Search for a long-lived spin-0 mediator in $b \to s$ transitions at the Belle II experiment (supplementary information)

Supplementary information

This material is submitted as supplementary information for the Electronic Physics Auxiliary Publication Service.

Peaking background selection

The transverse distance requirement of the displaced vertex is increased to $d_v > 0.2$ cm in S mass regions in the vicinity of various known narrow SM states with two-body decays $D^0 \to K^+K^-$ and $D^0 \to K^-\pi^+$, $J/\psi \to e^+e^-$ and $J/\psi \to \mu^+\mu^-$, $\psi(2S) \to e^+e^-$ and $\psi(2S) \to \mu^+\mu^-$, hadronic decays of η_c, χ_{c1}, and $\eta_c(2S)$, and $\phi \to K^+K^-$. These backgrounds include missing or misidentified particles leading to wrong mass hypotheses and hence invariant masses shift with respect to the known particle masses. The resulting final-state dependent mass regions are summarized in Table I.

Axionlike particles with coupling to fermions

For the model-dependent search for axionlike particles with coupling to fermions, we perform a combined fit in all relevant and kinematically accessible analysis channels, again separately for various lifetimes. We follow the conventions of Refs. [1, 2]. For masses $m_a \leq 2m_\mu$ we use the pseudoscalar branching fractions calculation from Ref. [3], neglecting interference with the π^0 meson. For masses $2m_\mu < m_a \leq 1.2$ GeV/c^2 we use a chiral model and for $m_a > 1.2$ GeV/c^2, we employ the spectator approach [4]. The interference with the η_c is not included in this calculation. Our observed upper limits are shown in Fig. X 1.

Table I: Selection requirements on two-body masses (in GeV/c^2) to reject peaking backgrounds for the different S final states.

	e^+e^-	$\mu^+\mu^-$	$\pi^+\pi^-$	K^+K^-
D^0	[1.0, 1.3]	[1.7, 1.8]	[1.65, 1.75]	[1.75, 1.85]
J/ψ	[3.00, 3.15]		-	-
$\psi(2S)$	[3.65, 3.75]		-	-
η_c	-	-	[2.85, 3.15]	[2.80, 3.20]
$\chi_{c1}, \eta_c(2S)$	-	-	[3.4, 3.8]	
ϕ	-	-	-	[1.00, 1.04]

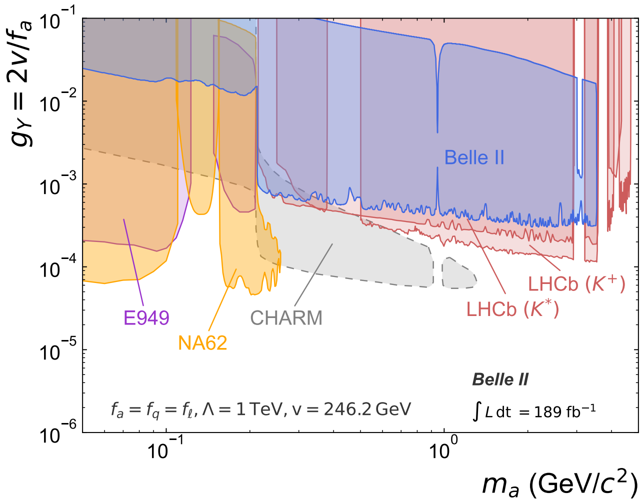
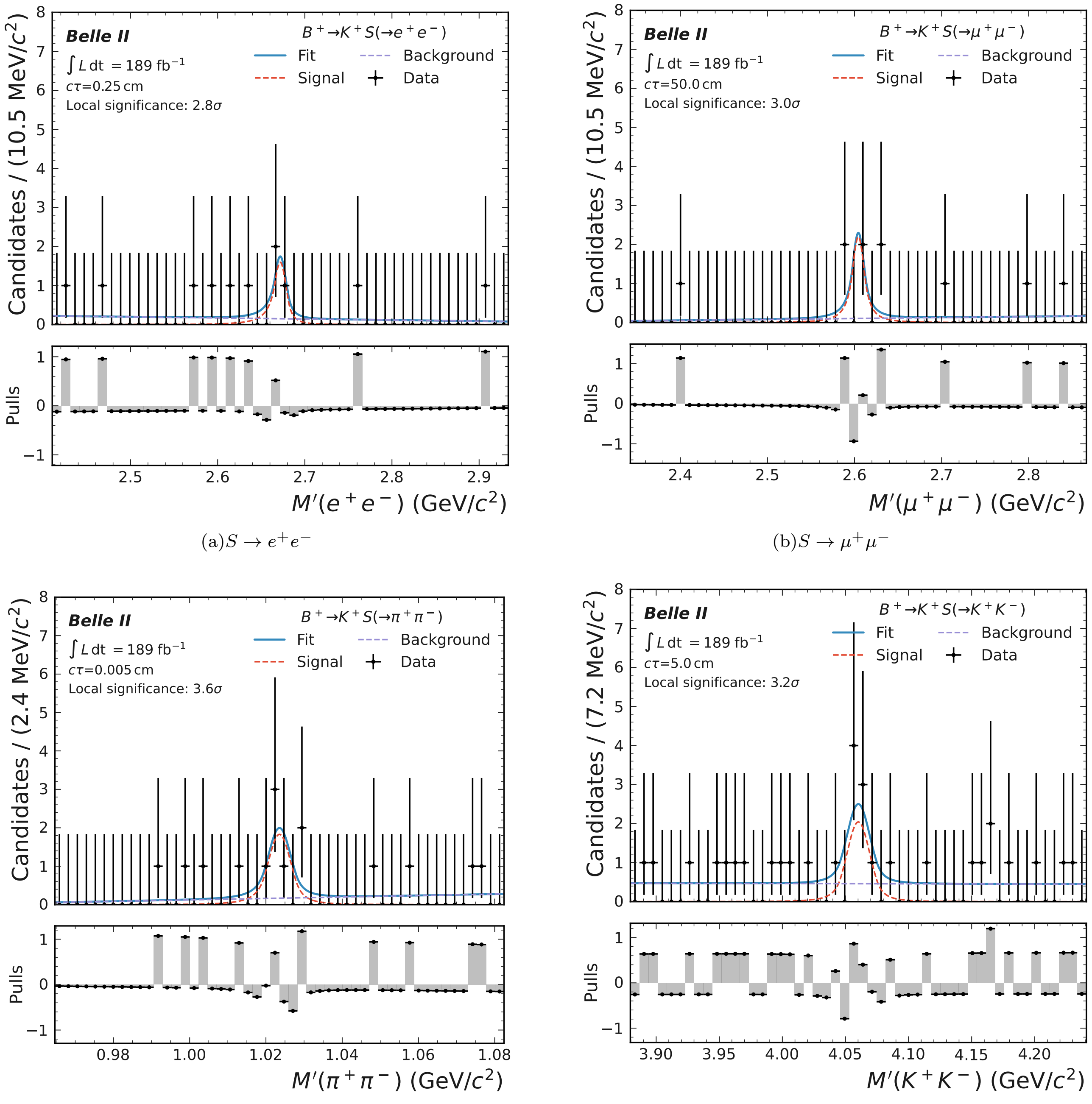


