Collapse Dynamics Are Diffusive

Sandro Donadi[*]
Centre for Quantum Materials and Technologies, School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom

Luca Ferialdi[†]
Department of Physics and Chemistry, University of Palermo, via Archirafi 36, I-90123 Palermo, Italy

Angelo Bassi[‡]
Department of Physics, University of Trieste, Strada Costiera 11, 34151 Trieste, Italy and INFN, Sezione di Trieste, Strada Costiera 11, 34126 Trieste, Italy

(Received 19 October 2022; accepted 9 May 2023; published 9 June 2023)

Noninterferometric experiments have been successfully employed to constrain models of spontaneous wave function collapse, which predict a violation of the quantum superposition principle for large systems. These experiments are grounded on the fact that, according to these models, the dynamics is driven by noise that, besides collapsing the wave function in space, generates a diffusive motion with characteristic signatures, which, though small, can be tested. The noninterferometric approach might seem applicable only to those models that implement the collapse through noisy dynamics, not to any model, that collapses the wave function in space. Here, we show that this is not the case: under reasonable assumptions, *any collapse dynamics (in space) is diffusive*. Specifically, we prove that any space-translation covariant dynamics that complies with the no-signaling constraint, if collapsing the wave function in space, must change the average momentum of the system and/or its spread.

DOI: 10.1103/PhysRevLett.130.230202

Introduction.—Searching for the potential limits of validity of the quantum superposition principle is of highest relevance for the foundations of quantum mechanics and in general for our understanding of nature [1–4], and more pragmatically also for the scalability of quantum technologies. One further motivation is provided by models of spontaneous wave function collapse [5–8], which predict a progressive breakdown of quantum linearity and the localization in space of physical systems as their size increases, thus justifying the emergence of a classical world from quantum constituents.

The most natural way of testing the superposition principle is via "interferometric experiments," and several platforms have been employed or proposed for this scope, including atoms [9,10], molecules [11,12], optomechanical systems [13], crystals [14], and nitrogen-vacancy centers [15]. The difficulty in performing such experiments is that it is hard to generate and maintain a macroscopic spatial superposition in a (almost) decoherence-free environment, and check whether over time it survives or decays. The current world record for a matter wave delocalized in space is about 0.5 m, obtained with cold atoms [10], while the largest mass that interfered with itself weighs about 10^5 amu [12]. We are still far away from probing the quantum nature of the macroscopic world, but impressive technological development makes the goal less far away in the future [16–18].

Meanwhile, a different strategy has been successfully employed to test models of spontaneous wave function collapse, which consists of "noninterferometric experiments" [19]. The basic principle lies in the observation that in all these models the collapse of the wave function is triggered by noise, which shakes the particles' dynamics, whether their wave function is localized or not. As such, particles undergo a characteristic diffusion process, which can be tested through high-precision position measurements; these, although demanding, are easier to perform than the interferometric ones. Examples of these kind of experiments are the precise monitoring of the motion of cold atoms [20], cantilevers [21,22], or gravitational wave detectors [23,24]. Another consequence of the collapse-induced diffusion is that atoms emit an extra radiation, which is not predicted by standard quantum mechanics; also this effect has been used to test collapse models [25–28]. The bounds on the phenomenological parameters of the continuous spontaneous localization model [6] resulting from noninterferometric experiments are 6 orders of magnitude stronger than the best bounds set by interferometric experiments [29]. Moreover, a recent experiment based on precise measurements of the radiation emitted from

 © 2023 American Physical Society

germanium ruled out the parameters-free version of the Diósi-Penrose model [31].

An apparently weak side of noninterferometric experiments is that they seem not to represent a direct test of the quantum superposition principle, but only of the models so far proposed, which explicitly violate it. This leaves the possibility open to formulate a model where the wave function does collapse, without inducing diffusion on the system. Here, we show that this is not possible: any dynamics that localizes the wave function in space also changes the momentum. We call this "diffusion" because in all current collapse models it manifests as such, but in this Letter it is meant to signify any change in the momentum of the system. Since a change in momentum can be (at least in principle) detected by noninterferometric experiments, our result shows that they represent a test of the quantum superposition principle in a stronger sense than one might suppose.

