Supplementary Materials of: 'Tiered social distancing policies and epidemic control'

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1 Description of contents

The following supplementary materials report a number of theoretical results and additional simulation findings. In particular, we report

- an approximate assessment of the endemic states of the model with tiered interventions for the special but noteworthy case where infection-related mortality is absent (Section 2).
- a pedagogic example (Section 3) showing in a simple manner the main result stated in the main body of the manuscript, namely that the steady states of an hereditary system as the one embedded in our model for tiered interventions will generally depend on the system initial conditions (Section 3).
- some additional simulation results and related figures (Section 4).

2 Approximation of equilibria

In this **section,** we illustrate an approximate assessment of endemic equilibria by focusing on the case where $\delta = 0$ and $N(t) = 1$, which allows the onset of a unique (stable) endemic state in the basic SEIRS model in the absence of governmental interventions. Given the richness of sub–cases provided by our model, we only investigate the case where the summary index M_G only includes the transmission dimension M_1, namely $c_1 = 1, c_2 = c_3 = c_4 = 0$ (Section 3.1.1 of main text), implying $M_G(t) = M_1(t)$. Let us assume that $\beta(t)$ converges to a steady state β_e.

Substituting this into the model and setting the derivatives of the state variables to zero, we easily obtain that the system has the following endemic equilibrium

$$EE = (S_e, E_e, I_e, M_1^e)\,, \tag{1}$$

such that

$$S_e = \frac{\gamma}{\beta_e}.$$

Note that this implies that at the endemic state it holds the well-known relationship $\mathcal{R}_t^e = 1$. This is a general feature of simple epidemiological models with waning immunity that we have systematically observed in the simulations reported in the main text.
Moreover:

$$E_e = \frac{\gamma}{\alpha} I_e;$$

$$M_1^e = \mathcal{R}_t^e = 1;$$

$$I_e = \frac{\theta}{\gamma + \theta\left(1 + \frac{\gamma}{\alpha}\right)}\left(1 - \frac{\gamma}{\beta_e}\right). \tag{2}$$

In principle, the determination of β_e is nontrivial due to the dependency of the system from its history. However, in the present case we have (see Remark 2.1 of main text)

$$\max_{t\in(0,+\infty)} M_1 = \mathcal{R}.$$

Thus

$$M_G^e = \frac{1}{\mathcal{R}},$$

yielding

$$\beta_e = \beta_H F\left(\frac{1}{\mathcal{R}}\right).$$

3 A simple Hereditary Model showing dependence of equilibria on initial conditions

In this **section, with pedagogical aims we provide a simple example showing how the class of models considered in this work can have equilibria that depends on the initial conditions. This feature arises in our model due to the hypothesis of *dynamic normalization* used to combine the various indicators M_i into the composite index M_G. Here we suggest that dynamic normalizations having the chosen form typically yield this dependency of equilibria on initial conditions. To this aim, let us consider the following hereditary model:**

$$\dot{U} = \frac{1}{1 + \frac{U}{W}} - U \tag{3}$$

$$W(t) = \max_{s\in[0,t)} U(s). \tag{4}$$

Formulation 3-4 describes a simple nonlinear differential equation including a dynamic normalization term. Using the representation adopted in the main text, equations 3-4 can be rewritten as

$$\dot{U} = \frac{1}{1+\frac{U}{W}} - U \tag{5}$$

$$\dot{W} = \texttt{Heav}(U-W)\left|\frac{1}{1+\frac{U}{W}} - U\right|_+ \tag{6}$$

$$W(0) = U(0) \tag{7}$$

If $U(0) = U_0 \geq 1/2$ then it is easy to show that the dynamics of $U(t)$ is decreasing and given by

$$\dot{U} = \frac{1}{1+\frac{U}{U_0}} - U$$

which tends to

$$U(t) \to U_e(U_0) = \frac{U_0}{2}(-1+\sqrt{1+\frac{4}{U_0}})$$

Note that in this first scenario $\dot{W} = 0$ thus $W(t) = U_0$. Thus defining $X(t) = (U, W)$ and $X(t) \to (U_e(U_0), U_0)$.
If $U(0) < 1/2$ then U grows by following the law

$$\dot{U} = \frac{1}{1+1} - U$$

that is

$$U(t) \to U_2 = \frac{1}{2}$$

Note that in this scenario $W(t) = U(t)$ and $X(t) \to (1/2, 1/2)$.
Summarizing, the equilibrium point for system (5)–(6)–(7) depends on the initial conditions.

4 Supplementary Simulations and Figures

In this section, we provide and comment some further simulations.

4.1 Supplementary Simulation 1: index M_G includes hospitalizations only

The figure 1 reports a long-term simulations over a 10 year-long horizon of the tiered intervention reported in figure 6 (for a 1-year horizon) of the main text. The figure shows that - thanks to the control enacted by Z_3 - the epidemic activity fully stabilises in less than one year on the controlled regimes of figure 6 of the main text.

