Metroeconomica

WILEY

METROECONOMICA
INTERNATIONAL REVIEW OF ECONOMICS

ORIGINAL ARTICLE OPEN ACCESS

Public Debt Dynamics in a Monetary Economy of Production

Lorenzo Di Domenico

University of Trieste, Trieste, Italy

Correspondence: Lorenzo Di Domenico (lorenzo.didomenico@deams.units.it)

Received: 20 September 2024 | **Revised:** 21 October 2025 | **Accepted:** 24 October 2025

Keywords: monetary economy of production | public debt-to-GDP ratio | stock-flow consistent models

ABSTRACT

This paper investigates the determinants and stability conditions of the public debt-to-GDP ratio within a theoretical framework representing the main characteristics of a monetary economy of production. To this end, we develop a dynamic Stock-Flow Consistent (SFC) model based on the Supermultiplier approach, incorporating both bank and fiat money, capital accumulation and endogenous public debt service. Steady-state values are derived, and stability is assessed through both analytical and simulation-based approaches. Our main findings show that the public debt-to-GDP ratio is positively influenced by the saving rate and negatively influenced by the growth rate of autonomous demand components and the capital intensity of the economy. The effects of the interest rate and tax rate are found to be non-linear, depending on the growth regime emerging in the economy. Under the "standard-regime", the tax rate has a negative impact, while the impact of the policy rate is positive. Given the exogenous parameters, and under the stability conditions, there exists a long-run level of public debt-to-GDP ratio toward which the economy converges. These results challenge the rationale for applying blanket regulations on public budgets, disregarding the distinct traits of each economic system.

JEL Classification: E12, E17, E42, E43, E52, E62

1 | Introduction

The public debt-to-GDP ratio is commonly used to assess the sustainability of government borrowing. Understanding the determinants and stability conditions of this ratio is central to macroeconomic policy, as it shapes both short-term fiscal strategies and long-term economic trajectories. In the existing literature, two major schools of thought—Post-Keynesian and Neoclassical economics—offer differing approaches to analyzing this ratio and its implications for economic stability.

From a Neoclassical perspective, public debt is often seen as a potential drag on long-term growth. High debt levels can crowd out private investments, increase interest rates, and impose future tax burdens, ultimately leading to lower economic efficiency (Gale and Orszag 2003; Baldacci and Kumar 2010; R. Barro 1979; R. J. Barro 1995; Dotsey 1994; Sargent and Wallace 1981; Cochrane 2011; Burnside et al. 2001; Hemming and Ter-Minassian 2003). In this regard, a large strand of literature claims the existence of a threshold above which the public debt-to-GDP ratio is detrimental to economic growth (Reinhart and Rogoff 2010a, 2010b; Abbas and Christensen 2010; Cecchetti et al. 2011; Minea and Parent 2012).

In this approach, savings have a purely real nature and are a direct result of production. For this reason, debt relationships can only be established ex-post, after the generation of savings, as depicted in the Loanable Funds theory (Bertocco and Kalajzić 2022). In this context, the financing of public debt and public spending necessarily represents a subtraction of real resources from the (private) capital market, thereby depressing production and implying that the (re)financing of public debt could be problematic, depending on the ex ante formation of household real savings and/or their willingness to purchase it.

These causal relationships clearly emerge from the Walrasian framework, where input-output exchanges are intended as simultaneously carried out in real terms, and the resolution of a simultaneous system of optimal conditions ideally depicts a world where the operation of the auctioneer makes all agents' plans mutually consistent (how much they will exchange with firms) before the production process starts (Gaffeo et al. 2007). The real process of exchange ideally follows in the meta-time the matching of demand and supply curves (Gallegati and Kirman 2012). Firms act as an intermediate box, receiving the real inputs from the representative agent (capital and labor) and, according to the marginal productivities, giving back the homogeneous output in real terms. A share of this is consumed, and the remaining part is saved and available for future investments. Accordingly, the natural role of banks could be exclusively that of intermediaries between the real savings of agents,[1] while the interest rate is the adjusting variable equalizing savings and investments. Money is not strictly required to perform the process of production and exchange; it can be attached secondarily through a money market that works as the market of real goods (as in the IS-LM model). As exposed by F. Hahn (1973), F. H. Hahn (1989), models based on Walrasian, Arrow-Debreu or time-0 auction have no role for money, credit, liquidity, central, and commercial banks.[2] Economic phenomena such as savings can be defined independently of money. Because in real economies, credit comes in monetary terms, monetary values can be obtained simply by multiplying the volume of real variables by a monetary price. In this sense, most of the neoclassical conclusions about the nature of public debt and its influence on GDP stem from this a-monetary conception of the economy or, as defined by Keynes (1933), from a market system understood as a *contractual barter economy* (Rogers 2018). In detail, this type of analytical framework gives rise to specific causal relationships, such as debts and loans that can only emerge as a consequence of a prior generation of (real) savings, or investments that are determined ex ante by savings. Since the financing of public spending implies a reduction in savings, an expansionary fiscal policy would depress capital accumulation. As a consequence, lower accumulation of savings would undermine the sustainability of public debt by reducing the very possibility of future debt refinancing and, due to lower economic growth, would result in a higher debt-to-GDP ratio.

Accordingly, the sustainability of the public debt-to-GDP ratio depends largely on the government's ability to maintain fiscal discipline, ensuring that public debt grows at a rate lower than or equal to economic growth over time. As such, Neoclassical models typically emphasize balanced budgets and low public debt levels. A common recommendation to ensure sustained long-run growth is to stabilize or reduce the public debt-to-GDP ratio by imposing public budget constraints. Stability is granted if a primary balance budget is imposed and the output growth rate equals the interest rate (Diamond 1965; Giavazzi and Pagano 1990; Aspromourgos et al. 2010; Barrett 2018; Blanchard 2019; Mehrotra and Sergeyev 2021). This result derives from simple arithmetic of the public debt-to-GDP ratio: $d_t = \frac{1+r_t}{1+g_t} d_{t-1} + x_t$, where x_t is the primary deficit-to-GDP ratio, d_t is the public debt-to-GDP ratio, r_t is the interest rate on public bonds and g_t is the nominal GDP growth rate. Following this arithmetic, by imposing d_t equal to d_{t-1}, the condition that stabilizes the debt-to-GDP ratio can be derived: $x = \left(\frac{g-r}{1+g}\right) d \approx (g-r)d$. If $x_t = 0$, the growth rate of public debt is equal to the interest rate and the debt-to-GDP ratio has an explosive trend if $r > g$. Thereby, the common policy recommendation to stabilize or reduce d^* is that imposing a primary balance budget constraint, the growth rate of the economy has to be equal to or higher than the interest rate (see Aspromourgos et al. 2010 for a review of the literature on the subject).

In general, this interpretation of the arithmetic of the public debt-to-GDP ratio derives from the hypothesis of an absence of interdependence between primary deficit (thus public spending), debt service, and GDP growth. This is conceivable in a framework where output and savings are independent of public spending and deficit and where, under flexible prices and wages, the market reaches the full-employment output.

This perspective stands in stark contrast to approaches that conceptualize the economy as a monetary economy of production (Keynes 1933; Dillard 1980; Lavoie 1984; F. H. Hahn 1989; Wray 1999; Graziani 2003; Lucarelli and Passarella 2012; Parguez and Seccareccia 2000). In such frameworks, savings are understood in monetary terms and emerge *ex post* as a consequence of credit or debt creation (Godley and Lavoie 2007a; Di Domenico et al. 2024). As Fontana et al. (2020) point out, financial or monetary assets—unlike real ones—always have a corresponding liability recorded on the balance sheet of another agent or sector.

Contrary to the neoclassical position, (monetary) savings are not the direct result of productive activity but originate from the lending activities of commercial and central banks. The lending process is not based on scarce or pre-existing resources; banks create liquidity ex nihilo and inject money into the economy by financing production.

In this context, money cannot be analyzed through traditional supply and demand functions, in a separate stage with respect to the production side. Money is endogenous (Kaldor 1970; Lavoie 1984; Rochon 1999), it enters into circulation when investments and autonomous components of demand are financed. The determination of real quantities goes hand in hand with the dynamics of money circulation. The exogenous injections of purchasing power into the system (autonomous component of demand and investments) trigger the dynamic of demand and production over time, while savings decisions symmetrically determine the net debt of those sectors that have realized such spending. It is the initial finance of investments and autonomous components through endogenous money that allows them to be ex-post founded by household savings (Cesaratto 2017; Di Domenico et al. 2024). Aggregate savings equal aggregate debt so much so that the former could not exist without the latter.

In summary, while investments or public spending in a contractual-barter economy require prior savings, in a monetary economy, savings depend on prior investment decisions or debt-financed injections of purchasing power. Among other things, the nature of these mechanisms is of vital importance in order to affirm or deny the existence of crowding-out or crowding-in effects between private investment and public spending.

Within this approach, the stability of the public debt-to-GDP ratio does not depend solely on balancing the budget, but rather on a complex interaction between fiscal policy, output growth, and public debt dynamics (Godley and Lavoie 2007a; Ryoo and Skott 2013; Hein 2018). Several works based on the notion of "functional finance" (Lerner 1943) have focused on the role of fiscal policies and Government deficit in stabilizing the economy and stock-flow ratios (Arestis and Sawyer 2003; Fontana 2009; Hein and Stockhammer 2010). Within this literature, the Stock-Flow Consistent approach (SFC) (Godley and Lavoie 2007a) has been demonstrated to be particularly fruitful in depicting the macroeconomics accounting regarding money creation, circulation, and destruction, and highlighting the liability/asset relations across sectors forming the macroeconomy (see Caverzasi and Godin 2015; Nikiforos and Zezza 2017 for a revision).

Although the power of the SFC approach in explaining the nature of debt and savings and the determination of stock and flow ratios, the factors influencing the long-run behavior of the public debt-to-GDP ratio have not been investigated in much detail. Some attempts to study the stability conditions of the public debt-to-GDP ratio have been carried out by Godley and Lavoie (2007b), Dos Santos and Zezza (2008), Ryoo and Skott (2013), Hein (2018), and Morlin (2022). However, a theoretical depiction, explicitly based on a dynamic SFC model, of the determinants and stability conditions of the public debt-to-GDP ratio is still missing. To our knowledge, the closest attempts using SFC models have been pursued in Godley and Lavoie (2007a) and Dos Santos and Zezza (2008). In the first contribution, the authors derive the steady-state values of the stock-flow ratios within an SFC model based on the income-expenditure scheme. However, their model does not consider investments, capital accumulation, bank money, public debt service, and interest rates on the stock of savings and debts. In the second contribution, the authors develop an SFC post-Keynesian growth model that integrates capital accumulation through a neo-Kaleckian investment function, studying the long-run stability conditions. However, the model by Dos Santos and Zezza imposes steady-state conditions rather than deriving them from the dynamic system. As a result, the system's stability properties, as well as the values of steady-state stock/flow ratios, are not explicitly analyzed in relation to exogenous parameters (such as the tax rate, propensity to consume, policy rate, etc.), but rather in terms of endogenous variables such as the deficit ratio.

This paper aims to contribute to this literature by conducting a dynamic stability analysis and deriving the analytical steady-state values of public debt- and private wealth-to-GDP ratios as a function of the exogenous parameters. This makes it possible to assess how the system would respond to parametric shocks.

To this end, we perform this kind of analysis by developing an SFC model based on the Supermultiplier approach (Serrano 1995; Freitas and Serrano 2015; Cesaratto et al. 2003; Girardi and Pariboni 2016). To this extent, this contribution can be understood as an extension of the income-expenditure model proposed by Godley and Lavoie (2007a, 2007b), also including capital accumulation, bank money, interest rates, and endogenous public debt service.

We follow two lines of analysis: first, identifying the key determinants of long-term public debt-to-GDP ratio values and their specific correlations; second, demonstrating that, given the exogenous pattern of primary public spending, the stability condition is fulfilled through an endogenous adjustment of the primary deficit/surplus such that $x = \left(\frac{g-r}{1+g}\right)d$. This implies that the condition of $r > g$ jointly with a primary balance budget does not have per se any particular relevance for the stability of the public debt-to-GDP ratio in the long term.

Our main findings can be summarized as follows: (i) the public debt-to-GDP ratio is determined by the saving rate, the growth rate of primary public spending, the tax rate, the capital intensity of the production process and the interest rate, while two different regimes emerge. Within the standard regime, the growth rate of the public debt service converges to the growth rate of the primary public spending. Within the interest-led regime, the growth rate of the public debt service is higher than the growth rate of primary public spending. Under the "standard regime", the public debt-to-GDP ratio is positively affected by the saving rate and the interest rate, and negatively affected by the tax rate, the capital intensity, and the growth rate of the autonomous component. Under the interest-led regime, the relationship with the tax rate and the interest rate is reversed. (ii) Given an exogenous path of primary public spending and the absence of budget constraints inconsistent with stock-flow norms, the condition for the stability of the public debt-to-GDP ratio in the standard regime is a positive growth rate of primary public spending or a positive propensity to consume out of wealth. Within the interest regime, the condition for stabilization is the absence of full hoarding of the income generated by interest on government bonds, or a non-zero propensity to consume out of wealth. (iii) The introduction of fiscal rules that are not compatible with "normal" values may, through cuts in primary public spending, have a counterproductive effect on the public debt-to-GDP ratio, resulting in a shift of the system's state from a standard to an interest-led regime.

The rest of the paper is organized as follows. Section 2 presents the model and its derivations, discussing the results regarding the stationary levels of the public debt-to-GDP ratio and the stability conditions. Section 3 discusses the impact of different fiscal rules. Section 4 concludes.

2 | Determinants and Stability of the Public Debt-to-GDP Ratio: A Dynamic SFC Supermultiplier Model

This section presents the SFC-SM developed in the paper. We solve the model analytically to study the stability conditions of the public debt-to-GDP ratio and its determinants. In the first part, we perform our analysis assuming no fiscal rules and considering that, given an exogenous pattern of primary public expenditure, the Government pays the endogenously determined public debt service. Appendix D reports the R code to reproduce model results both through analytical solutions and simulations, and checking their consistency.

The SM model (Freitas and Serrano 2015; Cesaratto et al. 2003; Girardi and Pariboni 2016) is a demand-led growth model characterized by fully induced investments and one (or more) autonomous component of demand. Firms produce based on demand and invest expanding productive capacity to satisfy expected demand at the target (or normal) degree of capacity utilization. In the considered model, there are two sources of money creation and related injection of purchasing power into the system: public spending and firm investments. Money is endogenously created when the public deficit is financed, and bank loans are created (see e.g. McLeay et al. 2014; Werner 2015). The autonomous component of demand and investments represents the exogenous injections of purchasing power, which give rise to the current income circulation. The latter is the result of the income-expenditure (or multiplier) sequences triggered by the injections realized in each period. Since investments are fully endogenous, the long-run GDP is the result of the interaction between multiplier and accelerator triggered by the autonomous components of demand.

The following system of difference equations describes the out-of-equilibrium dynamics of the SFC-SM model. Appendix A reports the balance sheet and transaction matrix of the economy.

$$\begin{cases}
Y_t^p = C_t + G_{t-1}(1+g_G) + I_t & (1)\\
Y_t = C_t + G_{t-1}(1+g_G) + I_t + B_{t-1}^h r^b + M_{t-1} r^m + \Pi_t^B - L_{t-1}(\delta + r^l) & (2)\\
C_t = \left[Y_{t-1} - (B_{t-2}^h r^b + M_{t-2} r^m)\alpha\right](1-\theta)c_1 + S_{t-1}c_2 & (3)\\
I_t = Y_t^e v - K_t(1-\delta) & (4)\\
Y_t^e = Y_{t-1}^p(1+g_t^e) & (5)\\
\Pi^B = L_{t-1}r^l + H_{t-1}r^m - M_{t-1}r^m & (6)\\
\Pi^{CB} = B_{t-1}^{CB} r^b - H_{t-1} r^m & (7)\\
B_t = B_{t-1}(1+r^b) + G_t - \theta Y_t - \Pi^{CB} & (8)\\
S_t = S_{t-1} + Y_{t-1}(1-\theta) - C_t & (9)\\
K_t = K_{t-1}(1-\delta) + I_{t-1} & (10)\\
L_t = L_{t-1}(1-\delta) + I_t & (11)\\
B_t^h = \min(\beta S_t, B_t) & (12)\\
M_t^h = S_t - B_t^h & (13)\\
B_t^{CB} = B_t - B_t^h & (14)\\
H_t = M_t^h - L_t & (15)\\
H_t = B_t^{CB} & (16)
\end{cases}$$

where Y_t is the income including financial yields, Y_t^P is the income generated from production, C_t is the consumption, G_t is the primary public expenditure, I_t that is the investment demand, Π^{CB} and Π^B are CB and commercial bank profits, respectively; B_t is the public debt, S_t is the stock of household savings, K_t is the stock of capital, L_t is the stock of firms' debt, B_t^h and B_t^{CB} are, respectively, the stock of public bonds held by households and CB, M_t is the amount of deposits, H_t is the stock of reserves held by commercial bank at CB, r^b is the interest rate on public bonds, r^m is the interest rate on deposits and reserves, θ is the average tax rate, c_1 is the propensity to consume out-of-income, c_2 is the propensity to consume out-of-wealth, α is the share of income generated by the interests and hoarded by households, δ is the capital depreciation rate and the debt reimbursement rate,[3] v is the normal capital-to-output ratio and β is the share of household savings held in form of public bonds.