We consider a general situation: we assume that physical systems are associated with a wave function ψ, which is subject to a generic norm-preserving (possibly, nonlinear) dynamics. The requirements on the dynamics are as follows: (i) it does not allow for superluminal signaling, and (ii) it is space-translation covariant, at least at the statistical level. The first assumption implies that the dynamics for the wave function ψ also provides a well-defined dynamics for the density matrix $\hat{\rho}$, which in general is not true for a nonlinear evolution [32]. Then, by construction, the dynamics for $\hat{\rho}$ is linear, completely positive, and trace preserving (see Supplemental Material [33]). The second assumption amounts to requiring that the map for $\hat{\rho}$ is space-translation covariant and has the same physical motivation on which all physical fundamental theories are based. We will prove our result for a single particle, where by particle we also (and mostly) mean the center of mass of a composite system.

To fix the notation, we will consider a particle in a box of size L, with periodic boundary conditions; this choice will avoid potential problems when dealing with plane waves. Let $\hat{p}_i$ be the momentum operator along direction $i(=x, y, z)$, and $(2\pi\hbar/L)n_i$ its eigenvalues, with $n_i \in \mathbb{Z}$; let $\boldsymbol{n} = (n_x, n_y, n_z)$. The average value of the momentum operator for a given state $\hat{\rho}$ is denoted as $\bar{p}_{i,\hat{\rho}} = \mathrm{Tr}(\hat{p}_i\hat{\rho})$ and its variance as $\Delta p_{i,\hat{\rho}} = \mathrm{Tr}(\hat{p}_i^2\hat{\rho}) - [\mathrm{Tr}(\hat{p}_i\hat{\rho})]^2$. Last, let $\hat{\boldsymbol{n}} = |\boldsymbol{n}\rangle\langle\boldsymbol{n}|$ be the state of definite momentum $(2\pi\hbar/L)\boldsymbol{n}$.

We will prove the following theorem. Consider a particle in a box of size L with periodic boundary conditions for its wave function. Consider a dynamical map for the wave function satisfying the conditions (i) and (ii), and let $\boldsymbol{\Phi}$ be the associated map for the density matrix $\hat{\rho}$ [49]. Assume that the average momentum is conserved along the three directions: $\bar{p}_{i,\boldsymbol{\Phi}[\hat{\rho}]} = \bar{p}_{i,\hat{\rho}}$ for any $\hat{\rho}$. Then, $\Delta p_{i,\boldsymbol{\Phi}[\hat{\boldsymbol{n}}]} = \Delta p_{i,\hat{\boldsymbol{n}}}\ \forall\ \boldsymbol{n}$ if and only if $\boldsymbol{\Phi}$ is a function of the momentum operator only, in which case plane waves do not collapse in space. Moreover, if $\Delta p_{i,\boldsymbol{\Phi}[\hat{\boldsymbol{n}}]} \neq 0$ for some $\boldsymbol{n}$, then $\Delta p_{i,\boldsymbol{\Phi}[\hat{\rho}]} > \Delta p_{i,\hat{\rho}}$ for any $\hat{\rho}$ such that $\langle\boldsymbol{n}|\hat{\rho}|\boldsymbol{n}\rangle \neq 0$.

Before proceeding with the proof, some comments are at order. We assume that the average momentum $\bar{p}_i$ is conserved for all states because, if this is not true for a certain $\hat{\rho}$, then a noninterferometric experiment is immediately available, namely to measure the change of $\bar{p}_i$ induced by $\boldsymbol{\Phi}$ on the $\hat{\rho}$. Most collapse models in the literature conserve the average momentum. An exception are the dissipative models, as for example the dissipative continuous spontaneous localization model [50]; however, also in this case, for generic states $\Delta p_{i,\hat{\rho}}$ changes due to the collapse, i.e., there is diffusion. This is discussed in detail in the Supplemental Material, Sec. C [33].

Plane waves are the most delocalized states, and a sensible collapse model is expected to collapse them in space. In that case, Δp_i changes for all of them (it is 0 before the collapse, and not 0 after); then the theorem tells us that Δp_i will increase for any $\hat{\rho}$. This means that any sensible collapse model must induce an increase of Δp_i of the system for any state (delocalized or not). When the map acts repeatedly over time, the increase of Δp_i amounts to some form of diffusion.

