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Advantages and Disadvantages of Different Solvers for Preliminary DP Capability and Operability Predictions

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Abstract. With the constant movement of DP operations through deep water areas, nowadays the DP system is installed on all kinds of offshore units. Therefore, it is of utmost importance to predict the DP system's capability since the early stages of the design. In this stage, it is impossible to perform accurate time domain simulations as many vessel and system parameters are unknown. Then, it is only possible to perform quasi-static simulations to perform capability and operability predictions. In the literature, plenty of methods are suitable for executing a quasi-static calculation, employing different algorithms for the problem of thrust allocation. The thrust allocation problem has straightforward resolutions, up to complex and sophisticated options based on differential evolution algorithms. Besides the definition of the thrust allocation algorithm, there is a problem associated with the environmental loads. Also in this case the representations are multiple and can cause a problem in interpreting the capability and operability predictions. The present work compares different resolution methods for the thrust allocation procedure by keeping the estimation process for the environmental loads constant. The results presented for a reference vessel highlight the peculiarities and abilities of each type of thrust allocation solution, providing insight to an offshore designer in selecting the proper tool for their preliminary predictions.

Keywords. Dynamic Positioning, Offshore units, Thrust allocation, capability plots, operability

1. Introduction

Dynamic positioning (DP) is a technology employed primarily in the offshore industries [1] that, nowadays, is also utilised in other maritime industry sectors, like yachting [2]. Such technology enables the vessel to keep a precise location and heading without needing mooring lines and anchors, relying only on the onboard thruster and propulsive systems. DP is crucial for operations in challenging environments, such as deep-sea drilling, underwater construction and positioning of floating platforms and vessel stationing in environmentally protected areas.

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Then, the DP system is mounted on almost all offshore units operating in deep waters, becoming also a plus for many modern large yachts. As such, it is relevant to consider the issues related to DP since the early stages of the design, employing proper tools to predict the DP system's capability [3] or operability [4]. Between the possibilities available for predicting DP performances, early calculations necessitate a fast tool that doesn't need too many inputs. Thus, preliminary predictions rely on quasi-static calculations [5,6].

Even though a quasi-static calculation is less complex than a full DP simulation in the time domain, some issues may confuse a designer approaching the calculations. The quasi-static process finds the equilibrium between the thruster forces and the environmental loads acting on the ship employing a thrust allocation algorithm. Therefore, the key points of a preliminary prediction are the form of the thrust allocation algorithm and the methods used for defining environmental loads.

The paper deals with the thrust allocation algorithm issue, leaving the environmental loads problem to future research. In this sense, the literature gives multiple thrust allocation options, from simple procedures to complex allocation processes based on differential evolution algorithms [7]. Each method has advantages and disadvantages in predicting the DP system's capability and operability. Here, the discussion compares a set of thrust allocation algorithms, ranging from the easiest solution to the most employed in the DP applications.

After describing the quasi-static DP problem and the differences between capability and operability, the work describes the selected thrust allocation algorithms and compares the different processes with the aid of a worked example. The results highlight how different the prediction could be in terms of both capability and operability, providing a hint to the designer in selecting the most balanced solution for a preliminary DP prediction.

2. Dynamic Positioning predictions

The prediction of dynamic positioning (DP) performance is crucial for designers, as it provides an overview of the system's capabilities and operability. There are two main approaches for performing calculations: time domain simulations and quasi-static calculations.

Time domain simulations require knowledge of multiple attributes related to both the hull and the onboard systems, making them most suitable for advanced stages of vessel design [8]. However, these calculations demand considerable computational time since multiple repetitions of the same environmental conditions are necessary.

In contrast, quasi-static calculations require fewer inputs and significantly less computational time. This allows designers to easily alter input parameters, offering high flexibility. Therefore, the quasi-static approach is ideal for the initial design stage, as it enables the concurrent evaluation of multiple design options.

This work focuses on solving a quasi-static DP problem. When addressing a quasi-static DP challenge, the primary task is to resolve equilibrium on the horizontal plane, balancing the environmental forces against the thrust generated by the actuators.

