Cooperative Adaptive Cruise Control in the Presence of Communication and Radar Stochastic Data Loss

Hamed Rezaee, *Senior Member, IEEE*, Kangkang Zhang, Thomas Parisini, *Fellow, IEEE*, and Marios M. Polycarpou, *Fellow, IEEE*

***Abstract*— Control of a platoon of connected vehicles with nonlinear longitudinal dynamics subject to failure in receiving communicated and radio data is addressed in this paper. We consider a scenario when due to malfunction of communication and radio devices, required data for cooperative adaptive cruise control may be stochastically unavailable. We develop a control scheme such that under certain conditions, the regulation of the intervehicle distances in desired values can be guaranteed. Specifically, we rigorously show that if the probability of receiving the required data for each vehicle is not zero, then under the proposed control strategy, the intervehicle distances almost surely converge to the desired values. Accurate performance of radars is a key assumption in the existing results in the literature on cooperative adaptive cruise control. Hence, the main contribution of this paper is that under the proposed control strategy, the robustness of the platoon against stochastic data loss in both communication network and vehicles radars is guaranteed. Simulation results verify the acceptable performance of the proposed control strategy.**

***Index Terms*— Adaptive control, adaptive cruise control, data loss, stochastic systems, vehicular platoons.**

I. Introduction

ADAPTIVE cruise control is a radar-based technology in today driver-assistance systems based on which each vehicle adjusts its speed to keep a desired distance from its preceding vehicle. Accordingly, one of the vehicles (driven by a human, semi-autonomously, or autonomously) may serve as a leader for a platoon of vehicles, while the other vehicles follow their preceding vehicles in desired relative distances. Cooperative adaptive cruise control (CACC) is an enhanced version of adaptive cruise control, based on which in addition to radar-based information, the vehicles exchange more state information via a communication network. Accordingly, the performance/accuracy of each vehicle in following its preceding vehicles can be improved [1], [2], [3], [4], [5].

Due to stochastic properties of electronic and communication devices, communication and radio subsystems are subject to data lose, and this issue can affect the performance of closed-loop control systems. Thus, one of challenging problems in control of vehicular platoons is guaranteeing regulation of intervehicle distances in desired values when information flow via communication or receiving data via radars are stochastically failing over time.

A. State of the Art and Motivation of the Study

In the CACC problem, *spacing errors* (error of keeping desired distances) are the variables to be regulated, and hence in the literature, CACC has relied on the accurate measurement of the distances among the vehicles. Therefore, in the existing results on CACC in the presence of data loss, accurate performance of the radars is an assumption, and just communication loss is considered. For instance, control of connected linear vehicles in the presence of cyber-attacks is investigated in [6], [7], and [8], in which attacks are modeled as random data loss in communication among the vehicles. In [9], to reduce the impact of communication loss in linear connected vehicles, an observation strategy is proposed to estimate the state information of the preceding vehicle. In [10] and [11], switching strategies for adaptive control of linear vehicular platoons in the presence of communication loss are developed. In [12], a model with capability of adaptation to changes in platoons topology due to communication failures is developed. Control of linear vehicular platoons in the presence of random data loss is studied in [13] and [14]. In [15], the stochastic $\mathcal{L}_2$ stability of linear connected vehicles under unreliable communication channels is addressed, and to increase the robustness of linear connected vehicles against data loss, the idea of using model predictive control is used in [16] and [17]. However, in practice, radars are also subject to data loss, and data loss is an inevitable issue in radars [18], [19], [20], [21], [22]. Hence, assumption on receiving data from radars constantly may not be always realistic.

Manuscript received 5 July 2022; revised 4 March 2023, 24 July 2023, and 28 October 2023; accepted 2 November 2023. Date of publication 1 January 2024; date of current version 31 May 2024. This work was supported in part by the European Union's Horizon 2020 Research and Innovation Program under the Marie Skłodowska-Curie Agreement 101027980 and the Agreement 739551 (KIOS Center of Excellence) and in part by the Italian Ministry for Research in the framework of the 2017 Program for Research Projects of National Interest (PRIN) under Grant 2017YKXYXJ. The Associate Editor for this article was S. Santini. *(Corresponding author: Hamed Rezaee.)*

Hamed Rezaee is with the Department of Mechanical Engineering, Imperial College London, SW7 2AZ London, U.K. (e-mail: h.rezaee@imperial.ac.uk).

Kangkang Zhang is with the Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ London, U.K. (e-mail: kangkang.zhang@imperial.ac.uk).

Thomas Parisini is with the Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ London, U.K., also with the Department of Engineering and Architecture, University of Trieste, 34127 Trieste, Italy, and also with the KIOS Research and Innovation Center of Excellence, University of Cyprus, 1678 Nicosia, Cyprus (e-mail: t.parisini@imperial.ac.uk).

Marios M. Polycarpou is with the KIOS Research and Innovation Center of Excellence and the Department of Electrical and Computer Engineering, University of Cyprus, 1678 Nicosia, Cyprus (e-mail: mpolycar@ucy.ac.cy).

Digital Object Identifier 10.1109/TITS.2023.3335310

In addition to the aforementioned studies, CACC in the presence of delays is also considered in several studies in recent years (for instance, see [23], [24], [25], [26]). The main focus of those studies is on just guaranteeing the local stability and string stability of the platoon in the presence of time-delays in the communication network. For instance, in [23] decreasing minimum string-stable time gap in linear connected vehicles is considered. To achieve this goal, based on knowledge of the upper bound of the communication delays, a control architecture based on the Smith Predictor is proposed. In [24], the problem of CACC in the presence of communication delays is addressed. In that study, by developing an optimal CACC strategy, the local stability and string stability of the platoon with any time gap greater than the communication delay are guaranteed. By developing a μ-synthesis approach, a similar problem is addressed in [25]. In that study, based on knowledge of the range of time delays, string stability in the presence of communication delays is studied, and in [26], results of simulation studies of the robustness of nonlinear connected vehicles in the presence of communication delays are presented.

B. Objectives and Contributions

By considering *the constant time-gap spacing policy* [27], [28], a distributed adaptive control strategy for a platoon of vehicles is proposed in this paper. Since the main objective in CACC is regulation of intervehicle distances in desired values, access to the information of the intervehicle distances is an important requirement. Accordingly, in the existing results on CACC (dealing with data loss) [6], [7], [8], [9], [10], [11], [13], [15], [16], [17], the constant measurement of the distances among the vehicles is a key assumption such that the radars should measure the intervehicle distances constantly. In other words, the design and analysis of the CACC strategy in the aforementioned studies is based on correct performance of the vehicles radars. The main contribution of this paper is to propose a CACC strategy which does not rely on the constant availability of distance information measured by radars, and guarantees the robustness of the platoon against both communication and radar data loss. We develop a control strategy such that in the presence of communication and radar stochastic data loss, the convergence of some surfaces of the vehicles states to zero is guaranteed. On the surfaces, we show that if the probability of receiving the required data for CACC is not zero, the intervehicle distances almost surely converge to the desired values.

In addition to robustness against data loss, we show that under certain conditions, the proposed CACC strategy is robust against delays in the platoon of vehicles as well. Despite the existing results in the literature devoted to CACC in the presence of communication delays [23], [24], [25], [26], under the proposed control strategy, the robustness of the platoon against delays in both the vehicles radars and the communication network is guaranteed. Moreover, based on the control structure proposed in this paper, the effect of time-delays is rejected. In this condition, for unknown communication delay (with unknown upper bound), the spacing errors almost surely converge to zero and local stability and string stability of the platoon are also guaranteed.

It is worth mentioning that a preliminary version of this paper was presented in the conference paper [29]. The obtained results in [29] are based on conditions on the control gains with respect to the probability of receiving data. However, in the proposed control strategy in this paper, no knowledge of the probability of receiving data is needed. Moreover, in [29], the vehicles are modeled as linear double-integrators. However, when the model parameters are unknown, feedback linearization may not be possible such that the vehicles may not be described by integrator models. Hence, another difference between the current study and [29] is that the nonlinear model of the longitudinal dynamics of the vehicles with unknown parameters is considered in this paper. It should be noted that in the presence of data loss, some closed-loop control variables of the vehicles are not continuous and differentiable, and this issue makes the design of an adaptive control strategy more challenging.

The rest of the paper is organized as follows. Preliminaries are provided in the next section. The problem is stated in Section III. The control strategy is proposed in Section IV. Section V provides simulation results, and some conclusions are given in Section VI.

II. Preliminaries

Notations and some concepts and definitions on stochastic processes are presented in this section.

A. Notations

$\mathbb{R}$ is the set of real numbers. $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$ denote the sets of positive and non-negative real numbers, respectively. $\mathbb{E}\{\cdot\}$ and $\mathbb{P}\{\cdot\}$ are the expected value and probability of a stochastic variable, respectively. $\mathbb{E}\{X|E\}$ denotes the conditional expected value of X given E. $\mathcal{K}$ denotes the class of continuous and strictly increasing functions from $[0, a)$ to $[0, \infty)$ such that $k(0) = 0$. $|\cdot|$ denotes the absolute value, and $\mathrm{sgn}(\cdot)$ denotes the sign function defined as follows:

$$\mathrm{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0. \end{cases}$$

Moreover, $\equiv$ denotes equivalency such that $x(t) \equiv 0$ implies that $x(t)$ is identical to zero. $\sup\{\cdot\}$ denotes supremum. $\|\cdot\|_{\mathcal{L}_p}$ stands for the $\mathcal{L}_p$ norm. 'a.s.' means 'almost surely', and 'w.p.' means 'with probability'.

B. Stochastic Processes

We describe a stochastic process by the probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω denotes the space of events, $\mathcal{F}$ is a σ-algebra on Ω, and $\mathbb{P}$ is a probability measure on $(\Omega, \mathcal{F})$ where $0 \leq \mathbb{P}\{\cdot\} \leq 1$ and $\mathbb{P}\{\Omega\} = 1$ [30]. A filtration $\{\mathcal{F}_t, t \geq 0\}$ on $(\Omega, \mathcal{F}, \mathbb{P})$ is a family of sub-σ-algebras of $\mathcal{F}$ where

$$\mathcal{F}_q \subset \mathcal{F}_t, q < t.$$

Accordingly, a stochastic process $X = \{X(t), t \geq 0\}$ is said to be adapted to the filtration $\{\mathcal{F}_t\}$ if $X(t)$ is $\mathcal{F}_t$-measurable for all $t \geq 0$. Now, X is said to be a *super-martingale* relative to $\{\mathcal{F}_t\}$ and $\mathbb{P}$ if [30], [31]:

i) X is adapted to the filtration $\{\mathcal{F}_t\}$,

ii) $\mathbb{E}\{|X(t)|\} < \infty, \forall t$,

iii) $\mathbb{E}\{X(t)|\mathcal{F}_q\} \leq X(q), t > q$.

