

# Determination of $|V_{cb}|$ using $B \rightarrow D\ell\nu_\ell$ Decays at Belle II

The Belle II Collaboration

## SUPPLEMENTAL MATERIAL

### Validation of kinematic variables

To validate the modelling of key variables of the signal component, we reconstruct control samples that resemble certain features of  $B \rightarrow D\ell\nu_\ell$  signal decays. In particular, we partially reconstruct  $B \rightarrow K^*\ell$  within a sample of fully reconstructed  $B \rightarrow K^*(\rightarrow K\pi)J/\psi(\rightarrow \ell\ell)$  decays. The unreconstructed second lepton introduces missing energy and momentum, allowing validation of the ROE and the kinematic variable reconstruction.

We study distributions for electron and muon modes separately. We show distributions of the lepton helicity angle  $\cos\theta_{\ell,W}$ , the fit variable  $\cos\theta_{BY}$ , the angle between the  $D$  meson and the lepton  $\theta_{D,\ell}$ , the invariant mass of the ROE  $m_{\text{ROE}}$ , the ROE momentum  $p_{\text{ROE}}$ , and the CMS frame energy of the reconstructed  $Y = D\ell$  system. The figures are shown for electrons and muons in Fig. 1 and Fig. 2 respectively.

We observe good agreement between simulation and data, verifying our construction of the ROE and the signal efficiency.

### Bin-wise extracted values of $|V_{cb}|$

As a consistency check, we extract the value of  $V_{cb}$  of each bin in every mode individually, by comparing the measured differential decay rate to the predicted one. The breakup is shown in Fig. 3.

### Fitted form factor

In Fig. 4, we show the form factors  $f_+(w)$  and  $f_0(w)$  obtained from the fit to our experimental  $B \rightarrow D\ell\nu$  data. Lattice QCD results from FNAL/MILC 15C [1] and HPQCD 15 [2] are overlaid for comparison. The FNAL/MILC values include synthetic points at three  $w$  values with full correlations, while the synthetic points for HPQCD are generated by FLAG [3] from the provided parameterization. The fit is performed using a BCL parameterization with  $N_+ = N_0 = 3$  parameters. We find consistent agreement between form factor fitted to the measured decay rates and the lattice inputs.

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- [1] J. A. Bailey *et al.* (MILC), Phys. Rev. D **92**, 034506 (2015), arXiv:1503.07237 [hep-lat].  
[2] H. Na, C. M. Bouchard, G. P. Lepage, C. Monahan, and J. Shigemitsu (HPQCD), Phys. Rev. D **92**, 054510 (2015), [Erratum: Phys. Rev. D **93**, 119906 (2016)], arXiv:1505.03925 [hep-lat].  
[3] Y. Aoki *et al.* (Flavour Lattice Averaging Group (FLAG)), (2024), arXiv:2411.04268 [hep-lat].

