositionality, stressing problematic cases like intensional contexts or the role of pragmatics. However, notwithstanding its general importance as the standard and default option, it should be noted that the case for compositionality is particularly strong here, given that we can restrict our attention to merely extensional propositional fragments (where compositionality amounts to truth functionality) and to logics that are already equipped with an intended compositional semantics, so that compositionality is somehow *de facto* assumed.

The reason why I bring compositionality up is a striking feature of Suszko's reduction. In fact, in some cases, the two-valued 'semantics' obtained by Suszko's reduction is, remarkably, not compositional. Indeed, what is at stake is not just a lack of compositionality, but a loss of compositionality, since Suszko's reduction may force the passage from a compositional many-valued semantics to a non-compositional two-valued one. An example of this forced loss is provided by the application of the reduction to Łukasiewicz's three-valued logic (see the Appendix). Łukasiewicz's three-valued logic has an intended three-valued semantics that is compositional, but when Suszko's reduction is applied, what is obtained is a two-valued semantics that is no longer compositional. This potential loss of compositionality is what usually makes Suszko's theorem lose relevance in the eyes of logicians, leading them to a hasty dismissal. The reason should be clear. If the structure provided by Suszko's reduction, besides not being intended, can also impose a loss of compositionality, then it reveals itself to be a formal artefact obtained by brute mathematical force rather than a genuine semantic interpretation. The obtained model amounts to a formal manipulation to be judged by its possible mathematical merits, but void of semantic significance and hardly bearing on the foundation of many-valued logic. Suszko's attack quickly stops looking as a serious menace.

V. RESCUING TRUTH PLURALISM

This outcome has notable consequences for strong pluralism, since the loss of compositionality and the lack of an authentic semantic status of the obtained

²⁶ A different example of the role of compositionality in adjudicating the legitimacy of a formal semantics is provided by Hintikka's game-theoretic semantics for IF-logic, accused of not being compositional. See Hodges (1997) and Hintikka and Sandu (2011).

structure neutralises the commitment to a generic property of truth seemingly carried by designation. Let me repeat the point for the sake of clarity. The key point is captured by Suszko's thesis, which hinges on Suszko's reduction: if the pluralist semantics can be reduced to a two-valued semantics, then designation can be turned into a semantic value and, given the logical role of such a value, it can be eventually identified with (generic) truth. However, since treating designation as a sentential value can give rise to semantically unacceptable structures, the truth pluralist can deny that designation is to be identified with a semantic value and with (generic) truth. Having a designated value is not a property of truth because it fails to provide a value in a compositional semantics. Note also that the possible coincidence of designation with generic truth in some models, where the reduction possibly goes through, just shows that the two notions extensionally coincide in some cases. Occasional extensional coincidence, however, is insufficient to demonstrate that two properties are identical.²⁷ This remark should be familiar from debates on the metaphysics of properties, being an analogous case, in a formal context, of basic metaphysical distinctions.²⁸ Possible failures of compositionality in some models are thus enough to show that designation and generic truth are not the same property in general. Indeed, that the identification of designation and truth is too hasty becomes even more evident the moment it is noticed that logicians can and sometimes do designate values other than truth in order to define and study different logical relations. Choices on what values are to be designated in many-valued frameworks are far from trivial and give rise to alternative notions of validity that are both legitimate and interesting. This gives further support to the view that designation and generic truth (or the disjunction of all truth properties) extensionally coincide in particular cases, but such a coincidence is not the effect of an identity. If having a designated value cannot be identified with truth, let alone with generic truth, then designation is available to truth pluralism, even strong. Indeed, the only legitimate truth properties are those represented in the plurality of algebraic elements of the many-valued semantics, which embody the only values traded in valid inferences and stand for the area-specific truth properties of the pluralist. This is the conclusion that should be drawn from a careful analysis of many-valued logic and the role that designation plays in it.