To set the stage, we consider a reference system centered on the vessel's midpoint (Point O), with the x-axis pointing towards the bow, the y-axis pointing towards the starboard side, and clockwise rotations considered positive. The equilibrium system of the ship can then be described as follows:

$$\begin{cases} \sum_{i=1}^{N_T} T_{x_i} & = F_{x_{ext}} \\ \sum_{i=1}^{N_T} T_{y_i} & = F_{y_{ext}} \\ \sum_{i=1}^{N_T} (T_{y_i} x_{T_i} - T_{x_i} y_{T_i}) & = M_{z_{ext}} \end{cases} \tag{1}$$

where T_x and T_y are the longitudinal and transversal components of the thrust delivered by the N_T actuators installed onboard, x_T and y_T are the locations of the thrusters and $F_{x_{ext}}$, $F_{y_{ext}}$ and $M_{z_{ext}}$ are the external forces and moments acting on the system. System (1) is valid either in case of azimuth thrusters, tunnel thrusters or propellers, as, accordingly to the actuator type, some components may be equal to zero.

The external forces present in the equilibrium system are composed of environmental forces (i.e. wind, waves and current) and additional forces related to the vessel operation (e.g. drilling force, pipe tension force, etc.). The additional forces are usually considered constant between the calculation sets. Multiple methods are available for the environmental load estimation. A designer can use experimental data, numerical calculations or empirical formulations. In the case of the initial design stage, it is necessary to use simple formulations, opting for empirical formulations. In this study, the choice is to estimate all the loads according to the empirical indications provided by DNV [9].

According to the quasi-static calculation, the resolution of system (1) is sufficient to find the thruster forces that satisfy the equilibrium in a given environmental scenario. If a solution to the system exists, then the vessel holds the position; otherwise, the position is lost. The conditions to test change according to the methodology employed for the DP system evaluation, i.e. the capability or operability. The following sections describe these two alternative concepts.

2.1. DP capability

The capability is the standard way to assess the performances of a DP system. The process involve the execution of several recursive calculations to obtain a DP capability plot. The conventional capability plot reports the maximum sustainable wind speed at each encounter angle of the environmental loads [10]. The environmental loads are supposed to be collinear, the current is constant and wind and waves changes according to a wind-wave correlation curve.

The wind wave correlation associates for each wind speed a combination of significant wave height H_s and a zero crossing period T_z. In the literature, multiple wind-wave correlations are present. Some of these correlations are representative of a specific sea area of interest, others are purely referential conditions that could be used in an early design stage for comparison purposes between different design alternatives. The important thing to underline is that a wind-wave correlation curve is representative of a single set of tuples (H_s, T_z, V_w), where V_w is the wind speed, means it is not covering all the possible combination of the three parameters.

Figure 1 shows an example of capability plot. This example refers to the traditional way of representing the capability plot. However, it is possible to adopt the same kind of diagram to represent also other quantities, e.g. the thrust absorbed at each encounter angle or the power absorbed by each actuator.

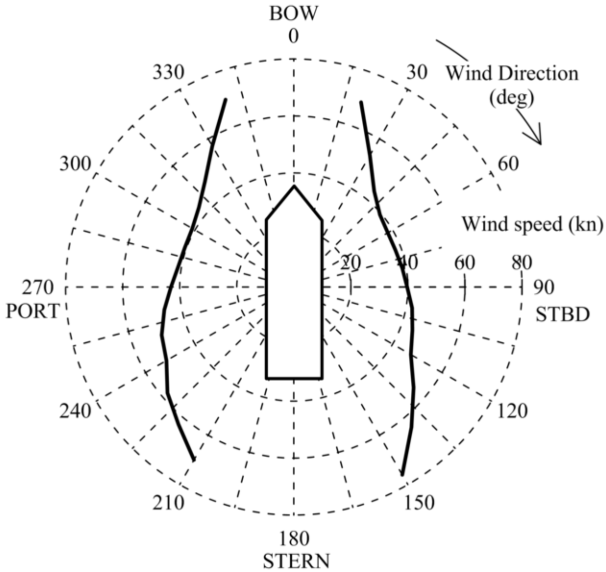

Figure 1. Example of a conventional DP capability plot.