We say the stochastic variable $X(t)$ converges to X_f *in the sense of probability* if for any $\epsilon \in \mathbb{R}_{>0}$ [32],

$$\lim_{t\to\infty} \mathbb{P}\{|X(t) - X_f| \geq \epsilon\} = 0.$$

Moreover, we say the stochastic variable $X(t)$ *almost surely* converges to X_f if [32]

$$\mathbb{P}\{\lim_{t\to\infty} X(t) = X_f\} = 1.$$

If this holds, we write

$$\lim_{t\to\infty} X(t) \xrightarrow{\text{a.s.}} X_f.$$

In general, we say that an event happens almost surely if it happens with probability 1. Now, based on the concepts of super-martingales and almost sure convergence, the *Super-Martingales Convergence Theorem* can be stated as follows.

Theorem 1: [31, Th. 4.1] Let $X = \{X(t), t \geq 0\}$ be a non-negative super-martingale. Then, $X(t)$ almost surely converges to a non-negative finite value.

III. Problem Statement

Consider a platoon of vehicles consisting of a leader indexed by $i = 0$ and N followers indexed by $i \in \mathcal{S} = \{1, 2, \ldots, N\}$, where the mathematical model of the longitudinal dynamics of the ith vehicle is as follows [1], [33]:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t)\\ \dot{v}_i(t) &= \frac{1}{M_i}\Big(F_{ia}(t) - F_{id}(t) - \frac{1}{2}C_i\eta_i A_i v_i(t)^2\\ &\quad - M_i g f_i \cos\phi_i(t) - M_i g \sin\phi_i(t)\Big), \end{aligned} \tag{1}$$

where $x_i(t)$ denotes the displacement of the rear-bumper, $v_i(t)$ is the speed, $\phi_i(t)$ is the road grade, $F_{ia}(t)$ denotes the accelerating input and $F_{id}(t)$ denotes the deaccelerating/braking input. For simplicity of notation, in the rest of the paper we consider $F_i(t) = F_{ia}(t) - F_{id}(t)$ as the control input. In this condition, the positive part of the input signal is associated with $F_{ia}(t)$ and the negative part of the input signal is associated with $F_{id}(t)$. Moreover, g is the gravitational acceleration and M_i, C_i, η_i, A_i, and f_i are unknown constant parameters where M_i is the mass, C_i is the coefficient of the aerodynamic drag, η_i is the air density, A_i is the frontal area, and f_i is the coefficient of the rolling resistance.

Let Vehicle $i-1, i \in \mathcal{S}$, be the preceding vehicle of Vehicle i. Accordingly, we define the distance of Vehicle i from Vehicle $i-1$ as follows:

$$d_i(t) = x_{i-1}(t) - L_i - x_i(t), \tag{2}$$

where L_i denotes the length of Vehicle i. Note that $x_i(t)$ may not be an accessible information. Thus, we assume that Vehicle i just has access to the relative distance $d_i(t)$ from Vehicle $i-1$. In this condition, the objective is to develop a local control strategy for each vehicle such that by considering a spacing policy, Vehicle $i, i \in \mathcal{S}$, reaches a desired distance from Vehicle $i-1$. A common spacing policy is the *constant time-gap spacing policy* based on which the desired distance of Vehicle i from Vehicle $i-1$ is a linear function of the speed of Vehicle i determined as follows [27]:

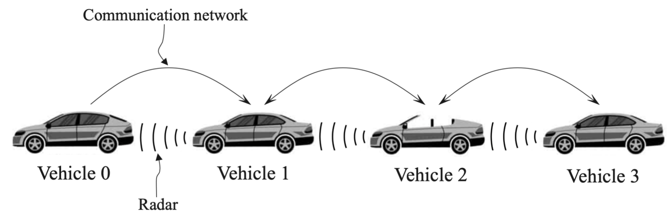


Fig. 1. A platoon of four connected vehicles.

$$d_{\text{des},i}(t) = r_i + h_i v_i(t), \quad i \in \mathcal{S}, \tag{3}$$

where $d_{\text{des},i}(t)$ is the desired distance of Vehicle i from Vehicle $i-1$, r_i is the standstill distance, and h_i is the time headway. Therefore, the spacing error of the ith vehicle is described as follows:

$$e_i(t) = d_{\text{des},i}(t) - d_i(t), \quad i \in \mathcal{S}. \tag{4}$$

In order to design a proper control strategy for Vehicle i such that the spacing errors $e_i(t), i \in \mathcal{S}$, converge to zero, it is necessary for the vehicle to have access to some information of the preceding vehicle. We assume that $d_i(t)$ is measured by a local radar, and more required state information of the preceding vehicle is obtained via the communication network connecting the vehicles. In Fig. 1, a platoon of connected vehicles equipped with radars under a communication network is depicted. However, due to malfunction of electronic and communication devices, the radars and the communication network may be subject to data loss. Accordingly, by assuming the data loss to be stochastic, for Vehicle $i, i \in \mathcal{S}$, we define $p_i(t) \in [0, 1]$ as the probability of receiving all the required radio/communication data, that is, $1 - p_i(t)$ is the probability of losing some radio/communication data. In the presence of stochastic communication and radar failure, the objective is to develop a control strategy such that the spacing errors almost surely converge to zero as follows:

$$\lim_{t\to\infty} e_i(t) \xrightarrow{\text{a.s.}} 0, \quad i \in \mathcal{S}. \tag{5}$$

Since the vehicles parameters are considered unknown, an adaptive control scheme is needed. To design a suitable adaptive control strategy, the following assumption is considered.

Assumption 1: The leader speed and acceleration are bounded.

We should note that the stability analysis of the vehicular platoon under the proposed control strategy relies on the boundedness of the leader speed and acceleration. However, such requirement is typically satisfied in real practical scenarios.

Remark 1: Since the vehicles dynamics depend on the road slope, the precise performance of the proposed control strategy in this paper is based on the measurement of the road slope by possible slope meters. Such sensors are also required in several other techniques for control of autonomous vehicles (for instance, see [1] and [34]).

IV. Proposed Cooperative Adaptive Cruise Control Strategy

A control strategy for regulation of the intervehicle distances is developed in this section. The base of the proposed control strategy is designing surfaces of the follower vehicles states on which the spacing errors almost surely converge to zero. Accordingly, an adaptive control strategy for driving the follower vehicles behaviors toward the designed surfaces (and then remaining on it) is designed. For each follower, a surface of the vehicle states is proposed as follows:

$$\begin{aligned}\dot{s}_i(t) &= v_i(t) - \xi_{1i}(t), \quad i \in \mathcal{S}\\ \dot{\xi}_{1i}(t) &= \xi_{2i}(t)\\ \dot{\xi}_{2i}(t) &= \frac{1}{h_i}\Big(- \xi_{2i}(t) - k_i(\xi_{1i}(t) + h_i\xi_{2i}(t))\\ &\qquad - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))\Big),\end{aligned} \tag{6}$$

where $s_i(t)$ is the surface, and $\xi_{1i}(t)$ and $\xi_{2i}(t)$ are auxiliary states with arbitrary initial values, $k_i > 0$ is a constant, $\chi_i(t) > 0$ is a robustifying gain to be designed later, and $\zeta_i(t)$ is as follows:

$$\begin{aligned}\zeta_i(t) = \xi_{1i}(t) - \xi_{1,i-1}(t) + h_i\xi_{2i}(t)\\ + k_i(-d_i(t) + d_{\mathrm{des},i}(t)).\end{aligned} \tag{7}$$

Moreover, $a_i(t) \in \{0, 1\}$, where we set $a_i(t) = 1$ if Vehicle i receives all the required communication and radar data at time t, otherwise we set $a_i(t) = 0$. In this condition, $a_i(t)$ will be switching with the following probabilities:

$$a_i(t) = \begin{cases} 1 & \text{w.p.} \quad p_i(t)\\ 0 & \text{w.p.} \quad 1 - p_i(t).\end{cases}$$

Indeed, $p_i(t)$ is a joint probability that Vehicle i receives all the required information at time t. In practice, the probability of receiving the communication data and the probability of receiving the radar data can be different, but $a_i(t)$ is 1 when both signals are available at time t. This means that $a_i(t)$ is 1 with the joint probability $p_i(t)$.

Note that as the discontinuous term $a_i(t)\mathrm{sgn}(\zeta_i(t))$ is measurable and locally bounded, Filippov solutions for (6) exist [35]. Hence in the rest of the paper, all the solutions are in the sense of Filippov. Moreover, regarding the leader vehicle, since it does not follow the other vehicles and $F_0(t)$ can be determined arbitrarily, we define $\xi_{10}(t)$ and $\xi_{20}(t)$ as follows:

$$\begin{aligned}\xi_{10}(t) &= v_0(t)\\ \xi_{20}(t) &= \dot{v}_0(t).\end{aligned} \tag{8}$$

There are two considerations behind designing $s_i(t)$ in the form of (6) which are explained as follows:

1) Since the information received from the preceding vehicle is subject to failing, $\zeta_i(t)$ may not be constantly available, and this is a source of error for the control system. As the sign function changes among $\{-1, 0, 1\}$, by using $\mathrm{sgn}(\zeta_i(t))$ instead of $\zeta_i(t)$ in the control law, it is possible to limit the magnitude of the related signals. Hence, by considering the definition of $a_i(t)$, by proper design of the sign function gain $\chi_i(t)$, it is possible to provide robustness against erroneous signals.
2) One important concern is nonlinear robust control is the chattering phenomenon in control inputs due to using discontinuous robustifying terms such as the sign function. In this paper, the main idea of using the auxiliary states $\xi_{1i}(t)$ and $\xi_{2i}(t)$ is that they increase the order of the vehicles closed-loop models. In this condition, the robust nonlinear control law of (6) is designed for the auxiliary states such that the chattering due to the sign functions just affects $\dot{\xi}_{2i}(t)$. Hence, according to (6), the existing integrator between $\dot{\xi}_{2i}(t)$ and $\xi_{2i}(t)$ filters the chattering in $\xi_{1i}(t)$ and $\xi_{2i}(t)$. Therefore, by proper design of the control strategy based on chattering-free signals, possible input chattering can be avoided. Note that $s_i(t)$ is deigned based on the auxiliary states as (6) such that by the convergence of $s_i(t)$ and $\dot{s}_i(t)$ to zero, the almost sure convergence of the spacing errors to zero is guaranteed. The above-mentioned issues will be investigated in Theorem 2.

To achieve the aforementioned objectives, since the vehicles parameters are considered unknown, an adaptive control strategy is considered. First by considering the definition of $\xi_{1i}(t)$ and $s_i(t)$ in (6), we define the following variable:

$$\rho_i(t) = v_i(t) - \xi_{1i}(t) + \sigma_{1i}s_i(t), \tag{9}$$

where σ_{1i} is a positive constant. Now, we propose an adaptive control strategy for the ith follower vehicle as follows:

$$F_i(t) = -s_i(t) - \varphi_i(t)^\top\hat{\theta}_i(t) - \sigma_{2i}\rho_i(t), \tag{10}$$

where σ_{2i} is a positive constant and $\hat{\theta}_i(t) \in \mathbb{R}^3$ is obtained from the following adaptation law:

$$\dot{\hat{\theta}}_i(t) = \gamma_i\rho_i(t)\varphi_i(t), \tag{11}$$

in which γ_i is a positive constant and

$$\begin{aligned}\varphi_i(t) = \Big[- v_i(t)^2 \quad - g\cos\phi_i(t)\\ - g\sin\phi_i(t) - \xi_{2i}(t) + \sigma_{1i}(v_i(t) - \xi_{1i}(t))\Big]^\top.\end{aligned} \tag{12}$$

The proposed control strategy is rigorously analyzed in Theorem 2.