Although I believe that this already gives an effective general strategy to rescue truth pluralism, it is worth discussing some possible rejoinders. A first potential one comes from a paper of Andy Yu,²⁹ in which he argues that, if a certain pluralist account is adopted, the truth-pluralist semantics is not itself

²⁷ This also shows that the issue is (hyper)intensional. The point is accounting for the nature of truth, not for its extension.

²⁸ Consider, for example, the property of being cordate and the property of being renate, which are different properties even though in the actual world their extensions coincide.

²⁹ Yu (2017b).

compositional. Clearly, if this claim held in general, then the above strategy would be a non-starter. It would be no use trying to delegitimise the two-valued semantics pointing to its lack of compositionality, if the pluralist semantics was already not compositional. However, Yu's argument can also be read as providing a reason not to adopt the particular pluralist account he assumes, rather than concluding that pluralism as such yields a non-compositional semantics. Indeed, this is the very option Yu himself favours, as he proposes to keep compositionality by revising the non-compositional pluralist account initially adopted. Note also that such an initial account was arguably not the best choice in any case. For example, according to it, a sentence may be true by having more than one area-specific truth property, even if it belongs to a single area of discourse.³⁰ This concession is crucial to run the argument for non-compositionality, but it is not endorsed by the majority of truth pluralists, nor is or must be assumed in this paper. Thus, while the issue raised by Yu may be interesting for the general understanding of pluralism, it does not seriously threaten the present project.

A more pressing objection could be moved, though, by focusing on the two-valued semantics. The key point of the strategy proposed above on behalf of strong pluralism is that, in some cases, Suszko's reduction outputs a non-compositional semantics. A critic might then retort that if compositionality is not lost when the particular pluralist semantics is reduced, then a commitment to generic truth keeps looming upon strong pluralism. If the pluralist proposes a many-valued model that, when subject to Suszko's reduction, yields a two-valued semantics that remains compositional, the reduction goes through and its semantic significance cannot be undermined. Thus, despite what happens in other models, in the pluralist one designation does stand for a generic property of truth. I offer two replies to counteract this concern. One is a weakening of the criticism and the other takes it on board and gestures towards some suitable pluralist semantics.

Let me start with the weakening. According to the objection, as long as Suszko's reduction goes through in the pluralist model (or a class of models), then there is co-extension where it matters. If designation and truth coincide in all *relevant* cases, then they can be identified. To claim that designation and truth diverge, the strong pluralist should show that the loss of compositionality occurs exactly in the relevant models, not in others. Stressing failures of the reduction in irrelevant cases is pointless. However, even if focusing only on pluralist models was legitimate, such a rejoinder would not yet be fatal to the pluralist. The move, in fact, can be tamed again with considerations about the metaphysics of properties. The reason is that not even necessary co-extension is arguably enough for property identity, as recent discussions on

³⁰ The proposal is thus also different from Cotnoir (2013).

hyper-intensionality have enlightened.³¹ Thus, even if there was extensional coincidence in all cases—intended as coincidence in all *relevant* cases—, one might still be suspicious of the identity of designation and truth. The objection is thus not completely conclusive.

An obstinate critic, however, does not need to embark in such disputes. The general distinction between designation and truth, in fact, may be enough to secure the legitimacy of many-valued logic, but as long as a property of generic truth emerges in some relevant cases, the viability of strong truth pluralism remains blocked. In other words, the very problem for truth pluralism is the commitment to generic truth, not its eventual identification with designation. Occasional failures of Suszko's reduction, and considerations about hyper-intensionality, might prevent the latter, but not the former. This is why, to be safe, strong pluralists should show that it is precisely in their semantics that Suszko's reduction gives rise to an unacceptable (namely non-compositional) two-valued structure. In order to tame this reformulation of the rejoinder, I briefly sketch a more elaborate move on behalf of strong pluralism.