Each period corresponds to a production cycle, and incomes are paid ex-post, at the end of the period. Total production is the sum of public spending, households consumption and investments Equation (1). The total gross income of households also include financial yields and equals the income generated by production, net of corporate debt servicing,[4] plus the income from interest on deposits, government bonds, and commercial bank profits Equation (2). Disposable income is distributed at the end of the period and serves as the basis for consumption in the following period. It depends on both income and the stock of wealth. A portion of financial income, denoted by α, may be entirely hoarded by households and does not enter into disposable income available for consumption Equation (3). Investments correspond to the difference between the capital required to satisfy expected demand at the target level of capacity utilization and the residual capital in the case firms would not invest Equation (4). Since we are not including equities, stocks or the possibility for commercial bank to buy public bonds, the commercial bank holds the difference between deposits and loans as reserves at the CB Equation (14). The profits of a commercial bank are equal to the difference between the interest earned on loans and reserves, and the interest paid on deposits Equation (6). The profits of the CB are equal to the difference between the interest earned on public bonds and the interest paid on reserves Equation (7). CB profits are redistributed to the Government. Equations (8) and (9) express the dynamics of public debt and savings over time, respectively. Equations (10) and (11) describe the dynamics of capital stock and firms' debt. Households hold their wealth in the form of deposits and public bonds Equations (12) and (13). CB acts as lender of last resort, purchasing the amount of public bond not acquired by households Equation (14). Finally, Equation (16) is the redundant equation, which serves as a check of the stock-flow consistency, stating that the amount of reserves is equal to the amount of public bonds held by CB.

For the sake of analytical tractability, we are assuming that investments are fully financed by loans, $\alpha = 0$, and $\beta = 1$, namely, the public debt is fully held by households. The last assumption allows the analysis to proceed by considering only one interest rate in the model.[5] Alternatively, the same analytical formulation is produced when considering that the interest rate on public bonds is equal to the interest rate on deposits.

Public expenditure and investments financed out of debt creation set in motion the economic system, they are the exogenous injection of purchasing power realized in each period that triggers the income-expenditure sequence realized over the next periods. that is, if we analyze separately these two components, in the first period, the Government demands a certain amount of goods from the production sector, which satisfies this demand and gets paid by the Government. At the end of the period, the production sector distributes the corresponding income (net of taxation) to households; this amount constitutes the base for consumption in the next period. In the second period, the demand will correspond to public spending plus the consumption originating from the income distributed at the end of the previous period, and so on.[6]

The repeated unfolding of this sequence over time represents the multiplier effect triggered by autonomous spending realized in each period. Each income–expenditure cycle reflects the ongoing interaction — the "ping-pong"—between the production sector and households, through income payments, induced consumption, and firms' revenues, namely the circular flow of income. In this sense, each autonomous spending realized in each period triggers a multiplier sequence that is realized over the following periods.

Then, the income-expenditure sequences triggered by public spending overlap with the income-expenditure sequences triggered by investments. In particular, through the accelerator process, investments have a dual role. On the one hand, investments are a component of aggregate demand and, by generating income, they become a second source of induced consumption. On the other hand, according to the principle of capacity adjustment, investments also react to changes in aggregate demand. As such, variations in investment feed back into aggregate demand, production, and income, leading to further rounds of investment adjustment. To this extent, as mentioned above, the long-run income is the result of the interaction between the multiplier and the accelerator.

The residual stock of money is determined by households' saving decisions (both in terms of saving rate and portfolio preferences). The Government initially finances its spending through CB overdraft, while its final debt is determined by the combination of fiscal revenues generated by such spending and fiscal revenues induced by previous spending and originating from the multipliers of private investments. The end-of-period debt held by CB will be residually determined by the public bonds demand of households.[7]

In the proceeding of the section, we first determine the long-run growth rate of macro variables and the steady-state values of public debt- and wealth-to-GDP ratios. Secondly, we derive stability conditions of the system.

2.1 | The Long-Run Dynamic

The system can be rewritten in a reduced form as a system of fourth-order difference equations with three dynamic variables (income, savings, and public debt) and three equations:

$$\begin{cases} Y_t = [Y_{t-1} + B_{t-2}r(1-\alpha) - Y_{t-3}v\delta](1-\theta)c_1 + S_{t-1}c_2 \\ \qquad + G_t(1+g_t) + Y_{t-1}v - Y_{t-2}v(1-\delta) \\ B_t = B_{t-1}(1+r) + G_t - \theta(Y_t + B_{t-1}r - Y_{t-2}v\delta) \\ S_t = S_{t-1}(1-c_2) + (Y_{t-1} + B_{t-1}r - Y_{t-2}v\delta)(1-\theta)(1-c_1) \end{cases} \tag{17}$$

By substituting the values of public debt with a rewritten expression of the redundant equation, we can rewrite the system as a fourth-order Vector Autoregressive (VAR) model[8]:

$$\begin{cases} Y_t = Y_{t-1}[c_1(1-\theta) + v] + S_{t-1}c_2 - Y_{t-2}v(1-\delta) \\ \qquad + B_{t-2}rc_1(1-\theta) - Y_{t-3}v\delta c_1(1-\theta) + G_t \\ B_t = Y_{t-1}\theta[(1-\theta)(1-c_1) - v] + B_{t-1}(1+r)(1-\theta) \\ \qquad + \theta S_{t-1}(1-c_2) - \theta Y_{t-2}v\delta(1-\theta) + \theta B_{t-2}r(1-\theta)(1-c_1) + \\ \qquad + \theta Y_{t-3}v\delta(1-\theta)c_1 + G_t(1-\theta) \\ S_t = Y_{t-1}(1-\theta)(1-c_1) + S_{t-1}(1-c_2) + B_{t-2}r(1-\theta)(1-c_1) \\ \qquad - Y_{t-3}v\delta(1-\theta)(1-c_1) \end{cases} \tag{18}$$

After several manipulations, the entire system can be re-expressed as a fourth-order difference equation describing the level of income as an autoregressive function:

$$\begin{aligned} &Y_{t+1} - Y_t[(1-\theta)(c_1+r) + v + 1 - c_2] + Y_{t-1}[vr(1-\theta) \\ &\quad + v(2-\delta-c_2) + (1-\theta)(1+r)(c_1-c_2)] \\ &\quad + Y_{t-2}v\{\delta[(1-\theta)(c_1+r) + 1 - c_2] - (1-c_2)[1 + r(1-\theta)]\} \\ &\quad - Y_{t-3}(1+r)(1-\theta)(c_1-c_2) \\ &\quad = G_0(1+g_G)^{t-2}[r(1-\theta)(c_2-c_1) + (1+g)[(1-c_2) \\ &\qquad + r(1-\theta)(1-c_1)]] \end{aligned} \tag{19}$$

The solution of the difference equation expressing the level of income in each period is:

$$Y_t = C_1 x_1^t + C_2 x_2^t + C_3 x_3^t + C_4 x_4^t - \frac{G(1+g_G)^{1+t}\left[c_2 + g_G(1 + c_2 + g_G) - (1-\theta)r(1 - c_1 - c_2 - g_G)\right]}{(1 - g_G + x_1)(1 + g_G - x_2)(1 + g_G - x_3)(1 + g_G - x_4)} \tag{20}$$

were $x_{1,2,3,4}$ stands for the solution of the following fourth-degree polynomial:

$$x^4 + bx^3 + cx^2 + dx + \mathrm{e} = 0 \tag{21}$$

where:

$$b = c_2 - (1-\theta)(c_1 + r) - v - 1 \tag{22}$$

$$c = (1-\theta)r(c_1 - c_2)(1 + r) + v[2 - c_2 - \delta + (1-\theta)r] \tag{23}$$

$$d = -v\{1 - c_2 - \delta[1 - c_2 + c_1(1-\theta)(1+r)] + (1-\theta)r(1 - c_2)\} \tag{24}$$

$$e = c_2\delta v(1-\theta)(1+r)(c_2 - c_1) \tag{25}$$

The solutions of x are:

$$x_{1,2} = -\frac{b}{4} - Q \pm \sqrt{-4Q - \frac{8ac - 3b^2}{4} + \frac{8d - 4bc + b^3}{3Q}} \tag{26}$$

$$x_{3,4} = -\frac{b}{4} + Q \pm \sqrt{-4Q - \frac{8ac - 3b^2}{4} - \frac{8d - 4bc + b^3}{3Q}} \tag{27}$$

where:

$$Q = \frac{1}{2}\sqrt{-\frac{8a - 3b^2}{12} + \frac{1}{3}\,\frac{\sqrt[3]{\left(27d^2 - 72ce + 27b^2e - 9bcd + 2c^3 + \sqrt{\frac{s^2 - 4q^3}{2}}\right)^2} + 12e - 3bd + c^2}{\sqrt[3]{27d^2 - 72ce + 27b^2e - 9bcd + 2c^3 + \sqrt{\frac{\left(27d^2 - 72ce + 27b^2e - 9bcd + 2c^3\right)^2 - 4\left(12e - 3bd + c^2\right)^3}{2}}}}} \tag{28}$$

By studying the value of the roots x, it is possible to determine the long-term growth rate of stocks and flows. In particular, the maximum value of the root x determines the long-run growth rate of GDP. If the values of exogenous parameters (tax rate, propensities to consume, interest rate, and the capital-to-output ratio) are such that the module of x is lower than $1 + g_G$, the growth rate of the economy converges to the growth rate of the primary public spending. Otherwise, it converges to the maximum value of x, that is, the growth rate of the public debt service. In this case, the growth rate of the public debt service becomes higher than the growth rate of primary public spending, and the stock and flow variables converge to such a growth rate. Indeed, the GDP growth rate converges to the highest value of the growth rate of the two autonomous components (Pariboni 2016; Allain 2022). In this sense, depending on the value of c_1, c_2, θ, g_G, v and r, two different regimes emerge: (i) a "standard regime", where the growth rate of public debt service converge to the growth rate of primary public spending, ; (ii) an "interest-led regime", where the growth rate of the public debt service is higher than the growth rate of primary spending.

Equations (29) and (30) highlight the threshold combinations of parameters beyond which the two distinct regimes emerge.

$$if -\frac{b}{4} - Q + \sqrt{-4Q - \frac{8ac - 3b^2}{4} + \frac{8d - 4bc + b^3}{3Q}} - 1 < g_G : g^* = g_Y = g_B = g_S = g_G \tag{29}$$

$$else :: g^* = g_Y = g_B = g_S = -\frac{b}{4} - Q + \sqrt{-4Q - \frac{8ac - 3b^2}{4} + \frac{8d - 4bc + b^3}{3Q}} - 1 \tag{30}$$

Once the public debt service needed to maintain the exogenously determined level of primary public expenditure is included in the model, another source of autonomous expenditure comes into play.[9] Indeed, the financing of primary public spending necessarily implies a variation in the stock of debt and, thus, an impact on the public debt service. The latter is a component of demand which is independent of current income, but, differently from primary public expenditure (which can be assumed as exogenous), it has an endogenous path and is positively affected by the interest rate, the saving rate, and negatively affected by the tax rate. The interest accrued on the public debt becomes part of the income and consumption of households. Thus, public debt service, adding new purchasing power into the system, triggers a multiplier effect, like other autonomous components of demand. On the one hand, under the interest-led regime, given the interest rate, the higher the saving rate or the lower the tax rate, the higher the accumulation of savings and the higher the growth rate of public debt. On the other hand, given the saving rate and the tax rate, the higher the interest rate, the higher the flow of income generated by the stock of savings. Then, the higher growth rate of public debt service is the result of the combination of both mechanisms: higher growth rate of the stock and higher growth rate of the flows. Of course, both dynamics interact. Anyhow, it is worth noticing that the "interest-led regime", although theoretically possible, requires peculiar combinations of the values of above mentioned parameters. The growth rate of the public debt service can be higher than the growth rate of primary public spending only in the case of extremely low values of propensities to consume, low tax rates, low growth rate of the primary public spending and/or extremely high interest rates. Figure 1 shows the cases in which the interest-led regime emerges as the combinations of the propensity to consume out of income, interest rate, and tax rate vary. Moreover, its low plausibility also relies on the social and political unsustainability of a situation in which the long-run share of public spending for goods and services tends to zero in favor of the interest payment on public debt. However, because of the theoretical aim of this paper, we keep studying all the possible configurations of the economic system.

Finally, within the standard regime, a third sub-scenario can be identified. If the growth rate of primary public spending is zero (and if the parameters' combination does not give rise to the interest-led regime), the GDP reaches a stationary-state where the level of income is constant and is equal to:

$$Y^* = \frac{\overline{G}[c_2 - (1-\theta)r(1-c_1+c_2)]}{c_2\theta(1-\delta v) + (1-\theta)r[(1-c_1+c_2)(\delta v - 1) + c_2 v]} \tag{31}$$

Notice that this is the value of the SM model, including the propensity to consume out-of-wealth and endogenous public debt service. Instead, equilibrium income when $c_2 > 0$ and $r = 0$ is the following:

$$Y^* = \frac{\overline{G}}{\theta(1-\delta v)} \tag{32}$$

From a reduced form of the general model, it is also possible to derive the stationary income of the traditional SM (when $c_2 = 0$, $r = 0$ and $g_G = 0$)[10]:

$$Y^* = \frac{\overline{G}}{1 - c_1(1-\theta)(1-\delta v) - \delta v} \tag{33}$$

In the proceeding of the section, we study the determinants of the public debt-to-GDP ratio separately, in the stationary state (zero growth) and the steady-state economy (positive growth).

2.2 | Stock-Flow Ratios in the Stationary Economy

In this section, we analyze the long-run values of stock-flow ratios in the stationary economy. After some derivation, it is possible to determine the values as a function of the exogenous parameters:

$$\frac{B^*}{Y^*} = \frac{(1-\theta)(1-c_1+c_2)(1-\delta v) - c_2 v}{c_2 - r(1-\theta)(1-c_1+c_2)} \tag{34}$$

$$\frac{S^*}{Y^*} = \frac{(1-c_1)(1-\theta)[(\delta + r)v - 1]}{(1-c_1)r(1-\theta) + c_2[r(1-\theta)-1]} \tag{35}$$

By applying the partial derivatives, we can assess the influence of each parameter on the public-to-GDP ratio:

$$\frac{\partial d^*}{\partial c_1} = -\frac{(1-v(r+\delta))(1-\theta)c_2}{\{r(1-\theta)(1-c_1) - [1-r(1-\theta)]c_2\}^2} < 0 \tag{36}$$

$$\frac{\partial d^*}{\partial c_2} = -\frac{[1-v(r+\delta)](1-\theta)(1-c_1)}{\{r(1-\theta)(1-c_1) - [1-r(1-\theta)]c_2\}^2} < 0 \tag{37}$$

$$\frac{\partial d^*}{\partial \theta} = -\frac{(1-v(r+\delta))(1-c_1+c_2)c_2}{\{r(1-\theta)(1-c_1) - [1-r(1-\theta)]c_2\}^2} < 0 \tag{38}$$

$$\frac{\partial d^*}{\partial r} = \frac{(1-c_1+c_2)[-c_2 v + (1-c_1+c_2)(1-v\delta)(1-\theta)](1-\theta)}{[c_2 - (1-c_1+c_2)r(1-\theta)]^2} > 0 \tag{39}$$

$$\frac{\partial d^*}{\partial v} = -\frac{(1+g)(c_2+g) + (1-c_1+c_2+g)\delta(1-\theta)}{(1+g)\{(c_2+g)[1+g-r(1-\theta)] - (1-c1)r(1-\theta)\}} < 0 \tag{40}$$

The public debt-to-GDP ratio is positively affected by the interest rate, while it is negatively affected by the propensity to consume, the tax rate, and the capital intensity of the economic system. Through the multiplier, the propensity to consume positively affects both the output and the dynamics of fiscal revenues. Given 1 $ of public spending, the higher the propensity to consume, the higher the amount of money that flows back to the Government through fiscal revenues, and the lower the deficit. Therefore, the propensity to consume reduces the public debt-to-GDP by lowering the numerator and increasing the denominator. In this sense, the accumulation of private

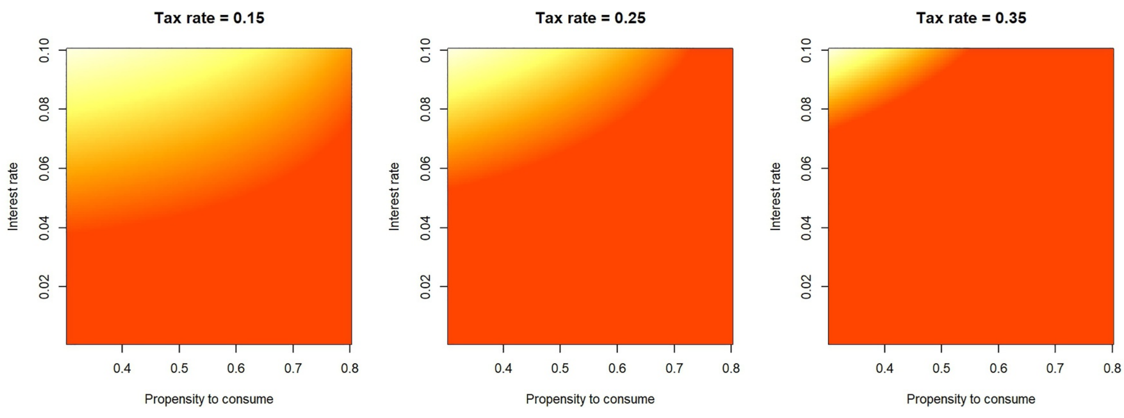

FIGURE 1 | Growth regimes emerging from different combinations of the propensity to consume, interest rate, and tax rate. The standard regime is represented by the red zone. Other colors indicate the interest-led regime. [Colour figure can be viewed at wileyonlinelibrary.com]

savings (in whatever form) symmetrically determines the accumulation of public debt.