2.2. *DP operability*

An alternative approach for the evaluation of DP performances is given by the concept of operability. The process is based on a scatter diagram approach, means a DP calculation is performed for each couple of H_s and T_z that may be encountered in a reference sea area.

The scatter diagram approach assumes that the wind speed V_w is changing according to the different values of H_s and T_z assuming a correlation derived from the definition of Pierson-Moskowitz wave spectrum. Then, the correlation between wind and wave parameters is as follows:

$$\begin{cases} T_z = \frac{2\pi}{g\sqrt[4]{\pi\beta}} V_w \\ H_s = \frac{2}{g}\sqrt{\frac{\alpha}{\beta}} V_w^2 \end{cases} \tag{2}$$

where α and β are two constants of the Pierson-Moskowitz spectrum having values 0.0081 and 0.74, respectively.

The correlation (2) assumes a linear correlation between V_w and T_z and a quadratic correlation between V_w and H_s. Therefore, there are two possible values for the V_w for each couple H_s, T_z. According to the process, the V_w value to select for the calculations is the maximum between the two formulations of Eq. (2). Alternatively, use can be made of joint distributions for H_s, T_z and V_w, resulting, in this case, in an unique relation between the parameters.

The final goal of the scatter diagram approach is the evaluation of the operability of the DP system. This calculation is performed according to the following relation:

$$OP_{DP} = \sum_{i=1}^{N_h} p_h \sum_{j=1}^{N_w} p_{w_j} I_{DP_{ij}} \tag{3}$$

where p_h is the probability associated to each one of the N_h calculation headings (supposed to be equiprobable in absence of detail data on the vessel operative profile) and p_w is the joint probability of the H_s and T_z (i.e. the numbers on the scatter diagram [11]). The functional I_{DP} is evaluated at each combination of heading and wave parameters and is equal to 1 when the vessel hold the position, 0 otherwise.

The operability allows for having a wider overview of the DP performances of an offshore unit, giving a direct evaluation of the downtime period of the DP system and not only an associated wind speed like for the capability case. Therefore, DP operability is a powerful indicator for estimating DP performances.

3. Thrust allocation algorithms

The core of a quasi-static DP prediction is the resolution of the equilibrium of system (1). The system admits an unique solution only in the case of three unknowns, which means considering a DP system composed of 3 fixed thrusters or 1 fixed thruster and 1 azimuth thruster. Both cases are not typical layouts for an offshore ship or a DP equipped ship in general, where the number of actuators composing the DP system is higher. Therefore, system (1) admits infinite possible solutions and needs proper techniques to be solved. In this section, the most common algorithm employed for the thrust allocation problem resolution are briefly presented in order to introduce the different options available for a designer.

3.1. Pseudo-inverse matrix

One of the most simple way to solve the thrust allocation problem is given by the application of the pseudo-inverse matrix. This is a simple optimisation technique and for its understanding it is better to write system (1) in matrix form:

$$\begin{pmatrix} 1\cdots 1 & 0\cdots 0 \\ 0\cdots 0 & 1\cdots 1 \\ -y_{T_1}\cdots -y_{T_{N_T}} & x_{T_1}\cdots x_{T_{N_T}} \end{pmatrix} \begin{pmatrix} T_{x_1} \\ \vdots \\ T_{x_{N_T}} \\ T_{y_1} \\ \vdots \\ T_{y_{N_T}} \end{pmatrix} = \begin{pmatrix} F_{x_{ext}} \\ F_{y_{ext}} \\ M_{z_{ext}} \end{pmatrix} \quad (4)$$

In this way, Eq. (4) represent the equilibrium system in the classical form $A \cdot x = b$, where A is a $(3, 2N_T)$ matrix. Matrix A has rank 3, that means system (4) admits infinite possible solutions. This overestimation can be solved by adopting the Moore-Penrose pseudo-inverse matrix A^+. Therefore, an optimal solution of the thrust allocation problem can be found in the least-square sense by the following form:

$$\bar{x} = A^+ b \quad (5)$$

This solution is particular as it is the one with minimum norm $||x||$ between the infinite possible solutions of the system. The method is extremely simple but has some significant limitations. In fact, it is not possible to consider directly the thrusters saturation and an iterative approach should be used to try to overcome the issue.