Assume that if compositionality was lost in other cases but not in the pluralist semantics, then a commitment to generic truth would still be a genuine threat. To escape this more limited difficulty, the pluralist should adopt a specific many-valued semantics that loses compositionality when reduced to a two-valued one. In this way, the pluralist could explicitly hold that exactly in the specific case of the pluralist semantics designation does not stand for generic truth.³² The challenge then becomes that of checking whether the pluralist semantics does the job. Unfortunately, since truth pluralists are still discussing what semantics should be adopted, the issue cannot be definitely settled. This is not too bad, however, since, while some proposals might not work, at the same time room for manoeuvre is left available. Before giving some hints on how suitable pluralist semantics could be devised, let me point out that the critics face the same burden. To attack pluralism complaining that designation carries a commitment to generic truth, they must show that the pluralist is committed to a specific semantics such that, when reduced, it yields a two-valued structure that is semantically significant and, in particular, compositional. Providing this is far from trivial for both parties, so that, under the light of Suszko's reduction, the accusation is in any case much less straightforward and decisive than one might initially think.

³¹ For example, triangularity and trilaterality are necessarily co-extensive but distinct properties. On hyper-intensionality, see, for example, Nolan (2014).

³² The assumption that truth must be treatable as a value in a compositional semantics seems to conflict with non-compositional approaches such as supervaluationism. Given that in supervaluationism compositionality fails for supertruth, according to my argument supertruth should not be considered a kind of truth. While I cannot enter this debate here, let me point out that the status of supertruth is already contentious on its own, as, for example, supertruth fails to validate the truth schema. Moreover, and notably, failure of compositionality has indeed been employed to question the legitimacy of the identification of supertruth with truth (Smith 2016).

Let me now briefly sketch how a pluralist could proceed to elaborate a semantics resistant to Suszko's reduction. First of all, she can exploit the fact that pluralism usually assumes that different areas of discourses can have different truth properties and thus different semantics. Since the account of validity based on designation is supposed to take care of all possible cases, it is supposed to work for all areas of discourse, also singularly taken. Thus, it is enough that the semantics of a single area of discourse is resistant to Suszko's reduction to undermine the commitment to generic truth allegedly carried by designation. This already eases the task to a large extent. Accordingly, it is enough that pluralists endorse (possibly on independent grounds) a logic like Łukasiewicz's three-valued logic (L_3) in at least one area to be safe. The issue then becomes whether pluralists can find independent reasons to embrace a Suszko-resistant logic (like, but not limited to, L_3) in at least one area of discourse. Below, I show not just that they can, but that some of them already did. To this end, let me briefly rehearse the proposal advanced in recent works by Filippo Ferrari and Sebastiano Moruzzi.³³

Remarkably for the present project, Ferrari and Moruzzi's view has nothing to do with the puzzle raised by designation, since, instead, it is the result of a careful criticism of Crispin Wright's inflationary argument against truth deflationism.³⁴ A key step of such an inflationary argument, which is criticised by Ferrari and Moruzzi, is that truth and justification diverge in epistemically neutral cases of information. They oppose this step arguing that, if a reasonable epistemology and metaphysics for taste are endorsed, in the taste domain there is a perfect alignment between truth and justification. According to the most refined variant of their approach, in a (radically) neutral state of information, propositions do not fall either within the extension or within the anti-extension of both justification and truth. Such truth value gaps then block the inflationary argument, making room for a deflationary truth in the taste domain. Allowing truth to be gappy, however, also endangers the general validity of the disquotational truth schema, which is essential for truth and truth deflationism in particular. It is to fix this, and secure the schema, that a logic like L_3 is introduced.³⁵ According to Ferrari and Moruzzi, an alethic pluralist treatment of the taste domain eventually calls for a deflationary property of truth and a particular, L_3 -like, many-valued logic.

With such an outlined proposal at hand, we can return to our issue. According to the rejoinder, if the commitment to truth carried by designation is to be avoided, Suszko's reduction must be blocked exactly in the pluralist model. It is nonetheless clear that, if Ferrari and Moruzzi considerations are correct, pluralists do have independent grounds to maintain that at least one

³³ Ferrari & Moruzzi (2019, 2020).