Figure X 1: Exclusion regions in the plane of the coupling $g_Y = 2v/f_a$ with the vacuum expectation value v and the ALP mass m_a from this work (blue) together with existing constraints from LHCb [5, 6], E949 [7], and CHARM [8]. The exclusion regions from Belle II, LHCb, and CHARM correspond to 95% CL, while E949 correspond to 90% CL. All existing limits are taken from Ref. [1]. The constraint colored in gray with dashed outline is a reinterpretation not performed by the experimental collaboration and without access to raw data.

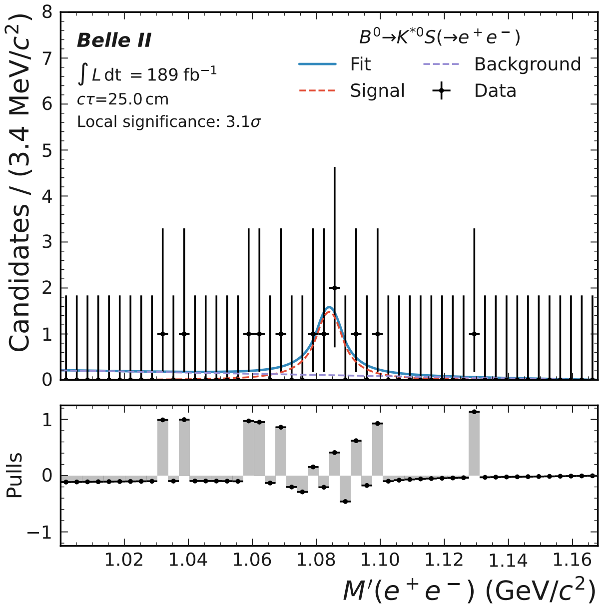
Example fits

Figures X 2 and X 3 show the fits that resulted in the largest local significance observed in the single channel fits. Figure X 4 shows the fit for $m_S = 2.619$ GeV/c^2 for a lifetime of $c\tau = 100$ cm that resulted in the largest local significance observed in the combined fits.

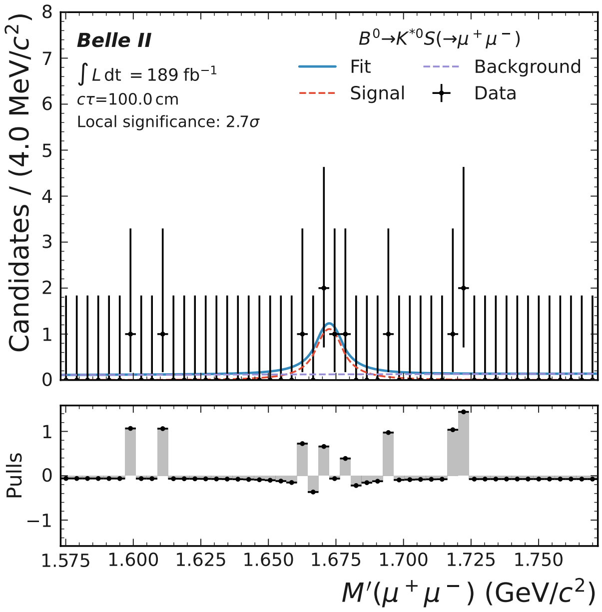


(a)$S \to e^+e^-$ (b)$S \to \mu^+\mu^-$ (c)$S \to \pi^+\pi^-$ (d)$S \to K^+K^-$

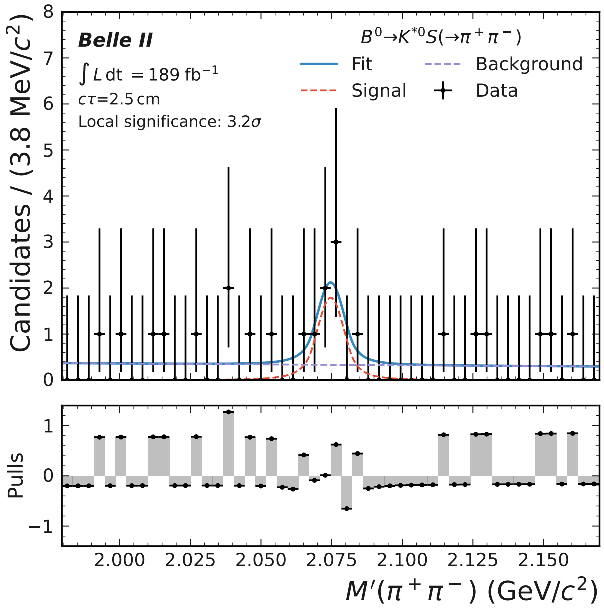
Figure X 2: Single channel fits with highest local significances for $B^+ \to K^+S(\to x^+x^-)$. The bottom panels show the pulls per bin, defined as the difference between data and simulation, divided by the statistical uncertainty of the data.