3.2. Quadratic Programming

The mostly applied method for the resolution of the equilibrium consists in the application of quadratic programming techniques (i.e an optimisation problem), which are used not only for preliminary predictions but also for applications onboard. The main peculiarity of quadratic programming is the structure of the optimisation problem, means the objective function and constraints form. The general objective function of quadratic programming is as follows:

$$\min z = \frac{1}{2} x H x^T + f^T x \tag{6}$$

subject to:

$$\begin{cases} Ax \leq b \\ A_{eq} x = b_{eq} \\ x_{min} \leq x \leq x_{max} \end{cases} \tag{7}$$

where H, f, A and A_{eq} are matrices defining the objective function and the equality and inequality constraints, b and b_{eq} are vectors defining the constraints and x is the vector of unknowns. For the specific case of dynamic positioning, the unknowns are the thrust components of each thruster (F_{x_i}, F_{y_i}), H is a diagonal matrix of 2 and f is equal to zero.

Particular attention should be given to the constraints definition. The equality constraints are suitable to describe the equilibrium between internal and external forces and, therefore, are identified by system (1). Inequality constraints identify the thrust domain of each single thruster unit. In the case of a fixed tunnel thruster it is simply necessary to identify the thrust thresholds that should be respected. For a steerable thruster the process is more complicated as the thrust domain is represented by a circle. Because the quadratic programming process admits only linear constraints, the circle should be linearised. In the specific, each circle could be approximated with a set of lines, typically in a number between 8 or 16. Here, 16 lines are considered to solve the allocation problem.

The thrust allocation algorithm can handle also the interactions between steerable thruster, through an iterative process. Each interaction zone is identified by a main angle τ_{int_i} and a spreading angle $\delta\tau_i$. In case, after the first optimisation process, the orientation angle τ_i of one or more thrusters is in the interval $[\tau_{int_i} - \delta\tau_i, \tau_{int_i} + \delta\tau_i]$, the procedure automatically fix the thruster orientation to the nearest boundary (by adding an additional equality constraints) and rerun the optimisation. The process continues iteractively until no interaction is more detected.

3.3. Non-linear Programming

To further enhance the solution of the thrust allocation problem more complicated and efficient algorithms can be applied. This is the case of non-linear programming, an optimisation technique that allows for using both non-linear objective function and constraints. For the specific case of DP, it is relevant to have the possibility to describe the problem with non-linear function, as, for example, the definition of the thrust constraints is a circle. Then, to avoid the linearisation, it will be possible to describe these constraints with a single function. The non-linear optimisation has the following form:

$$\min z = f(x) \tag{8}$$

subject to:

$$l \leq \begin{pmatrix} x(i) \\ c[x(i)] \end{pmatrix} \leq u \tag{9}$$

where $f(x)$ is the objective function of n variables$x(i)$ and $c[x(i)]$ is a vector of m constraint functions. The sole requirement for the objective function and constraints is that the functions should be continuous and differentiable at points that satisfy the bounds l and u for the given $x(i)$, thus allowing the evaluation of the gradient vector $g(x) = \nabla f(x)$ and the Jacobian matrix $J(x) = \nabla c(x)$.

According to this process, the objective function can be non-linear. This allows to consider a non-quadratic objective function, which reflects the modeling of absorbed power as a function of thrust. The function has the following form:

$$f(x) = \sum_{i=1}^{N_T} x_i^{3/2} \tag{10}$$

where x_i is the thrust of each single device. In the specific, the optimisation resolution is adopting an extended version of Robinson method, based on filtering and employing a trust region obtained by adopting Ritz values [12].