³⁴ Wright (1992).

³⁵ See also Greenough (2010).

domain is governed by a many-valued logic resistant to Suszko's reduction. Although I do not intend to defend Ferrari and Moruzzi's approach here, the overall strategy of my paper clearly receives robust support from the fact that it can be successfully executed.

VI. FINAL REMARKS

Before ending the paper, let me add two final remarks. First, truth pluralism is often saddled with a set of truisms that are supposed to characterise truth. The basic idea is that in order to count as truth, a notion must meet some core platitudes any competent speaker should recognise. Examples can be the disquotational truth schema, the normative role of truth with respect to belief, the objectivity of truth, etc. Accordingly, to determine if a property is a truth property, we should check if it satisfies these truisms. The basic idea of truth pluralism is that in different areas of discourse, the truth platitudes are satisfied by different properties, so that there are many properties of truth. Given that meeting the truisms is necessary for a property to count as truth, one might propose a much simpler and straightforward strategy to argue that designation is not a truth property. Avoiding any detour through many-valued logic and Suszko's reduction, we could just assess whether designation meets the platitudes or not. However, pursuing a truism-based strategy might be harder than initially thought. Indeed, discussions on the status of notions similar to designation (like the second-order property of playing the truth role) proved contentious and somehow open to opposite verdicts.³⁶ Adopting an alternative strategy allows us to avoid similar complications. Moreover, even if another strategy exists, the one I have proposed here is a legitimate one nonetheless and it enriches the debate. Finally, the present strategy actually moves the debate to a deeper level. Instead of focusing on principles characterising truth, it relies on more basic and general constraints about semantic properties in general, such as the possibility of being treated as a reference of an expression in a formally acceptable semantics. This approach is more general than the one based on truth platitudes, given that it is not able to discriminate between truth and other semantic properties, like falsehood. Hence, it provides more basic evidence against the identification of designation with truth (or with falsehood for what matters).

Secondly, while this paper is intended to show, on behalf of strong pluralism, that designation should not be confused with generic truth, at the same time, this move might lead to a serious worry. Once designation and generic truth are disentangled, it might be worried that the truth-theoretic account of validity, which was assumed in the entire debate, has just been replaced

³⁶ Lynch (2008, 2009: 57, 63) and Pedersen & Wright (2013: 121).

with a different one. The idea goes like this: if designation is not truth, and validity is preservation of designation, then validity is not preservation of truth. Since the initial challenge for the pluralist was to make sense of the standard truth-theoretic account, it seems that the present strategy eventually misses the target. Given the above remarks, however, it should already be clear why the worry is misplaced. First of all, since also values other than truth can be designated, validity as preservation of designation is not standard logical validity, but it stands for a more general notion whose validity as truth preservation is a special case. The accusation would thus make sense only if restricted to the case in which only values for truth are designated. Even so, however, all efforts so far have been intended exactly to prevent a replacement of truth as the preserved value with designation. The discussion undermining the semantic significance of Suszko's reduction shows that designation cannot be turned into a semantic value, so that it cannot be the preserved value. The only semantic values had by sentences and preserved are the algebraic values standing for truth. Speaking of preservation of designation is just a loose way of speaking, which is harmless as long as it is not taken too seriously.³⁷ This kind of reflections also demonstrates how discussing Suszko's thesis from the perspective of the philosophy of truth sheds important light on crucial notions employed in the foundation of logic.

VII. CONCLUSION

By exploiting the comparison of truth pluralism and many-valued logic through the lens of Suszko's thesis, the paper has offered two main contributions. One is about truth pluralism and its ability to vindicate the standard understanding of logical validity. In this respect, I have argued that strong pluralists can afford the notion of designation, since it should not (or at least need not) be identified with truth. The basic reason is that designation may fail to provide a semantic value for an acceptable (compositional) semantics. The other, strictly related, is about the foundation of many-valued logic. In both cases, I hope that the fruitfulness of a combined treatment of truth pluralism and many-valued logic has emerged.³⁸

³⁷ Gamester (2019).