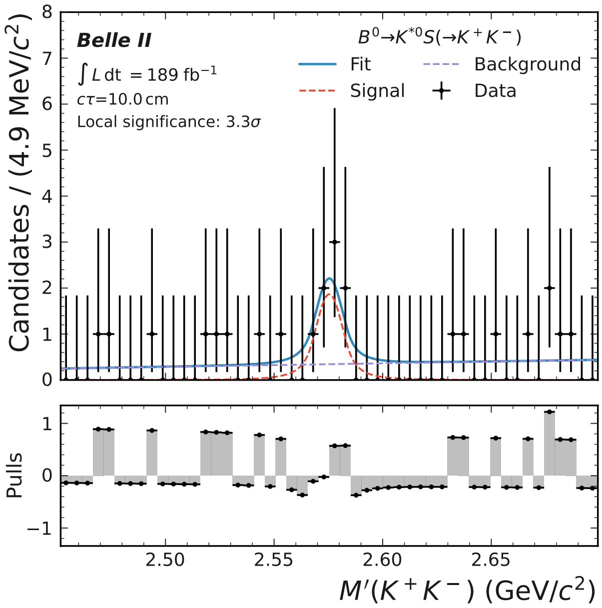
(a)E



(b)F



(c)G



(d)H

Figure X 3: Single channel fits with highest local significances for $B^0 \to K^{*0}(\to K^+\pi^-)S(\to x^+x^-)$. The bottom panels show the pulls per bin, defined as the difference between data and simulation, divided by the statistical uncertainty of the data.

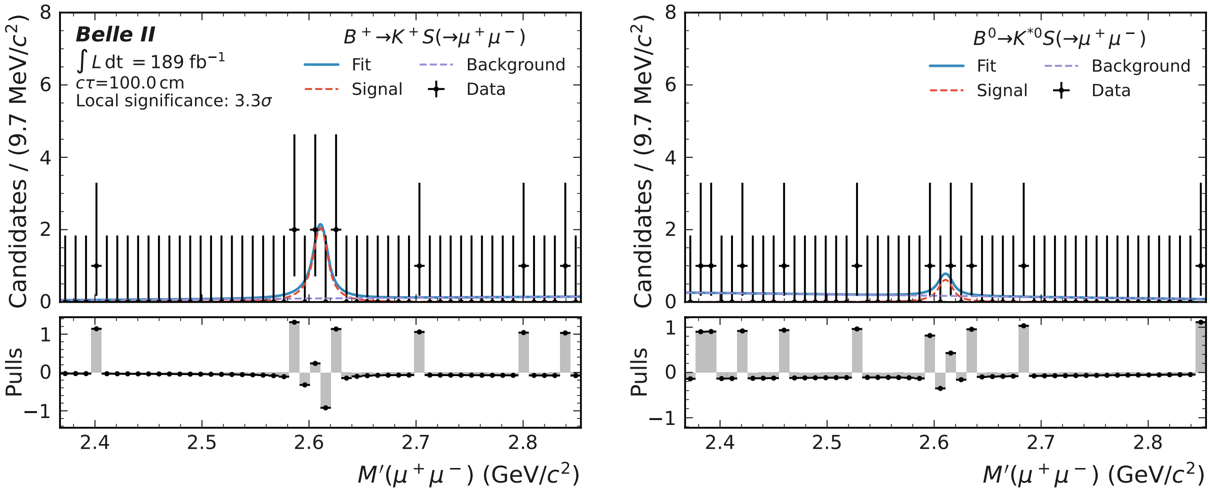
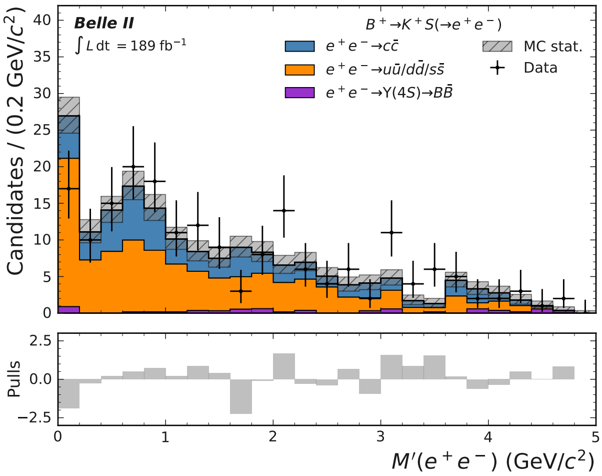


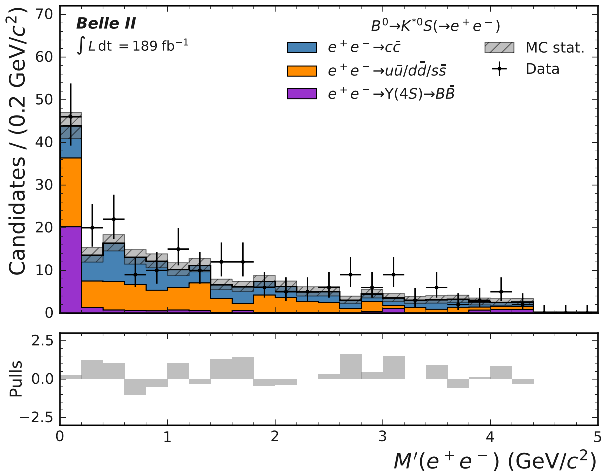
Figure X 4: Combined fit for $m_S = 2.619$ GeV/c^2 in the (left) $B^+ \to K^+ S, S \to \mu^+\mu^-$ and (right) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$ states with background pdfs (purple dashed), signal pdfs (red dashed) and total pdfs (blue). The combined signal correspond to a local significance including systematic uncertainties of $S = 3.3\sigma$. The bottom panels show the pulls per bin, defined as the difference between data and simulation, divided by the statistical uncertainty of the data.