Per each 1 $ of public spending, the higher the tax rate, the higher the amount of fiscal revenues that flow back to the government, and the lower the deficit. Although higher taxation reduces GDP, the higher amount of fiscal revenues per unit of GDP always leads to a reduction in fiscal deficit. The deficit effect always dominates the output effect, and the reduction in GDP is always lower than the reduction in the public deficit. A rise in the interest rate has the same impact as an increase in the saving rate. Indeed, a higher interest rate reduces the average fiscal multiplier.[11]

Finally, the public debt-to-GDP ratio is negatively affected by the normal capital-to-output ratio $(v)Q()$. This parameter has to be intended as a technical coefficient of production. A rise in the capital intensity of the production process implies that the amount of private investments per unit of public spending and the elasticity of private indebtedness with respect to a 1% increase in public spending raises. If the capital-to-output ratio increases, the amount of investments per unit of public spending rises, and fiscal revenues generated by the injection of purchasing power financed by private debt increase as well. Parallelly, through the interaction between the accelerator and multiplier, GDP expands. This leads to an increase in the private debt-to-GDP ratio and a corresponding reduction in the public debt-to-GDP ratio. That is, the higher the normal capital-to-output ratio, the higher the share of household savings covered by private debt and the lower the share covered by public debt.

2.3 | Stock-Flow Ratios in the Steady-State Economy (Standard and Interest-Led Regimes)

In this section, we consider the case of a growing economy. As discussed previously, depending on the values of parameters, the economy reaches a steady state where the growth rate of stock and flow variables converges to the higher between the growth rate of primary public spending and the growth rate endogenously determined by the public debt service. The steady-state values of the public debt-to-GDP ratio, household wealth-to-GDP ratio and deficit-to-GDP ratio are[12]:

$$d^* = \frac{B^*}{Y^*} = \frac{(1-\theta)(1+c_2-c_1+g)\left[(1+g^*)^2-\delta v\right]-v(1+g)(c_2+g)}{(1+g)\{(c_2+g)[g-r(1-\theta)+1]-(1-c_1)r(1-\theta)\}} \tag{41}$$

$$s^* = \frac{S^*}{Y^*} = \frac{(1-c_1)\left\{(1-\theta)(1+g)^2-v[r(1-\theta)+\delta(1-\theta)]\right\}}{(1+g)[r(1-\theta)(-1+c_1-c_2-g)+(1+g)(c_2+g)]} \tag{42}$$

$$x^* = \frac{(g-r)(1-\theta)(1+c_2-c_1+g)\left[(1+g)^2-\delta v\right]-v[(1+g)(c_2+g)]}{(1+g)^2\{(c_2+g)[g-r(1-\theta)+1]-(1-c_1)r(1-\theta)\}} \tag{43}$$

Figure 2 displays the relation between the propensities to consume and the public debt-to-GDP ratio as the interest rate increases.

The combination of the propensities to consume for which an increase in the interest rate leads to an expansion in the public debt-to-GDP ratio corresponds to the standard regime. The combinations of the propensities to consume for which a rise in the interest rate reduces the public debt-to-GDP ratio correspond to the interest-led regime. Of course, the lower the interest rate, the lower the values of the propensity to consume and the tax rate required to generate an "interest-led economy". To this extent, the range of the propensities to consume for which a restrictive monetary policy led to a reduction in the public debt-to-GDP ratio is wider when the interest rate is higher (i.e., the upper surface). Let's further analyze these aspects by looking at the analytical solutions. Computing the partial derivatives of the composite function of the public debt-to-GDP ratio, we can analytically identify the sign of each relationship and differentiate between the two scenarios. The partial derivatives of the public debt-to-GDP ratio with respect to each parameter are:

$$\frac{\partial d^*}{\partial c_1} = -\frac{(c_2+g)[1+g(2+g)-v(r+\delta)](1-\theta)}{\{g(1+g)+c_2[1+g-r(1-\theta)]-gr(1-\theta)-r[1-c_1(1-\theta)-\theta]\}^2} ><0 \tag{44}$$

$$\frac{\partial d^*}{\partial c_2} = -\frac{(1-c_1)[1+g(2+g)-v(r+\delta)](1-\theta)}{\{g(1+g)+c_2[1+g-r(1-\theta)]-gr(1-\theta)-r[1-c_1(1-\theta)-\theta]\}^2} ><0 \tag{45}$$

$$\frac{\partial d^*}{\partial \theta} = -\frac{(c_2+g)(1-c_1+c_2+g)[1+2g(1+g)-v(r+\delta)]}{\{g(1+g)+c_2[1+g-r(1-\theta)]-gr(1-\theta)-r[1-c_1(1-\theta)-\theta]\}^2} ><0 \tag{46}$$

$$\frac{\partial d^*}{\partial r} = \frac{(1-c_1+c_2+g)\left\{-(1+g)(c_2+g)v+(1-c_1+c_2+g)\left[(1+g)^2-v\delta\right](1-\theta)\right\}(1-\theta)}{(1+g)\{(c_2+g)[1+g-r(1-\theta)]-(1-c_1)r(1-\theta)\}^2} ><0 \tag{47}$$

$$\frac{\partial d^*}{\partial v} = -\frac{(1-\theta)(1+c_2-c_1+g)\delta-(1+g)(c_2+g)}{(1+g)\{(c_2+g)[g-r(1-\theta)+1]-(1-c_1)r(1-\theta)\}} < 0 \tag{48}$$

$$\frac{\partial d^*}{\partial g} = -\frac{(1+g)^2[v-2(1+g)(1-\theta)]-(c_1+c_2)\left[v\delta-(1+g)^2\right](1-\theta)}{(1+g)^2\{g(1+g)+c_2(1+g-r(1-\theta))-gr(1-\theta)-r(1-c1(1-\theta)-\theta)\}} - \frac{[1+c_2+2g-r(1-\theta)]\left\{-(1+g)(c_2+g)v+(1-c_1+c_2+g)\left[(1+g)^2-v\delta\right](1-\theta)\right\}}{(1+g)\{(c_2+g)[1+g-r(1-\theta)]-(1-c_1)r(1-\theta)\}^2} < 0 \tag{49}$$

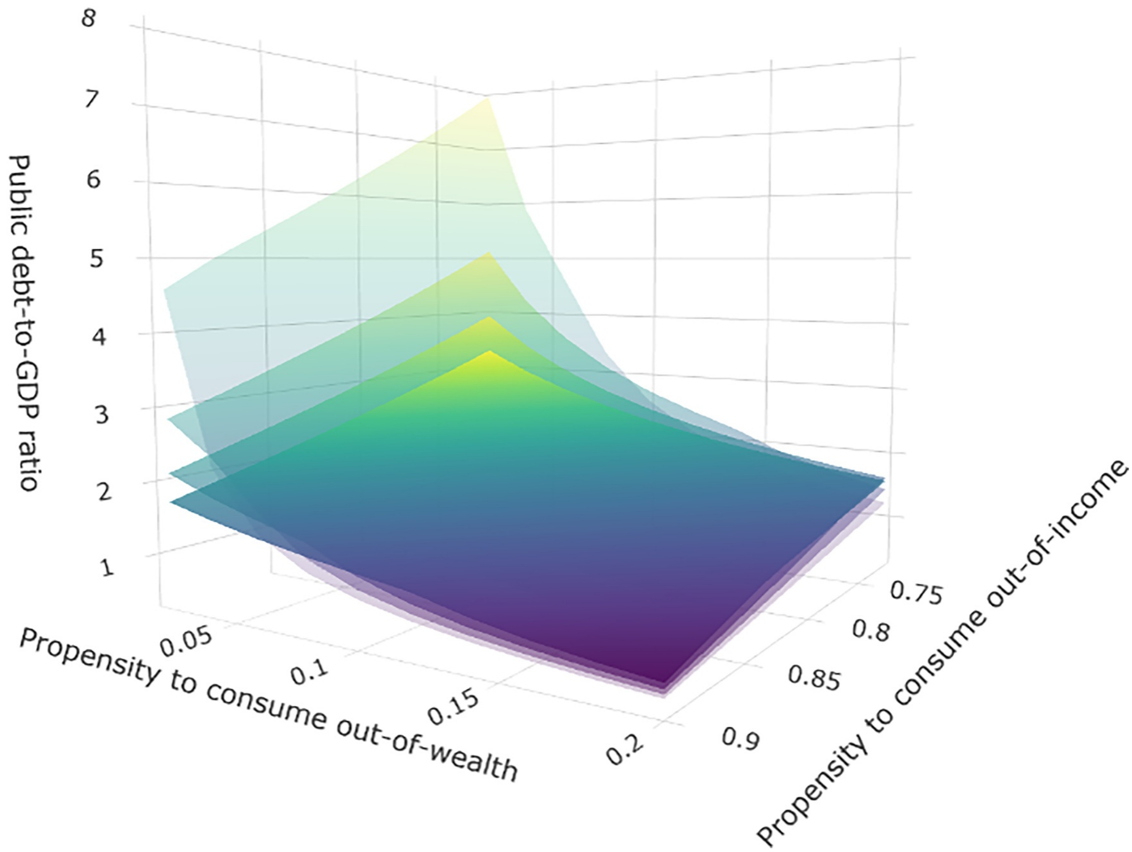

FIGURE 2 | Steady-state public debt-to-GDP ratio for different combinations of the propensity to consume out of income (*x*-axis) and wealth (*y*-axis). Each surface represents increasing levels of the interest rate. The transparency of surfaces decreases as the interest rate increases. [Colour figure can be viewed at wileyonlinelibrary.com]

With the exception of the capital-to-output ratio and the GDP growth rate, since each parameter enters also in the determination of the growth rate (g), the overall effect needs to be computed using the composite function. First, under the standard scenario, we can verify that the partial derivatives of the public debt-to-GDP ratio with respect to the propensities to consume, the tax rate are and the capital intensity are negative, while the partial derivative with respect to the interest rate is positive. Equations (44) and (45) are always negative if:

$$1 + g(2 + g) > v(r + \delta) \tag{50}$$

Such a condition is always verified in the standard regime,[13] then, the propensities to consume negatively affect the public debt-to-GDP ratio. The relationship between the public debt-to-GDP ratio and the tax rate is negative if:

$$(1 - c_1 + c_2 + g)[1 + 2g(1 + g) - v(r + \delta)] > 0 \tag{51}$$

Alternatively, we can say that the rise in the tax rate increases the public debt-to-GDP ratio if one of the following conditions is verified:

$$c_2 + g - c_1 < -1 \wedge 1 + 2g(1 + g) - v(r + \delta) > 0 \tag{52}$$

or

$$c_2 + g - c_1 > -1 \wedge 1 + 2g(1 + g) - v(r + \delta) < 0 \tag{53}$$

Since $0 < c_2, g, c_1 < 1$, $c_2 + g - c_1$ cannot be lower than -1 and condition (53) is never fulfilled. Condition (54) is verified if $c_2 + g - c_1 > -1$ and $1 + 2g(1 + g) < v(r + \delta)$. $c_2 + g - c_1$ is always higher than -1, however since $1 + 2g(1 + g)$ cannot be lower than $v(r + \delta)$, also condition (54) is never verified. As a result, the tax rate has a negative impact on the public debt-to-GDP ratio.

The partial derivative of the public debt-to-GDP ratio with respect to the interest rate is positive if:

$$(1 + g)(c_2 + g)v < (1 - c_1 + c_2 + g)\left[(1 + g)^2 - v\delta\right] \tag{54}$$

This condition is always verified and the public debt-to-GDP ratio is a positive function of the interest rate, in the standard regime. Finally, since the growth rate in the standard regime is determined by the growth rate of public spending, the latter has a negative impact on the public debt-to-GDP ratio.

Figure 3 displays the relationship between the public debt-to-GDP ratio and the propensities to consume for different values of the tax rate. Figure 4 displays the relationship between the public debt-to-GDP ratio and the propensities to consume for different values of the interest rate.

Within the interest-led regime since each parameter affect also the growth rate of the economy, we have to compare the derivative of the compound function (e.g., $\frac{\partial g^*}{\partial c_1}\ \frac{\partial d^*}{\partial g}$) with the associated partial derivatives (e.g., $\frac{\partial d^*}{\partial c_1}$):

$$\frac{\partial g^*}{\partial c_1}\frac{\partial d^*}{\partial g} < \frac{\partial d^*}{\partial c_1} \rightarrow \frac{\partial d^*}{\partial c_1} < 0 \tag{55}$$

$$\frac{\partial g^*}{\partial c_2}\frac{\partial d^*}{\partial g} < \frac{\partial d^*}{\partial c_2} \rightarrow \frac{\partial d^*}{\partial c_2} < 0 \tag{56}$$

$$\frac{\partial g^*}{\partial \theta}\frac{\partial d^*}{\partial g} < \frac{\partial d^*}{\partial \theta} \rightarrow \frac{\partial d^*}{\partial \theta} > 0 \tag{57}$$

$$\frac{\partial g^*}{\partial r}\frac{\partial d^*}{\partial g} > \frac{\partial d^*}{\partial r} \rightarrow \frac{\partial d^*}{\partial r} < 0 \tag{58}$$

As in the standard regime, the propensities to consume negatively affect the public debt-to-GDP ratio. Conversely to the standard regime, the tax rate has a positive effect on the public debt-to-GDP ratio, while the impact of the interest rate is negative. From an analytical perspective, the partial derivative of the public debt ratio with respect to the tax rate (negative sign) is smaller than the composite derivative of the debt-to-GDP ratio with respect to the tax rate ($\frac{\partial g^*}{\partial \theta}\ \frac{\partial d^*}{\partial g}$) (positive sign). The same applies for the comparison between derivative of the public debt with respect to the interest rate and the composite derivative ($\frac{\partial g^*}{\partial r}\ \frac{\partial d^*}{\partial g}$).

In both cases, the growth effect offsets the reduction in fiscal multipliers[14] and, as a result, the decrease in the tax rate or the rise in the interest rate leads to a reduction in the public debt-to-GDP ratio.

Concluding, Figures 5 and 6 summarize the general non-linear relationships between the tax rate, the interest rate and the public debt-to-GDP ratio.