³⁸ A primordial idea of this paper took form during a series of seminars on many-valued logic at Scuola Normale Superiore, organised by Massimo Mugnai and Hykel Hosni. I thank them, and the other people attending the seminars, for calling my attention to Suszko's thesis. I am deeply in debt also to Rossella Marrano with whom for some time I worked on a similar paper in a different direction. I must thank audiences at seminars at Lingnan University, Singapore National University and University of Torino. A special thanks to Filippo Ferrari, Sebastiano Moruzzi, and Nikolaj Pedersen, for involving me in several activities on truth pluralism in Bologna. Giorgio Lando, Andy Yu, Tianqun Pan, Yong Cheng and Zhang Lifeng gently commented on previous

VIII. APPENDIX

A. Suszko's Theorem

I illustrate the theorem with respect to the simplified case discussed in the paper and refer to Shramko & Wansing (2008, 2018) for a general treatment.

Assume that there are only two ways of being true, represented by values t_1 and t_2 , and one way of being false, f . Let \mathcal{L} be a propositional language inductively defined as usual from a set of atomic sentences \mathcal{AL} by connectives \neg, \vee (the other connectives being possibly defined as usual). In order to give \mathcal{L} a many-valued semantics, let \mathbb{V} be a set of valuation functions $v : \mathcal{AL} \rightarrow \{t_1, t_2, f\}$.

The resulting semantics is *truth-functional* if these functions extend uniquely to $v : \mathcal{L} \rightarrow \{t_1, t_2, f\}$, namely if there are fixed functions f_{\neg}, f_{\vee} defined on the set of truth values.

Let $D = \{t_1, t_2\}$ be the set of designated values. The relation of logical consequence is defined as follows:

$$\phi_1, \dots, \phi_n \models_{\mathbb{V}} \theta \Leftrightarrow \forall v \in \mathbb{V} \text{ if } \forall i \in \{1, \dots, n\} v(\phi_i) \in D \text{ then } v(\theta) \in D,$$

for all $\phi_1, \dots, \phi_n, \theta \in \mathcal{L}$.

B. Suszko's Reduction

Let $D' = \{1, 0\}$, and, for each $v \in \mathbb{V}$, define a new, *two-valued* valuation function $v^* : \mathcal{L} \rightarrow \{1, 0\}$ in such a way that the value of a sentence ϕ under v^* is 1 just in case ϕ takes a designated value under the original three-valued valuation v , namely $v^*(\phi) = 1$ if $v(\phi) \in D$; otherwise, $v^*(\phi) = 0$.

$$v^*(\phi) = \begin{cases} 1, & \text{if } v(\phi) \in D \\ 0, & \text{otherwise.} \end{cases}$$

Let \mathbb{V}^* be the set of these valuations. We can then define a new notion of logical consequence based on this set:

$$\phi_1, \dots, \phi_n \models_* \theta \Leftrightarrow \forall v^* \in \mathbb{V}^* \text{ if } \forall i \in \{1, \dots, n\} v^*(\phi_i) = 1 \text{ then } v^*(\theta) = 1.$$

By generalising this construction, one can define the desired class of bivalent models characterising the logic. It follows from the definitions that the two consequence relations, $\models_{\mathbb{V}}$ and \models_* , are equivalent, since an inference is valid under $\models_{\mathbb{V}}$ iff it is also valid under \models_* .