Our proof is valid for free particles as well as for particles interacting with an external potential. In the second case, in general, the change in Δp_i cannot be easily separated into the contribution coming from the interaction potential and that coming from the collapse. However, all experiments are such that this separation is possible, either because the particle is free or because the effect of the interaction potential can be estimated.

The theorem might seem a manifestation of Heisenberg's uncertainty principle: a collapse in position must increase the spread in momentum; this is true for plane waves, but it is not necessary when the state is not a minimum uncertainty state (the vast majority of them are not). Yet the theorem says that also in that case the spread in momentum increases [33].

Proof.—As discussed in the Supplemental Material [33], the map $\boldsymbol{\Phi}$ is linear, trace preserving, and completely positive. Then Kraus' theorem [51] states that it is of the form

$$\boldsymbol{\Phi}[\hat{\rho}] = \sum_k \hat{A}_k\hat{\rho}\hat{A}_k^\dagger, \tag{1}$$

where the operators $\hat{A}_k$ satisfy the condition $\sum_k \hat{A}_k^\dagger\hat{A}_k = 1$.

The structure of translation covariant maps is characterized by Holevo's theorem [52], whose essence is the following. A map is covariant under a space translation amounting to a displacement $\boldsymbol{x}$ if, for any $\hat{\rho}$,

$$e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}\boldsymbol{\Phi}[\hat{\rho}]e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}} = \boldsymbol{\Phi}[e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}\hat{\rho}e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}]. \tag{2}$$

By multiplying the two sides of this equation on the left by $e^{(i/\hbar)\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}$ and on the right by $e^{-(i/\hbar)\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}$ and using Eq. (1), one finds that

$$\boldsymbol{\Phi}[\hat{\rho}] = \sum_k \hat{A}_k(\boldsymbol{x})\hat{\rho}\hat{A}_k^\dagger(\boldsymbol{x}) \tag{3}$$

with

$$\hat{A}_k(\boldsymbol{x}) = e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}\hat{A}_k e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}. \tag{4}$$

By requiring the covariance in Eq. (2) to hold for any possible displacement $\boldsymbol{x}$ with $x_j \in [-L/2, L/2]$, one eventually finds

$$\boldsymbol{\Phi}[\hat{\rho}] = \frac{1}{L^3}\int_{-\frac{L}{2}}^{+\frac{L}{2}} d\boldsymbol{x} \sum_k A_k(\boldsymbol{x})\hat{\rho}A_k^\dagger(\boldsymbol{x}), \tag{5}$$

with $A_k(\boldsymbol{x})$ given by Eq. (4) and

$$\sum_{\boldsymbol{m}}\sum_k |\langle\boldsymbol{m}|A_k|\boldsymbol{n}\rangle|^2 = 1 \qquad \boldsymbol{m},\boldsymbol{n} \in \mathbb{Z}^3. \tag{6}$$

Equation (5) represents the general space-translation covariant Kraus map inside a box.

Since we are assuming that the average momentum does not change, the change of its spread is given by

$$D_{i,\hat{\rho}} \coloneqq \mathrm{Tr}(\hat{p}_i^2\boldsymbol{\Phi}[\hat{\rho}]) - \mathrm{Tr}(\hat{p}_i^2\hat{\rho}), \tag{7}$$

which, according to Eq. (5), is equal to (see Supplemental Material [33])

$$D_{i,\hat{\rho}} = \sum_{\boldsymbol{m},\boldsymbol{n}} P(\boldsymbol{m},\boldsymbol{n})\tilde{m}_i^2\langle\boldsymbol{n}|\hat{\rho}|\boldsymbol{n}\rangle, \tag{8}$$

where $\tilde{m}_i \coloneqq (2\pi\hbar/L)m_i$ and

$$P(\boldsymbol{m},\boldsymbol{n}) \coloneqq \sum_k |\langle\boldsymbol{m}+\boldsymbol{n}|A_k|\boldsymbol{n}\rangle|^2. \tag{9}$$

The requirement that the map Φ does not lead to diffusion is equivalent to asking that $D_{i,\hat{\rho}} = 0$ for any $\hat{\rho}$. We now prove that a map fulfilling this condition cannot collapse the wave function in space. By assuming that $D_{i,\hat{\rho}} = 0$ for any statistical operator of the form $\hat{\rho} = |\boldsymbol{n}_0\rangle\langle\boldsymbol{n}_0|$, we conclude that

$$\sum_{\boldsymbol{m}} P(\boldsymbol{m},\boldsymbol{n}_0)\tilde{m}_i^2 = 0 \quad \forall\ \boldsymbol{n}_0. \tag{10}$$