Distributions

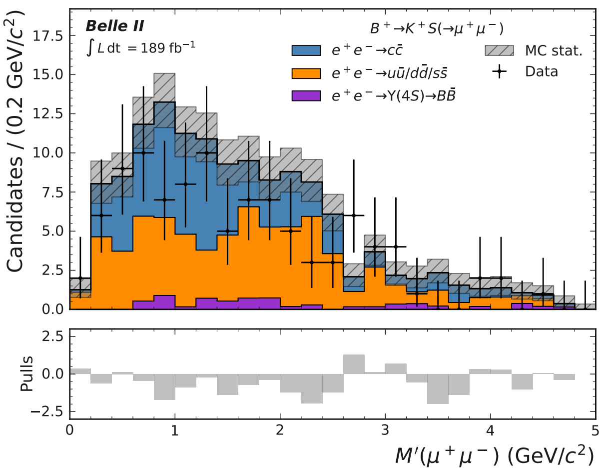
Figures X 5 and X 6 show the $M'(x^+x^-)$ distributions together with the stacked contributions from the different simulated SM background samples. Normalization discrepancies observed in some of the following plots are not a concern since backgrounds are floating in all fits.



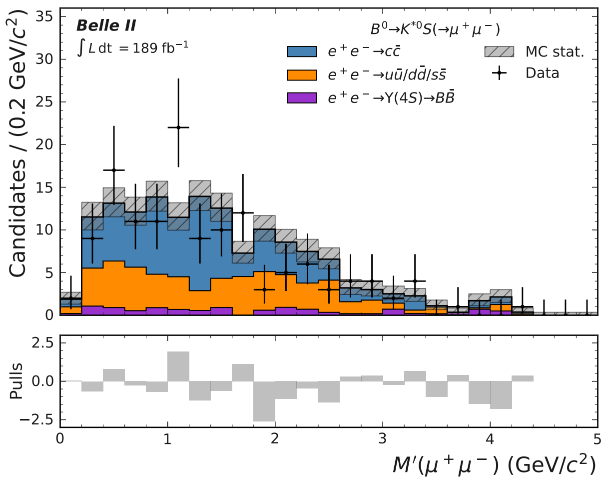
(a)$B^+ \to K^+S, S \to e^+e^-$.



(b)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$.

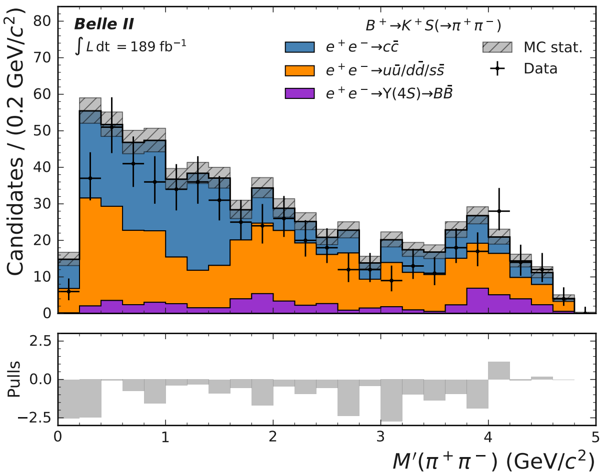


(c)$B^+ \to K^+S, S \to \mu^+\mu^-$.

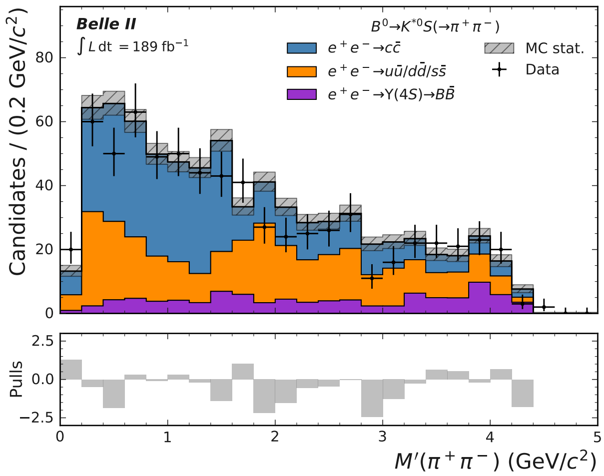


(d)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$.

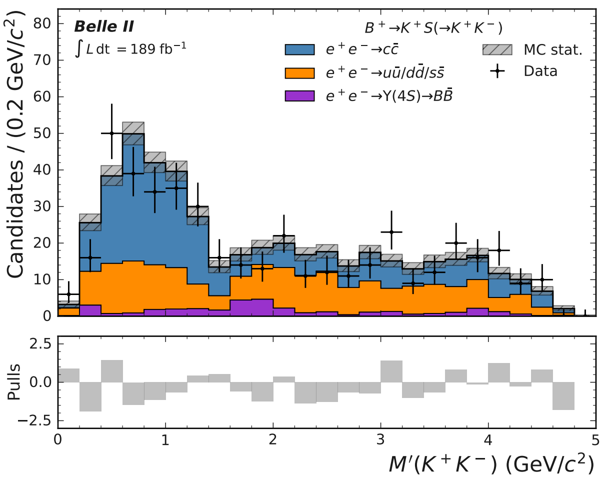
Figure X 5: Distribution of $M'(x^+x^-)$ together with the stacked contributions from the various simulated SM background samples. Simulation is normalized to a luminosity of 189 fb^{-1}. The hatched area represents the statistical uncertainty of the SM background prediction. The bottom panels show the pulls per bin, defined as the difference between data and simulation, normalized to the statistical uncertainties added in quadrature.



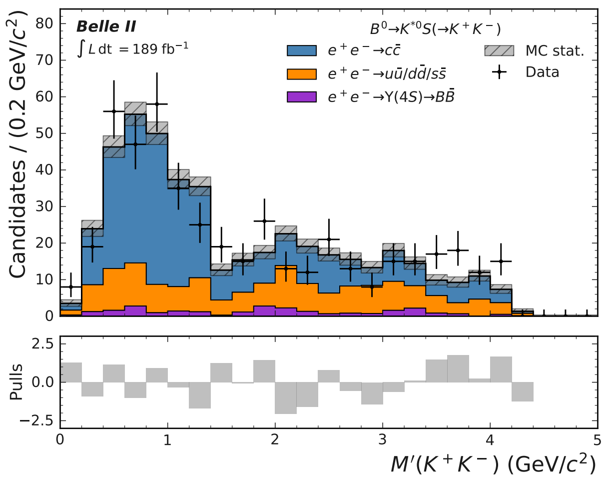
(a) $B^+ \to K^+S, S \to \pi^+\pi^-$.