Adopting this kind of resolution it is possible to adopt two different strategies for the problem definition. In fact, it is possible to write the objective function and constraints considering the thrust directly or its longitudinal and lateral components. In the first case each thruster is defined by the thrust and the orientation, in the latter the components are sufficient. Once the thrust and orientation are selected for describing the problem, it is possible to automatically consider the interactions between thrusters by setting the bounds on the orientations.

3.4. Differential evolution algorithm

Another technique that can be employed to solve the thrust allocation problem is represented by genetic algorithms. Genetic algorithms are adaptive heuristic search algorithms designed to simulate processes in natural systems necessary for evolution. This means that they represent an intelligent exploitation of a random search within a defined design space to solve a specific problem. In case a problem is difficult to model with conventional methods, then genetic algorithm are an ideal solution. Therefore, thrust allocation problems can be solved with this kind of methods, especially when the maximum capability or operability of the DP system is investigated and not only a single environmental condition is considered.

In the specific, the present work employs a differential evolution algorithm for global optimisation, where, basically, the identification of the operations is not made on binary function but directly with real values.

The initial issue to start a differential evolution optimisation procedure is to create a sample population of N_{pop} individuals, characterising each individual with M parameters. The initial size of the population can be set arbitrarily; however, it is suggested to set it as a function of M. A good practice is to assume $N_{pop} \geq 8M$.

The initial population is generated randomly, covering the entire feasibility range, i.e. the design space. In case a more likely subspace for the optimal solution could be identified, the solutions may be seeded in a particular area, setting proper values to the M parameters of each individual. After setting the initial population, the objective function must be evaluated for each member of the population. The differential evolution algorithm allows for using non-linear objective functions; thus, the process employs the same function of the non-linear optimisation technique, i.e. Eq. (10). Same is for the constraints; thus, by considering thrust and orientations as for the non-linear optimisation case, the final problem to solve is the following:

$$\min z = f(x) \tag{11}$$

subject to:

$$l \leq \begin{pmatrix} x_i \\ c\left[(x_i)\right] \end{pmatrix} \leq u \tag{12}$$

which is absolutely equal to the problem of the non-linear optimisation. After the objective function evaluation of each individual, which are automatically respecting the constraints, a population portion is selected for the new population generation. Here, the best individual is selected at each generation round, the other members are randomly and distinctly selected across the rest of the population. It is not advisable to select only the best members for this procedure as it could be that the objective function remains in a local optimum point. From each member of the population portion, a crossover procedure is carried out. This process is mixing the properties of the individuals. The process selects 4 distinct individuals, proceeding with the following crossover scheme:

$$x_{new_i} = x(1)_i + k\left(x(2)_i - x(3)_i\right) + k_1\left(x(3)_i - \frac{1}{2}x(4)_i - \frac{1}{2}x(1)_i\right) \tag{13}$$

where k and k_1 are the crossover probability factors. Alternatively, an exponential scheme can be used.

The crossover procedure generates a set of offsprings. The objective function should be evaluated for each offspring member, comparing each individual with his parent. Afterwards, the best individuals between offspring and old population are selected for the next step of the process. Therefore, at each iteration the new population is composed by the best parent and the best child. The process continues until the the objective function reaches the maximum or minimum value of the problem (i.e. when a convergence in the iterations is found).

4. Comparisons on a reference ship

The present section analyses the effect of the previously described thrust allocation algorithms on the capability and operability of the DP system. To facilitate the discussion and the graphical representation the following abbreviations are adopted:

- PIO: Pseudo-inverse optimisation.
- QPO: Quadratic Programming optimisation.
- NLO: Non-linear optimisation.
- DEA: Differental Evolution Algorithm optimisation.

Table 1. Thruster configuration of the reference HLCV.