At this point, it is convenient to introduce the marginal distributions of $P(\boldsymbol{m},\boldsymbol{n}_0)$ given by

$$P_i(m_i,\boldsymbol{n}_0) \coloneqq \sum_{m_{j\neq i}} P(\boldsymbol{m},\boldsymbol{n}_0), \tag{11}$$

which allows one to rewrite Eq. (10) as

$$\sum_{m_i} P_i(m_i,\boldsymbol{n}_0)m_i^2 = 0 \tag{12}$$

for all $\boldsymbol{n}_0$. Equations (6) and (9) imply that $P(\boldsymbol{m},\boldsymbol{n})$ is a probability distribution for the variables $\boldsymbol{m}$, for any fixed $\boldsymbol{n}$. It follows from Eq. (12) that the marginals $P_i(m_i,\boldsymbol{n}_0)$ are probability distributions with zero variance, which implies

$$P_i(m_i,\boldsymbol{n}_0) = \delta_{m_i,0} \quad \forall\ \boldsymbol{n}_0. \tag{13}$$

Since this is true for all the marginals of $P(\boldsymbol{m},\boldsymbol{n}_0)$, then

$$P(\boldsymbol{m},\boldsymbol{n}) = \sum_k |\langle\boldsymbol{m}+\boldsymbol{n}|A_k|\boldsymbol{n}\rangle|^2 = \delta_{\boldsymbol{m},0} \quad \forall\ \boldsymbol{n}. \tag{14}$$

The equation above implies that

$$|\langle\boldsymbol{m}+\boldsymbol{n}|A_k|\boldsymbol{n}\rangle|^2 = c_k(\boldsymbol{n})\delta_{\boldsymbol{m},0} \quad \forall\ \boldsymbol{n}, \tag{15}$$

with $c_k(\boldsymbol{n})$ generic non-negative functions such that $\sum_k c_k(\boldsymbol{n}) = 1$. By writing the matrix element in the form $\langle\boldsymbol{m}+\boldsymbol{n}|\hat{A}_k|\boldsymbol{n}\rangle = R_k(\boldsymbol{m},\boldsymbol{n})e^{i\varphi_k(\boldsymbol{m},\boldsymbol{n})}$ one finds that $R_k(\boldsymbol{m},\boldsymbol{n}) = \sqrt{c_k(\boldsymbol{n})}\delta_{\boldsymbol{m},0}$ and therefore

$$\begin{aligned}\hat{A}_k &= \sum_{\boldsymbol{m},\boldsymbol{n}} |\boldsymbol{m}+\boldsymbol{n}\rangle\langle\boldsymbol{m}+\boldsymbol{n}|\hat{A}_k|\boldsymbol{n}\rangle\langle\boldsymbol{n}| \\ &= \sum_{\boldsymbol{n}} \sqrt{c_k(\boldsymbol{n})}\, e^{i\varphi_k(0,\boldsymbol{n})}|\boldsymbol{n}\rangle\langle\boldsymbol{n}| = \hat{A}_k(\hat{\boldsymbol{p}}),\end{aligned} \tag{16}$$

where the last equality signifies that the operators $\hat{A}_k$ are functions of the momentum operator only. As such, the map $\boldsymbol{\Phi}$ becomes

$$\boldsymbol{\Phi}[\hat{\rho}] = \sum_k \hat{A}_k(\hat{\boldsymbol{p}})\hat{\rho}\hat{A}_k^\dagger(\hat{\boldsymbol{p}}). \tag{17}$$

Typical examples of maps of this kind contain only one Kraus operator and are the free evolution $[\hat{A}(\hat{\boldsymbol{p}}) \sim \exp[-i\hat{p}^2/2m]]$ and spatial translations $[\hat{A}(\hat{\boldsymbol{p}}) \sim \exp[i\hat{\boldsymbol{p}}\cdot\boldsymbol{a}]]$, which do not modify the spread in momentum.