Theorem 2: Consider the vehicular platoon given in (1) and the control law given in (6)-(12) with the gain $\chi_i(t), i \in \mathcal{S}$, being designed as follows:

$$\dot{\chi}_i(t) = a_i(t)\lambda_i|\zeta_i(t)|, \tag{13}$$

where $\lambda_i \in \mathbb{R}_{>0}$ is any positive scalar. If $p_i(t) \neq 0, i \in \mathcal{S}$, then we have

$$\lim_{t\to\infty} e_i(t) \xrightarrow{\text{a.s.}} 0. \tag{14}$$

Moreover, all the variables $s_i(t)$, $\xi_{1i}(t)$, $\xi_{2i}(t)$, $\hat{\theta}_i(t)$, and $\chi_i(t)$ and the states $d_i(t)$ and $v_i(t), i \in \mathcal{S}$, almost surely remain bounded.

Proof: We prove the theorem in two steps. In the first step, we show that $s_i(t)$ and $\dot{s}_i(t)$ remain bounded and converge to zero. Then, in the next step, along $s_i(t) = 0$ and $\dot{s}_i(t) = 0$, achievement of (14) is concluded.

a) Step 1: From (6), we recall that

$$\dot{s}_i(t) = v_i(t) - \xi_{1i}(t). \tag{15}$$

To analyze the evolution of $s_i(t)$ in (15), let us consider the following positive definite function of $s_i(t)$:

$$V_{1i}(t) = \frac{1}{2}s_i(t)^2. \tag{16}$$

The time derivative of $V_{1i}(t)$ along (15) yields

$$\dot{V}_{1i}(t) = s_i(t)v_i(t) - s_i(t)\xi_{1i}(t). \tag{17}$$

Accordingly, the following control law as the desired value of $v_i(t)$ can be considered:

$$\delta_i(t) = \xi_{1i}(t) - \sigma_{1i}s_i(t). \tag{18}$$

From (17) and (18), it follows that

$$\dot{V}_{1i}(t) = s_i(t)v_i(t) - s_i(t)(\delta_i(t) + \sigma_{1i}s_i(t)). \tag{19}$$

According to (9) and (18), one gets

$$\rho_i(t) = v_i(t) - \delta_i(t), \tag{20}$$

and therefore (19) can be rewritten as follows:

$$\dot{V}_{1i}(t) = s_i(t)\rho_i(t) - \sigma_{1i}s_i(t)^2. \tag{21}$$

However, because of the term $s_i(t)\rho_i(t)$, (21) does not guarantee the convergence of $V_{1i}(t)$ to zero. We show that the control law given in (10) compensates this term. By considering (20), one gets

$$\dot{\rho}_i(t) = \dot{v}_i(t) - \dot{\delta}_i(t),$$

which by considering (1) and (18), can be restated with respect to the control command $F_i(t)$ as follows:

$$\begin{aligned}\dot{\rho}_i(t) = {}& \frac{1}{M_i}\Big(F_i(t) - \frac{1}{2}C_i\eta_iA_iv_i(t)^2 \\ & - M_igf_i\cos\phi_i(t) - M_ig\sin\phi_i(t)\Big) \\ & - \dot{\xi}_{1i}(t) + \sigma_{1i}\dot{s}_i(t).\end{aligned} \tag{22}$$

From (6) and (15), (22) can be rewritten as

$$\begin{aligned}\dot{\rho}_i(t) = {}& \frac{1}{M_i}\Big(F_i(t) - \frac{1}{2}C_i\eta_iA_iv_i(t)^2 \\ & - M_igf_i\cos\phi_i(t) - M_ig\sin\phi_i(t)\Big) \\ & - \xi_{2i}(t) + \sigma_{1i}(v_i(t) - \xi_{1i}(t)).\end{aligned}$$

Therefore, we can restate $M_i\dot{\rho}_i(t)$ as the summation of $F_i(t)$ and another term obtained by the multiplication of a known vector and an unknown parameter vector as follows:

$$M_i\dot{\rho}_i(t) = \varphi_i(t)^\top\theta_i + F_i(t), \tag{23}$$

where $\varphi_i(t)$ is given in (12), and

$$\theta_i = \left[\tfrac{1}{2}C_i\eta_iA_i \;\; M_if_i \;\; M_i\right]^\top.$$

To analyze $\rho_i(t)$ along the differential equitation (23) and under the control law given in (10), we consider the following Lyapunov function:

$$V_{2i}(t) = \frac{1}{2}M_i\rho_i(t)^2 + \frac{1}{2\gamma_i}(\hat{\theta}_i(t) - \theta_i)^\top(\hat{\theta}_i(t) - \theta_i). \tag{24}$$

The time derivation of $V_{2i}(t)$ along (23), by considering $F_i(t)$ given in (10), yields

$$\begin{aligned}\dot{V}_{2i}(t) = {}& \rho_i(t)\Big(\varphi_i(t)^\top(\theta_i - \hat{\theta}_i(t)) - s_i(t) \\ & - \sigma_{2i}\rho_i(t)\Big) + \frac{1}{\gamma_i}(\hat{\theta}_i(t) - \theta_i)^\top\dot{\hat{\theta}}_i(t).\end{aligned} \tag{25}$$

Therefore, if we consider the adaptation law (11), from (25), we have

$$\dot{V}_{2i}(t) = -\rho_i(t)\Big(s_i(t) + \sigma_{2i}\rho_i(t)\Big). \tag{26}$$

Now, let us consider the following Lyapunov function:

$$V_i(t) = V_{1i}(t) + V_{2i}(t). \tag{27}$$

By considering (21) and (26), one gets

$$\dot{V}_i(t) = -\sigma_{1i}s_i(t)^2 - \sigma_{2i}\rho_i(t)^2.$$

Since $\dot{V}_i(t) \leq 0$, from (16), (24), and (27), it follows that $s_i(t)$, $\rho_i(t)$, and $\hat{\theta}_i(t) - \theta_i$ are bounded which implies the boundedness of $\hat{\theta}_i(t)$ as well. Therefore, by considering the nonautonomous system described by (11), (15), and (23), according to [36], one concludes that

$$\lim_{t\to\infty} s_i(t) \equiv 0,$$

that is, $s_i(t)$ and $\dot{s}_i(t)$ converge to zero. From (18) and (20), one gets

$$\begin{aligned}\xi_{1i}(t) &= \delta_i(t) + \sigma_{1i}s_i(t) \\ v_i(t) &= \rho_i(t) + \delta_i(t).\end{aligned} \tag{28}$$

Therefore, from (15) and (28), it follows that

$$\dot{s}_i(t) = \rho_i(t) - \sigma_{1i}s_i(t). \tag{29}$$

We have shown that $s_i(t)$ and $\rho_i(t)$ are bounded. Thus, from (29), the boundedness of $\dot{s}_i(t)$ can be concluded as well.

b) Step 2: From (15), it follows that

$$v_i(t) = \dot{s}_i(t) + \xi_{1i}(t), \tag{30}$$

implying that there exists a constant c_i such that

$$x_i(t) = s_i(t) + \int_0^t \xi_{1i}(\tau)d\tau + c_i. \tag{31}$$

Therefore, from (30) and (31), $d_i(t)$ and $d_{\text{des},i}(t)$ defined in (2) and (3) can be restated based on the auxiliary states as follows:

$$\begin{aligned}d_i(t) = {}& s_{i-1}(t) + \int_0^t \xi_{1,i-1}(\tau)d\tau + c_{i-1} - L_i - s_i(t) \\ & - \int_0^t \xi_{1i}(\tau)d\tau - c_i \\ d_{\text{des},i}(t) = {}& r_i + h_i\dot{s}_i(t) + h_i\xi_{1i}(t).\end{aligned} \tag{32}$$

In order to show the almost sure convergence of $e_i(t)$ to zero, according to (4) and (32) and since $s_i(t)$, $s_{i-1}(t)$, and $\dot{s}_i(t)$

converge to zero, it is sufficient to show that the following modified error, denoted by $\varepsilon_i(t)$, almost surely converges to zero:

$$\varepsilon_i(t) = r_i + h_i\xi_{1i}(t) - \int_0^t \xi_{1,i-1}(\tau)d\tau - c_{i-1} + L_i + \int_0^t \xi_{1i}(\tau)d\tau + c_i. \quad (33)$$

Now, to show the almost sure convergence of $\varepsilon_i(t)$ to zero, we consider the following variable:

$$z_i(t) = \dot{\varepsilon}_i(t) + k_i\varepsilon_i(t). \quad (34)$$

Since $z_i(t)$ is a Hurwitz polynomial of $\varepsilon_i(t)$, if we show that $z_i(t)$ is almost surely bounded and converges to zero; then, we can conclude that $\varepsilon_i(t)$ and $\dot{\varepsilon}_i(t)$ almost surely remain bounded and converge to zero (due to the input-to-state stability of Hurwitz linear systems [37]). Let us define

$$\Xi_i(t) = \frac{1}{h_i}(\xi_{2,i-1}(t) + k_i\xi_{1,i-1}(t)), \quad i \in \mathcal{S}. \quad (35)$$

According to (8) and Assumption 1, $\Xi_1(t)$ is bounded. Based on the boundedness of $\Xi_1(t)$, first we prove the theorem for Vehicle 1. Accordingly, we will conclude that $\Xi_2(t)$ is bounded as well. Then, with the same procedure we can prove the theorem for Vehicle 2, and respectively it will be proved for all Vehicles $i \in \{3, \dots, N\}$. First we consider the case when $i = 1$. In this condition, according to the boundedness of $\Xi_i(t)$, first we define a positive constant χ_i^* where

$$\sup\left\{\frac{h_i|\Xi_i(t)|}{\mathbb{E}\{a_i(t)\}}\right\} < \chi_i^*. \quad (36)$$

Now, we consider a positive definite function of $z_i(t)$ and $\chi_i(t) - \chi_i^*$ as follows:

$$\bar{V}_i(t) = \frac{1}{2}z_i(t)^2 + \frac{1}{2\lambda_i}(\chi_i(t) - \chi_i^*)^2. \quad (37)$$

By considering (6) and (33), one observes that

$$\begin{aligned} \dot{\varepsilon}_i(t) &= h_i\xi_{2i}(t) - \xi_{1,i-1}(t) + \xi_{1i}(t) \\ \ddot{\varepsilon}_i(t) &= h_i\dot{\xi}_{2i}(t) - \xi_{2,i-1}(t) + \xi_{2i}(t). \end{aligned} \quad (38)$$

From (6), we recall that

$$\dot{\xi}_{2i}(t) = \frac{1}{h_i}\Big(- \xi_{2i}(t) - k_i(\xi_{1i}(t) + h_i\xi_{2i}(t)) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))\Big). \quad (39)$$