#	Thruster name	x_T (m)	y_T (m)	P_S (kW)
1	Azimuth Forward 1	57.00	4.20	2050
2	Azimuth Forward 2	52.30	-4.20	2050
3	Azimuth Middle 1	27.50	-15.00	1400
4	Azimuth Middle 2	-22.50	15.00	1400
5	Azimuth Aft 1	-60.00	15.00	2050
6	Azimuth Aft 2	-60.00	-15.00	2050

To analyse the differences among the procedures, a reference vessel has been selected for the study. The vessel is a Heavy Lift Crane Vessel (HLCV), which represent an interesting test case as it has a totally asymmetric disposition of the actuators. The HLCV has a length of 165 metres, a displacement of 24000 tonnes, a breadth of 38 metres and is equipped with 6 steerable thruster devices. The thruster configuration is reported in Table 1 according to the reference system employed for the calculations, together with the shaft power of each device.

On this reference vessel capability and operability calculations have been performed according to the environmental condition of the North Sea (Area 11 in the Global Wave Statistics [11]), as this area is representative for the initial design stage of an offshore vessel. Consequently, a dedicated wind-wave correlation provided by IMCA has been employed for the capability and the joint distribution of waves for Area 11 to determine operability. The current has been considered constant and concurrent with wind and waves for both the calculations.

For the environmental loads use has been made of experimental results available for this vessel.

4.1. *Capability results*

First of all, capability calculations have been performed on the reference HLCV. As a result, the capability plots have been obtained for all the four thrust allocation strategies described in the previous section.

Fig. 2 reports the capability plots for the four cases, facilitating a direct comparison between the tested thrust allocation methodologies. From the results, it is evident that the thrust allocation algorithm providing the maximum capability is the DEA approach. The DEA process allows for determining the equilibrium of the DP system up to higher conditions in all the tested heading angles. This is true for all the tested headings, also in conditions close to bow and stern directions. The second best performer is the NLO method. Compared to DEA is giving less capability in all the directions. This is due to the difficulties of finding a feasible solution once the thruster are closed to saturation, something that is achievable with the DEA approach. The QPO process gives similar results compared to NLO methodology. QPO has lower capability only for directions coming from starboard side, probably due to the non satisfaction of the linear constraints for thrust saturation. The worst performances are provided by the PIO approach. This was advisable as the method is extremely simple and has many limitations.

The capability plots obtained as a result of the study are an easy way to directly compare the thrust allocation methodologies. However, the capability plot is not sufficient to quantify the improvement achievable with each one of the algorithms. In fact, at each

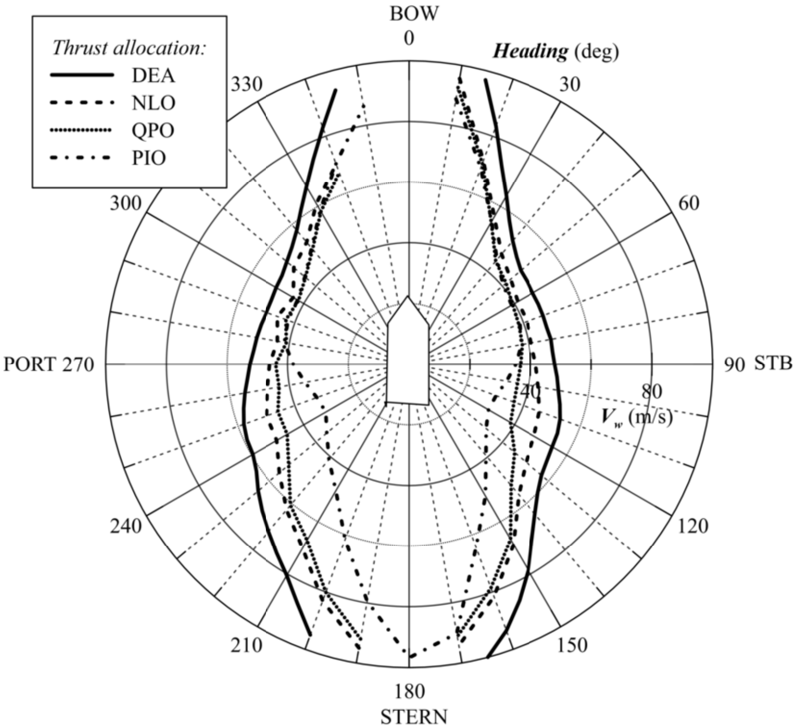

Figure 2. DP capability plot comparing the tested thrust allocation algorithms.

