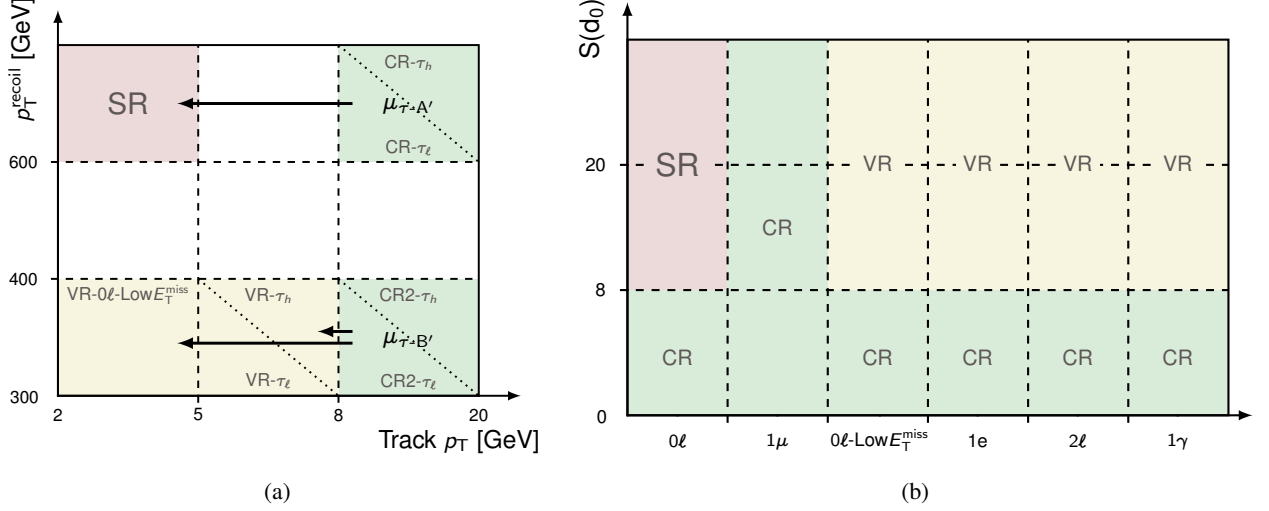# Supplemental Material

Figure 1: Schematics illustrating the selection of the CRs and VRs used to estimate (a) the tau track background and (b) the QCD track background, along with the orthogonality with respect to the SR. The arrows in (a) indicate the direction of extrapolation in the kinematic phase space which is validated in the VRs.

Table 1: Event selection for the control and validation regions relevant for estimating the tau track background, and that provide orthogonality with the SR. Selections that are shared with SR are omitted. Additional quality requirements are applied for leptons in the relevant regions, as described in the Appendix. Parentheses are used to group similar control and validation regions together. The selections inside the parentheses apply to the control regions and indicate the criteria that enforces orthogonality with respect to the corresponding validation region.

Variable	SR	CR-τ_h	CR-τ_ℓ	VR(CR2)-τ_h	VR(CR2)-τ_ℓ
N_ℓ	$= 0$	$= 0$	$= 1$	$= 0$	$= 1$
m_T [GeV]	–	–	< 50	–	< 50
p_T^{recoil} [GeV]	> 600	> 600		[300,400]	
Track p_T	[2,5]	[8,20]		[5,8] ([8,20])	
Track $S(d_0)$	> 8	> 3		> 3	

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Table 2: Event selection for the control and validation regions relevant for estimating the QCD track background, and that provide orthogonality with the SR. Selections that are shared with SR are omitted. The transverse mass (m_T) is defined by $m_\mathrm{T} = \sqrt{2p_\mathrm{T}(\ell)E_\mathrm{T}^\mathrm{miss}(1 - \cos\Delta\phi(\ell, \mathbf{p}_\mathrm{T}^\mathrm{miss}))}$. Additional quality requirements are applied for leptons and photons in the relevant regions, as described in the Appendix. Parentheses are used to group similar regions together. The selections inside the parentheses apply to the control regions and indicate the criteria that enforces orthogonality with respect to the corresponding signal or validation region.

Variable	SR (CR-0ℓ)	CR-1μ	VR(CR)-0ℓ−low$E_\mathrm{T}^\mathrm{miss}$	VR(CR)-1e	VR(CR)-2ℓ	VR(CR)-1γ
Trigger	$E_\mathrm{T}^\mathrm{miss}$	$E_\mathrm{T}^\mathrm{miss}$	$E_\mathrm{T}^\mathrm{miss}$	Single-e	$E_\mathrm{T}^\mathrm{miss}$ or Single-e	Single Photon
$N(e)$	= 0	= 0	= 0	= 1	–	= 0
$N(\mu)$	= 0	= 1	= 0	= 0	–	= 0
$N(e$ or $\mu)$	= 0	= 1	= 0	= 1	= 2	= 0
N_γ	= 0	= 0	= 0	= 0	= 0	= 1
$p_\mathrm{T}(\ell_1)$ [GeV]	–	> 10	–	> 30	$p_\mathrm{T}(\mu) > 10$ ($p_\mathrm{T}(e) > 30$)	–
$p_\mathrm{T}(\ell_2)$ [GeV]	–	–	–	–	> 10	–
m_{ll} [GeV]	–	–	–	–	[66.2, 116.2]	–
m_T [GeV]	–	[56, 106]	–	[56, 106]	–	–
$p_\mathrm{T}^\mathrm{recoil}$ [GeV]	> 600	> 300	[300, 400]	> 300	> 300	> 600
Track $S(d_0)$	> 8 (< 8)	–			> 8 (< 8)	