## Search for an invisible Z' in a final state with two muons and missing energy at Belle II

Belle II Collaboration

This material is submitted as supplementary information for the Electronic Physics Auxiliary Publication Service We provide a text file with numerical results for the observed 90% CL upper limit on the cross section of  $e^+e^- \rightarrow \mu^+\mu^- Z'$ , with  $Z' \rightarrow$  invisible as well as of the observed 90% CL upper limit on g' as functions of  $M_{Z'}$ .



FIG. S1: Squared recoil mass spectrum for the  $\mu^+\mu^-$  sample before the Punzi-net selection, compared with the stacked contributions from the various simulated background samples normalized to the integrated luminosity.

We also show the limits on g' as functions of  $M_{Z'}$  on a logarithmic scale in Fig. S3 for the  $L_{\mu} - L_{\tau}$  vanilla model and in Fig. S5 for the  $L_{\mu} - L_{\tau}$  fully invisible model.



FIG. S2: Distribution of (top) expected background events and (bottom) data across the  $\theta_{\text{recoil}}^{\text{c.m.}}$  versus  $M_{\text{recoil}}^2$  plane after all the analysis selections.



FIG. S3: Observed 90% CL upper limits on the coupling g' for the  $L_{\mu} - L_{\tau}$  vanilla model as functions of  $M_{Z'}$  on a logarithmic scale. Existing limits from BaBar [1], Belle [2], CMS [3] (95% CL), NA64-e [4], and Belle II [5] are shown, along with constraints (95% CL) derived from the trident production in neutrino experiments [6–8]. The red band shows the region that could explain the muon anomalous magnetic moment  $(g-2)_{\mu} \pm 2\sigma$  [9].



FIG. S4: Observed 90% CL upper limits on the cross section  $\sigma(e^+e^- \rightarrow \mu^+\mu^- Z', Z' \rightarrow \text{invisible})$  as functions of the Z' mass for  $\Gamma_{Z'} = 0.1M_{Z'}$ , including the  $\pm 1$  and  $\pm 2\sigma$  bands around the expected limits. Also shown are previous limits from Belle II [5] and the observed limits for the negligible width case.



FIG. S5: Observed 90% CL upper limits on the coupling g' for the  $L_{\mu} - L_{\tau}$  fully invisible model as functions of  $M_{Z'}$  in logarithmic scale. Existing limits from NA64-e [4] and Belle II [5] are shown, along with constraints (95% CL) derived from the trident production in neutrino experiments [6–8]. The vertical dashed line indicates the limit beyond which the hypothesis  $\mathcal{B}(Z' \to \chi \bar{\chi}) \approx 1$  is not valid in the negligible  $\Gamma_{Z'}$  case. The red band shows the region that could explain the muon anomalous magnetic moment  $(g - 2)_{\mu} \pm 2\sigma$  [9].



FIG. S6: Observed 90% CL upper limits on the coupling g' for the  $L_{\mu} - L_{\tau}$  fully invisible model as functions of  $M_{Z'}$ . Existing limits from NA64-e [4] and Belle II [5] are shown, along with constraints (95% CL) derived from the trident production in neutrino experiments [6–8]. The vertical dashed line indicates the limit beyond which the hypothesis  $\mathcal{B}(Z' \to \chi \bar{\chi}) \approx 1$  is not valid in the negligible  $\Gamma_{Z'}$  case. The red band shows the region that could explain the muon anomalous magnetic moment  $(g-2)_{\mu} \pm 2\sigma$  [9].



FIG. S7: Observed 90% CL upper limits on the coupling g' for the  $L_{\mu} - L_{\tau}$  vanilla model as functions of  $M_{Z'}$ . Existing limits from BaBar [1], Belle [2], CMS [3] (95% CL), NA64-e [4], and Belle II [5] are shown, along with constraints (95% CL) derived from the trident production in neutrino experiments [6–8]. The red band shows the region that could explain the muon anomalous magnetic moment  $(g - 2)_{\mu} \pm 2\sigma$  [9].

- [1] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 94, 011102 (2016).
- [2] T. Czank et al. (Belle Collaboration), Phys. Rev. D 106, 012003 (2022).
- [3] A. M. Sirunyan et al. (CMS Collaboration), Phys. Lett. B 792, 345 (2019).
- [4] Y. M. Andreev et al. (NA64), Phys. Rev. D 106, 032015 (2022).
- [5] I. Adachi et al. (Belle II Collaboration), Phys. Rev. Lett. 124, 141801 (2020).
- [6] W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin, Phys. Rev. Lett. 113, 091801 (2014).
- [7] G. Bellini et al. (Borexino Collaboration), Phys. Rev. Lett. 107, 141302 (2011).
- [8] A. Kamada, K. Kaneta, K. Yanagi, and H.-B. Yu, J. High Energy Phys. 06, 117 (2018).
- [9] B. Abi et al. (Muon g 2 Collaboration), Phys. Rev. Lett. **126**, 141801 (2021).