Table 2. DP operability results for area 11 according to the different thrust allocation methods.

Optimisation method	ID	OP_{DP}
Pseudo inverse matrix	PIO	93.15%
Quadratic programming	QPO	97.72%
Non-linear programming	NLO	98.13%
Diff. evolution algorithm	DEA	99.08%

angle, only the maximum sustainable wind speed is available and the advantage is changing for each heading. To this end, a useful comparison can be reached by employing a different indicator for the DP performances, i.e. the operability.

4.2. Operability results

To quantify the benefits in term of DP performances between the different thrust allocation process, operability calculations have been carried out on the reference HLCV. The results of the analysis are reported in Table 2.

From the table it is evident that the best approach is given by the DEA solution, followed by NLO, QPO and finally PIO. Therefore,the ranking is the same obtained with the capability plot representation. However, the evaluation of operability allows to consider the downtime period obtained for Area 11 with each one of the algorithm. The results are as follows:

- PIO algorithm: 340 working days per year.
- QPO algorithm: 357 working days per year.
- NLO algorithm: 358 working days per year.
- DEA algorithm: 362 working days per year.

Therefore, the difference between DEA and PIO (the best and the worst) are of about 2.5 weeks per year, which is a significant amount of working days in terms of operations. Considering the other methods, the difference between QPO and NLO is of 1 day per year which is a negligible difference. For the tested case, the difference between NLO and DEA is of a bout 4 days, which means it is not so high for a preliminary design stage prediction.

The fact that DEA is not overperforming so much NLO and QPO in therms of operability is due to the calculation conditions. According to the capability results, the improvements of DEA are at significantly high wind speed values, which are associated to wave parameters not present in the scatter diagram of Area 11. Therefore, such an advantage is not visible in terms of operability.

Concerning the calculation time, there is no practical implication for the preliminary design stage as the predictions are all obtained in timing ranging from few seconds and several minutes. Of course, more advanced allocation algorithms require larger computational time that may be a problem only for real-time applications, especially if DEA solution want to be pursued. However, this is going outside the purposes of the present study.

As a conclusion it can be stated that advanced optimisation techniques are not really suitable for the preliminary design stage as they are not giving high changes in the estimated operability.

5. Conclusions

The present work reports a comparison between different thrust allocation procedures suitable for the preliminary design stage of a DP equipped ship. The procedures are ranging multiple levels of simplification, starting from really simple methods up to more complicated process that requires more knowledge for the implementation. The processes that have been considered are: Pseudo-inverse matrix resolution, Quadratic Programming optimisation, non-linear optimisation and Differential Evolution Algorithm optimisation. After a description of the different processes, the procedures have been tested on a reference HLCV vessel, with the aim of obtaining the DP capability and the operability. The calculation results allow for ranking the different DP allocation techniques. According to the capability, the best process is the one based on differential evolution algorithm, followed by non-linear optimisation and quadratic programming. The method based of the pseudo-inverse matrix is giving the lower capability according to the resulting capability plots. However, the capability is not able to provide an objective value to properly quantify the benefits of each one of the processes. To this end, the operability calculations are necessary, allowing to determine the downtime period evaluated for each one of the processes in a given operational area. Operability results confirm the ranking obtained with capability calculations. However, the comparisons in terms of downtime period shows really low differences among the DEA, QPO and NLO processes. For such a reason, during an early design stage, the QPO can be applied without loosing to much reliability in the estimation of the effective operability and capability of the vessel. More complicated processes like DEA and NLO could be applied in more advanced phases of the design, when designers need a more reliable estimation of the DP performances.

The present work is a first study in the field of comparison between operability and capability results for different thrust allocation processes. Further study are needed to achieve a final ranking between procedures, considering more vessels and different reference areas for the calculations.

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