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C. Loss of Compositionality

Suszko's reduction can output a semantics that is not compositional, even if applied to a compositional many-valued semantics. Consider Łukasiewicz three-valued logic. There we have three-valued valuations $v : \mathcal{L} \rightarrow \{0, \frac{1}{2}, 1\}$ and a set of designated values $D = \{1\}$. Accordingly, Suszko's reduction yields bivaluations $v^* : \mathcal{L} \rightarrow \{1, 0\}$, such that for any sentence $\phi \in \mathcal{L}$, $v^*(\phi) = 1$ if $v(\phi) = 1$, $v^*(\phi) = 0$ otherwise. Thus, given Łukasiewicz's truth table for negation

	¬
1	0
$\frac{1}{2}$	$\frac{1}{2}$
0	1

if a sentence ϕ is given value $\frac{1}{2}$ by a certain valuation v , its negation $\neg\phi$ is also $\frac{1}{2}$ under v . Since $\frac{1}{2}$ is not designated, we have that both ϕ and $\neg\phi$ are given value 0 under v^* . However, if ϕ is given value 0 under v' , its negation is 1 under v' . Thus, we have that ϕ is 0 under another valuation v'^* , whereas $\neg\phi$ is 1 under v'^* . This is a case where, although ϕ is given the same value (0) under these two bivaluations, its negation receives different values (0 and 1). In other words, the value of the sentence is not a function of the values of its compounding sentences and truth-functionality fails.

Note that the loss of compositionality is also forced if the set of designated values is $D = \{1, \frac{1}{2}\}$, thus including more than one value. In this case, if a sentence ϕ is given value $\frac{1}{2}$ by a certain valuation v , its negation $\neg\phi$ is also $\frac{1}{2}$ under v . Since $\frac{1}{2}$ is now designated, we have that both ϕ and $\neg\phi$ are given value 1 under v^* . However, if ϕ is given value 1 under v' , its negation is 0 under v' . Thus, we have that ϕ is 1 under v'^* , whereas $\neg\phi$ is 0 under v'^* . This is a case where, although ϕ is given the same value (1) under these two bivaluations, its negation receives different values (1 and 0).

REFERENCES

- Beall, J. C. (2000) 'On Mixed Inferences and Pluralism about Truth Predicates', *Philosophical Quarterly*, 50: 380–82.
- Caret, C. and Cotnoir, A. J. (2008) 'True, False, Paranormal, and Designated: A Reply to Jenkins', *Analysis*, 68: 3.
- Chemla, E. and Égré, P. (2019) 'Suszko's Problem: Mixed Consequence and Compositionality', *Review of Symbolic Logic*, 12: 736–67.
- Cotnoir, A. J. (2013) 'Validity for Strong Pluralists', *Philosophy and Phenomenological Research*, 86: 563–79.
- Dummett, M. (1959) 'Truth', *Proceedings of the Aristotelian Society*, 59: 141–62.
- (1978) *Truth and Other Enigmas*, Cambridge, MA: HUP.
- Ferrari, F. and Moruzzi, S. (2019) 'Ecumenical Alethic Pluralism', *Canadian Journal of Philosophy*, 49: 368–93.