If we add and subtract the right-hand side of (39) by $\Xi_i(t)$, from (38), it follows that

$$\ddot{\varepsilon}_i(t) = -h_i\Xi_i(t) - k_i\dot{\varepsilon}_i(t) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t)). \quad (40)$$

Then, according to the definition of $z_i(t)$, from (40) we have

$$\dot{z}_i(t) = -h_i\Xi_i(t) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t)). \quad (41)$$

Therefore, the time derivative of $\bar{V}_i(t)$ along (13) and (41) yields

$$\dot{\bar{V}}_i(t) = z_i(t)\Big(- h_i\Xi_i(t) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))\Big) + (\chi_i(t) - \chi_i^*)a_i(t)|\zeta_i(t)|. \quad (42)$$

Let us consider a filtration $\mathcal{F}_t$ as follows:

$$\begin{aligned} \mathcal{F}_t &= \Big\{F_i(\varrho), i \in \mathcal{S} \cup \{0\}, 0 \le \varrho \le t\Big\} \\ F_0(\varrho) &= \{x_0(\varrho), v_0(\varrho), \xi_{20}(\varrho)\} \\ F_i(\varrho) &= \{x_i(\varrho), v_i(\varrho), \xi_{1i}(\varrho), \xi_{2i}(\varrho), \hat{\theta}_i(t), \chi_i(t)\}, \quad i \in \mathcal{S}. \end{aligned}$$

Accordingly, from (42), it follows that

$$\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\} = z_i(t)\Big(- h_i\Xi_i(t) - \chi_i(t)\mathbb{E}\{a_i(t)\}\mathrm{sgn}(\zeta_i(t))\Big) + (\chi_i(t) - \chi_i^*)\mathbb{E}\{a_i(t)\}|\zeta_i(t)|, \quad (43)$$

where $\mathbb{E}\{a_i(t)\}$ is positive as $p_i(t)$ is nonzero. From (32) and (33), we have

$$-d_i(t) + d_{\mathrm{des},i}(t) = \varepsilon_i(t) + s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t). \quad (44)$$

Now, from (38) and (44), (7) yields

$$\zeta_i(t) = \dot{\varepsilon}_i(t) + k_i\varepsilon_i(t) + k_i(s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t)),$$

which according to the definition of $z_i(t)$, can be restated as follows:

$$\zeta_i(t) = z_i(t) + k_i(s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t)). \quad (45)$$

Therefore, as $s_i(t)$, $s_{i-1}(t)$, and $\dot{s}_i(t)$ are bounded and converge to zero, for $z_i(t) \neq 0$, there exists a finite time t_{fi} such that for $t \ge t_{fi}$,

$$\mathrm{sgn}(\zeta_i(t)) = \mathrm{sgn}(z_i(t)). \quad (46)$$

By considering (45), it follows that

$$z_i(t)\mathrm{sgn}(\zeta_i(t)) = \zeta_i(t)\mathrm{sgn}(\zeta_i(t)) - k_i(s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t))\mathrm{sgn}(\zeta_i(t)),$$

which can be simplified as

$$z_i(t)\mathrm{sgn}(\zeta_i(t)) = |\zeta_i(t)| - k_i(s_i(t) - s_{i-1}(t) + h_i\dot{s}_i(t))\mathrm{sgn}(\zeta_i(t)). \quad (47)$$

According to (47) and since $s_i(t)$, $s_{i-1}(t)$, and $\dot{s}_i(t)$ are bounded and converge to zero, $z_i(t)\mathrm{sgn}(\zeta_i(t)) - |\zeta_i(t)|$ is bounded and converges to zero. Based on the above-mentioned issues, for $t \ge t_{fi}$, there exists an almost surely bounded and vanishing variable $\epsilon_i(t)$ such that (43) can be simplified as follows (note that (13) and (41) imply the almost sure boundedness of $\chi_i(t)$):

$$\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\} = z_i(t)\Big(- h_i\Xi_i(t) - \chi_i^*\mathbb{E}\{a_i(t)\}\mathrm{sgn}(z_i(t))\Big) + \epsilon_i(t), \quad (48)$$

which implies that

$$\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\} \le -\Big(\chi_i^*\mathbb{E}\{a_i(t)\} - h_i|\Xi_i(t)|\Big)|z_i(t)| + \epsilon_i(t). \quad (49)$$

By considering (36) and (49) and since $\mathbb{E}\{a_i(t)\} > 0$, there exists a finite time $\bar{t}_{fi} \ge t_{fi}$ such that $\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\}$ is negative semidefinite for $z_i(t) \neq 0$, $t \ge \bar{t}_{fi}$. According to (6) and (13) and the definition of $z_i(t)$, one observes that $\bar{V}_i(t)$ remains bounded in the finite time $t < \bar{t}_{fi}$. Now, since $\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\}$ is negative semidefinite for $t \geq \bar{t}_{fi}$, $\mathbb{E}\{\bar{V}_i(t)\}$ is bounded for all time. Based on the above-mentioned issues and by considering the filtration $\mathcal{F}_t$, $\bar{V}_i(t)$ satisfies the conditions of super-martingales given in Section II-B for $t \geq \bar{t}_{fi}$. Therefore, by invoking Theorem 1, there exists $\bar{V}_{if} \geq 0$ such that

$$\lim_{t\to\infty} \bar{V}_i(t) \xrightarrow{\text{a.s.}} \bar{V}_{if}. \tag{50}$$

From the boundedness of $\mathbb{E}\{\bar{V}_i(t)\}$, it follows that the probability of the unboundedness of $\bar{V}_i(t)$ is zero. Thus, $\bar{V}_i(t)$ is almost surely bounded, which implies the almost sure boundedness of $z_i(t)$ as well. In this condition, since $\mathbb{E}\{\bar{V}_i(t)\}$ is lower bounded and $\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\}$ is negative semidefinite, according to [36] and (49), one concludes that $z_i(t)$ converges to zero in the sense of probability (specifically, see the proof of [36, Th. 2.11] with stochastic arguments). Note that according to (13), $\chi_i(t)$ is nondecreasing. Hence, the almost sure boundedness of $\chi_i(t)$ implies the almost sure convergence of $\chi_i(t)$ to a constant value. From (50) and since $z_i(t)$ converges to zero in the sense of probability, one concludes that

$$\lim_{t\to\infty} z_i(t) \xrightarrow{\text{a.s.}} 0.$$

In this condition, since $z_i(t)$ is a Hurwitz polynomial of $\varepsilon_i(t)$, one gets $\varepsilon_i(t)$ and $\dot{\varepsilon}_i(t)$ almost surely remain bounded and converge to zero. Since $\varepsilon_i(t)$ almost surely remains bounded and converges to zero and $s_i(t) - s_{i-1}(t) + h_i \dot{s}_i(t)$ is bounded and converges to zero, by considering (4) and (44), we conclude that $e_i(t)$ almost surely remains bounded and converges to zero. Now, we show that $\xi_{1i}(t)$ and $\xi_{2i}(t)$, $v_i(t)$, and $d_i(t)$ almost surely remain bounded. From (33), we have

$$\begin{aligned}\xi_{1i}(t) = \frac{1}{h_i}\Big(&-\int_0^t \xi_{1i}(\tau)d\tau + \int_0^t \xi_{1,i-1}(\tau)d\tau + \varepsilon_i(t)\\ &- r_i + c_{i-1} - L_i - c_i\Big),\end{aligned}$$

implying that

$$\dot{\xi}_{1i}(t) = \frac{1}{h_i}\big(-\xi_{1i}(t) + \xi_{1,i-1}(t) + \dot{\varepsilon}_i(t)\big). \tag{51}$$

By considering (8) and Assumption 1, one gets $\xi_{10}(t)$ is bounded. Hence, as $\dot{\varepsilon}_i(t), i \in \mathcal{S}$, almost surely remain bounded, by considering (51), the almost sure boundedness of $\xi_{1i}(t)$ and $\dot{\xi}_{1i}(t)$ can be concluded (due to the input-to-state stability of Hurwitz linear systems). Moreover, from (6), the almost sure boundedness of $\xi_{2i}(t)$ can be concluded. Now, by considering (30), (32), and (33), the almost sure boundedness of $v_i(t)$ and $d_i(t)$ is concluded as well. Based on the all the above-mentioned issues, the proof is completed for Vehicle 1. Now, by considering (35), we can conclude that $\Xi_2(t)$ is bounded as well. Based on the boundedness of $\Xi_2(t)$, we consider the same function as (37) for $i = 2$, whose derivative along (13) and (41) proves the theorem for Vehicle 2. Then in the same procedure, the theorem will be proved for Vehicles $i \in \{3, \ldots, N\}$, respectively. ■

According to the results of Theorem 2, by employing the proposed control strategy, the intervehicle distances almost surely converge to the desired values (3), while the leader vehicle is moving with a desired speed. One complementary analysis for the proposed control strategy is to investigate the response of the platoon to a perturbed leader. One of criteria to investigate such behavior is the $\mathcal{L}_p$ string stability property of the platoon. The basis for the $\mathcal{L}_p$ string stability is the $\mathcal{L}_p$-boundedness of the vehicles states with respect to the disturbance/input of the leader [38]. In this condition, by considering the platoon states $e_i(t)$ and $\dot{e}_i(t), i \in \mathcal{S}$, let us define

$$y(t) = \big[e_1(t)\ e_2(t)\ \ldots\ e_N(t)\ \dot{e}_1(t)\ \dot{e}_2(t)\ \ldots\ \dot{e}_N(t)\big]^\top.$$

According to [38] and the stochastic behavior of the vehicular platoon addressed in this paper, the vehicular platoon (1) is defined to be almost surely $\mathcal{L}_p$ string stable if there exist class $\mathcal{K}$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ such that for any initial state $y(0) \in \mathbb{R}^{2N}$,

$$\|y(t)\|_{\mathcal{L}_p} \leq \alpha_1\big(\|u_0(t)\|_{\mathcal{L}_p}\big) + \alpha_2\big(\|y(0)\|\big), \quad \text{a.s.}, \tag{52}$$

where $u_0(t) \in \mathbb{R}$ is the $\mathcal{L}_p$-bounded inputs/disturbances of the leader including $F_0(t)$ and the effect of gravity. By considering $p = \infty$, the string stability of the platoon under the proposed CACC strategy is analyzed in the following theorem.

Theorem 3: Consider the vehicular platoon given in (1) under the control law given in (6)-(13). If $p_i(t) \neq 0, i \in \mathcal{S}$, then the platoon is stochastic $\mathcal{L}_\infty$ string stable, that is, there exist class $\mathcal{K}$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ such that for any initial state $y(0) \in \mathbb{R}^{2N}$, (52) is satisfied.