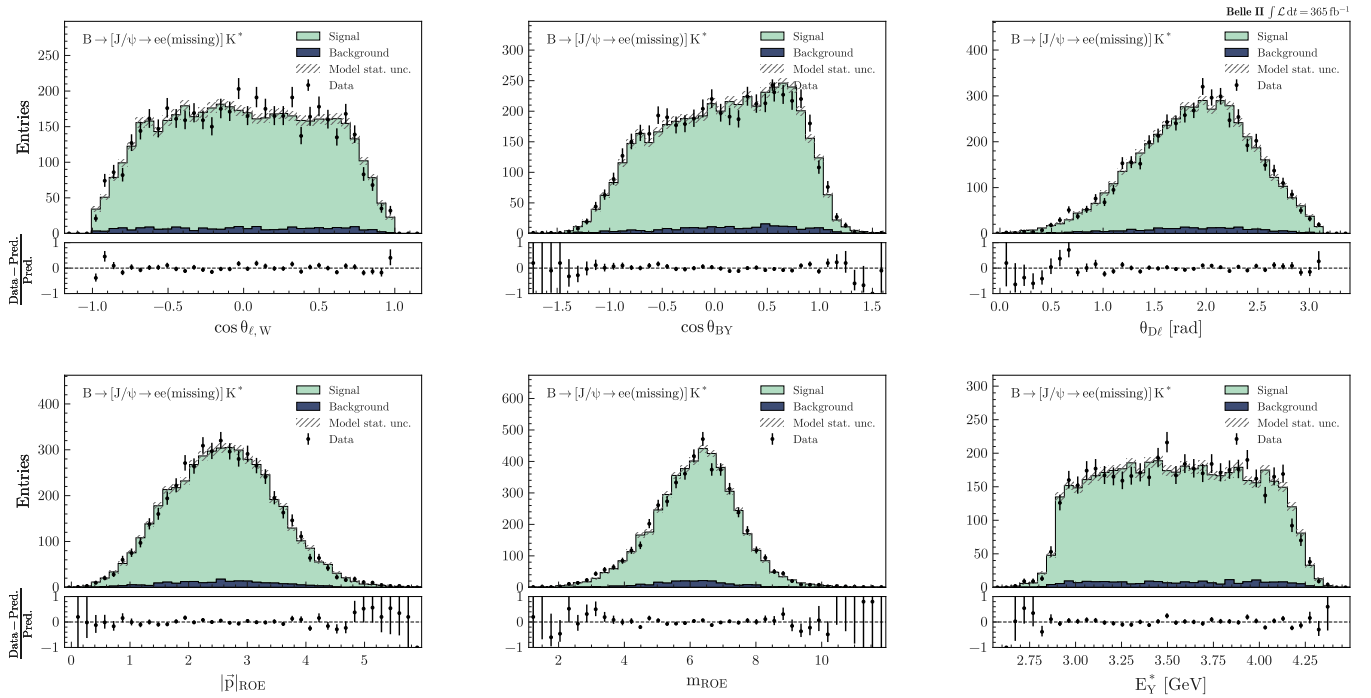


FIG. 1. Comparison of data (points) and simulation (histograms) for key kinematic variables in the electron mode of the  $B \rightarrow J/\psi K^*$  control sample.