Taking as given the value of the interest rate, variations in the tax rate can both reduce or increase the public debt-to-GDP ratio. The lower the interest rate and the lower the switching value of the tax rate above which the latter has a positive effect on the public debt-to-GDP ratio. Then, the higher the interest rate, the higher the range of the tax rate over which an increase in the tax rate causes an expansion

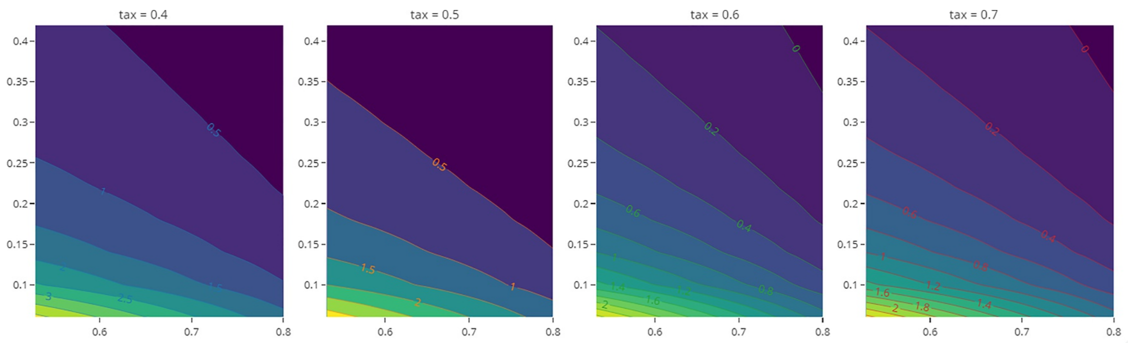

FIGURE 3 | Contour plots of the steady-state public debt-to-GDP ratio (z-axis) for different combinations of the propensity to consume out of income (x-axis) and the propensity to consume out of wealth (y-axis). Each plot presents a different level of the tax rate. [Colour figure can be viewed at wileyonlinelibrary.com]

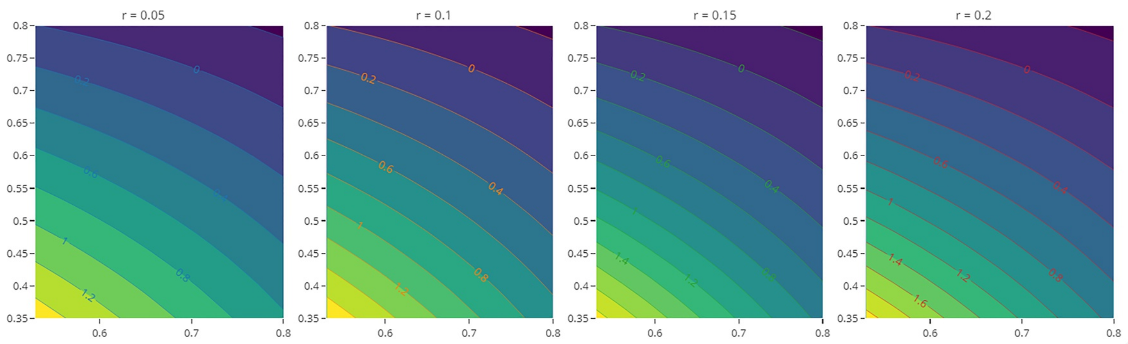

FIGURE 4 | Contour plots of the steady-state public debt-to-GDP ratio (z-axis) for different combinations of the propensity to consume out of income (x-axis) and tax rate (y-axis). Each plot presents a different level of the interest rate. [Colour figure can be viewed at wileyonlinelibrary.com]

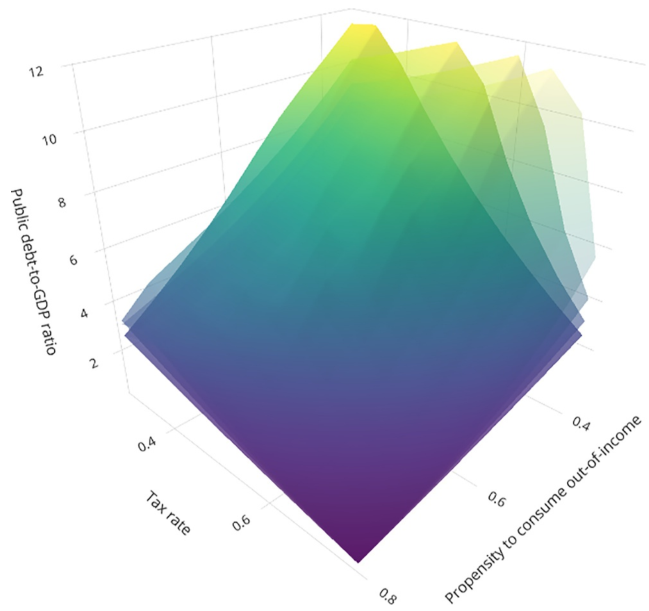

FIGURE 5 | Steady-state public debt-to-GDP ratio (z-axis) for different combinations of the propensity to consume out of income (x-axis) and the tax rate (y-axis). Each surface represents a different level of the interest rate. Transparency decreases when the interest rate increases. [Colour figure can be viewed at wileyonlinelibrary.com]

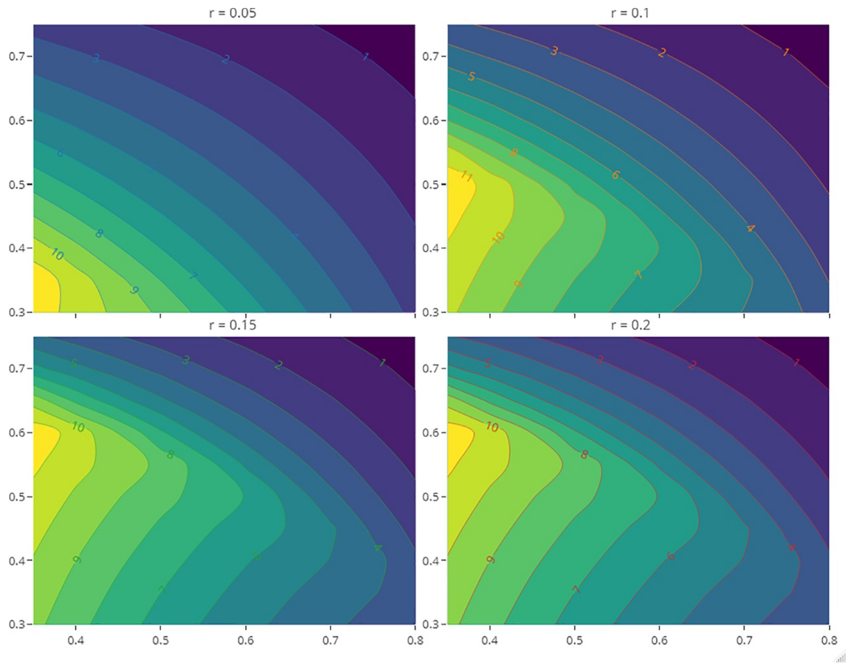

FIGURE 6 | Contour plots of the steady-state public debt-to-GDP ratio (z-axis) for different combinations of the propensity to consume out of income (x-axis) and tax rate (y-axis). Each plot presents a different level of the interest rate. [Colour figure can be viewed at wileyonlinelibrary.com]

in the public debt-to-GDP ratio. Taking as given the value of the tax rate, a specular argument can be applied to the analysis of the non-linear effect of the interest rate (see Figures 7 and 8).

The dynamics of the non-linearity strictly depend on the emerging growth regime, as depicted in Figure 9. Shaded areas represent the interest-led regime. The public debt-to-GDP ratio is a negative function of the tax rate when the growth rate of public debt service converges to the growth rate of primary public spending (standard regime). Conversely, when the growth rate is higher than the growth rate of primary public spending, and then it results endogenously determined by public debt service, the public debt-to-GDP ratio is a positive function of the tax rate. On the one hand, the switching point moves to the right as the interest rate increases. Indeed, the higher the interest rate, the higher the range for the tax rate below which the "interest-led regime" can arise. On the other hand, the switching point moves to the left as the propensity to consume out-of-income increases. A mirrored argument can be applied to the study of the interest rate impacts for different values of the tax rate (see Figure 10).

Table 1 summarizes the relationships across parameters and the public debt-to-GDP ratio in the two regimes.

2.4 | Stability Analysis of the Public Debt-to-GDP Ratio in the Long-Run

In this section, we assess the stability conditions of the public debt-to-GDP ratio. Namely, we study under which conditions the system asymptotically converges to the solutions presented in Sections 2.2 and 2.3. In the remainder of the paper, we will refer to asymptotic values as normal values. The ratio B_t/Y_t is stable in the long run if both B_t and Y_t share the same principal growth component (i.e., the same dominant eigenvalue), and if the ratio of their corresponding values in the dominant eigenvector is finite and well-defined. The companion-form matrix is:

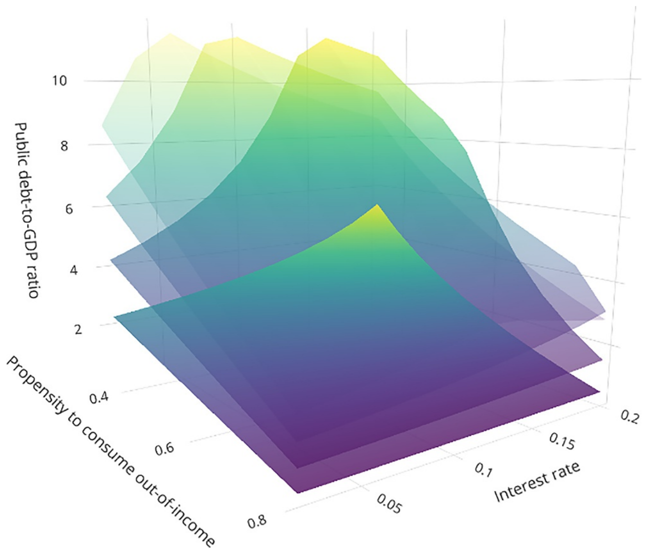

FIGURE 7 | Steady-state public debt-to-GDP ratio (z-axis) for different combinations of the propensity to consume out of income (x-axis) and the interest rate (y-axis). Each surface represents a different level of the tax rate. Transparency decreases as the tax rate increases. [Colour figure can be viewed at wileyonlinelibrary.com]

$$\Gamma = \begin{pmatrix} c_1(1-\theta)+v & 0 & c_2 & -v(1-\delta) & rc_1(1-\theta)(1-\alpha) & 0 & -v\delta c_1(1-\theta) & 0 & 0 \\ \theta[(1-\theta)(1-c_1)-v] & (1+r)(1-\theta) & \theta(1-c_2) & -\theta v\delta(1-\theta) & r\theta(1-\theta)(1-c_1) & 0 & v\theta\delta(1-\theta)c_1 & 0 & 0 \\ (1-\theta)(1-c_1) & 0 & (1-c_2) & 0 & r(1-\theta)(1-c_1) & 0 & -v\delta(1-\theta)(1-c_1) & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad (59)$$

$$\begin{bmatrix} Y_t \\ B_t \\ S_t \\ Y_{t-1} \\ B_{t-1} \\ Y_{t-2} \end{bmatrix} = \begin{bmatrix} c_1(1-\theta)+v & 0 & c_2 & -v(1-\delta) & rc_1(1-\theta)(1-\alpha) & -v\delta c_1(1-\theta) \\ \theta[(1-\theta)(1-c_1)-v] & (1+r)(1-\theta) & \theta(1-c_2) & -\theta v\delta(1-\theta) & r\theta(1-\theta)(1-c_1) & v\theta\delta(1-\theta)c_1 \\ (1-\theta)(1-c_1) & 0 & 1-c_2 & 0 & r(1-\theta)(1-c_1) & -v\delta(1-\theta)(1-c_1) \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Y_{t-1} \\ B_{t-1} \\ S_{t-1} \\ Y_{t-2} \\ B_{t-2} \\ Y_{t-3} \end{bmatrix} \quad (60)$$

The eigenvalues of the matrix Γ are represented by λ in the expression:

$$|\Gamma - \lambda \mathrm{I}| = 0 \quad (61)$$

The roots of the eigenvalues can be derived from the following equation:

$$x^6 + x^5 A + x^4 B + x^3 C + x^2 D + Ex + F = 0, \quad (62)$$

where:

$$A = (1-\theta)(c_1 - r) + c_2 + \theta - v - 2 \quad (63)$$

$$\begin{aligned} B = {} & 1 + r\left(1 + v + \theta^2\right) + 3v - c_2(2 + r + v) - v\delta \\ & - c_1(1 + r - \theta)(-2 + \theta) + c_2(2 + r)\theta \\ & + \{-1 - v + r[-2 + v(-1 + \alpha)]\}\theta \end{aligned} \quad (64)$$

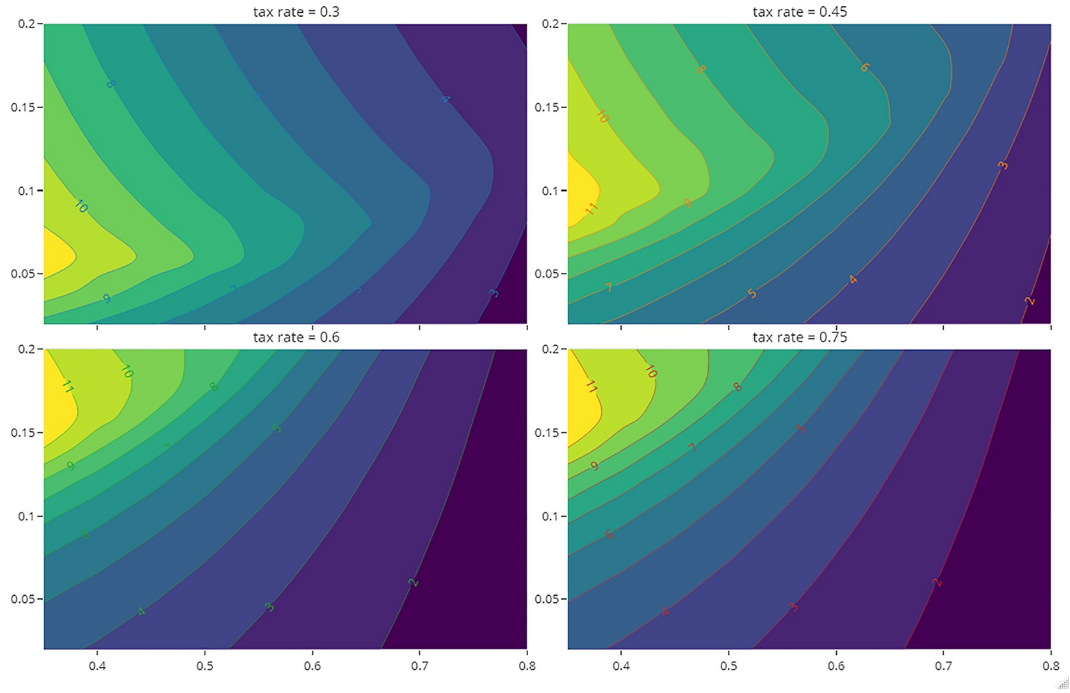

FIGURE 8 | Contour plots of the steady-state public debt-to-GDP ratio (z-axis) for different combinations of the propensity to consume out of income (x-axis) and interest (y-axis). Each plot presents a different level of the tax rate. [Colour figure can be viewed at wileyonlinelibrary.com]

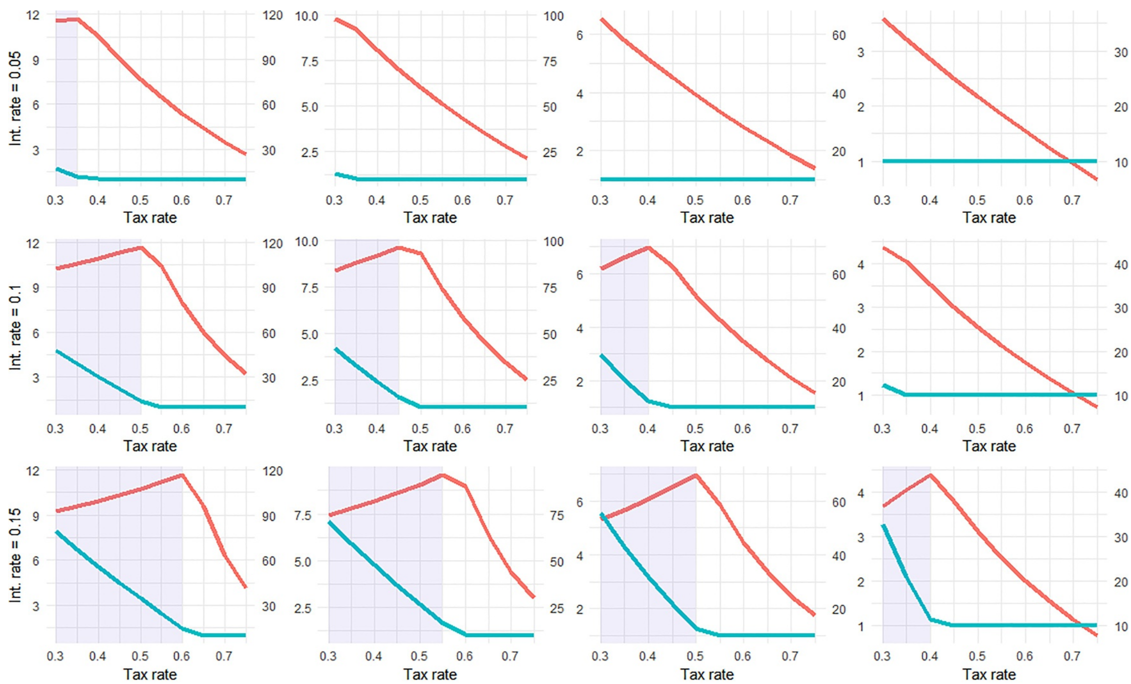

FIGURE 9 | Each plot reports the growth rate of the economy (blue line) and the public debt-to- GDP ratio (red line) as the tax rate changes. Each plot moving from the right side to the left side reports such relationship for an increasing value of the propensity to consume. In the results reported in the first line, the interest rate is equal to 5%, in the second line it is 10% and in the third one it is 15%. [Colour figure can be viewed at wileyonlinelibrary.com]

$$
\begin{aligned}
C = & -\nu[3+\delta(-2+\theta)-2\theta+r(-1+\theta)(-2+\delta+\theta)] \\
& -(-1+\theta)[-1+\nu\delta+r(-1+\theta)+\theta]c_1 \\
& +\left[-r(1+\nu-\theta)(-1+\theta)+(-1+\theta)^2-\nu(-2+\delta+\theta)\right]c_2
\end{aligned} \tag{65}
$$

$$
D = \nu(-1+\theta)\left\{\begin{array}{c}(1-\delta)[-1+r(-1+\theta)]+\delta[2-\theta(1+r(1-\theta))+r]c_1 \\ +[1+r(1-\delta)-2\delta]c_2\end{array}\right\} \tag{66}
$$

$$
\begin{aligned}
E = & \ \delta-\nu\delta(-1+\theta)^2\{[-1+r(-1+2\theta)]c_1 \\
& +[1+r(1-\theta)]c_2\}
\end{aligned} \tag{67}
$$

$$
F = r\nu\delta(-1+\theta)^2\theta(c_1-c_2) \tag{68}
$$

The dominant eigenvector (λ^*) is:

$$(\Gamma - \lambda^* \mathrm{I})\mathrm{v} = 0 \tag{69}$$

The dominant eigenvalue λ^* gives the growth rate of the system ($g = \lambda^* - 1$) and the value of the public debt and income over time is:

$$B_T \approx \lambda^* v_B \tag{70}$$

$$Y_t \approx \lambda^* v_Y \tag{71}$$

Although a sixth-degree polynomial equation does not have an analytical solution, it is possible to numerically verify that if all parameters are higher than 0 and lower than 1, the ratio v_b/v_Y is finite. We have computed local solutions by performing a multivariate analysis in the space vector covering all possible values of parameter combinations (see Appendix D):

$$\begin{cases} c_1 \in (0,01 : 0.99;\ 0.25) \\ c_2 \in (0 : 0.99; 0.25) \\ \theta \in (0.01 : 0.99;\ 0.25) \\ \delta \in (0.01 : 0.99; 0.25) \\ v \in (0.01 : 0.99; 0.25) \\ r \in (0 : 0.99; 0.25) \end{cases} \tag{72}$$