(b) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$.



(c) $B^+ \to K^+S, S \to K^+K^-$.



(d) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$.

Figure X 6: Distribution of $M'(x^+x^-)$ together with the stacked contributions from the various simulated SM background samples. Simulation is normalized to a luminosity of 189 fb^{-1}. The hatched area represents the statistical uncertainty of the SM background prediction. The bottom panels show the pulls per bin, defined as the difference between data and simulation, normalized to the statistical uncertainties added in quadrature.

Expected and observed limits

Expected and observed 95% upper limits for various scalar masses, final states, and lifetimes are shown in Figures X 7, X 8, X 9, X 10 (e^+e^-), X 11, X 12, X 13, X 14 ($\mu^+\mu^-$), X 15, X 16, X 17, X 18 ($\pi^+\pi^-$), X 19, X 20, X 21, X 22 (K^+K^-).

[1] J. Beacham et al., J. Phys. G **47**, 010501 (2020), arXiv:1901.09966.

[2] B. Batell, M. Pospelov, and A. Ritz, Phys. Rev. D **83**, 054005 (2011), arXiv:0911.4938.

[3] M. J. Dolan, F. Kahlhoefer, C. McCabe, and K. Schmidt-Hoberg, JHEP **03**, 171 (2015), [Erratum: JHEP 07, 103 (2015)], arXiv:1412.5174.

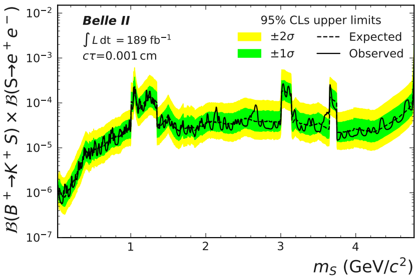
[4] F. Domingo, JHEP **03**, 052 (2017), 1612.06538.

[5] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. **115**, 161802 (2015), arXiv:1508.04094.

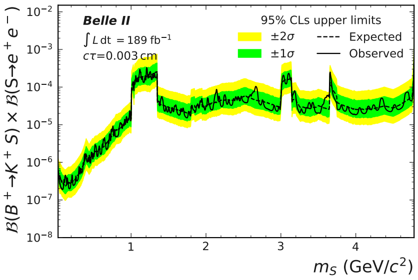
[6] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D **95**, 071101 (2017), arXiv:1612.07818.

[7] A. V. Artamonov et al. (BNL-E949 Collaboration), Phys. Rev. D **79**, 092004 (2009), arXiv:0903.0030.

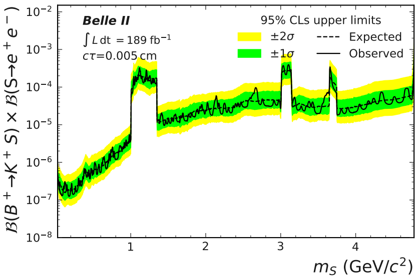
[8] F. Bergsma et al. (CHARM Collaboration), Phys. Lett. B **157**, 458 (1985).



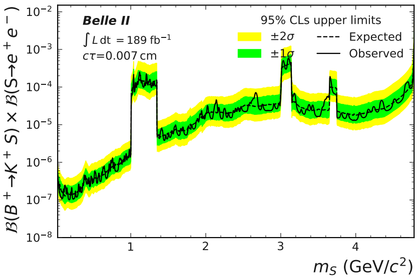
(a) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.001$ cm.



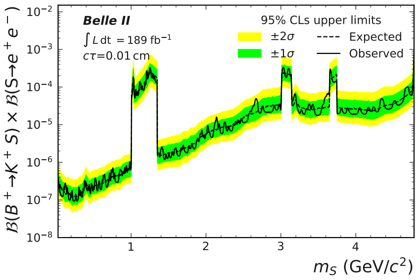
(b) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.003$ cm.



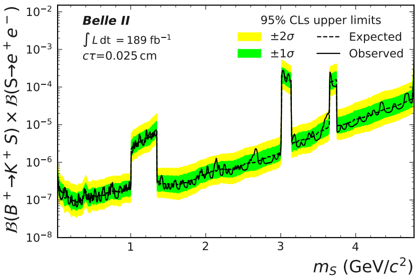
(c) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.005$ cm.



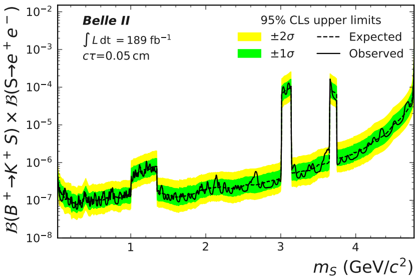
(d) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.007$ cm.



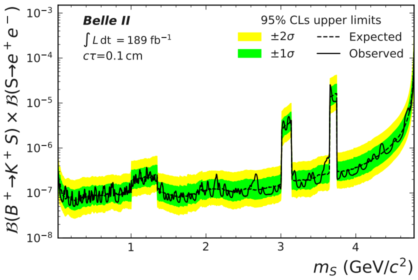
(e) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.01$ cm.



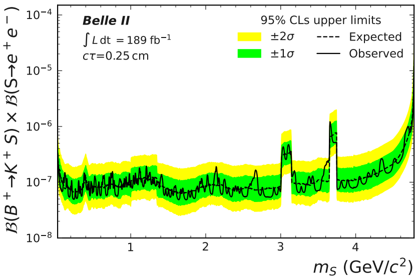
(f) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.025$ cm.



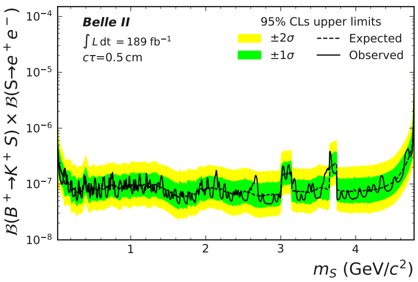
(g) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.05$ cm.