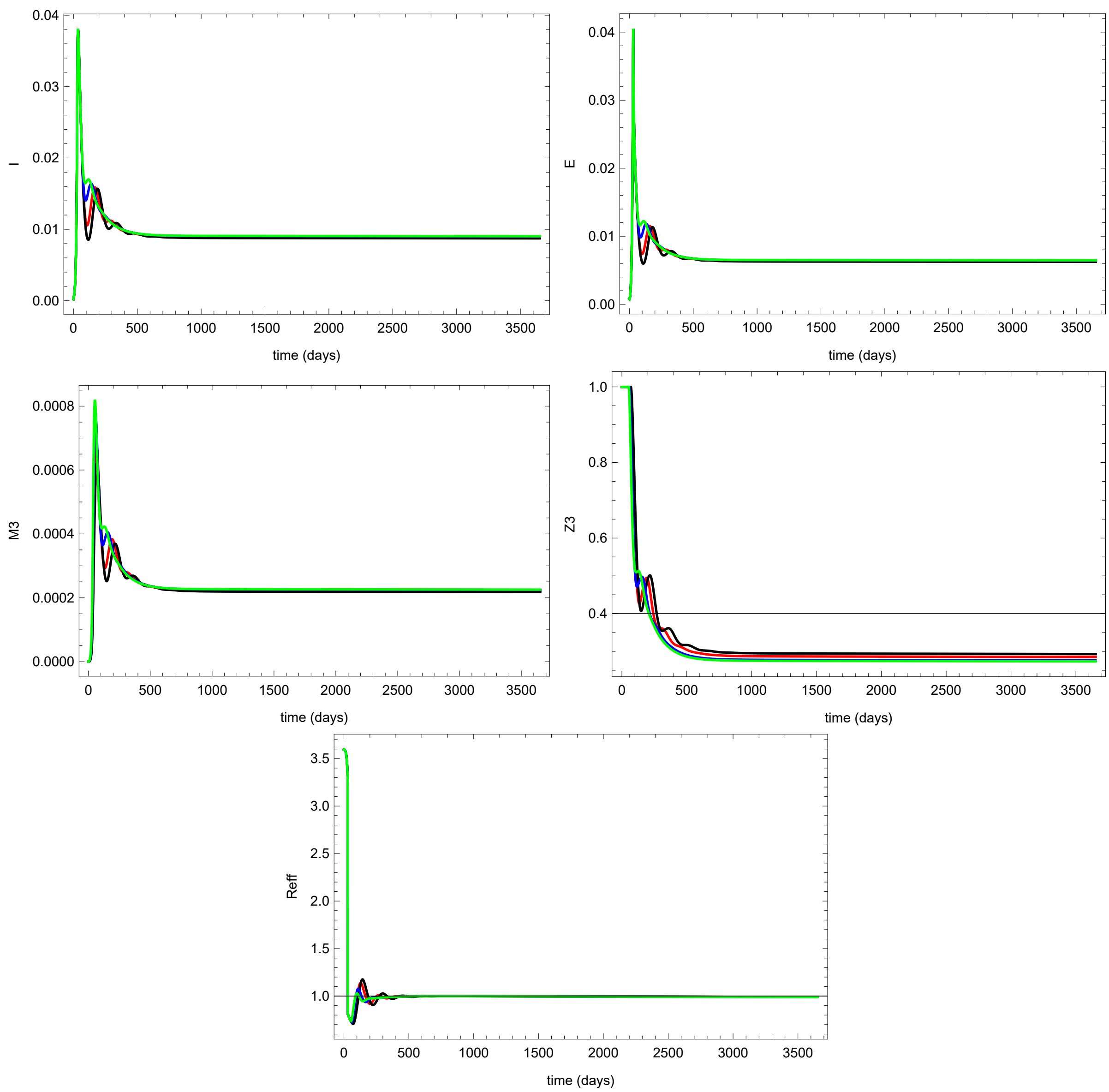

Figure 1: Temporal trend of a controlled epidemic where activation of tiered interventions (with $\omega = 3$) is uniquely informed by the hospitalization indicator M_3. **The graphs display the effects of different levels of the delay rate** a_3**:** $a_3 = 1/21$ **(black),** $a_3 = 1/14$ **(red),** $a_3 = 1/7$ **(blue),**$a_3 = 1/3$ **(green).** Intervention starts at $t_{st} = 30$. Time horizon: 10 years. Upper Left panel: prevalence $I(t)$; Upper Right panel: exposed $E(t)$; Lower Left panel: $M_3(t)$; Lower Central panel: $Z_3(t)$; Lower Right panel: $\mathcal{R}_E(t) = S(t)\beta(t)/\gamma$. Other parameters and initial conditions described in the main text.