It is trivial to check that a map like Eq. (17) does not change the momentum distribution, and that plane waves $\hat{\rho} = |\boldsymbol{n}_0\rangle\langle\boldsymbol{n}_0|$ are stationary states, which implies that the map $\boldsymbol{\Phi}$ is not capable of collapsing such fully delocalized states in position.

Coming to the second part of the theorem, let us assume that the map changes the spread in momentum of a given momentum eigenstate $|\boldsymbol{n_0}\rangle$; this implies that

$$\sum_{\boldsymbol{m}} P(\boldsymbol{m},\boldsymbol{n}_0)\tilde{m}_i^2 > 0. \tag{18}$$

Then from Eq. (8) it is clear that for all states $\hat{\rho}$ such that $\langle \boldsymbol{n_0}|\hat{\rho}|\boldsymbol{n}_0\rangle \neq 0$ the spread in momentum also increases under the action of the map. This completes the proof.

The proof presented here is applied to a single quantum operation, but clearly holds for a sequence of them. A proof of the theorem for Lindblad's dynamics is given in the Supplemental Material [33].

Discussion.—The no-faster-than-light signaling requirement, which implies that the dynamical map for the density matrix is uniquely identified and is linear, sets a very strong constraint on the possible collapse dynamics. In particular, it tells us that the dynamics must be such that, at the *statistical level* (i.e., $\boldsymbol{\Phi}$), it acts on the density matrix in the same way, whether the underlying mixture is made of delocalized states (which should collapse) or of localized states (which are not expected to further collapse). This excludes the possibility of having a collapse dynamics that takes place only when the system is in a superposition and is suspended when the system is not; in this second case the effect might be small or null for some specific states, but the dynamics as such is there.

Since the time when a superposition is created is not specified *a priori*, and since $\boldsymbol{\Phi}$ is "blind" to the states forming a statistical mixture, the dynamics must act repeatedly in time with a sufficiently high rate to make sure that superpositions do not live too long (in the Markovian limit, one typically has a Lindblad dynamics).

Note that such dynamics implies a time directionality (see Ref. [53] for further discussion). At the statistical level this is clear since pure states evolve into statistical mixtures. At the wave function level, time directionality arises because spatial superpositions collapse to localized states, while the opposite does not occur. The collapse occurring in position implies energy nonconservation.

Space-translation covariance enters as follows. Consider (in one dimension) a partition of the real line into small enough intervals and let $\hat{A}_k$ be the projection operators associated to the intervals. The associated Kraus map $\boldsymbol{\Phi}$ in Eq. (1) is not covariant under space translations. When applied to a generic superposition, it collapses it and changes the spread in momentum, while preserving its average; when the map is applied any other time, the state does not change anymore (here we are neglecting the Hamiltonian dynamics). In this case diffusion does not occur, apart from the change in momentum in the very first instance. Space-translation covariance requires that there are no privileged points in space, so no privileged partition of space; therefore no state, however localized (except for the pathological—and unstable under the free evolution—case of a Dirac delta), can remain unaltered by a repeated application of the map because there is always the chance that it is further localized by the action of an operator $\hat{A}_k$ associated to an interval that does not entirely contain the state when the map acts. This is the source of diffusion.

We implicitly framed our theorem in a nonrelativistic setting, but we do not see any fundamental obstacle in extending it to a relativistic scenario.

Conclusions.—Modifying the quantum dynamics provided by the Schrödinger equation is tricky and easily generates nonphysical situations. One of the most common problems is superluminal signaling with arbitrarily high speed; collapse models are designed to avoid this problem. Our theorem shows why the no-signaling constraint, together with space-translation covariance, requires that collapse in position always comes with diffusion. For this reason, noninterferometric experiments are equally good as interferometric ones for testing these models; as anticipated in the introduction, the first type of tests are easier to perform and have already set significantly stronger bounds on the collapse parameters, ruling out some of them.

The same logic applies to any open-quantum-system dynamics: typical environments induce decoherence in position [54] and the resulting dynamics is space-translational covariant because most interactions depend on the relative distances among particles—all fundamental ones do; as such they must also generate diffusion. This is reflected by the Lindblad structure of the most common master equations: when the Lindblad operators depend on position, the expectation value of $\hat{p}^2$ is not constant.