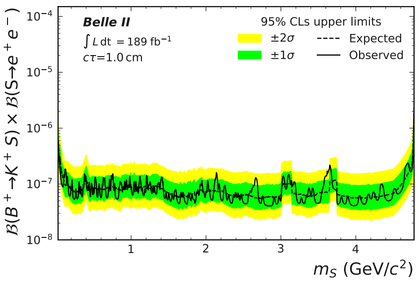
(h) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.100$ cm.



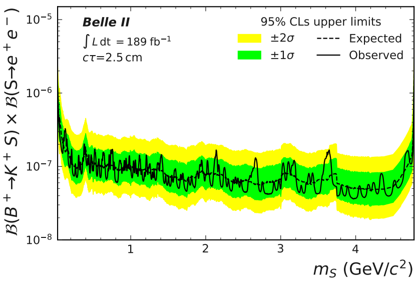
(i) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.25$ cm.



(j) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 0.5$ cm.

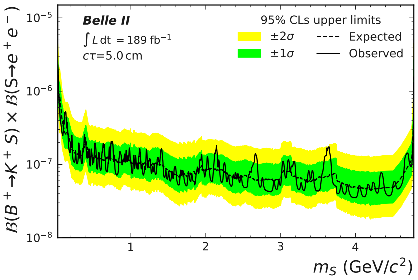


(k) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 1$ cm.

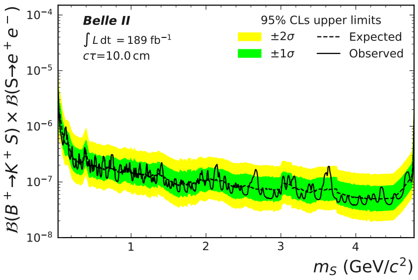


(l) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 2.5$ cm.

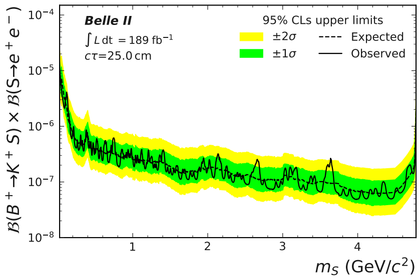
Figure X 7: Expected and observed limits on the product branching fractions $\mathcal{B}(B^+ \to K^+S) \times \mathcal{B}(S \to e^+e^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm.



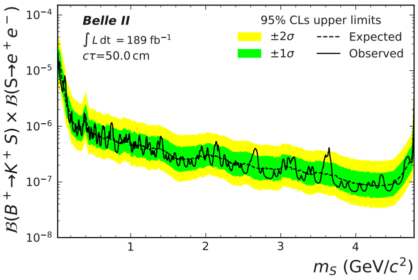
(a) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 5\,\mathrm{cm}$.



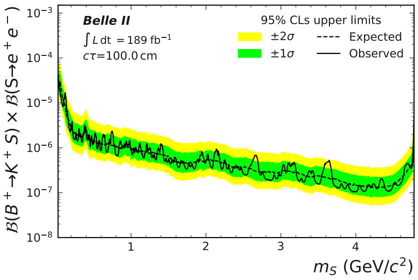
(b) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 10\,\mathrm{cm}$.



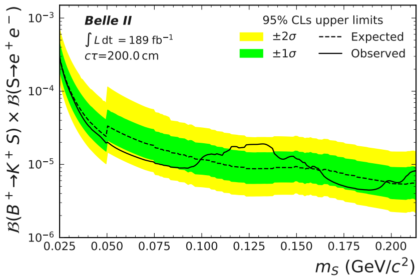
(c) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 25\,\mathrm{cm}$.



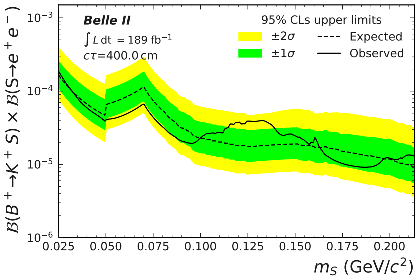
(d) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 50\,\mathrm{cm}$.



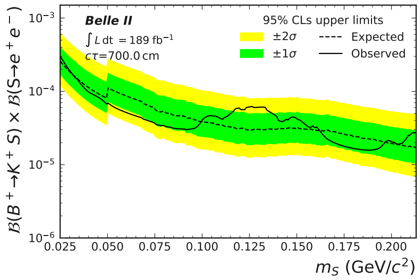
(e) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 100\,\mathrm{cm}$.



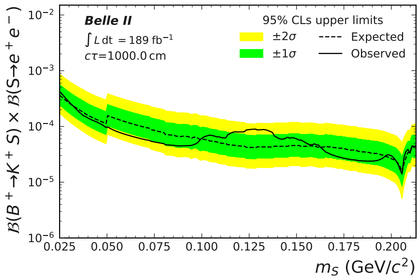
(f) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 200\,\mathrm{cm}$.



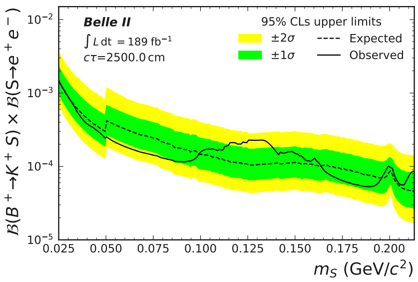
(g) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 400\,\mathrm{cm}$.



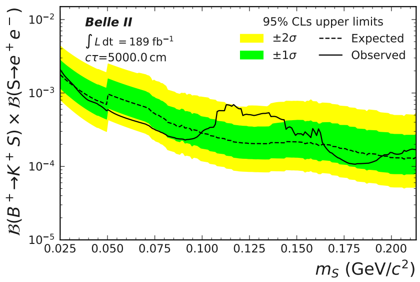
(h) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 700\,\mathrm{cm}$.