- (2020) ‘Deflating Truth about Taste’, *American Philosophical Quarterly*, 75: 389–402.
- Ferrari, F., Moruzzi, S. and Pedersen, N. J. (2020) ‘Austere Truth Pluralism’, in J. Wyatt, J. Kim, M. P. Lynch and N. Kellen (eds) *The Nature of Truth*, 2nd edn. Cambridge, Massachusetts: MIT Press.
- Font, J. M. (2009) ‘Taking Degrees of Truth Seriously’, *Studia Logica*, 91: 383–406.
- Gamester, W. (2019) ‘Logic, Logical Form and the Disunity of Truth’, *Analysis*, 79: 34–43.
- Greenough, P. (2010) ‘Deflationism and Truth Value Gaps’, in C. D. Wright and N. J. L. Pedersen (eds) *New Waves in Truth. New Waves in Philosophy*, 115–25. London: Palgrave Macmillan.
- Hintikka, J. and Sandu, G. (2001) ‘Aspects of Compositionality’, *Journal of Logic, Language and Information*, 10: 49–61.
- Hodges, W. (1997) ‘Compositional Semantics for a Language of Imperfect Information’, *Logic Journal of the IGPL*, 5: 539–63.
- Keefe, R. (2018) ‘Pluralisms: Logic, Truth and Domain-Specificity’, in N. Kellen, N. Wyatt and N. Kellen (eds) *Pluralisms in Truth and Logic. Palgrave Innovations in Philosophy*, 429–52. Cham: Palgrave Macmillan.
- Langford, C. H. and Lewis, C. I. (1932) *Symbolic Logic*. New York: Dover Publications.
- Łukasiewicz, J. (1930) ‘Philosophische Bemerkungen zu Mehrwertigen Systemen Des Aussagenkalküls’, *Comptes rendus de la Société des Sciences et des Lettres de Varsovie, cl. iii*, 23, 51–77.
- Lynch, M. P. (2009) *Truth as One and Many*. Oxford: Clarendon Press.
- Malinowski, G. (1993) *Many-Valued Logics*. Oxford: Clarendon Press.
- (1994) ‘Inferential Many-Valuedness’, in J. Wolenski (ed.) *Philosophical Logic in Poland*, 75–84. Dordrecht: Kluwer Academic Publishers.
- Nolan, D. (2014) ‘Hyperintensional Metaphysics’, *Philosophical Studies*, 171: 149–60.
- Pedersen, N. J. (2006) ‘What Can the Problem of Mixed Inferences Teach Us about Alethic Pluralism?’, *The Monist*, 89: 103–17.
- Pedersen, N. J. and Wright, C. D. (2013) ‘Varieties of Alethic Pluralism (and Why Alethic Disjunctivism Is Relatively Compelling)’, in N. J. L. Pedersen and C. D. Wright (eds) *Truth Pluralism: Current Debates*. Oxford: OUP.
- Shramko, Y. and Wansing, H. (2008) ‘Suszko’s Thesis, Inferential Many-Valuedness, and the Notion of a Logical System’, *Studia Logica*, 88: 405–29.
- Shramko, Y. and Wansing, H. (2018) ‘Truth Values’, in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy* (Spring 2018 Edition). <<https://plato.stanford.edu/archives/spr2018/entries/truth-values/>> Winter 2020.
- Smith, N. J. J. (2016) ‘Truthier than Thou: Truth, Supertruth and Probability of Truth’, *Nous*, 50: 740–58.
- Stroilo, A. (2018a) ‘A Simple Notion of Validity for Alethic Pluralism’, *Synthese*, 195: 1529–46.
- (2018b) ‘Algebraic Semantics and Mixed Validities’, *Logique et Analyse*, 61: No 244.
- Suszko, R. (1977) ‘The Fregean Axiom and Polish Mathematical Logic in the 1920’s’, *Studia Logica*, 36: 373–80.
- Tappolet, C. (1997) ‘Mixed Inferences: A Problem for Pluralism about Truth Predicates’, *Analysis*, 57: 209–10.
- (2000) ‘Truth Pluralism and Many-Valued Logics: A Reply to Beall’, *Philosophical Quarterly*, 50: 382–85.
- Williamson, T. (1994) ‘A Critical Study of ‘Truth and Objectivity’’, *International Journal of Philosophical Studies*, 30: 130–44.
- Wright, C. (1992) *Truth and Objectivity*. Cambridge, MA: HUP.
- (1999) ‘Truth: A Traditional Debate Reviewed’, in S. Blackburn and K. Simmons (eds) *Truth*. Oxford: OUP.
- (2013) ‘A Plurality of Pluralisms?’, in N. J. Pedersen and C. Wright (eds) *Truth and Pluralism: Current Debates*, 123–53. New York: Oxford University Press.
- Yu, A. D. (2017a) ‘Logic for Alethic Pluralists’, *Journal of Philosophy* 114: 277–302.
- (2017b) ‘Can Alethic Pluralists Maintain Compositionality?’, *The Philosophical Quarterly*, 67: 625–63.

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