Proof: According to the proof of Theorem 2 presented in two steps, the proposed control strategy has two parts just one of which is influenced by interconnection among the vehicles. Indeed, independent to interconnection among the vehicles, $s_i(t)$ and $\dot{s}_i(t), i \in \mathcal{S}$, remain bounded and converge to zero, whether the intervehicle distances are regulated or not. Hence, let us consider the second step when $s_i(t) \equiv 0$. Since each vehicle just follows its preceding vehicle, we show the string stability by analysis of the interaction among Vehicles i and $i-1$ first when $i = 1$. Then, with the similar analysis for cases when $i \in \{2, \ldots, N\}$, the string stability of all the platoon can be concluded. It should be mentioned that an aerodynamic drag depends on the forward speed of a vehicle. Therefore, the drag force $\frac{1}{2}C_i\eta_i A_i v_i(t)^2$ cannot be larger than the supremum of the resultant of other forces/inputs. Hence, by considering (1), for $i = 0$ we have

$$\frac{1}{2}C_0\eta_0 A_0 v_0(t)^2 \leq \|u_0\|_{\mathcal{L}_\infty},$$

where according to the definition of the $\mathcal{L}_\infty$ norm it implies that

$$\left\|\frac{1}{2}C_0\eta_0 A_0 v_0(t)^2\right\|_{\mathcal{L}_\infty} \leq \|u_0\|_{\mathcal{L}_\infty}. \tag{53}$$

Therefore, from (53) one concludes that

$$\|v_0(t)\|_{\mathcal{L}_\infty} \leq \left\|\frac{2u_0(t)}{C_0\eta_0 A_0}\right\|_{\mathcal{L}_\infty}^{\frac{1}{2}}. \tag{54}$$

Moreover, from (1) it follows that (note that for two variables $\mathrm{v}_1(t)$ and $\mathrm{v}_2(t)$, we have $\|\mathrm{v}_1(t) + \mathrm{v}_2(t)\|_{\mathcal{L}_\infty} \leq \|\mathrm{v}_1(t)\|_{\mathcal{L}_\infty} +$

$\|v_2(t)\|_{\mathcal{L}_\infty})$

$$\|\dot{v}_0(t)\|_{\mathcal{L}_\infty} \leq \frac{1}{M_0}\left(\|u_0(t)\|_{\mathcal{L}_\infty} + \left\|\frac{1}{2}C_0\eta_0 A_0 v_0(t)^2\right\|_{\mathcal{L}_\infty}\right). \tag{55}$$

Now, by considering (53) and (55), one gets

$$\|\dot{v}_0(t)\|_{\mathcal{L}_\infty} \leq \frac{2}{M_0}\|u_0(t)\|_{\mathcal{L}_\infty}. \tag{56}$$

From (35), (54), and (56), it follows that

$$\left\|\Xi_i(t)\right\|_{\mathcal{L}_\infty} \leq \frac{1}{h_i}\left(\frac{2}{M_0}\|u_0(t)\|_{\mathcal{L}_\infty} + k_i\left\|\frac{2u_0(t)}{C_0\eta_0 A_0}\right\|_{\mathcal{L}_\infty}^{\frac{1}{2}}\right). \tag{57}$$

According to (6) and (13), $\dot{z}_i(t)$ is bounded for bounded $z_i(t)$, and hence for nonzero $z_i(t)$, there exists a finite time $t'_{fi} \geq t_{fi}$ such that $\chi_i(t) \geq \chi_i^*, t \geq t'_{fi}$. In this condition, from (41) and (46), it follows that for $t \geq t'_{fi}$,

$$z_i(t)\dot{z}_i(t) \leq -h_i\Xi_i(t)z_i(t) - \chi_i^*\mathbb{E}\{a_i(t)\}|z_i(t)|. \tag{58}$$

Since $\mathbb{E}\{a_i(t)\} > 0$, from (36) and (58) one gets

$$\|z_i(t)\|_{\mathcal{L}_\infty} \leq h_i t'_{fi}\left\|\Xi_i(t)\right\|_{\mathcal{L}_\infty}, \quad \text{a.s.} \tag{59}$$

According to the definition of $z_i(t)$ in (34), we have

$$e_i(t) = e^{-k_i t}e_i(0) + \int_0^t e^{-k_i(t-\tau)}z_i(\tau)d\tau$$

implying that

$$\|e_i(t)\|_{\mathcal{L}_\infty} \leq e^{-k_i t}\|e_i(0)\| + \frac{1}{k_i}\|z_i(t)\|_{\mathcal{L}_\infty}. \tag{60}$$

Now, by considering (59) and (60), one concludes that

$$\left\|e_i(t)\right\|_{\mathcal{L}_\infty} \leq e^{-k_i t}\|e_i(0)\| + \frac{1}{k_i}h_i t'_{fi}\left\|\Xi_i(t)\right\|_{\mathcal{L}_\infty}, \quad \text{a.s.} \tag{61}$$

Moreover, by considering (34), (59), and (60), one observes that

$$\left\|\dot{e}_i(t)\right\|_{\mathcal{L}_\infty} \leq k_i e^{-k_i t}\|e_i(0)\| + 2h_i t'_{fi}\left\|\Xi_i(t)\right\|_{\mathcal{L}_\infty}, \quad \text{a.s.} \tag{62}$$

Therefore, from (61) and (62) and by considering (57), there exist class $\mathcal{K}$ functions $\alpha_{1i}(\cdot)$ and $\alpha_{2i}(\cdot)$ such that

$$\left\|[e_i(t) \quad \dot{e}_i(t)]\right\|_{\mathcal{L}_\infty} \leq \alpha_{1i}\left(\|u_0(t)\|_{\mathcal{L}_\infty}\right) + \alpha_{2i}\left(\|y(0)\|\right), \quad \text{a.s.}$$

Hence, by obtaining similar inequalities for Vehicles $2, \ldots, N$, (52) can be concluded. ■

Remark 2: A nonzero $p_i(t)$ is the sufficient condition on the proposed control strategy to guarantee the almost sure convergence of the spacing errors to zero and the string stability of the platoon, because if $p_i(t) > 0$, the robustifying term $z_i(t)\mathrm{sgn}(z_i(t))$ appears with negative gain in (48). Based on the same argument, the obtained results are also extendable to platoons of vehicles subject to stochastic delays in receiving communication and radar data if the probability of such delays is less than $p_i(t)$. Let $\bar{p}_i(t)$ be the unknown probability that Vehicle $i, i \in \mathcal{S}$, receives information with delays where $p_i(t) > \bar{p}_i(t)$. By considering the worst-case scenario when the delays always reverse the sign of $z_i(t)$, (48) becomes

$$\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\} \leq z_i(t)\Big(-h_i\Xi_i(t) - \chi_i^*(p_i(t) - \bar{p}_i(t))\mathrm{sgn}(z_i(t))\Big) + \epsilon_i(t). \tag{63}$$

Hence, for the unknown communication delay $T_i(t) \in \mathbb{R}_{\geq 0}$, if in the Lyapunov function (37) we select χ_i^* as

$$\sup\left\{\frac{h_i\left|\Xi_i(t - T_i(t))\right|}{p_i(t) - \bar{p}_i(t)}\right\} < \chi_i^*,$$

since $p_i(t) - \bar{p}_i(t) > 0$, (63) implies that $\mathbb{E}\{\dot{\bar{V}}_i(t)|\mathcal{F}_t\}$ is negative semidefinite for $\bar{t}_{fi} \geq t_{fi}$ (note that $p_i(t)$, $\bar{p}_i(t)$, and $T_i(t)$ are unknown and just used in defining χ_i^*, not in the design). Then, in a procedure similar to that in the proof of Theorem 2, the almost sure convergence of the spacing errors to zero, and in a procedure similar to that in the proof of Theorem 3, the $\mathcal{L}_\infty$ string stability of the platoon can be concluded.

According to Remark 2, the main difference between the current study and the existing results in the literature devoted to CACC in the presence of communication delays [23], [24], [25], [26] is that in addition to guaranteeing local stability and string stability of the platoon, the effect of time-delays is rejected, and for any communication delay (with unknown bounds), the spacing errors almost surely converge to zero. This achievement is the consequence of using the nonlinear robustifying term $\chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))$ in the controller with the adaptive gain $\chi_i(t)$. Moreover, in the existing results in the literature just delays in communication network are considered and the vehicles radars are assumed to be free of delays in receiving data. However, according to Remark 2, in the current study, time-delays in both communication network and the vehicles radars are considered.

Another important concern in control of vehicular platoons is collision avoidance, which is more serious in the presence of data loss. Any control strategy for autonomous collision avoidance relies on timely detection of the possibility of a collision. If due to any data loss, the distance from the preceding vehicle is not measurable over a long time period, autonomous collision avoidance may not be possible. For emergency actions in such scenarios, it is needed for the driver to take control or partial control of the vehicle. In the current study, if $p_i(t)$ is not zero, the probability of such scenarios is zero. Hence, the proposed control strategy can be extended to help collision avoidance among the vehicles. Using repulsive/braking functions is a common idea to avoid collision [39], [40], [41]. According to this idea, if a vehicle approaches a predetermined safety distance $d_{si}(t) < d_{\mathrm{des},i}(t)$ from a preceding vehicle, the vehicle input force will be increased in the opposite direction of motion such that at the distance $d_{si}(t)$, the repulsive function will be increased sharply. Based on this idea, let $d_{ai}(t)$, where $d_{si}(t) < d_{ai}(t) < d_{\mathrm{des},i}(t)$, be a distance from the preceding vehicle where the repulsive force will be active. Hence, in order to address the collision avoidance issue, the auxiliary state $\xi_{2i}(t)$ described in (6) can be modified as

$$\dot{\xi}_{2i}(t) = \frac{1}{h_i}\Big(-\xi_{2i}(t) - k_i(\xi_{1i}(t) + h_i\xi_{2i}(t)) \\ - (\chi_i(t) + f_{ri}(t))a_i(t)\mathrm{sgn}(\zeta_i(t))\Big), \qquad (64)$$

where $f_{ri}(t)$ is an additional term which represents the repulsive forces designed as follows:

$$f_{ri}(t) \\ = \begin{cases} 0 & d_i(q) > d_{ai}(t) \\ k_{ri}\Big(\frac{1}{d_i(q)} - \frac{1}{d_{ai}(t)}\Big)\frac{1}{(d_i(q) - d_{si}(t))^2} & d_i(q) \le d_{ai}(t), \end{cases}$$

where k_{ri} is a positive constant and q is the last time where $d_i(\cdot)$ has been available ($q = t$ if there is no data loss). In the design of $f_{ri}(t)$, the term $1/d_i(q) - 1/d_{ai}(t)$ is zero if $d_i(q) = d_{ai}(t)$, and will be increased when $d_i(q)$ approaches the distance $d_{si}(t)$. On the other hand, the term $1/(d_i(q) - d_{si}(t))^2$ will be increased sharply if $d_i(q)$ approaches the distance $d_{si}(t)$. Hence, $\chi_i(t) + f_{ri}(t)$ is increased, and thus according to the proof of Theorem 2, the distance among the vehicles will be led toward $d_{\mathrm{des},i}(t)$. More specifically, by considering (41), larger $\chi_i(t) + f_{ri}(t)$ increases the rate of almost sure convergence of $z_i(t)$ and $\varepsilon_i(t)$ toward zero such that the spacing error will be led to zero (the rate of the convergence of $\varepsilon_i(t)$ to zero depends on k_i as well). Note that the performance of the adaptive control strategy of Theorem 2 to regulate $s_i(t)$ and $\dot{s}_i(t)$ is independent to $\chi_i(t)$, and before any collision scenario, $s_i(t)$ and $\dot{s}_i(t)$ should be small enough. Moreover, the accuracy of the results in Theorem 2 just relied on a large enough $\chi_i(t)$, and by adding the positive term $f_{ri}(t)$ to $\chi_i(t)$ as (64), the accuracy of the analysis provided in the proof of the theorem will not be affected.