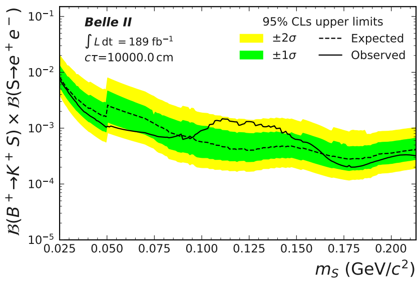
(i) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 1000\,\mathrm{cm}$.



(j) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 2500\,\mathrm{cm}$.

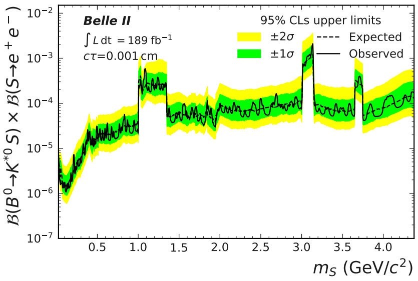


(k) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 5000\,\mathrm{cm}$.

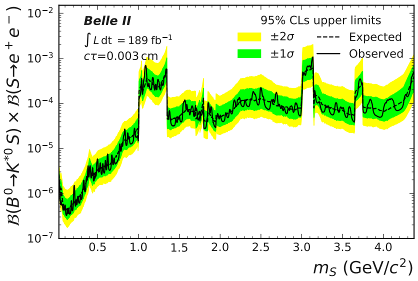


(l) $B^+ \to K^+S, S \to e^+e^-$, lifetime of $c\tau = 10000\,\mathrm{cm}$.

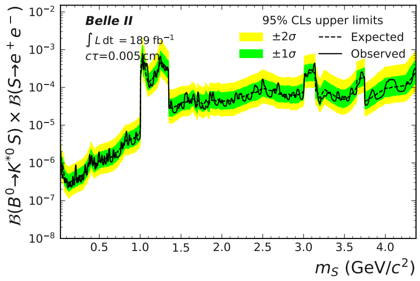
Figure X 8: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to K^+S) \times \mathcal{B}(S \to e^+e^-)$ for lifetimes $5 < c\tau < 10000$ cm.



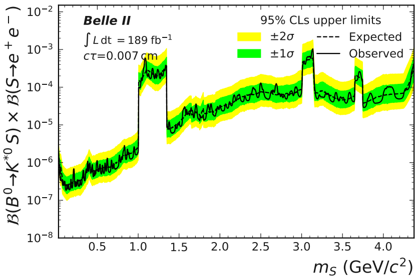
(a) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.001\,\text{cm}$.



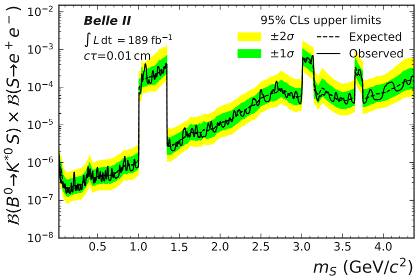
(b) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.003\,\text{cm}$.



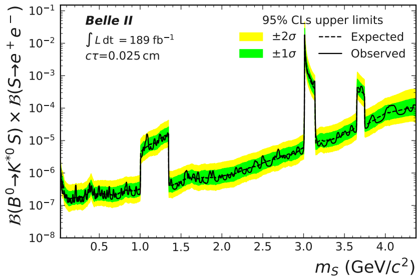
(c) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.005\,\text{cm}$.



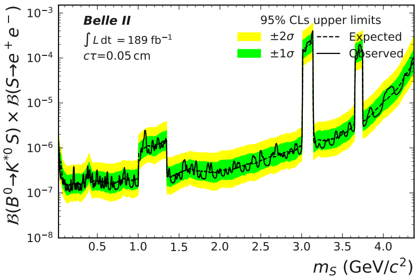
(d) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.007\,\text{cm}$.



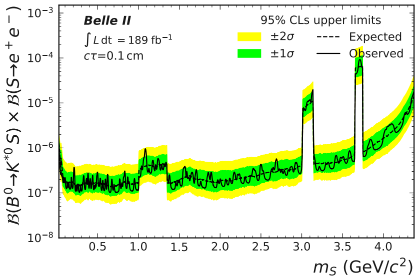
(e) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.01\,\text{cm}$.



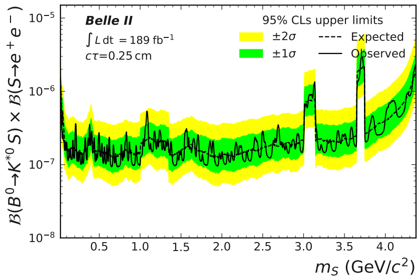
(f) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.025\,\text{cm}$.



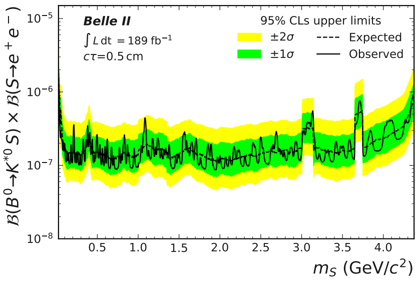
(g) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.05\,\text{cm}$.



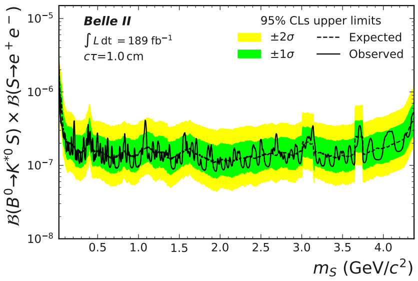
(h) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.100\,\text{cm}$.



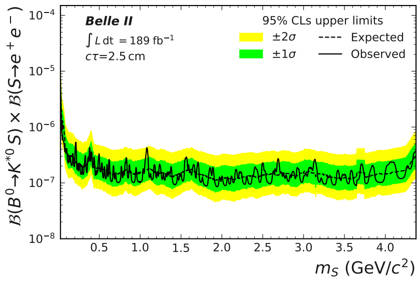
(i) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.25\,\text{cm}$.



(j) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 0.5\,\text{cm}$.