4.2 Supplementary Simulation 2: index M_G includes societal costs

Figure 2) reports the long-term behaviour (over 10 years) after the tiered intervention in figure 10 of the main text. The infection establishes on a trend coarsely overlapping that of a free epidemic, with a close period and amplitude of oscillations because rising societal costs force the government to rapidly lift most restrictions.

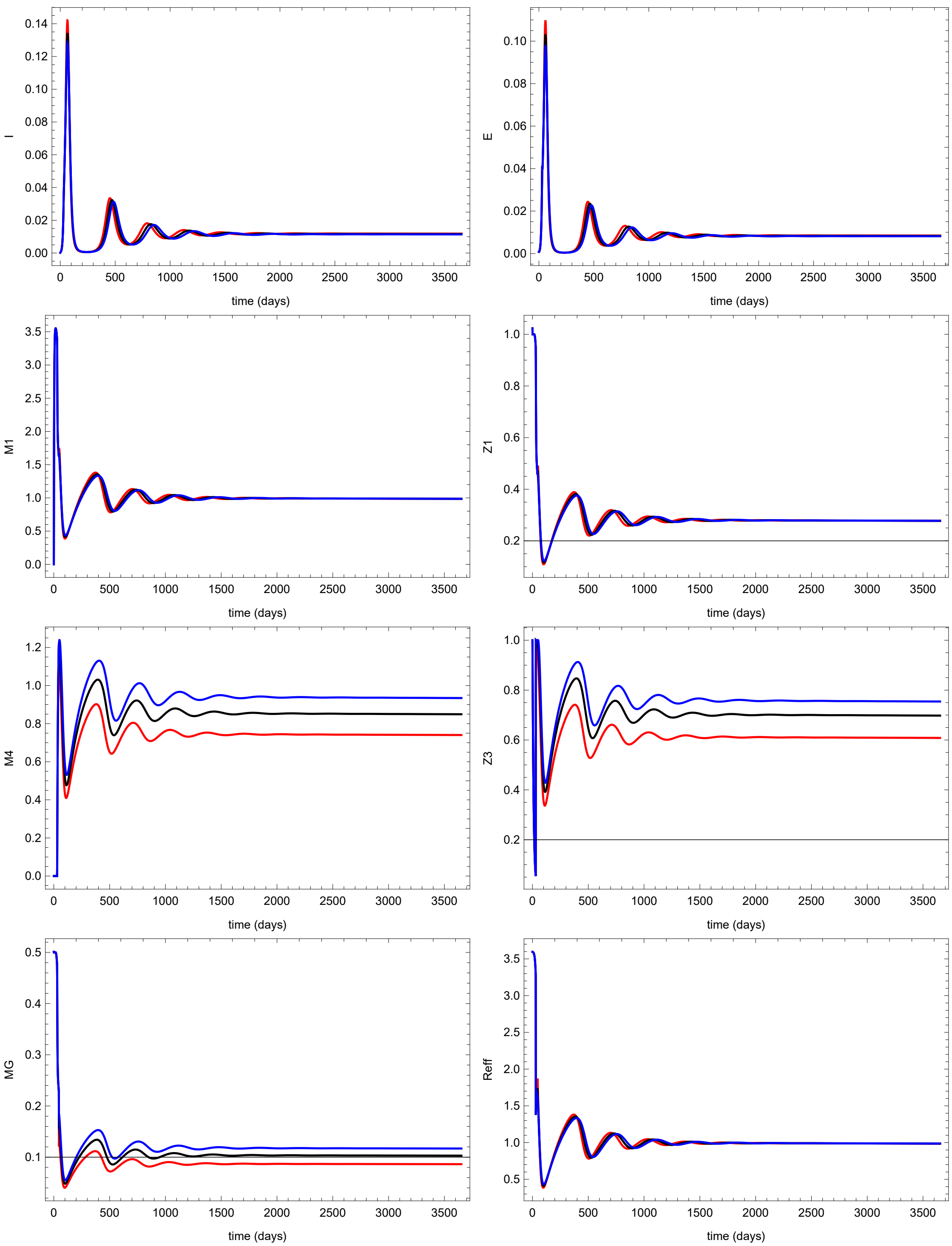

Figure 2: Trend of a controlled epidemic where interventions (with $\omega = 3$) are informed by both the transmission indicator M_1 and societal costs (M_4) **with** $c_1 = 0.5$ **and three values of** c_4**:** $c_4 = 0.5$ **(black),** $c_4 = 1$ **(red),** $c_4 = 0.25$ **(blue)**. Time horizon: 10 year. Intervention starts at $t_{st} = 30$, while social costs are included from $t = t_* = 45$. Left upper panel: $I(t)$; right upper panel: $E(t)$; second row, left panel: $M_1(t)$; second row, right panel: $Z_1(t)$; third row left panel: $M_4(t)$; third row, right panel: $Z_4(t)$; left lower panel: $M_G(t)$; right lower panel: $\mathcal{R}_E(t)$. Other parameters and initial conditions described in the main text.