Remark 3: The base of describing the dynamics of the vehicles by second-order models as (1) is assuming that the dynamics of the actuators are fast enough [1], [3], [33]. If such assumption on the actuators is not applicable, the dynamics of the actuators can be described as follows [2], [42]:

$$\dot{F}_i(t) = \frac{1}{\tau_i}(F_i^*(t) - F_i(t)),$$

where $\tau_i \in \mathbb{R}_{>0}$ is the actuators time constant and $F_i^*(t)$ is a desired input force. Accordingly, the total dynamical equations describing the vehicles are of order three. Now, since the vehicles dynamical equations still are in back-stepping form, it is possible to extend the obtained results in Theorem 2 to platoons of vehicles with third-order model as well, by adding another step to the design procedure. Indeed, the main step in the back-stepping control design is the first step, and the other steps are on deriving the control input to achieve the objective of the first step. Hence, to extend the obtained results in Theorem 2 to platoons of vehicles with third-order model, the surface $s_i(t)$ defined in (6) can be modified as follows:

$$\dot{s}_i(t) = v_i(t) - \xi_{1i}(t), \quad i \in \mathcal{S} \\ \dot{\xi}_{1i}(t) = \xi_{2i}(t) \\ \dot{\xi}_{2i}(t) = \xi_{3i}(t)$$

$$\dot{\xi}_{3i}(t) = \frac{1}{h_i}\Big(-\xi_{3i}(t) - k_{1i}(\xi_{2i}(t) + h_i\xi_{3i}(t)) \\ - k_{2i}(\xi_{1i}(t) + h_i\xi_{2i}(t)) - \chi_i(t)a_i(t)\mathrm{sgn}(\zeta_i(t))\Big),$$

where

$$\zeta_i(t) = \xi_{2i}(t) - \xi_{2,i-1}(t) + h_i\xi_{3i}(t) + k_{1i}(\xi_{1i}(t) - \xi_{1,i-1}(t) \\ + h_i\xi_{2i}(t)) + k_{2i}(-d_i(t) + d_{\mathrm{des},i}(t)),$$

where $k_{1i}, k_{2i} \in \mathbb{R}_{>0}$. Then, by following a procedure similar to the proof of Theorem 2, it is possible to derive $F_i^*(t)$ such that the objective (5) is achieved. In this condition, compared with the proof of Theorem 2, another step is needed such that by designing $F_i^*(t)$, the error $F_i(t) - \beta_i(t)$ converges to zero, where $\beta_i(t)$ is a virtual control law as (10).

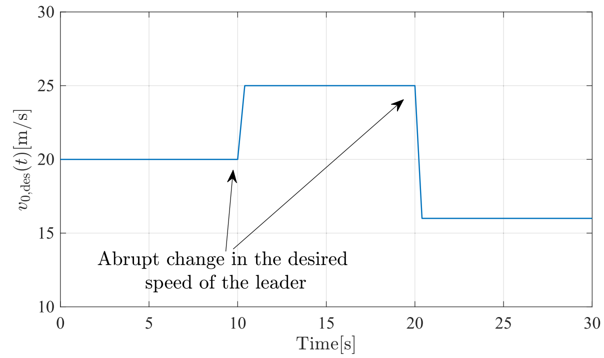


Fig. 2. Desired speed of the leader.

V. Simulation Results

We consider a platoon of five heterogeneous vehicles consisting of a leader and four followers whose the model parameters (unknown) are given in Table I. We suppose that the desired speed reference of the leader is as depicted in Fig. 2, implying abrupt change in the speed and acceleration of the leader. Such abrupt change in the speed of the leader also leads to abrupt change in the speeds and accelerations of the followers. Moreover, since a time-gap spacing policy is considered, the desired distances among the vehicles and the spacing errors change as well when the speeds of the followers change. The road slope is assumed to be zero, and the initial states of the followers are described by $(d_1(0), v_1(0)) = (19\mathrm{m}, 20.5\mathrm{m/s})$, $(d_2(0), v_2(0)) = (4.5\mathrm{m}, 19\mathrm{m/s})$, $(d_3(0), v_3(0)) = (4\mathrm{m}, 18.5\mathrm{m/s})$, and $(d_4(0), v_4(0)) = (6.5\mathrm{m}, 15\mathrm{m/s})$. The objective is to employ the CACC law of Theorem 2 such that each follower keeps the desired distance (3) from the preceding vehicle. Accordingly, we set $r_i = 2\mathrm{m}$ and $h_i = 0.2\mathrm{s}, i \in \mathcal{S}$. It should be noted that according to Theorem 2, all the control gains just have to be positive. Therefore, the control gains are set arbitrarily to $k_i = 1$, $\lambda_i = 10$, $\sigma_{1i} = 1000$, $\sigma_{2i} = 1$, and $\gamma_i = 1, i \in \mathcal{S}$. Indeed, most of the control parameters are set to 1, and just σ_{1i} and λ_i are selected to large values. A large σ_{1i} leads to faster convergence of the surface $s_i(t)$ to zero and a large λ_i leads to faster convergence of the robustifying gain $\chi_i(t)$. Accordingly, the convergence of spacing errors to zero will be faster.

TABLE I

Model Parameters of the Follower Vehicles

	M_i[kg]	C_i	$\eta_i\left[\frac{\text{kg}}{\text{m}^3}\right]$	$A_i[\text{m}^2]$	f_i	L_i[m]
1	1500	0.001	1.2	1.5	0.015	4
2	1200	0.0008	1	1.4	0.013	3.5
3	1000	0.001	1	1.1	0.011	3
4	1300	0.0011	0.9	1.3	0.01	3.5

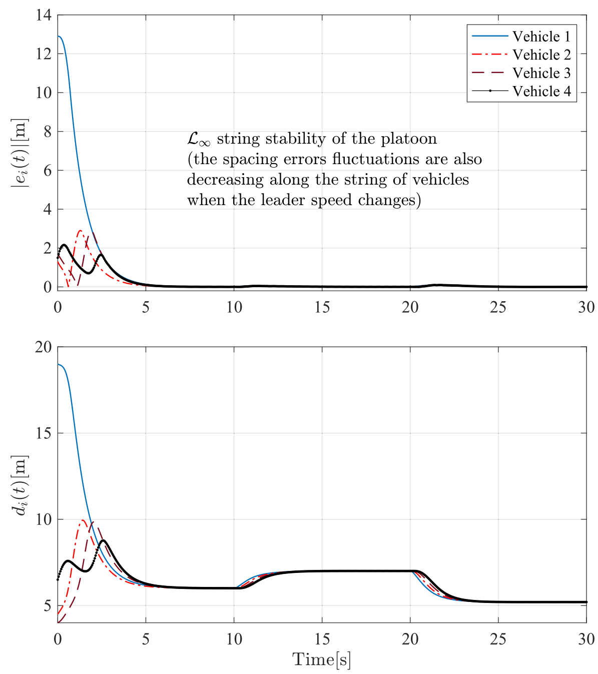


Fig. 3. Spacing errors and intervehicle distances in Scenario 1.

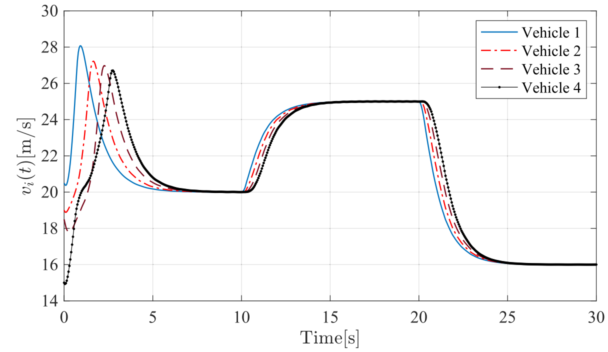


Fig. 4. Speeds of the follower vehicles in Scenario 1.

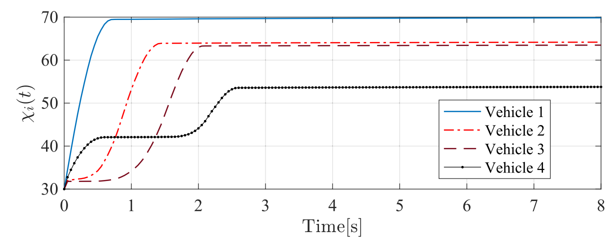


Fig. 5. Adaptive robustifying gains in in Scenario 1 where $\chi_i(0), i \in \mathcal{S}$, are set to 30.

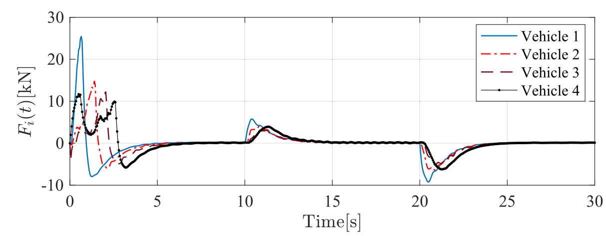


Fig. 6. Control efforts of the follower vehicles in Scenario 1.

We present two simulation scenarios. In the first scenario, we investigate the accuracy of the proposed control strategy in the presence of data loss and delays, and in the second one, we specifically focus on the performance of the proposed CACC strategy in dealing with unknown and large communication and radar delays.

Scenario 1: Without loss of generality and for simplicity, the probability of the data loss in a communication link is supposed to be independent to that in a radar. Hence, by considering the probability of the data loss for all the communication links as $0.2 + 0.1\sin(t)$ and the probability of the data loss for all the radars as 0.1, we have $p_i(t) = (1-0.2-0.1\sin(t))(1-0.1) = 0.72-0.09\sin(t)$. We assumed that the vehicles exchange information with stochastic delays with bound of 50ms where $\bar{p}_i(t) = 0.1$ (we have used a uniform random number to model such delays). Moreover, for simulation purpose, the actuators time constant is 0.1s.

Under these conditions, the spacing errors and the intervehicle distances are depicted in Fig. 3 and the speeds of the followers are depicted in Fig. 4. According to Fig. 3, after about 7s, the spacing errors converge to zero, and the platoon reaches a steady state determined by the constant time-gap spacing policy. In this steady state, as shown in Fig. 4, the followers reached the speed of the leader which is 20m/s. At $t = 10$s, the speed of the leader abruptly increases to 25m/s, and since the desired distances among the vehicles depend on their speeds, the platoon converges to another steady state with speed of 25m/s. There is another change in the speed of the leader at $t = 20$s, where the speed of the leader abruptly decreases to 16m/s that results in another steady state with speed of 16m/s. The abrupt change in the speed of the leader lead to acceleration disturbances affecting the platoon. As shown in Fig. 3 and Fig. 4, in this situation, while the platoon remains $\mathcal{L}_\infty$ string stable, the effect of such disturbance, that leads to fluctuations in the spacing errors and the states of the follower vehicles, is not amplified and is attenuated along the string of vehicles. In Fig. 5, it is shown that the adaptive robustifying gains $\chi_i(t), i \in \mathcal{S}$, remain bounded. Moreover, the control efforts of the follower vehicles are depicted in Fig. 6.