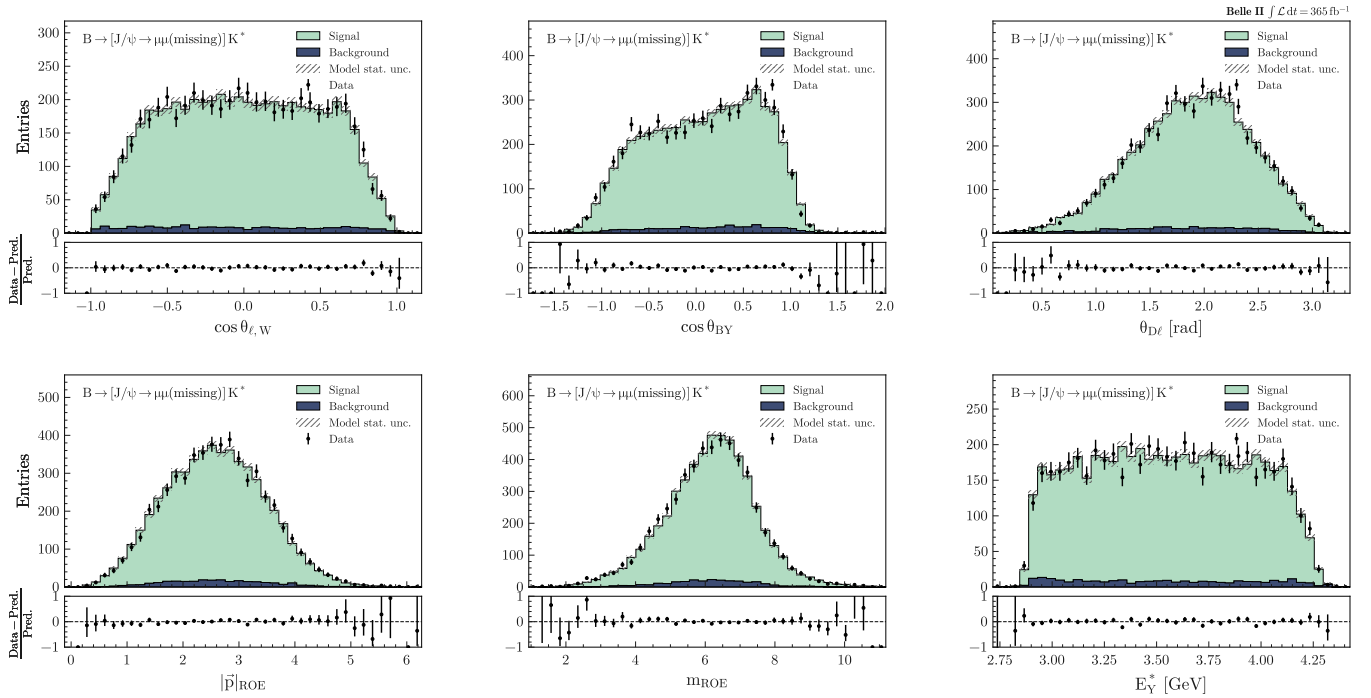


FIG. 2. Comparison of data (points) and simulation (histograms) for key kinematic variables in the muon mode of the  $B \rightarrow J/\psi K^*$  control sample.