Given that $\alpha \neq 0$, there is one specific case in which the dominant eigenvalue is equal to one and v_B/v_Y is not finite, that is, when the growth rate of public spending, the interest rate, and the propensity to consume out-of-wealth are simultaneously equal to zero. In this case, GDP reaches a stationary state with zero growth while the accumulation of savings and debt proceeds. As a result, the public debt-to-GDP ratio explodes. The second case of instability is when $\alpha = 1$ (full-hoarding on the interests accrued on the stock of savings), within the interest-led regime. In such a case, it is required that the propensity to consume out of wealth is higher than zero. Table 2 summarizes the stability conditions.

In short, given the exogenous trend in primary public spending, the public debt-to-GDP ratio stabilizes for any parameter values within the standard regime, as long as the growth rate of primary public spending is greater than zero. In the stationary regime, however, it is necessary for the propensity to consume

TABLE 1 | The impact of exogenous parameters on the public debt-to-GDP ratio.

Parameters	Standard regime	Interest-led regime
Propensity to consume out-of-income	↓	↓
Propensity to consume out-of-wealth	↓	↓
Tax rate	↓	↑
Capital-to-output ratio	↓	↓
Interest rate	↑	↓
Growth rate of primary public spending	↓	–

TABLE 2 | Stability conditions and growth regimes.

	Standard regime (stationary)	Standard regime (growth)	Interest-led regime
Stability conditions	$c_2 > 0$	∀	$\alpha \neq 1$ *or* $c_2 > 0$

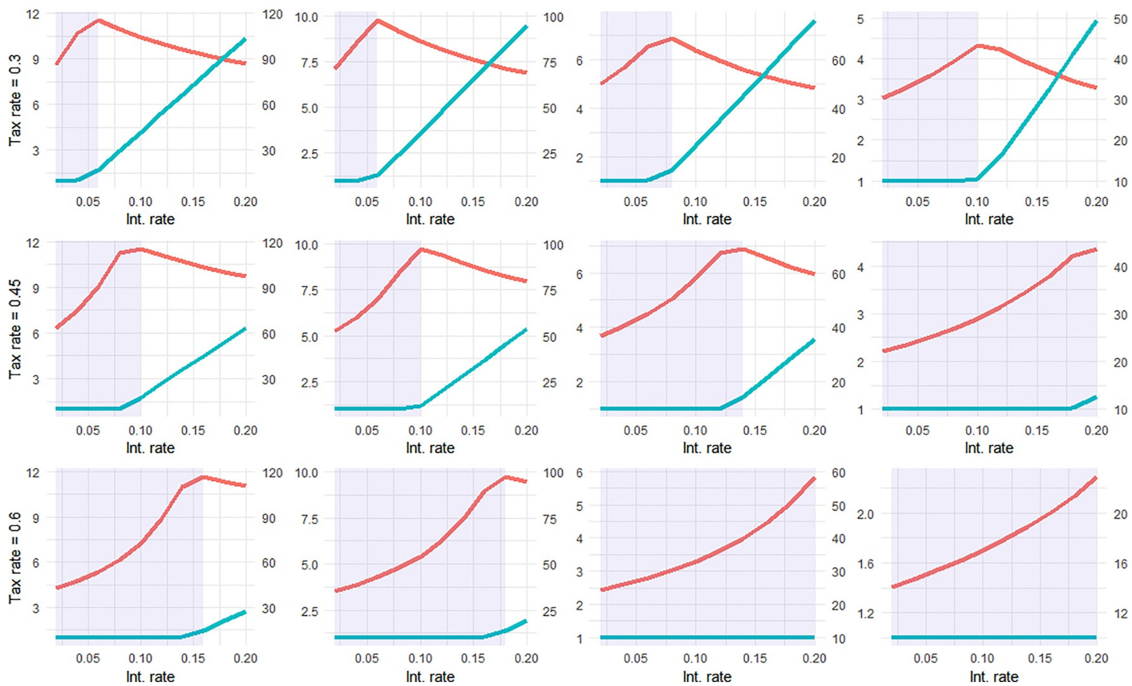

FIGURE 10 | Each plot reports the growth rate of the economy (blue line) and the public debt-to-GDP ratio (red line) as the interest rate changes (x-axis). Each plot moving from the left side to the right side reports such a relationship for an increasing value of the propensity to consume. In the results reported in the first line, the tax rate is equal to 30%, in the second line it is 45%, and in the third one it is 60%. [Colour figure can be viewed at wileyonlinelibrary.com]

out of wealth to be greater than zero. In the interest-led regime, it is required either that there is no full hoarding of the income generated by interest payments or alternatively, that the propensity to consume out of wealth is greater than zero. Indeed, in such a case, the component of public spending which is triggering the long-run dynamic does not generate any fiscal multiplier effect.

It is worth noticing that, in this scheme, the stability condition is fulfilled through an endogenous adjustment of the primary deficit-to-GDP ratio. In this regard, it is useful to recall the traditional arithmetic describing the dynamics of the public debt-to-GDP ratio, which is the following:

$$d_t = \frac{1+r}{1+g_t} d_{t-1} + x_t \tag{73}$$

where x_t is the primary deficit-to-GDP ratio, d_t is the public debt-to-GDP ratio, r is the interest rate on public bonds and g_t is the nominal GDP growth rate. Usually, by imposing that $x_t = 0$, the traditional argument concludes that the stability of the public debt-to-GDP ratio is granted if the GDP growth rate is equal to the interest rate. Alternatively, the stability condition is verified if the primary deficit-to-GDP ratio is the following:

$$x_t = \left(\frac{g-r}{1+g}\right) d_t. \tag{74}$$

It is possible to verify that, along the steady-state, the public deficit-to-GDP ratio corresponds to such a value:

$$x^* = \frac{(g-r)(1-\theta)(1+c_2-c_1+g^*)\left[(1+g^*)^2-\delta v\right] - v[(1+g^*)(c_2+g^*)]}{(1+g^*)^2[(c_2+g^*)(g^*-r(1-\theta)+1)-(1-c_1)r(1-\theta)]} = \left(\frac{g-r}{1+g}\right) d^* \tag{75}$$

In the stationary-state economy, the public deficit-to-GDP ratio is equal to the public debt-to-GDP ratio multiplied by the interest rate (with a negative sign):

$$x_t = -\frac{(1-\theta)(1-c_1+c_2)(1-\delta v)-c_2 v}{c_2 - r(1-\theta)(1-c_1+c_2)} r = -d_{t-1} r. \tag{76}$$

3 | Policy Experiments

This section analyses the impact of the introduction of fiscal rules, where the target threshold is lower than the "normal rate". We perform the policy experiments in the scenario with positive growth. In this regard, it is meaningless to assess the impact of a fiscal rule in the case of a stationary economy with zero accumulation of savings ($c_2 > 0$) and public deficit-to-GDP ratio equal to zero or where the ratio is persistently growing ($c_2 = 0$). Within the "standard regime", we consider two types of fiscal policy: (a) the Government attempts to reach the target deficit-to-GDP ratio by cutting primary public spending. The equation describing the path of public spending is:

$$\begin{cases} if\ d_t > d^* : G_t = \max\left(0, Y_{t-1}(d+\theta)(1+g_e) - rB_{t-1} + F_{CB}\right) \\ else : G_t = G_{t-1}(1+g_G) \end{cases} \tag{77}$$

(b) The Government tries to achieve the target ratio by managing the tax rate, as follows:

$$\begin{cases} if\ d_t > d^* : \theta_t = \dfrac{G_t + rB_{t-1} - F_{CB}}{Y_{t-1}(1+g_e)} - d^* \\ else : \theta_t = \theta_{t-1} \end{cases} \tag{78}$$

As reported in Figure 11, under scenario (a), the introduction of the fiscal rule is effective in achieving the target deficit-to-GDP ratio, however, it led to an expansion in the public debt-to-GDP ratio. In particular, it depresses the growth rate of primary spending, giving rise to an interest-led regime. As a result, the impact on GDP growth is higher than the reduction in deficit, and the public debt-GDP ratio increases.[15]

Under scenario (b), the introduction of the fiscal rule led to a persistent reduction in public deficit and public debt-to-GDP

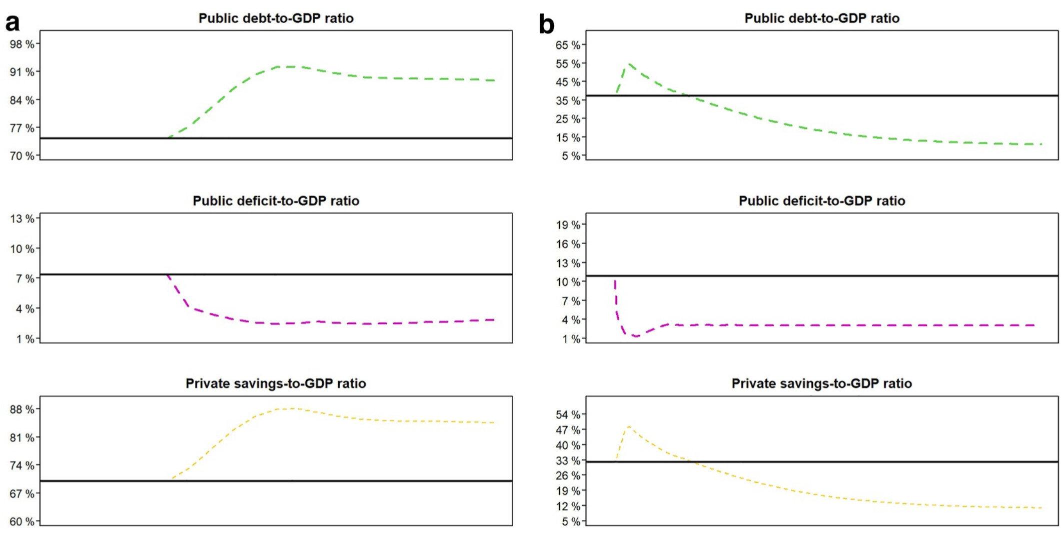

FIGURE 11 | The impact of the introduction of different fiscal rules. (a) Display the impact of fiscal rule and (b) fiscal rule. [Colour figure can be viewed at wileyonlinelibrary.com]

ratios. This result is not surprising, as shown in Section 2, the stationary level of the public debt-to-GDP ratio is negatively affected by the tax rate. This is exactly what happens in such a scenario: the Government raises the tax rate to reduce the public deficit.

Finally, it is unnecessary to mention that the introduction of a fiscal rule with a threshold higher than the normal rate is non-binding and therefore does not affect the autonomous path of public spending, nor the stock-flow ratios in the steady state. For this reason, we did not incorporate such a scenario in our analysis.

4 | Conclusions

In this paper, we have outlined that, in a framework reproducing the main features of a monetary economy of production and based on the Supermultiplier approach, the long-run or "normal" level of the public debt-to-GDP ratio is determined by the saving rate, the tax rate, the technical coefficients influencing the capital intensity of the economic system, the growth rate of primary spending and the policy rate.

While the saving rate has a positive impact on the public debt-to-GDP ratio, the capital intensity and the growth rate of the autonomous component have a negative effect. The interest rate and the tax rate have a non-linear impact. The non-linearity depends on the growth regime emerging in the economy. Within the standard regime, the growth rate of the public debt service converges to the growth rate of primary public spending, and the tax rate has a negative effect, while the interest rate has a positive impact. Within the interest-led regime, the growth rate of the public debt service is higher than the growth rate of the primary public spending, and the tax rate has a positive effect, while the interest rate has a negative impact. The stability analysis indicates that, given the exogenous trend of the primary public spending, the public debt-to-GDP ratio always stabilizes within the standard regime with positive growth (growth rate of primary public spending higher than zero). In the case of a stationary economy, the propensity to consume out of wealth has to be higher than zero in order to ensure the long-run stability. Indeed, in a stationary state, continued accumulation of savings entails a symmetrical accumulation of public debt, ultimately leading to the explosion of the debt-to-GDP ratio. Under the interest-led regime, the stability condition requires that interests accrued on the stock of savings are not fully hoarded, or that the propensity to consume out of wealth is greater than zero.

Finally, the introduction of fiscal rules in achieving an exogenous target lower than the "normal rate" is effective only in the case that the Government uses the tax rate as leverage. If the fiscal rule is implemented through the reduction of primary public spending, it results self-defeating in terms of a public debt-to-GDP ratio, leading to a shift of the system toward the interest-led regime.

Moreover, a normative conclusion can be drawn from such findings: the values of exogenous and policy parameters must be such a way that condition (31) is fulfilled. Namely, a low level of the interest rate with respect to the value of the propensity to consume or tax rate is required to prevent the economy from falling into the trap of the interest-led regime, in which the share of primary public spending tends to zero in the long run, in favor of the public debt service.

The results presented can generally be considered valid even under the assumption that the central bank is not allowed to purchase public bonds, provided that bond demand is interest-elastic and/or that agents, having sufficient liquidity, always demand the bonds issued by the government (note that, in the representation adopted here, public debt is entirely held by the private sector). However, the case in which the government is not allowed to spend in advance or to use an overdraft at the CB has not been examined. In any case, such a scenario lies outside the scope of the analysis conducted within the Supermultiplier framework, as it would undermine its foundational logic. In such a case, the autonomous component could no longer be considered truly autonomous; on the contrary, public spending would become endogenous and tied to investment dynamics. Indeed, public expenditure would then need to be ex ante financed by deposits created through loans issued to finance firms' investments. By contrast, when an overdraft facility is available, the deficit can always be ex-post financed by households through the conversion of central bank–created deposits into government bonds.

The main limitation of the paper is the absence of other autonomous components of demand, such as exports, credit to consumption or residential investments, and the closed-economy setting. In general, the growth rate of such components should negatively affect the public debt-to-GDP ratio. Regarding the closed economy setup, in the model, public bonds are held by domestic households and, therefore, the resulting interests flow back into it via consumption expenditure. However, at the current stage, the full-hoarding parameter emulates the share of government bonds that would be held abroad. Future extensions of the model should consider the incorporation of these two features. In particular, the future development of this work involves two lines of research: (i) the integration of the system presented in the paper, and aimed at determining quantities, with the system voted to determine relative prices. This makes it possible to assess the effect that changes in the interest rate have on the capital intensity of the economic system through the value dimension and, in turn, on the public debt-to-GDP ratio. (ii) The development of a theory-based (S)VAR model, as presented in Section 2, to empirically estimate and validate the relationships elucidated by the theoretical analysis. This type of development could constitute an important contribution to the debate on the nature and sustainability of public debt since, apart from the theoretical level, there is an absence of empirical work explicitly devoted to estimating the relationship between the above-mentioned parameters (such as the saving rate) and the public debt-to-GDP ratio.

Finally, in the paper, the policy implications of different time lags for convergence have not been discussed. The transition period is characterized by an increasing public debt-to-GDP ratio, while the length of such time depends on the value of parameters and the growth regime considered. In general, the interest-led regime, requiring very low values of the propensity to consume, needs sufficiently long time frames for stabilization. Of course, the length of the transition can have significant policy implications.