Scenario 2: Knowledge of the range of communication delays is the basis for the existing results in the literature dealing with robustness of vehicular platoons against communication delays [23], [24], [25]. For instance, one of the existing techniques is using Smith Predictor in the closed-loop control system [23]. The base of Smith Predictor is using the information of the range of time-delays in the control system (for instance, see Figure 2 in [23]). Therefore, for a designed

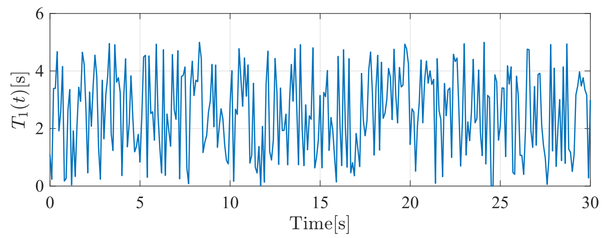


Fig. 7. Communication delay regarding Vehicle 1 in Scenario 2.

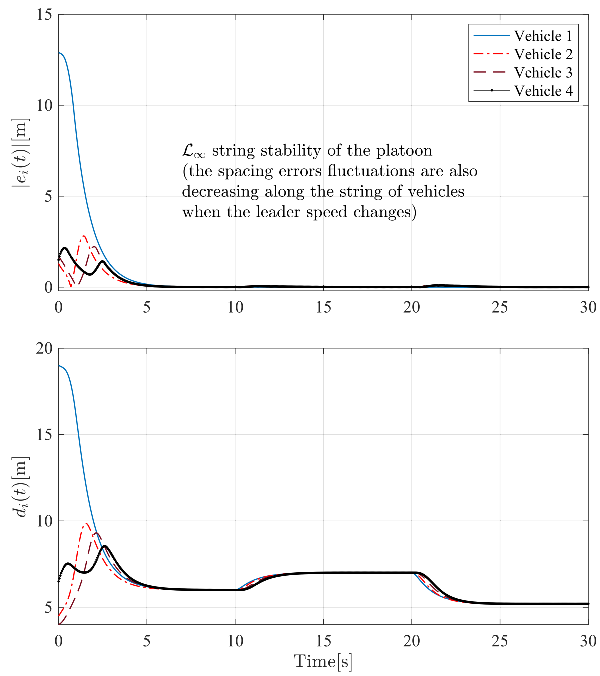


Fig. 8. Spacing errors and intervehicle distances in Scenario 2 by increasing the time-delays.

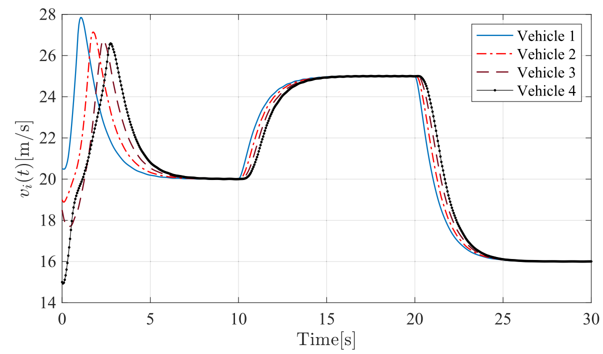


Fig. 9. Follower vehicles speeds in Scenario 2 by increasing the time-delays.

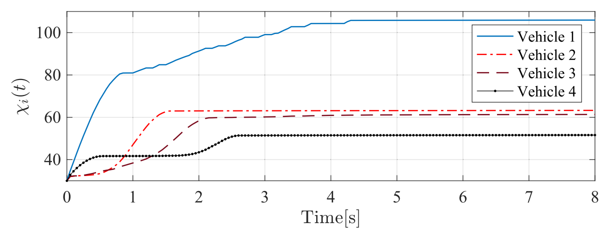


Fig. 10. Adaptive robustifying gains in Scenario 2 by increasing the time-delays where $\chi_i(0), i \in \mathcal{S}$, are set to 30.

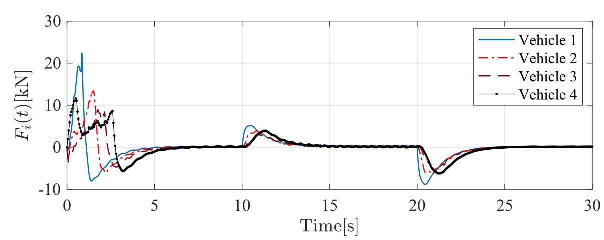


Fig. 11. Control efforts of the follower vehicles in Scenario 2 by increasing the time-delays.

Smith Predictor, if in practice time-delays become larger than the considered range, the assumptions of the analysis are not satisfied (the controller still can have a good performance, but there is no guarantee for that). Instead, unlike [23], [24], and [25], in our methodology, as far as the condition of Remark 2 is satisfied, the stability and string stability of the platoon is mathematically guaranteed without assuming the knowledge of the bound on the delays. To show this aspect of the proposed CACC strategy, we repeat the previous scenario and we just increase the time delays bound to 5s (which is a very large time-delay). For instance, the time-delay regarding receiving communication data by Vehicle 1 is depicted in Fig. 7. In this situation, the spacing errors and the intervehicle distances are depicted in Fig. 8 and the speeds of the followers are depicted in Fig. 9, which are very similar to those in Scenario 1 implying the high robustness of the proposed strategy to increasing the time-delays. From Fig. 8 one observes that the intervehicle distances track desired values determined by the speed of the leader which is changing over time, and from Fig. 9, one observes that the vehicles speeds track the time-varying speed of the leader. As shown in Fig. 8 and Fig. 9, whereas the time-delays are increased and are relatively large, the platoon is still $\mathcal{L}_\infty$ string stable. Moreover, according to the figures, the disturbing effect of the leader during its maneuver is not amplified along the string of vehicles. More specifically, when the leader speed abruptly changes at $t = 10$s and $t = 20$s, fluctuations in the states of the follower vehicles due the acceleration disturbance of the leader are being attenuated along the string of vehicles. Finally, in Fig. 10, the convergence of the adaptive robustifying gains $\chi_i(t), i \in \mathcal{S}$, to bounded values is demonstrated, and in Fig. 11, the control efforts of the follower vehicles are depicted.

VI. Conclusion and Future Work

Developing a control strategy to guarantee the robustness of a vehicular platoon against both communication and radar data loss was the main contribution of this paper. We proposed an adaptive control scheme such that in the presence of communication network and radars data loss, some surfaces of vehicles states converged to zero. Simultaneously on the designed surfaces, by compensating possible erroneous signals, almost sure regulation of the intervehicle distances in desired values was guaranteed. Moreover, the surfaces and the adaptive control laws were designed in a way that the chattering phenomenon (due to robust properties of the controllers) is avoided in the vehicles control inputs. This study was a primary step in considering radars data loss in control of vehicular platoons. Extension of the obtained results to platoons of vehicles in the presence of disturbances and stochastic noise and in the presence of cyber-attacks are challenging problems to be studied as future work.

REFERENCES

[1] B. Xu et al., "Cooperative method of traffic signal optimization and speed control of connected vehicles at isolated intersections," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 4, pp. 1390–1403, Apr. 2019.

[2] Y. Bian, Y. Zheng, W. Ren, S. E. Li, J. Wang, and K. Li, "Reducing time headway for platooning of connected vehicles via V2V communication," *Transp. Res. C, Emerg. Technol.*, vol. 102, pp. 87–105, May 2019.

[3] V. Turri, B. Besselink, and K. H. Johansson, "Cooperative look-ahead control for fuel-efficient and safe heavy-duty vehicle platooning," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 1, pp. 12–28, Jan. 2017.

[4] B. Besselink et al., "Cyber–physical control of road freight transport," *Proc. IEEE*, vol. 104, no. 5, pp. 1128–1141, May 2016.

[5] C. Zhai et al., "Ecological cooperative adaptive cruise control for heterogenous vehicle platoons subject to time delays and input saturations," *IEEE Trans. Intell. Transp. Syst.*, vol. 24, no. 3, pp. 2862–2873, Mar. 2023.

[6] A. Alipour-Fanid, M. Dabaghchian, and K. Zeng, "Platoon stability and safety analysis of cooperative adaptive cruise control under wireless Rician fading channels and jamming attacks," 2017, *arXiv:1710.08476*.

[7] A. Alipour-Fanid, M. Dabaghchian, and K. Zeng, "Impact of jamming attacks on vehicular cooperative adaptive cruise control systems," *IEEE Trans. Veh. Technol.*, vol. 69, no. 11, pp. 12679–12693, Nov. 2020.

[8] A. Alipour-Fanid, M. Dabaghchian, H. Zhang, and K. Zeng, "String stability analysis of cooperative adaptive cruise control under jamming attacks," in *Proc. IEEE 18th Int. Symp. High Assurance Syst. Eng. (HASE)*, Singapore, Jan. 2017, pp. 157–162.

[9] F. Acciani, P. Frasca, A. Stoorvogel, E. Semsar-Kazerooni, and G. Heijenk, "Cooperative adaptive cruise control over unreliable networks: An observer-based approach to increase robustness to packet loss," in *Proc. Eur. Control Conf. (ECC)*, Limassol, Cyprus, Jun. 2018, pp. 1399–1404.

[10] Y. A. Harfouch, S. Yuan, and S. Baldi, "An adaptive switched control approach to heterogeneous platooning with intervehicle communication losses," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 3, pp. 1434–1444, Sep. 2018.

[11] Y. A. Harfouch, S. Yuan, and S. Baldi, "Adaptive control of interconnected networked systems with application to heterogeneous platooning," in *Proc. 13th IEEE Int. Conf. Control Autom. (ICCA)*, Ohrid, Macedonia, Jul. 2017, pp. 212–217.

[12] R. Liu, Y. Ren, H. Yu, Z. Li, and H. Jiang, "Connected and automated vehicle platoon maintenance under communication failures," *Veh. Commun.*, vol. 35, Jun. 2022, Art. no. 100467.

[13] G. Guo and S. Wen, "Communication scheduling and control of a platoon of vehicles in VANETs," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1551–1563, Jun. 2016.

[14] M. A. Gordon, F. J. Vargas, and A. A. Peters, "Mean square stability conditions for platoons with lossy inter-vehicle communication channels," *Automatica*, vol. 147, Jan. 2023, Art. no. 110710.

[15] Z. Li, B. Hu, M. Li, and G. Luo, "String stability analysis for vehicle platooning under unreliable communication links with event-triggered strategy," *IEEE Trans. Veh. Technol.*, vol. 68, no. 3, pp. 2152–2164, Mar. 2019.