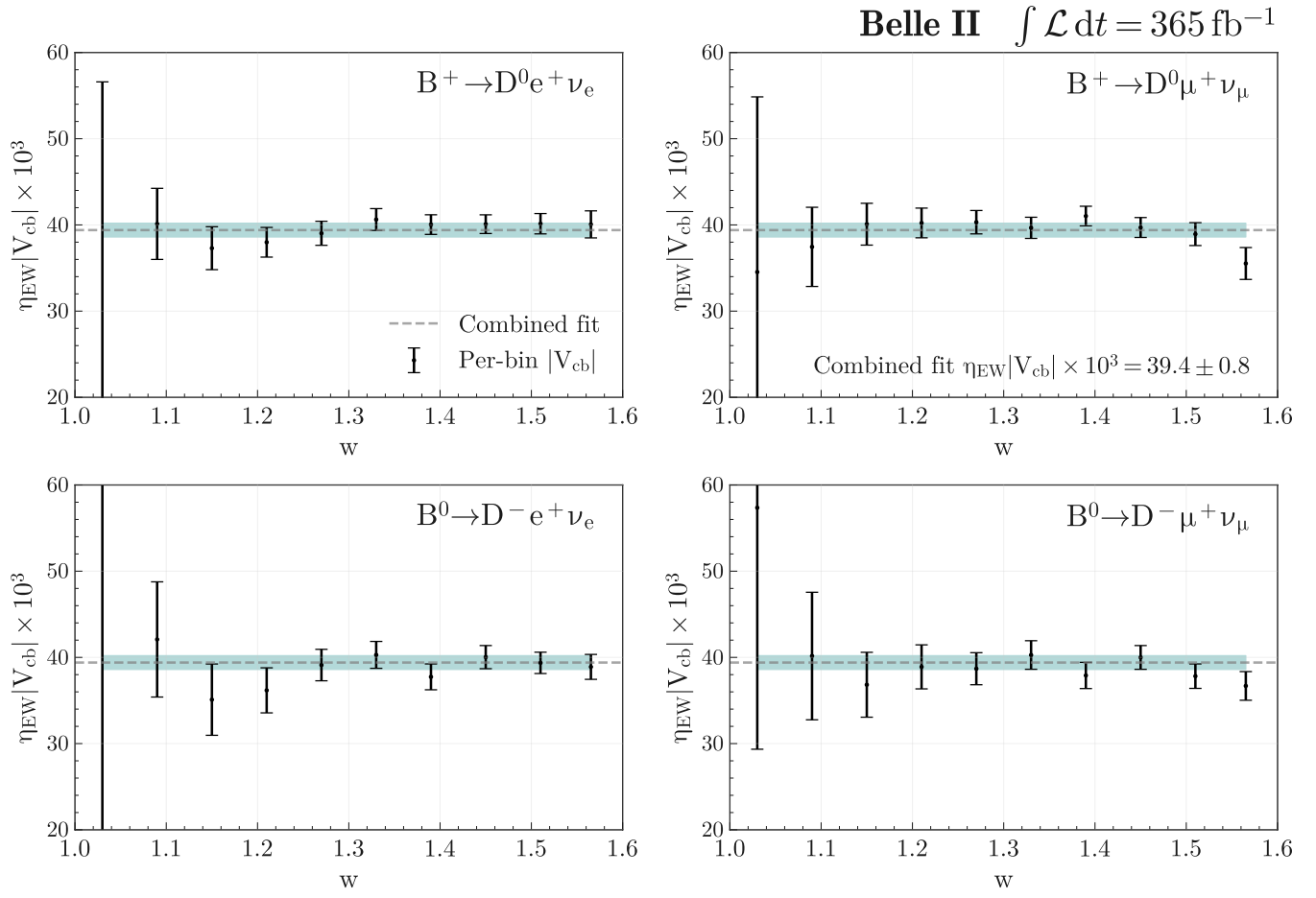


FIG. 3. Determination of  $\eta_{EW} \times |V_{cb}|$  using a single bin of the measured differential decay rates as a function of  $w$ , from the measured differential decay rates in each decay mode. The total uncertainties, shown by the error bars, are obtained as the quadratic sum of the statistical and systematic components. The results in individual  $w$  bins are compared to the combined fit result and uncertainty.

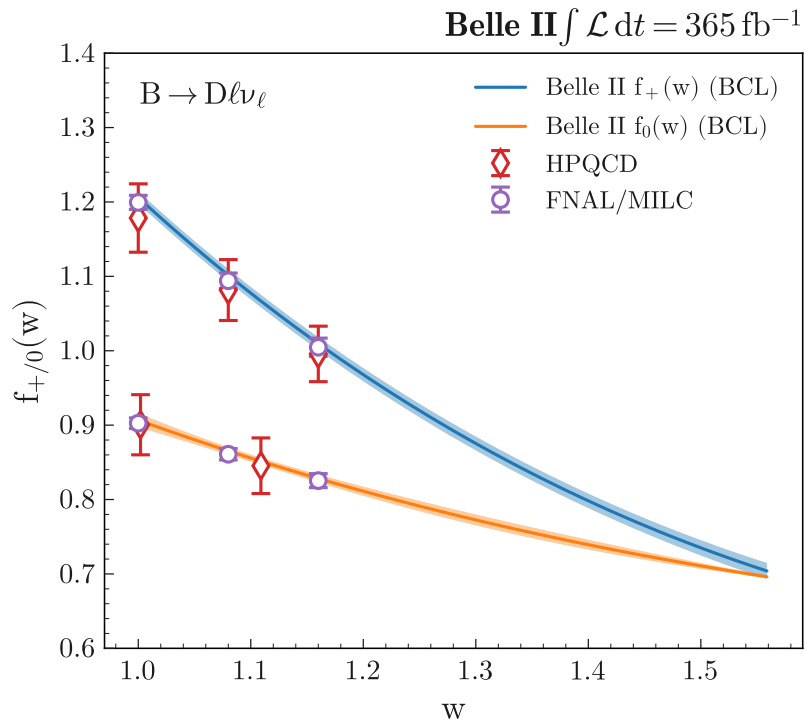


FIG. 4. Form factors  $f_+(w)$  and  $f_0(w)$  for  $B \rightarrow D \ell \nu$  resulting from the fit to our experimental data, with lattice QCD input overlaid. The FNAL/MILC [1] and HPQCD [2] results are shown for comparison. The BCL fit is performed using  $N_+ = N_0 = 3$  parameters. The band corresponds to the  $1\sigma$  uncertainty.