Acknowledgments

An earlier version of this research appeared as the working paper *Stability and determinants of the public debt-to-GDP ratio: a Stock-Flow Consistent investigation* (Di Domenico 2022). Open access publishing facilitated by Universita degli Studi di Trieste, as part of the Wiley - CRUI-CARE agreement.

Funding

The author has nothing to report.

Conflicts of Interest

The author declares no conflicts of interest.

Data Availability Statement

This study does not involve any empirical data.

Endnotes

[1] The core assumption in GE models is an all-purpose good (APG), which can be used interchangeably as a consumption good, an investment good, and a financial asset (see Mankiw 2020, 520). With the APG as the only financial asset, the saving of households provides the supply of "funds" on the capital market. If the households decide not to consume the APG, they make it available as a financial asset that can be supplied on the financial market and used as an investment good (Bofinger and Ries 2017).

[2] As warned by Hahn's problem (F. H. Hahn 1965), the attachment of money to the moneyless perfect barter GE brings paradoxical results converting money and monetary institutions into frictions in the model, contra the principle that monetary exchange is more efficient than barter (Rogers 2019).

[3] Capital depreciation and debt repayment are both assumed to occur at the same constant rate.

[4] Such approach is consistent with the traditional formulation of the SM model where retained profits are absent. Moreover, this allows us to streamline the mathematical treatment without altering the main conclusions of the model. Differently from the traditional SM, we include debt repayment. As a consequence, the income of households is lower than the value of GDP and corresponds to the NDP (Net Domestic Product). In detail, income before tax is equal to GDP net of capital allowances. If firm leverage to finance investments is equal to one and the useful life of capital goods is equal to the debt repayment time, the reimbursement in each period corresponds to the value of amortization of the capital stock. This reflects the fact that profits are net of capital amortization and the distributed income corresponds to NDP ($K_t = L_t$).

[5] The payment of interest on loans and on the resulting deposits sums to zero within the household-firm circuit. Since we are not distinguishing between the capitalist class and the working class, the payment of interest on loans does not affect aggregate dynamics and disappears from the system. The only additional financial income for households, in aggregate, comes from the interest accrued on public bonds and deposits created by the CB when purchasing public bonds. In this regard, although the Government apparently does not pay any interest on the stock of debt held by CB, it is actually paying an interest rate equal to the deposit interest rate (or the interest rate on reserves) on such an amount. Indeed, such a quantity is held as reserves by the commercial bank. Then, the effective interest rate paid by the Government on public debt is equal to a weighted average between the interest rate on public bonds and deposits (see Appendix B).

[6] In this sequential approach, the mutual causal relationship between income and consumption can be expressed only over the periods. Specifically, consumption in period t contributes to generate the aggregate demand and output in the same period. At the end of the period, the corresponding income is distributed, the latter forms the basis for consumption in the following period. Appendix C shows that the sequential approach dynamically represents the economic mechanisms that are implicit in the simultaneous scheme (where consumption simultaneously determines and is determined by the same income). In particular, it highlights how the simultaneous system represents the solution of the dynamic system under the imposition of the stationary state. At the same time, it warns against the feasibility and economic meaning of imposing simultaneity between the two variables within a dynamic framework—that is, in the representation of intraperiod dynamics.

[7] Public deficit is only ex-post founded by household's savings. It is the initial CB overdraft that allows for the creation of purchasing power, generation of income and savings. Only in a second step, through the deposits created by CB, households can buy back a share of public debt held by the CB (see Di Domenico et al. 2024).

[8] The redundant Equation (16) can be also rewritten as: $B_t = S_t - I_t - K_t(1-\delta) + (Y_t + B_{t-1}r - K_t\delta)(1-\theta)$ (17).

[9] Given the endogenous nature of this component, it would be more accurate to define it as a semi-autonomous component of demand.

[10] The value of the supermultiplier is slightly lower than traditional widespread formulations of the SM since we are considering also debt reimbursement.

[11] In a closed-economy, interest payment figures as endogenous Government transfers to households. Transfers have a lower multiplier than primary public spending.

[12] As above mentioned, for the sake of analytical tractability we are deriving the values for $\alpha = 0$.

[13] By construction, in the standard regime, the interest rate is necessarily enough low to not generate an endogenous growth rate higher than the growth rate of the autonomous component.

[14] A higher amount of income (and, therefore, consumption and investments) is generated from the stock of savings if the tax rate lowers or the interest rate expands.

[15] Given the same GDP growth rate, the interest-led regime generally presents higher values of the public debt-to-GDP ratio for two reasons: (i) the fiscal multiplier of primary public spending is higher than that of debt service; (ii) the interest-led regime requires low values for the propensities to consume and for taxation, and high values for the interest rate (which, excluding their effect on the growth rate, have a positive impact on the public debt-to-GDP ratio).

[16] Check deposits represents the disposable income that has been credited to households at the end of the period and that can be spent on consumption in the following period. In the end-of-period snapshot of agents' balance sheets, they appear under deposits, but they are not actual savings that accrue interest (as is the case with time deposits).

References

Abbas, S. A., and J. E. Christensen. 2010. "The Role of Domestic Debt Markets in Economic Growth: An Empirical Investigation for Low-Income Countries and Emerging Markets." *IMF Staff Papers* 57, no. 1: 209–255. https://doi.org/10.1057/imfsp.2009.24.

Allain, O. 2022. "A Supermultiplier Model With Two Non-Capacity-Generating Semi-Autonomous Demand Components." *Structural Change and Economic Dynamics* 63: 91–103. https://doi.org/10.1016/j.strueco.2022.09.008.

Arestis, P., and M. Sawyer. 2003. "Reinventing Fiscal Policy." *Journal of Post Keynesian Economics* 26, no. 1: 3–25. http://www.jstor.org/stable/4538859.

Aspromourgos, T., D. Rees, and G. White. 2010. "Public Debt Sustainability and Alternative Theories of Interest." *Cambridge Journal of Economics* 34, no. 3: 433–447. https://doi.org/10.1093/cje/bep010.

Baldacci, E., and M. Kumar. 2010. "Fiscal Deficits, Public Debt, and Sovereign Bond Yields." In *IMF Working Papers 2010/184*. International Monetary Fund.

Barrett, P. 2018. *Interest-Growth Differentials and Debt Limits in Advanced Economies*. International Monetary Fund.

Barro, R. 1979. "On the Determination of the Public Debt." *Journal of Political Economy* 87, no. 5: 940–971. https://doi.org/10.1086/260807.

Barro, R. J. 1995. "Inflation and Economic Growth." In *NBER Working Papers 5326*. National Bureau of Economic Research, Inc.

Bertocco, G., and A. Kalajzić. 2022. "A Critical Analysis of the Loanable Funds Theory: Some Notes on the Non-neutrality of Money." *Economia Politica* 40: 1–21. https://doi.org/10.1007/s40888-022-00286-4.

Blanchard, O. 2019. "Public Debt and Low Interest Rates." *American Economic Review* 109, no. 4: 1197–1229. https://doi.org/10.1257/aer.109.4.1197.

Bofinger, P., and M. Ries. 2017. "Excess Saving and Low Interest Rates: Theory and Empirical Evidence.".

Burnside, C., M. Eichenbaum, and S. Rebelo. 2001. *On the Fiscal Implications of Twin Crises*. CEPR Discussion Papers 2918, C.E.P.R. Discussion Papers.

Caverzasi, E., and A. Godin. 2015. "Post-Keynesian Stock-Flow-Consistent Modelling: A Survey." *Cambridge Journal of Economics* 39, no. 1: 157–187. https://doi.org/10.1093/cje/beu021.

Cecchetti, S., M. Mohanty, and F. Zampolli. 2011. *The Real Effects of Debt*. BIS Working Papers 352. Bank for International Settlements.

Cesaratto, S. 2017. "Initial and Final Finance in the Monetary Circuit and the Theory of Effective Demand." *Metroeconomica* 68, no. 2: 228–258. https://doi.org/10.1111/meca.12132.

Cesaratto, S., F. Serrano, and A. Stirati. 2003. "Technical Change, Effective Demand and Employment." *Review of Political Economy* 15, no. 1: 33–52. https://doi.org/10.1080/09538250308444.

Cochrane, J. H. 2011. "Understanding Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic." *European Economic Review* 55, no. 1 (January): 2–30: Elsevier. https://doi.org/10.1016/j.euroecorev.2010.11.002.

Diamond, P. A. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55, no. 5: 1126–1150. https://www.jstor.org/stable/1811114.

Di Domenico, L. 2022. "Stability and Determinants of the Public Debt-to-GDP Ratio: A Stock-Flow Consistent Investigation." *Università di Siena*.

Di Domenico, L., G. Ciaffi, and D. Romaniello. 2024. "The Monetary Theory of Production and the Supermultiplier: What Determines Savings?" *Review of Political Economy* 36, no. 5: 1–29. https://doi.org/10.1080/09538259.2024.2378373.

Dillard, D. 1980. "A Monetary Theory of Production: Keynes and the Institutionalists." *Journal of Economic Issues* 14, no. 2: 255–273. https://doi.org/10.1080/00213624.1980.11503744.

Dos Santos, C. H., and G. Zezza. 2008. "A Simplified, 'Benchmark' Stock-Flow Consistent Post-Keynesian Growth Model." *Metroeconomica* 59, no. 3: 441–478. https://doi.org/10.1111/j.1467-999x.2008.00316.x.

Dotsey, M. 1994. "Some Unpleasant Supply Side Arithmetic." *Journal of Monetary Economics* 33, no. 3: 507–524: ISSN 0304-3932. https://doi.org/10.1016/0304-3932(94)90041-8.

Fontana, G. 2009. "Fiscal Policy in Today's Endogenous Money World." In *Current Thinking on Fiscal Policy*. Palgrave Macmillan.

Fontana, G., R. Realfonzo, and M. V. Passarella. 2020. "Monetary Economics After the Global Financial Crisis: What Has Happened to the Endogenous Money Theory?" *European Journal of Economics and Economic Policies: Intervention* 17, no. 3: 339–355. https://doi.org/10.4337/ejeep.2020.0056.

Freitas, F., and F. Serrano. 2015. "Growth Rate and Level Effects, the Stability of the Adjustment of Capacity to Demand and the Sraffian Supermultiplier." *Review of Political Economy* 27, no. 3: 258–281. https://doi.org/10.1080/09538259.2015.1067360.

Gaffeo, E., M. Catalano, F. Clementi, D. D. Gatti, M. Gallegati, and A. Russo. 2007. "Reflections on Modern Macroeconomics: Can We Travel Along a Safer Road?" *Physica A: Statistical Mechanics and its Applications* 382, no. 1: 89–97. https://doi.org/10.1016/j.physa.2007.02.011.

Gale, W. G., and P. R. Orszag. 2003. "Economic Effects of Sustained Budget Deficits." *National Tax Journal* 56, no. 3: 463–485. https://doi.org/10.17310/ntj.2003.3.02.

Gallegati, M., and A. Kirman. 2012. "Reconstructing Economics: Agent Based Models and Complexity." *Complexity Economics* 1, no. 1: 5–31. https://doi.org/10.7564/12-coec2.

Giavazzi, F., and M. Pagano. 1990. *Can Severe Fiscal Contractions Be Expansionary? Tales of Two Small European Countries*, Vol. 5, 75–111. National Bureau of Economic Research Macroeconomics Annual. https://doi.org/10.3386/w3372.

Girardi, D., and R. Pariboni. 2016. "Long-Run Effective Demand in the US Economy: An Empirical Test of the Sraffian Supermultiplier Model." *Review of Political Economy* 28, no. 4: 523–544. https://doi.org/10.1080/09538259.2016.1209893.

Godley, W., and M. Lavoie. 2007a. *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. Palgrave Macmillan.

Godley, W., and M. Lavoie. 2007b. "Fiscal Policy in a Stock-Flow Consistent (SFC) Model." *Journal of Post Keynesian Economics* 30, no. 1: 79–100. https://doi.org/10.2753/pke0160-3477300104.

Graziani, A. 2003. *The Monetary Theory of Production*. Cambridge University Press.

Hahn, F. 1973. "On Transaction Costs, Inessential Sequence Economies and Money." *Review of Economic Studies* 40, no. 4: 449–461. https://doi.org/10.2307/2296580.

Hahn, F. H. 1965. "Equilibrium With Transaction Costs." *Econometrica* 39, no. 3: 417–439. https://doi.org/10.2307/1913257.

Hahn, F. H. 1989. "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy." In *General Equilibrium Models of Monetary Economies*, 297–306. Academic Press.

Hein, E. 2018. "Autonomous Government Expenditure Growth, Deficits, Debt, and Distribution in a neo-kaleckian Growth Model." *Journal of Post Keynesian Economics* 41, no. 2: 316–338. https://doi.org/10.1080/01603477.2017.1422389.

Hein, E., and E. Stockhammer. 2010. "Macroeconomic Policy Mix, Employment and Inflation in a Post-Keynesian Alternative to the New Consensus Model." *Review of Political Economy* 22, no. 3: 317–354. https://doi.org/10.1080/09538259.2010.491283.

Hemming, R., and T. Ter-Minassian. 2003. "VI Public Debt Dynamics and Fiscal Adjustment." *Managing Financial Crises* 65. https://www.elibrary.imf.org/view/book/9781589068506/ch06.xml.

Kaldor, N. 1970. "The New Monetarism." *Lloyds Bank Review* 97, no. 1: 18. https://public.econ.duke.edu/~kdh9/Courses/Graduate%20Macro%20History/Readings-1/Kaldor.pdf.

Keynes, J. M. 1933. A Monetary Theory of Production. The Collected Writings of John Maynard Keynes 13: 408–411.

Lavoie, M. 1984. "The Endogenous Flow of Credit and the Post Keynesian Theory of Money." *Journal of Economic Issues* 18, no. 3: 771–797. https://doi.org/10.1080/00213624.1984.11504274.

Lerner, A. P. 1943. *Functional Finance and the Federal Debt*, 38–51. Social research.

Lucarelli, S., and M. Passarella. 2012. *New Research Perspectives in the Monetary Theory of Production*. Bergamo University Press.

Mankiw, N. G. 2020. *Principles of Economics*. Cengage Learning.

McLeay, M., A. Radia, and R. Thomas. 2014. *Money Creation in the Modern Economy*, Q1. Bank of England quarterly bulletin.

Mehrotra, N. R., and D. Sergeyev. 2021. "Debt Sustainability in a Low Interest Rate World." Supplement, *Journal of Monetary Economics* 124: S1–S18. https://doi.org/10.1016/j.jmoneco.2021.09.001.

Minea, A., and A. Parent. 2012. "Is High Public Debt Always Harmful to Economic Growth? Reinhart and Rogoff and Some Complex Nonlinearities.".

Morlin, G. S. 2022. "Growth Led by Government Expenditure and Exports: Public and External Debt Stability in a Supermultiplier Model." *Structural Change and Economic Dynamics* 62: 586–598. https://doi.org/10.1016/j.strueco.2022.03.009.

Nikiforos, M., and G. Zezza. 2017. "Stock-Flow Consistent Macroeconomic Models: A Survey." *Journal of Economic Surveys* 31, no. 5: 1204–1239. https://doi.org/10.1111/joes.12221.

Parguez, A., and M. Seccareccia. 2000. "A Credit Theory of Money: The Monetary Circuit Approach." In *What is Money?*, edited by J. Smithin, 101–123. Routledge.

Pariboni, R. 2016. "Household Consumer Debt, Endogenous Money and Growth: A Supermultiplier-based Analysis." *PSL Quarterly Review* 69, no. 278. https://doi.org/10.13133/2037-3643_69.278_1.

Reinhart, C. M., and K. S. Rogoff. 2010a. "Growth in a Time of Debt." *American Economic Review: Papers & Proceedings* 100, no. 2: 573–578. https://doi.org/10.1257/aer.100.2.573.

Reinhart, C. M., and K. S. Rogoff. 2010b. *Growth in a Time of Debt*. Working Paper 15639. National Bureau of Economic Research.

Rochon, L.-P. 1999. *Credit, Money and Production: An Alternative Post-keynesian Approach*. Edward Elgar.

Rogers, C. 2018. "The Conceptual Flaw in the Microeconomic Foundations of Dynamic Stochastic General Equilibrium Models." *Review of Political Economy* 30, no. 1: 72–83. https://doi.org/10.1080/09538259.2018.1442894.

Rogers, C. 2019. "Misunderstanding Money, Banking and Finance in Dynamic Stochastic General Equilibrium (DSGE) Models." *Open Journal of Economics and Finance* 3: 11–17. https://www.researchgate.net/publication/332083514_Misunderstanding_Money_Banking_and_Finance_in_Dynamic_Stochastic_General_Equilibrium_DSGE_Models.