[16] E. van Nunen, J. Verhaegh, E. Silvas, E. Semsar-Kazerooni, and N. van de Wouw, "Robust model predictive cooperative adaptive cruise control subject to V2V impairments," in *Proc. IEEE 20th Int. Conf. Intell. Transp. Syst. (ITSC)*, Yokohama, Japan, Oct. 2017, pp. 1–8.

[17] E. van Nunen, J. Reinders, E. Semsar-Kazerooni, and N. van de Wouw, "String stable model predictive cooperative adaptive cruise control for heterogeneous platoons," *IEEE Trans. Intell. Vehicles*, vol. 4, no. 2, pp. 186–196, Jun. 2019.

[18] B. R. Mahafza, *Radar Systems Analysis and Design Using MATLAB*, 4th ed. New York, NY, USA: Taylor & Francis, 2022.

[19] G. S. N. Raju, *Rada Engineering*. New Delhi, India: I.K. International, 2008.

[20] P. Gerstoft, W. S. Hodgkiss, L. T. Rogers, and M. Jablecki, "Probability distribution of low-altitude propagation loss from radar sea clutter data," *Radio Sci.*, vol. 39, no. 6, pp. 1–9, Dec. 2004.

[21] M. Pham and K. Xiong, "A survey on security attacks and defense techniques for connected and autonomous vehicles," *Comput. Secur.*, vol. 109, Oct. 2021, Art. no. 102269.

[22] E. R. Yeh, C. R. Bhat, R. Heath, J. Choi, and N. G. Prelcic, "Security in automotive radar and vehicular networks," *Microw. J.*, vol. 60, no. 5, pp. 148–164, 2017.

[23] H. Xing, J. Ploeg, and H. Nijmeijer, "Compensation of communication delays in a cooperative ACC system," *IEEE Trans. Veh. Technol.*, vol. 69, no. 2, pp. 1177–1189, Feb. 2020.

[24] Y. Zhang, Y. Bai, M. Wang, and J. Hu, "Cooperative adaptive cruise control with robustness against communication delay: An approach in the space domain," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 9, pp. 5496–5507, Sep. 2021.

[25] H. Xing, J. Ploeg, and H. Nijmeijer, "Robust CACC in the presence of uncertain delays," *IEEE Trans. Veh. Technol.*, vol. 71, no. 4, pp. 3507–3518, Apr. 2022.

[26] A. Coppola, D. G. Lui, A. Petrillo, and S. Santini, "Cooperative driving of heterogeneous uncertain nonlinear connected and autonomous vehicles via distributed switching robust PID-like control," *Inf. Sci.*, vol. 625, pp. 277–298, May 2023.

[27] K. Santhanakrishnan and R. Rajamani, "On spacing policies for highway vehicle automation," *IEEE Trans. Intell. Transp. Syst.*, vol. 4, no. 4, pp. 198–204, Dec. 2003.

[28] S. Zhao and K. Zhang, "A distributionally robust stochastic optimization-based model predictive control with distributionally robust chance constraints for cooperative adaptive cruise control under uncertain traffic conditions," *Transp. Res. B, Methodol.*, vol. 138, pp. 144–178, Aug. 2020.

[29] H. Rezaee, T. Parisini, and M. M. Polycarpou, "Control of vehicular platoons: Stochastic robustness against jamming attacks," in *Proc. 21st IFAC World Congr.*, Berlin, Germany, Jul. 2020, pp. 17041–17046.

[30] D. Williams, *Probability With Martingales*. Cambridge, U.K.: Cambridge Univ. Press, 1991.

[31] M. Mahmoud, J. Jiang, and Y. Zhang, *Active Fault Tolerant Control Systems: Stochastic Analysis and Synthesis*, 1st ed. Berlin, Germany: Springer, 2003.

[32] R. Tempo, G. Calafiore, and F. Dabbene, *Randomized Algorithms for Analysis and Control of Uncertain Systems With Applications*, 2nd ed. London, U.K.: Springer, 2013.

[33] B. Asadi and A. Vahidi, "Predictive cruise control: Utilizing upcoming traffic signal information for improving fuel economy and reducing trip time," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 3, pp. 707–714, May 2011.

[34] J. C. Zegers, E. Semsar-Kazerooni, M. Fusco, and J. Ploeg, "A multi-layer control approach to truck platooning: Platoon cohesion subject to dynamical limitations," in *Proc. 5th IEEE Int. Conf. Models Technol. Intell. Transp. Syst. (MT-ITS)*, Naples, Italy, Jun. 2017, pp. 128–133.

[35] A. F. Filippov, *Differential Equations With Discontinuous Righthand Sides*. Dordrecht, The Netherland: Springer, 1988.

[36] I. Barkana, "Defending the beauty of the invariance principle," *Int. J. Control*, vol. 87, no. 1, pp. 186–206, Jan. 2014.

[37] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[38] J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Lp string stability of cascaded systems: Application to vehicle platooning," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 2, pp. 786–793, Mar. 2014.

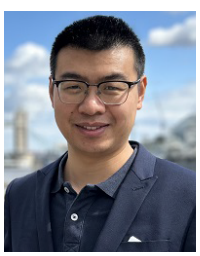
[39] L. Gao, D. Chu, Y. Cao, L. Lu, and C. Wu, "Multi-lane convoy control for autonomous vehicles based on distributed graph and potential field," in *Proc. IEEE Intell. Transp. Syst. Conf. (ITSC)*, Auckland, New Zealand, Oct. 2019, pp. 2463–2469.

[40] E. Semsar-Kazerooni, K. Elferink, J. Ploeg, and H. Nijmeijer, "Multi-objective platoon maneuvering using artificial potential fields," in *Proc. 20th IFAC World Congr.*, Toulouse, France, Jul. 2017, pp. 15006–15011.

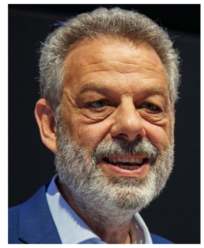
[41] M. Hu et al., “Coordinated collision avoidance for connected vehicles using relative kinetic energy density,” *Int. J. Automot. Technol.*, vol. 18, no. 5, pp. 923–932, Oct. 2017.

[42] S. E. Li, Y. Zheng, K. Li, and J. Wang, “Scalability limitation of homogeneous vehicular platoon under undirected information flow topology and constant spacing policy,” in *Proc. 34th Chin. Control Conf. (CCC)*, Hangzhou, China, Jul. 2015, pp. 8039–8045.

Hamed Rezaee (Senior Member, IEEE) received the B.Sc., M.Sc., and Ph.D. degrees in control engineering from the Department of Electrical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, in 2009, 2011, and 2016, respectively. From 2018 to 2022, he was a Research Associate with the Department of Electrical and Electronic Engineering, Imperial College London, London, U.K., where he is currently a Research Associate with the Department of Mechanical Engineering, with a research focuses on nonlinear dynamics and control, adaptive control, and robust control. He serves as an Associate Editor for the IEEE Control Systems Society Conference Editorial Board and the European Control Association Conference Editorial Board.



Kangkang Zhang received the B.Eng. degree in control science and engineering from the Henan University of Technology in 2012, the M.Sc. degree from Northeastern University in 2014, and the Ph.D. degree from the Nanjing University of Aeronautics and Astronautics (NUAA) in 2018. From 2017 to 2018, he was a Visiting Ph.D. Student with the University of Kent, U.K. From 2019 to 2022, he was a Research Associate with the KIOS Research and Innovation Center, University of Cyprus, Cyprus. He is currently a Marie Skłodowska-Curie Individual Fellow with the Control and Power Group, Imperial College London, London, U.K. His research interests include analysis, diagnosis, and control of complex and cyber-physical systems in the presence of cyber-physical threats. His thesis received the CAA Excellent Doctoral Dissertation Award from the Chinese Association of Automation.



Thomas Parisini (Fellow, IEEE) received the Ph.D. degree in electronic engineering and computer science from the University of Genoa in 1993. He was with Politecnico di Milano. He currently holds the Chair of industrial control and the Head of the Control and Power Research Group, Imperial College London. He is the Deputy Director of the KIOS Research and Innovation Centre of Excellence, University of Cyprus. Since 2001, he has been the Danieli Endowed Chair of automation engineering with the University of Trieste. From 2009 to 2012, he was the Deputy Rector of the University of Trieste. In 2016, he was awarded as the Principal Investigator at Imperial College of the H2020 European Union Flagship Teaming Project, KIOS Research and Innovation Centre of Excellence, led by the University of Cyprus. In 2023, he received the Knighthood of the Order of Merit of the Italian Republic for Scientific Achievements Abroad; and the Honorary Doctorate from the University of Aalborg, Denmark, in 2018. He held a ”Scholar-in-Residence” visiting position with Digital Futures-KTH, Stockholm, Sweden. He has authored or coauthored a research monograph in the *Communication and Control* series (Springer and Nature); and over 400 research papers in archival journals, book chapters, and international conference proceedings. He is a fellow of IFAC. He was a co-recipient of the IFAC Best Application Paper Prize of the *Journal of Process Control* (Elsevier), from 2011 to 2013, for three years; and the 2004 Outstanding Paper Award of IEEE TRANSACTIONS ON NEURAL NETWORKS. He was a recipient of the 2007 IEEE Distinguished Member Award. He has served as the 2021–2022 President for the IEEE Control Systems Society. Among other activities, he was the Program Chair of the 2008 IEEE Conference on Decision and Control and the General Co-Chair of the 2013 IEEE Conference on Decision and Control. From 2009 to 2016, he was the Editor-in-Chief of IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY. Since 2017, he has been an Editor of *Control Applications of Automatica* and the Editor-in-Chief of the *European Journal of Control*, since 2018.

Marios M. Polycarpou (Fellow, IEEE) received the B.A. degree in computer science and the B.Sc. degree in electrical engineering from Rice University, USA, in 1987, and the M.S. and Ph.D. degrees in electrical engineering from the University of Southern California, in 1989 and 1992, respectively. He is a Professor of electrical and computer engineering and the Director of the KIOS Research and Innovation Center of Excellence, University of Cyprus. He is also a Founding Member of the Cyprus Academy of Sciences, Letters, and Arts; an Honorary Professor of Imperial College London; and a member of Academia Europaea (The Academy of Europe). His research work has been funded by several agencies and industry in Europe and the United States, including the prestigious the European Research Council (ERC) Advanced Grant, the ERC Synergy Grant, and the EU-Widening Teaming Program. His teaching and research interests are in intelligent systems and networks, adaptive and learning control systems, fault diagnosis, machine learning, and critical infrastructure systems.

He is a fellow of IFAC. He was a recipient of the 2023 IEEE Frank Rosenblatt Technical Field Award and the 2016 IEEE Neural Networks Pioneer Award. He served as the President for the IEEE Computational Intelligence Society (2012–2013) and the European Control Association (2017–2019). He was the Editor-in-Chief of IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, from 2004 to 2010. He serves on the Editorial Boards for the PROCEEDINGS OF THE IEEE and the *Annual Reviews in Control*.