This fact has consequences, for example, regarding recent proposals for searching for dark matter signals using matter-wave interferometers [56–58], which are sensitive to the decoherence induced by dark matter particles. Equally well, one can propose noninterferometric experiments [57], which are sensitive to the diffusion generated by the particles; the application of noninterferometric techniques to collapse models proved that they hold the potential for providing stronger results.

The authors thank S. L. Adler, L. Diósi, A. Smirne, H. Ulbricht, and B. Vacchini for several useful comments. S. D. acknowledges the financial support from the Marie Sklodowska Curie Action through the UK Horizon Europe guarantee administered by UKRI and from INFN. L. F. acknowledges support from MUR through project PRIN (Project No. 2017SRN-BRK QUSHIP). A. B. acknowledges financial support from the University of Trieste, INFN, the EIC Pathfinder project QuCoM (GA No. 101046973), and the PNRR MUR Project No. PE0000023-NQSTI.

*s.donadi@qub.ac.uk
†luca.ferialdi@gmail.com
‡abassi@units.it

[1] A. J. Leggett, Prog. Theor. Phys. Suppl. **69**, 80 (1980).
[2] A. J. Leggett, Science **307**, 871 (2005).
[3] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy* (Cambridge University Press, Cambridge, England, 2004).

[4] S. Weinberg, Phys. Rev. A **85**, 062116 (2012).

[5] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[6] G. C. Ghirardi, R. Grassi, and A. Rimini, Phys. Rev. A **42**, 1057 (1990).

[7] A. Bassi and G. Ghirardi, Phys. Rep. **379**, 257 (2003).

[8] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. **85**, 471 (2013).

[9] M. Kasevich and S. Chu, Phys. Rev. Lett. **67**, 181 (1991).

[10] T. Kovachy, P. Asenbaum, C. Overstreet, C. Donnelly, S. Dickerson, A. Sugarbaker, J. Hogan, and M. Kasevich, Nature (London) **528**, 530 (2015).

[11] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. Van der Zouw, and A. Zeilinger, Nature (London) **401**, 680 (1999).

[12] Y. Y. Fein, P. Geyer, P. Zwick, F. Kiałka, S. Pedalino, M. Mayor, S. Gerlich, and M. Arndt, Nat. Phys. **15**, 1242 (2019).

[13] J. Bateman, S. Nimmrichter, K. Hornberger, and H. Ulbricht, Nat. Commun. **5**, 1 (2014).

[14] S. Belli, R. Bonsignori, G. D'Auria, L. Fant, M. Martini, S. Peirone, S. Donadi, and A. Bassi, Phys. Rev. A **94**, 012108 (2016).

[15] M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, and S. Bose, Phys. Rev. Lett. **111**, 180403 (2013).

[16] H. Pino, J. Prat-Camps, K. Sinha, B. P. Venkatesh, and O. Romero-Isart, Quantum Sci. Technol. **3**, 025001 (2018).

[17] G. Gasbarri, A. Belenchia, M. Carlesso, S. Donadi, A. Bassi, R. Kaltenbaek, M. Paternostro, and H. Ulbricht, Commun. Phys. **4**, 155 (2021).

[18] A. Belenchia, M. Carlesso, S. Donadi, G. Gasbarri, H. Ulbricht, A. Bassi, and M. Paternostro, Nature (London) **596**, 32 (2021).

[19] M. Carlesso, S. Donadi, L. Ferialdi, M. Paternostro, H. Ulbricht, and A. Bassi, Nat. Phys. **18**, 243 (2022).

[20] M. Bilardello, S. Donadi, A. Vinante, and A. Bassi, Physica (Amsterdam) **462A**, 764 (2016).

[21] A. Vinante, M. Bahrami, A. Bassi, O. Usenko, G. Wijts, and T. H. Oosterkamp, Phys. Rev. Lett. **116**, 090402 (2016).

[22] A. Vinante, M. Carlesso, A. Bassi, A. Chiasera, S. Varas, P. Falferi, B. Margesin, R. Mezzena, and H. Ulbricht, Phys. Rev. Lett. **125**, 100404 (2020).

[23] M. Carlesso, A. Bassi, P. Falferi, and A. Vinante, Phys. Rev. D **94**, 124036 (2016).

[24] B. Helou, B. J. J. Slagmolen, D. E. McClelland, and Y. Chen, Phys. Rev. D **95**, 084054 (2017).