Ryoo, S., and P. Skott. 2013. "Public Debt and Full Employment in a Stock-Flow Consistent Model of a Corporate Economy." *Journal of Post Keynesian Economics* 35, no. 4: 511–527. https://doi.org/10.2753/pke0160-3477350401.

Sargent, and N. Wallace. 1981. "Some Unpleasant Monetarist Arithmetic." *Quarterly Review* 5, no. 3: issue Fall. https://doi.org/10.21034/qr.531.

Serrano, F. 1995. "Long Period Effective Demand and the Sraffian Supermultiplier." *Contributions to Political Economy* 14, no. 1: 67–90. https://doi.org/10.1093/oxfordjournals.cpe.a035642.

Werner, R. A. 2015. "Do Banks Really Create Money Out of Nothing? Another Empirical Test of the Three Theories of Banking." *International Review of Financial Analysis*: 1–41.

Wray, L. R. 1999. *Theories of Value and the Monetary Theory of Production*. No. 261. Working Paper.

Appendix A

Table A1 reports the description of variables and parameters used in the model.

TABLE A1 | Variables and parameters description.

Description	Symbol
Income from production	Y^P
Total income (including financial rents)	Y
Public spending	G
Consumption	C
Investments	I
Central bank profits	Π^{CB}
Commercial bank profits	Π^B
Public debt	B
Savings	S
Deposits	M
Reserves	H
Check deposits	M^c
Public bonds held by HH	B^h
Public bonds held by CB	B^{CB}
Loans	L
Capital stock	K
Propensity to consume out-of-income	c_1
Propensity to consume out-of-wealth	c_2
Tax rate	θ
Interest rate on public bonds	r^b
Interest rate on deposits	r^m
Interest rate on loans	r^l
Share of wealth held in terms of bonds	β
Share of hoarded financial rent	α
Capital-to-output ratio	v
Growth rate of primary public spending	g_G

Tables A2 and A3 display the balance sheet and the transaction matrix, respectively.

TABLE A2 | Balance sheet matrix of the model.

Assets	Household	Production	Bank	Government	CB	$\sum$
Check deposits[16]	$+M_c$	$+M_c$	$-M_c$			0
Time deposits	$+M$		$-M$			0
HPM			$+H$		$-H$	0
Loans		$-L$	$+L$			0
Fixed capital		$+K_f$				$+K_f$
Public bonds	$+B^h$			$-B$	$+B_{CB}$	0
Net wealth	$-V_h$	$-V_f$	0	$+V_g$	0	$-K_f$
$\sum$	0	0	0	0	0	0

TABLE A3 | Transaction matrix of the model.

		Production			Bank		CB		
	Households	Current	Capital	Government	Current	Capital	Current	Capital	$\sum$
Consumption	$-C$	$+C$							0
Income	$+Y$	$-Y$							0
Investments		$+I$	$-I$						0
Public expenditure		$+G$		$-G$					0
Taxes	$-T$			$+T$					0
Depreciations		$-K_t\delta$	$+K_t\delta$						0
Profits BC				$+\Pi_{cb}$			$-\Pi_{cb}$		0
Profits bank	$+\Pi_b$				$-\Pi_b$				0
Int. deposit	$+rM_{t-1}$				$-rM_{t-1}$				0
Int. loans			$-rL_{t-1}$		$+rL_{t-1}$				0
Int. bond	$+rB^h_{t-1}$			$-rB_{t-1}$			$+rB^{bc}_{t-1}$		0
Int. reserves					$+rH_{t-1}$		$-rH_{t-1}$		0
Δ Deposit time	$-\Delta M$					$+\Delta M$			0
Δ Deposit check	$-\Delta M^c$					$+\Delta M^c$			0
Δ Loans			$+\Delta L$			$-\Delta L$			0
Δ Bonds	$-\Delta B_h$			$+\Delta B$			$-\Delta B_{bc}$		0
Δ Reserves						$-\Delta H$	$+\Delta H$		0
$\sum$	0	0	0	0	0	0	0	0	0

Appendix B

This appendix shows that, at the aggregate level, the only source of financial income for households is public spending on debt servicing. On the one hand, it is highlighted that the effective interest rate paid by the government on public debt is a weighted average of the interest rate on deposits and the interest rate on bonds, where the weights depend on households' preference for public bonds. On the other hand, the income generated by the banking system through the interest rates on loans and the interest on the amount of deposits created by those loans sums to zero.

The Government spending for the debt service, considering CB profits, is:

$$G^B = r^b B - r^b B_{CB} + r^m H \quad \text{(A1)}$$

Since reserves are equal to the amount of public bonds held by the CB, the latter can be rewritten as:

$$G^B = r^b B - B_{CB}\left(r^b - r^m\right) = r^b B^h + r^m B_{CB} \quad \text{(A2)}$$

This equation highlights that the government pays an interest rate equal to the bond rate on the amount of public debt held by households, while it pays an interest rate equal to the deposit rate on the amount of public bonds held by the CB. In fact, this amount corresponds to the deposits created by the CB and held as reserves by the commercial bank. If γ is the share of public debt held by households, the effective interest rate paid by the government on the public debt can be expressed as a weighted average of the interest rates on public bonds and deposits:

$$G^B = B\left[\gamma\left(r^b - r^m\right) + r^m\right] \quad \text{(A3)}$$

Let's consider the household's financial income at the aggregate level. Households generate financial income from the interests accrued on deposits and public bonds, commercial bank profits, while corporate profits are net of interest payments on loans.

$$Y^f = r^m M + r^b B^h + \Pi_B - r^L L \quad \text{(A4)}$$

$$\Pi_B = r^l L + r^m H - r^m M \quad \text{(A5)}$$

Substituting the expression of commercial bank profits, we get:

$$Y^f = r^b B^h + r^m H = r^b B^h + r^m B_{CB} \tag{A6}$$

That is, the only financial income generated by the household sector corresponds to the public debt service.

Appendix C

In this section, we briefly show how the sequential system explicitly represents the dynamics toward the stationary-state dynamics that, in typical income–expenditure models, are only captured instantaneously through their simultaneous resolution. This is done by modeling the production cycles and the mutually causal relationship between income and consumption, which can only be expressed across different production periods. Namely, the consumption in a given period determines the income of that period. The latter is distributed at the end of the period and will form the basis for the consumption of the following period. Let's consider a simple income–expenditure scheme with public expenditure as the only autonomous component and no taxation:

$$\begin{cases} Y_t = \overline{G} + C_t \\ C_t = cY_{t-1} \end{cases} \tag{A7}$$

This can be rewritten as a single difference equation:

$$Y_t = \frac{\overline{G}[1 - c^t]}{1 - c} \tag{A8}$$

The stationary-state income is:

$$Y^* = \lim_{t \to \to \infty} Y_t = \frac{\overline{G}}{1 - c_1} \tag{A9}$$

On the other hand, the solution of the traditional simultaneous system is:

$$\begin{cases} Y = \overline{G} + C \\ C = cY \end{cases} \rightarrow Y^* = \frac{\overline{G}}{1 - c_1} \tag{A10}$$

Likewise, the simultaneous system corresponds to the dynamic system when the stationary condition is imposed ($Y_t = Y_{t-1}$). In this sense, the resolution of the simultaneous system gives the stationary income reached by the system when public spending is constant for a sufficient number of periods.

Appendix D

The R-code to reproduce both analytical and simulated results is the following.

```
rm(list = ls())

# PARAMETERS

T = 2000 # Time periods

tau = 0.3 # Tax rate

c1 = 0.7 # Propensity to consume out-of-income

c2 = 0.05 # Propensity to consume out-of-wealth

Gin = 50 # Initial level of public spending

r = 0.04 # Interest rate on public bonds

rm = r*0.05 # Interest rate on deposits

alpha = 0 # Percentage of income generated by interests on public bonds which is hoarded

v_n = 0.6 # Normal capital/output

kin = Gin*v_n # Initial stock of capital

delta = 0.1 # Capital depreciation rate

gp = 0 # Growth rate of primary public spending

# VARIABLES

# Income from production or GDP

Y = matrix(data = 0, ncol = T)

# Net Income from production (NDP) plus financial income

YF = matrix(data = 0, nrow = T)

# Households consumption

C = matrix(data = 0, ncol = T)

# Households stock of savings

S = matrix(data = 0, ncol = T)

# Public spending

G = matrix(data = 0, ncol = T)

# Stock of public debt

B = matrix(data = 0, ncol = T)

# Disposable income

YD = matrix(data = 0, ncol = T)

# Investments

I = matrix(data = 0, nrow = T)

# Capital Stock

k = matrix(data = 0, nrow = T)

# Stock of loans

L = matrix(data = 0, nrow = T)

# Fiscal revenues

Tax = matrix(data = 0, nrow = T)

# Stock of public bonds held by households

Bh = matrix(data = 0, nrow = T)

# Stock of deposits

M = matrix(data = 0, nrow = T)

# Check deposits

Mc = matrix(data = 0, nrow = T)

# Stock of public bonds held by Central Bank

B_CB = matrix(data = 0, nrow = T)

# Public debt-to-GDP ratio

debpil = matrix(data = 0, ncol = T)

# Stock of Savings-to-GDP ratio

spil = matrix(data = 0, ncol = T)

# GDP growth rate

g = matrix(data = 0, ncol = T)

# Stock of savings growth rate

gs = matrix(data = 0, ncol = T)

# Variation in savings

deltasaving = matrix(data = 0, ncol = T)
```

```
# Public deficit
deficit = matrix(data = 0, ncol = T)
# Redundant equation
redundant = matrix(data = 0, nrow = T)
redundant2 = matrix(data = 0, nrow = T)
H = matrix(data = 0, nrow = T)
v = v_n
# Net Wealth of Sectors
NWCB = matrix(data = 0, nrow = T)
NWG = matrix(data = 0, nrow = T)
NWB = matrix(data = 0, nrow = T)
NWP = matrix(data = 0, nrow = T)
NWH = matrix(data = 0, nrow = T)
# BALANCE SHEET
namesSectors = c("HOUSEHOLDS","PRODUCTION", "BANKS", "GOVERNMENT", "CB", "SUM")
namesStocks = c("Deposits", "Loans", "Fixed Capital", "Bond", "HPM", "Net Worth","SUM")
data <-data.frame(matrix(0,ncol = length(namesSectors),nrow = length(namesStocks)))
BS <-array(data = 0,dim = c(length(namesStocks),length(namesSectors),T),dimnames = list(namesStocks, namesSectors))
colnames(BS) = namesSectors
rownames(BS) = namesStocks
# TRANSACTION MATRIX
namesSectors = c("HOUSEHOLDS", "PRODUCTION current", "PRODUCTION capital", "BANKS", "GOVERNMENT", "CB", "SUM")
namesAssets = c("Consumption", "Income", "Investments", "GovExp", "Tax", "Depreciation", "Interests bond", "deltadeposits tot", "deltaloan", "deltaBond", "deltaHb", "SUM")
data <-data.frame(matrix(0,ncol = length(namesSectors),nrow = length(namesAssets)))
TM <-array(data = 0,dim = c(length(namesAssets),length(namesSectors),T),dimnames = list(namesAssets, namesSectors))
colnames(TM) = namesSectors
rownames(TM) = namesAssets
# Model: Define time loop
for (t in 1:T){
G[t] = Gin
# Public spending
if(t > 1){
G[t] = G[t−1]*(1 + gp) }
if(t > 1){
# Updating capital stock
k[t] = k[t−1]*(1-delta) + I[t−1]
# Investment function
I[t] = Y[t−1]*v_n−k[t]*(1-delta)
if(t > 2){
# Consumption equation
C[t]=((Y[t−1]+B[t−2]*r−k[t−1]*delta)*(1-tau)-B[t−2]*r*alpha*(1-tau))*c1+S[t−1]*c2
}else{
C[t]=(Y[t−1]-k[t−1]*delta)*(1-tau)*c1+S[t−1]*c2
}
# Total production
Y[t] = C[t]+G[t]+I[t]
#Computing total income (NDP + financial income)
YF[t] = C[t]+G[t]+I[t]+B[t−1]*r−k[t]*delta
# Computing public debt
B[t] = B[t−1]*(1 + r) + G[t]-YF[t]*tau-(r*B_CB[t−1]t−rm*H[t−1])
# Computing Stock of Savings
S[t] = S[t−1]+YF[t−1]*(1-tau)-C[t]
# Computing the GDP and savings growth rate
g[t]=(Y[t]-Y[t−1])/Y[t−1]
gs[t]=(S[t]-S[t−1])/S[t−1]
# Computing public deficit
deficit[t] = G[t]-YF[t]*tau + r*B[t−1]
# Stock of loans
L[t] = I[t]+L[t−1]*(1-delta)
#redundant[t] = S[t]-L[t]+Mc[t]-B[t]
deltasaving[t] = S[t]-S[t−1]
# SFC check
redundant2[t] = S[t]-I[t]-k[t]*(1-delta)+(Y[t]+B[t−1]*r−k[t]*delta)*(1-tau)-B[t]
}else{
G[t] = Gin
I[t] = max(G[t]*v_n−k[t]*(1-delta),0)
Y[t] = G[t]+I[t]
YF[t] = G[t]+I[t]
B[t] = G[t]-YF[t]*tau
deficit[t] = G[t]-YF[t]*tau
L[t] = I[t]
redundant2[t] = S[t]-I[t]-k[t]*(1-delta)+(Y[t]-k[t]*delta)*(1-tau)-B[t]
}
# Public bonds held by Households
Bh[t] = B[t]
# Fiscal revenues
Tax[t] = YF[t]*tau
# Updating CB bonds
B_CB[t] = B[t]-Bh[t]
# Updating stock of deposits
M[t] = S[t]-Bh[t]
# Check deposits
Mc[t] = YF[t]*(1-tau)
```

```
H[t] = M[t] + Mc[t]-L[t]
# Disposable income
YD[t] = YF[t]*(1-tau)
# Net wealth of sectors
NWCB[t] = B_CB[t]−H[t]
NWG[t] = −B[t]
NWB[t] = L[t]-M[t]-Mc[t] + H[t]
NWP[t] = k[t]-L[t]
NWH[t] = Mc[t]+M[t] + Bh[t]
debpil[t] = B[t]/Y[t]
spil[t] = S[t]/Y[t]
redundant[t] = B_CB[t] − H[t]
if(t > 1){
if(TRUE){
#THE BALANCE SHEET OF THE ECONOMY ##################################################################################
#DEPOSITS
BS[which(rownames(BS) = = "Deposits"),which(colnames(BS) = = "HOUSEHOLDS"),t] = M[t]+Mc[t]
BS[which(rownames(BS) = = "Deposits"),which(colnames(BS) = = "PRODUCTION"),t] = 0
BS[which(rownames(BS) = = "Deposits"),which(colnames(BS) = = "BANKS"),t] = −(M[t]+Mc[t])
BS[which(rownames(BS) = = "Deposits"),which(colnames(BS) = = "GOVERNMENT"),t] = 0
#LOANS
BS[which(rownames(BS) = = "Loans"),which(colnames(BS) = = "PRODUCTION"),t] = −L[t]
BS[which(rownames(BS) = = "Loans"),which(colnames(BS) = = "BANKS"),t] = L[t]
#FIXED CAPITAL
BS[which(rownames(BS) = = "Fixed Capital"),which(colnames(BS) = = "PRODUCTION"),t] = k[t]
#HPM $ reserves
BS[which(rownames(BS) = = "HPM"),which(colnames(BS) = = "CB"),t] = −H[t]
BS[which(rownames(BS) = = "HPM"),which(colnames(BS) = = "BANKS"),t] = +H[t]
#Bonds
BS[which(rownames(BS) = = "Bond"),which(colnames(BS) = = "GOVERNMENT"),t] = −B[t]
BS[which(rownames(BS) = = "Bond"),which(colnames(BS) = = "CB"),t] = B_CB[t]
BS[which(rownames(BS) = = "Bond"),which(colnames(BS) = = "HOUSEHOLDS"),t] = Bh[t]
#NET WORTH
BS[which(rownames(BS) = = "Net Worth"),which(colnames(BS) = = "HOUSEHOLDS"),t] = −NWH[t]
BS[which(rownames(BS) = = "Net Worth"),which(colnames(BS) = = "PRODUCTION"),t] = −NWP[t]
BS[which(rownames(BS) = = "Net Worth"),which(colnames(BS) = = "BANKS"),t] = -NWB[t]
BS[which(rownames(BS) = = "Net Worth"),which(colnames(BS) = = "GOVERNMENT"),t] = −NWG[t]
BS[which(rownames(BS) = = "Net Worth"),which(colnames(BS) = = "CB"),t] = −NWCB[t]
#CONSISTENCY CHECK ROWs
BS[which(rownames(BS) = = "Deposits"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[which(rownames(BS) = = "Deposits"),t]), digits = 2)
BS[which(rownames(BS) = = "Net Worth"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[which(rownames(BS) = = "Net Worth"),t]), digits = 2)
BS[which(rownames(BS) = = "Loans"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[which(rownames(BS) = = "Loans"),t]), digits = 2)
BS[which(rownames(BS) = = "Bond"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[which(rownames(BS) = = "Bond"),t]), digits = 2)
BS[which(rownames(BS) = = "Fixed Capital"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[which(rownames(BS) = = "Fixed Capital"), t]), digits = 2)
BS[which(rownames(BS) = = "HPM"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[which(rownames(BS) = = "HPM"),t]), digits = 2)
#CONSISTENCY CHECK COLUMNSs
BS[which(rownames(BS) = = "SUM"),which(colnames(BS) = = "HOUSEHOLDS"),t] = round(sum(BS[,which(colnames(BS) = = "HOUSEHOLDS"),t]), digits = 2)
BS[which(rownames(BS) = = "SUM"),which(colnames(BS) = = "PRODUCTION"),t] = round(sum(BS[,which(colnames(BS) = = "PRODUCTION"),t]), digits = 2)
BS[which(rownames(BS) = = "SUM"),which(colnames(BS) = = "BANKS"),t] = round(sum(BS[,which(colnames(BS) = = "BANKS"),t]),2)
BS[which(rownames(BS) = = "SUM"),which(colnames(BS) = = "GOVERNMENT"),t] = round(sum(BS[,which(colnames(BS) = = "GOVERNMENT"),t]), digits = 2)
BS[which(rownames(BS) = = "SUM"),which(colnames(BS) = = "CB"), t] = round(sum(BS[,which(colnames(BS) = = "CB"),t]), digits = 2)
BS[which(rownames(BS) = = "SUM"),which(colnames(BS) = = "SUM"),t] = round(sum(BS[,which(colnames(BS) = = "SUM"),t]), digits = 2)
#TRANSACTION MATRIX#########
#Consumption
TM[which(rownames(TM) = = "Consumption"),which(colnames(TM) = = "HOUSEHOLDS"),t] = −C[t]
TM[which(rownames(TM) = = "Consumption"),which(colnames(TM) = = "PRODUCTION current"),t] = C[t]
TM[which(rownames(TM) = = "Income"),which(colnames(TM) = = "HOUSEHOLDS"),t]=(Y[t]−L[t−1]*delta)
TM[which(rownames(TM) = = "Income"),which(colnames(TM) = = "PRODUCTION current"),t] = -(Y[t]-L[t−1]*delta)
#Investments
TM[which(rownames(TM) = = "Investments"),which(colnames(TM) = = "PRODUCTION current"),t] = I[t]
```

```
TM[which(rownames(TM) = = “Investments”),which(colnames(TM) = = “PRODUCTION capital”),t] = −I[t]

#GOV EXPENDITURE

TM[which(rownames(TM) = = “GovExp”),which(colnames(TM) = = “GOVERNMENT”),t] = −G[t]

TM[which(rownames(TM) = = “GovExp”),which(colnames(TM) = = “PRODUCTION current”),t] = G[t]

#TAX

TM[which(rownames(TM) = = “Tax”),which(colnames(TM) = = “HOUSEHOLDS”),t] = −Tax[t]

TM[which(rownames(TM) = = “Tax”),which(colnames(TM) = = “GOVERNMENT”),t] = Tax[t]

#Depreciations

if(t > 1){

TM[which(rownames(TM) = = “Depreciation”),which(colnames(TM) = = “PRODUCTION current”),t] = −L[t−1]*delta

TM[which(rownames(TM) = = “Depreciation”),which(colnames(TM) = = “PRODUCTION capital”),t] = L[t−1]*delta

}

#Deltacash- > deltadeposit

if(t > 1){

TM[which(rownames(TM) = = “deltadeposits tot”),which(colnames(TM) = = “HOUSEHOLDS”),t] = −(M[t]+Mc[t]-M[t−1]−Mc[t−1])

TM[which(rownames(TM) = = “deltadeposits tot”),which(colnames(TM) = = “BANKS”),t] = M[t]+Mc[t]−M[t−1]−Mc[t−1]

}else{

TM[which(rownames(TM) = = “deltadeposits tot”),which(colnames(TM) = = “HOUSEHOLDS”),t] = −M[t]−Mc[t]

TM[which(rownames(TM) = = “deltadeposits tot”),which(colnames(TM) = = “BANKS”),t] = M[t]+Mc[t]

}

if(t > 1){

TM[which(rownames(TM) = = “Interests bond”),which(colnames(TM) = = “HOUSEHOLDS”),t] = +r*B[t−1]

TM[which(rownames(TM) = = “Interests bond”),which(colnames(TM) = = “GOVERNMENT”),t] = −r*B[t−1]

}

if(t > 1){

#DELTALOANS

TM[which(rownames(TM) = = “deltaloan”),which(colnames(TM) = = “PRODUCTION capital”),t] = L[t]−L[t−1]

TM[which(rownames(TM) = = “deltaloan”),which(colnames(TM) = = “BANKS”),t] = -(L[t]−L[t−1])

TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “GOVERNMENT”),t] = B[t]−B[t−1]

TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “CB”),t] = -(B_CB[t]-B_CB[t−1])

TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “HOUSEHOLDS”),t] = −(Bh[t]-Bh[t−1])

# TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “GOVERNMENT”),t] = stockdebtG[t]-stockdebtG[t−1]

# TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “CB”),t] = −(BondCB[t]-BondCB[t−1])

# TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “HOUSEHOLDS”),t] = −(sum(BondHH[t,])−sum(BondHH[t−1,]))

}else{

#DELTALOANS

TM[which(rownames(TM) = = “deltaloan”),which(colnames(TM) = = “PRODUCTION capital”),t] = L[t]

TM[which(rownames(TM) = = “deltaloan”),which(colnames(TM) = = “BANKS”),t] = −L[t]

TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “GOVERNMENT”),t] = B[t]

TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “CB”),t] = -B_CB[t]

TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “HOUSEHOLDS”),t] = −Bh[t]

}

#delta Hb

if(t > 1){

TM[which(rownames(TM) = = “deltaHb”),which(colnames(TM) = = “CB”),t] = H[t]−H[t−1]

TM[which(rownames(TM) = = “deltaHb”),which(colnames(TM) = = “BANKS”),t] = −(H[t]−H[t−1])

}else{

TM[which(rownames(TM) = = “deltaHb”),which(colnames(TM) = = “CB”),t] = H[t]

TM[which(rownames(TM) = = “deltaHb”),which(colnames(TM) = = “BANKS”),t] = −H[t]

}

#CONSISTENCY CHECK ROWs

TM[which(rownames(TM) = = “Consumption”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “Consumption”),t]),digits = 2)

TM[which(rownames(TM) = = “Income”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “Income”),t]),digits = 2)

TM[which(rownames(TM) = = “Investments”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “Investments”),t]),digits = 2)

TM[which(rownames(TM) = = “GovExp”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “GovExp”),t]),digits = 2)

TM[which(rownames(TM) = = “Tax”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “Tax”),t]),digits = 2)

TM[which(rownames(TM) = = “Depreciation”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “Depreciation”),t]),digits = 2)

TM[which(rownames(TM) = = “Interests bond”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “Interests bond”),t]),digits = 2)

TM[which(rownames(TM) = = “deltadeposits tot”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “deltadeposits tot”),t]),digits = 2)

TM[which(rownames(TM) = = “deltaloan”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “deltaloan”),t]),digits = 2)
```

```
TM[which(rownames(TM) = = “deltaBond”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “deltaBond”),t]),digits = 2)

TM[which(rownames(TM) = = “deltaHb”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[which(rownames(TM) = = “deltaHb”), t]),digits = 2)

#CONSISTENCY CHECK COLUMNSs

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “HOUSEHOLDS”),t] = round(sum(TM[,which(colnames(TM) = = “HOUSEHOLDS”),t]),2)

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “PRODUCTION current”),t] = round(sum(TM[,which(colnames (TM) = = “PRODUCTION current”),t]),2)

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “PRODUCTION capital”),t] = round(sum(TM[,which(colnames (TM) = = “PRODUCTION capital”),t]),2)

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “BANKS”),t] = round(sum(TM[,which(colnames(TM) = = “BANKS”),t]),2)

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “GOVERNMENT”),t] = round(sum(TM[,which(colnames(TM) = = “GOVERNMENT”),t]),2)

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “CB”),t] = round(sum(TM[,which(colnames(TM) = = “CB”),t]),2)

TM[which(rownames(TM) = = “SUM”),which(colnames(TM) = = “SUM”),t] = round(sum(TM[,which(colnames(TM) = = “SUM”),t]),2)

}

}}

c_1 = c1

c_2 = c2

f = 1-tau

m = rbind(c(1),c(2),c(3),c(4),c(5))
layout(m)

par(tcl = −0.2,mgp = c(2.0.4.0))

par(mar = c(1.4.1.8.3.6.2))

mini = min(Y)

maxi = max(Y)

plot(Y[1:T], ylim = range(mini, maxi),lwd = 2,main = “GDP”,type = “l“, xlab = “Tempo”,ylab = “”)

mini = min(debpil)

maxi = max(debpil)*1.1
plot(debpil[1:T],col = 3, lwd = 2,ylim = range(mini, maxi), main = “Public debt-to-GDP ratio”,type = “l”,xlab = “Tempo”,ylab = “”)

mini = min(spil)

maxi = max(spil)*1.1

plot(spil[1:T], col = 3,lwd = 2, ylim = range(mini, maxi),main = “Private savings-to-GDP ratio”,type = “l”,xlab = “Tempo”,ylab = “”)

mini = min(g[1:T])

maxi = max(g[1:T])

plot(g[1:T], col = 1,lwd = 2,ylim = range(mini, maxi),main = “GDP growth rate”,type = “l”,xlab = “Tempo”,ylab = “”)

mini = 1000

maxi = −1000

plot(redundant[1:T],col = 1,lwd = 2,lty = 4,ylim = range(mini, maxi), main = “Redundant equation“,type = “l”,xlab = “Tempo”,ylab = “”)

print(TM[,t])

print(BS[,t])

if(alpha = = 0){

if(round(g[T],6) = = 0){

#Stationary state income #

print(“Stationary GDP (analytical solution):”)

print((Gin*(c_2-(1-tau)*r*(1-c_1 + c_2)))/(c2*tau*(1-delta*v)+(1-tau)*r*((1-c_1 + c_2)*(delta*v−1) + c2*v)))

print(“Stationary GDP (simulated solution):”)

print(Y[T])

# Stock-flow ratio in stationary state:#######

print(“Private savings over GDP (analytical solution):”)

print(((1-c_1)*(1-tau)*((delta + r)*v−1))/((1-c1)*r*(1-tau) + c2*(r*(1-tau)−1)))

print(“Private savings over GDP (simulated solution):”)

print(spil[T])

print(“Public debt over GDP (analytical solution): ”)

print(((1-tau)*(1-c1+c2)*(1-delta*v)-c2*v)/(c2-r*(1-tau)*(1-c1+c2)))

print(“Public debt over GDP (analytical solution): ”)

print(B[T]/Y[T])

}else{

# The endogenous growth rate:

d = delta

f = 1-tau

p = gp

v1=(−v + c2* v + d* v - c2* d* v + c1* d* f* v - f* r* v + c2* f* r* v + d* f* r* v) #1 +

v2=(c1* f - c2* f + c1* f* r - c2* f* r + 2* v - c2* v - d* v + f* r* v) #1^2

v3=(−1 + c2 - c1* f - f* r - v)

v4 = -c1*d*f*v + c2*d*f*v-c1*d* f* r* v + c2* d* f* r* v

a = 1

b = v3

c = v2

d = v1

e = v4

q = 12*a*e−3*b*d + c^2

s = 27*a*d^2−72*a*c*e+27*b^2*e−9*b*c*d+2*c^3

z=(8*a*c−3*b^2)/(8*a^2)

d0=((s+(s^2−4*q^3)^0.5)/2)^(1/3)

Sp=(8*a^2*d−4*a*b*c + b^3)/(8*a^3)

Q = 0.5*(-(2/3)*z+1/(3*a)*(d0+q/d0))^0.5

x1 = -b/(4*a)-Q+0.5*(−4*Q^2−2*z + Sp/Q)^0.5

x2 = -b/(4*a)-Q−0.5*(−4*Q^2−2*z + Sp/Q)^0.5

x3 = -b/(4*a) + Q+0.5*(−4*Q^2−2*z-Sp/Q)^0.5

x4 = −b/(4*a) + Q−0.5*(−4*Q^2−2*z-Sp/Q)^0.5
```

```
roots = c(x1,x2,x3,x4)
sol = max(roots[which(is.nan(roots) = = FALSE)])
if((sol−1) < gp){
print("Standard regime: The growth rate of the public debt service converges to the growth rate of the primary public spending")
print("The growth rate is (analytical solution)")
print(round(gp, 3))
}else{
print("Interest-led regime: The growth rate of the public debt service is higher than the growth rate of the primary public spending")
print("The growth rate is (analytical solution)")
print(round(sol−1.3))
}
print("The growth rate is (simulated solution)")
print(round(g[T],3))
### Stock-flow ratios in steady growth #############
# Private savings over income from production
print("Private savings over GDP (analytical solution):")
print(((1 - c1)*((1 - tau)*(1 + g[T])^2 - v*(r*(1 - tau) + delta*(1 - tau))))/((1 + g[T])*(r*(1 - tau)*(−1 + c1 - c2 - g[T]) + (1 + g[T])*(c2 + g[T]))))
print("Private savings over GDP (simulated solution):")
print(spil[T])
# Public debt over GDP
print("Public debt over GDP (analytical solution): ")
print(((1 -tau)*(1 + c2 - c1 + g[T])*((1 + g[T])^2 -delta*v) - v*((1 + g[T])*(c2 + g[T])))/((1 + g[T])*((c2 + g[T])*(g[T] - r*(1 - tau) + 1) - (1 - c1)*r*(1 - tau))))
print("Public debt over GDP (simulated solution): ")
print(B[T]/Y[T])
}
}
stabilityAnalsysis = "no"
if(stabilityAnalsysis = = "yes"){
library(tibble)
library(purrr)
# VAR system
check_BY_ratio < - function(c1 = 0.7, c2 = 0.05, r = 0.04, alpha = 0, tau = 0.3, v = 0.6, delta = 0.1) {
Gamma < - matrix(0, 6, 6)
Gamma[1, 1] <- c1 * (1 - tau) + v
Gamma[1, 3] <- c2
Gamma[1, 4] <- -v * (1 - delta)
Gamma[1, 5] <- -(1−alpha) * r * c1 * (1 - tau)
Gamma[1, 6] <- -v * delta * c1 * (1 - tau)
Gamma[2, 1] <- tau * ((1 - tau) * (1 - c1) - v)
Gamma[2, 2] <- (1 + r) * (1 - tau)
Gamma[2, 3] <- tau * (1 - c2)
Gamma[2, 4] <- -v * delta * tau * (1 - tau)
Gamma[2, 5] <- r * tau * (1 - c1) * (1 - tau)
Gamma[2, 6] <- c1 * v * delta * tau * (1 - tau)
Gamma[3, 1] <- (1 - c1) * (1 - tau)
Gamma[3, 3] <- 1 - c2
Gamma[3, 5] <- r * (1 - c1) * (1 - tau)
Gamma[3, 6] <- -v * delta * (1 - c1) * (1 - tau)
Gamma[4, 1] <- 1
Gamma[5, 2] <- 1
Gamma[6, 4] <- 1
eig < - eigen(Gamma)
i_dom < - which.max(Mod(eig$values))
lambda_dom < - eig$values[i_dom]
v_dom < - eig$vectors[, i_dom]
vY < - round(Re(v_dom[1]),5)
vB < - round(Re(v_dom[2]),5)
ratio < - vB/vY
return(list(
lambda_dom = lambda_dom,
vY = vY,
vB = vB,
ratio = ratio
))
}
check_BY_ratio()
grid < - expand.grid(
c1 = seq(0.01, 0.99, 0.25),
c2 = seq(0, 0.99, 0.25),
r = seq(0, 0.99, 0.25),
alpha = seq(0, 1, 0.25),
tau = seq(0.02, 1, 0.25),
v = seq(0.02, 1, 0.25),
delta = seq(0.02, 1, 0.25)
)
grid < - grid[1:1000, ]
# Multivariate simultations
results < - pmap_dfr(grid, function(c1, c2, r, alpha, tau, v, delta) {
res < - tryCatch({
out < - check_BY_ratio(c1, c2, r, alpha, tau, v, delta)
tibble(
c1 = c1, c2 = c2, r = r, alpha = alpha, tau = tau, v = v, delta = delta, lambda_dom = Re(out$lambda_dom), ratio = out$ratio
)
}, error = function(e) {
tibble(
c1 = c1, c2 = c2, r = r, alpha = alpha, tau = tau, v = v, delta = delta, lambda_dom = NA, ratio = NA
)
```

```
})
})
print(head(results))
InstabilityConditions < - results[!is.finite(results$ratio), ]
print("Parameters combinations generating instable solutions:")
print(InstabilityConditions)
}
```