[25] Q. Fu, Phys. Rev. A **56**, 1806 (1997).

[26] S. L. Adler and F. M. Ramazanoğlu, J. Phys. A **40**, 13395 (2007); **42**, 109801 (2009).

[27] S. L. Adler, A. Bassi, and S. Donadi, J. Phys. A **46**, 245304 (2013).

[28] S. Donadi, D.-A. Deckert, and A. Bassi, Ann. Phys. (Amsterdam) **340**, 70 (2014).

[29] The best bound from interferometric experiments comes from macromolecule interferometry and for the reference value $r_c = 10^{-7}$ m it is $\lambda \leq 10^{-7}$ s^{-1} [see Fig. 3(b) of [12]]. This should be compared to the bound coming from x-rays emission from germanium, which is $\lambda \leq 5.2 \times 10^{-13}$ s^{-1} (see Fig. 4 of [30]).

[30] S. Donadi, K. Piscicchia, R. Del Grande, C. Curceanu, M. Laubenstein, and A. Bassi, Eur. Phys. J. C **81**, 773 (2021).

[31] S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, Nat. Phys. **17**, 74 (2021).

[32] N. Gisin, Helv. Phys. Acta **62**, 363 (1989).

[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.130.230202 for details, which includes References [34–48], See also the final comment in Sec. of the Supplemental Material.

[34] A. Bassi and K. Hejazi, Eur. J. Phys. **36**, 055027 (2015).

[35] M. Caiaffa, A. Smirne, and A. Bassi, Phys. Rev. A **95**, 062101 (2017).

[36] A. Bassi, E. Ippoliti, and B. Vacchini, J. Phys. A **38**, 8017 (2005).

[37] L. Ferialdi and A. Bassi, Phys. Rev. A **86**, 022108 (2012).

[38] L. Ferialdi and A. Bassi, Phys. Rev. Lett. **108**, 170404 (2012).

[39] A. Smirne, B. Vacchini, and A. Bassi, Phys. Rev. A **90**, 062135 (2014).

[40] J. Nobakht, M. Carlesso, S. Donadi, M. Paternostro, and A. Bassi, Phys. Rev. A **98**, 042109 (2018).

[41] A. Bassi, J. Phys. A **38**, 3173 (2005).

[42] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) **17**, 821 (1976).

[43] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

[44] A. S. Holevo, Rep. Math. Phys. **32**, 211 (1993).

[45] A. S. Holevo, Rep. Math. Phys. **33**, 95 (1993).

[46] B. Vacchini and F. Petruccione, Eur. Phys. J. Special Topics **159**, 135 (2008).

[47] B. Vacchini, J. Phys. A **40**, 2463 (2007).

[48] B. Vacchini and K. Hornberger, Phys. Rep. **478**, 71 (2009).

[49] We assume $\hat{\rho}$ belongs to the domain of $\hat{p}_i$ and $\hat{p}_i^2$, and such that its image under $\mathbf{\Phi}$ belongs to the same domain.

[50] A. Smirne and A. Bassi, Sci. Rep. **5**, 12518 (2015).

[51] K. Kraus, A. Böhm, J. D. Dollard, and W. Wootters, *States, Effects, and Operations Fundamental Notions of Quantum Theory: Lectures in Mathematical Physics at the University of Texas at Austin* (Springer, New York, 1983).

[52] A. Holevo, J. Math. Phys. (N.Y.) **39**, 1373 (1998).

[53] D. Z. Albert and H. C. von Baeyer, Am. J. Phys. **69**, 1285 (2001).

[54] See Ref. [55] for the difference between collapse models and decoherence in resolving the quantum measurement problem.

[55] S. L. Adler, Stud. Hist. Phil. Sci. Part B **34**, 135 (2003).

[56] C. J. Riedel, Phys. Rev. D **88**, 116005 (2013).

[57] J. Bateman, I. McHardy, A. Merle, T. R. Morris, and H. Ulbricht, Sci. Rep. **5**, 8058 (2015).

[58] I. Alonso, C. Alpigiani, B. Altschul, H. Araujo, G. Arduini, J. Arlt, L. Badurina, A. Balaz, S. Bandarupally, B. C. Barone *et al.*, Eur. Phys. J. Quantum Technol. **9**, 30 (2022).