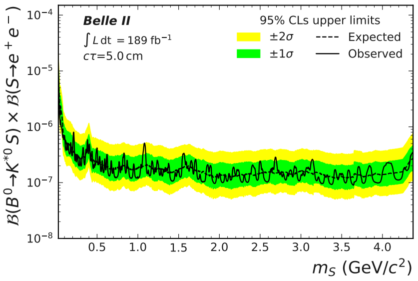


(k) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 1\,\text{cm}$.

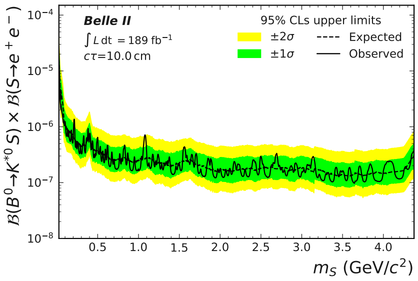


(l) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 2.5\,\text{cm}$.

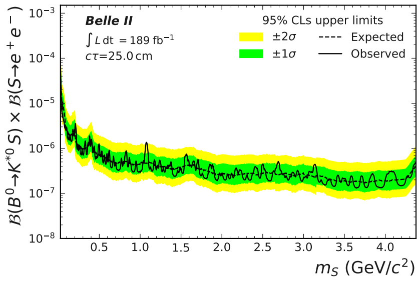
Figure X 9: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to e^+e^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm.



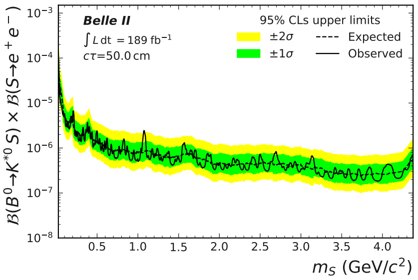
(a) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 5$ cm.



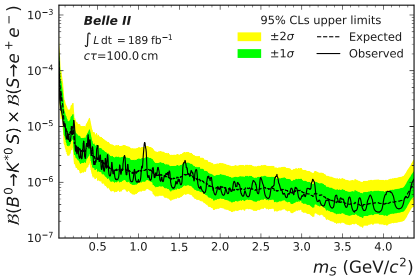
(b) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 10$ cm.



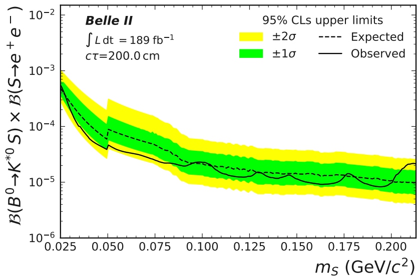
(c) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 25$ cm.



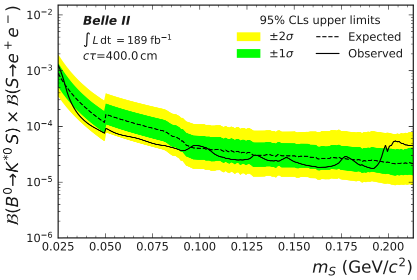
(d) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 50$ cm.



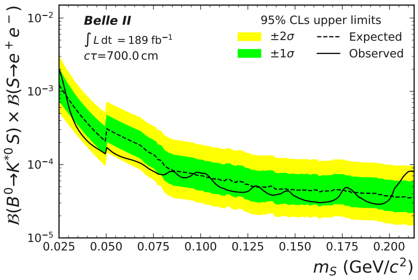
(e) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 100$ cm.



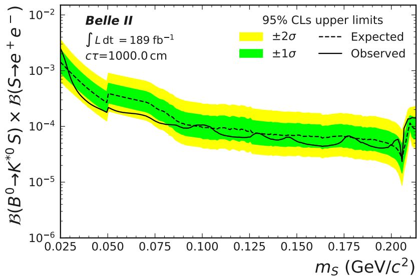
(f) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 200$ cm.



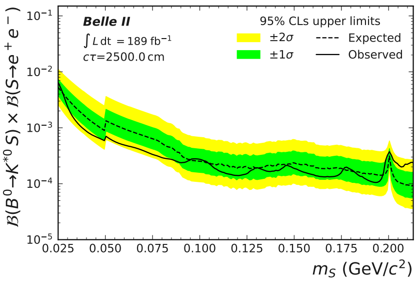
(g) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 400$ cm.



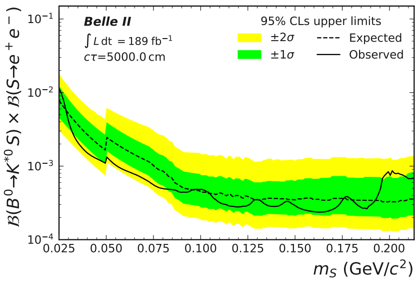
(h) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 700$ cm.



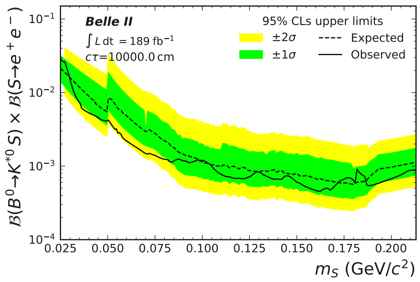
(i) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 1000$ cm.



(j) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 2500$ cm.

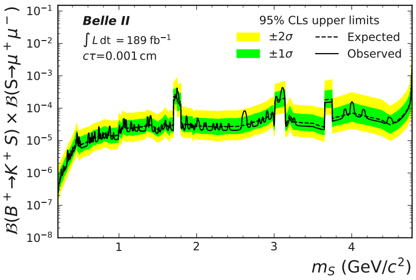


(k) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 5000$ cm.

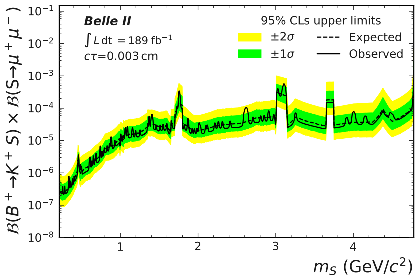


(l) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to e^+e^-$, lifetime of $c\tau = 10000$ cm.

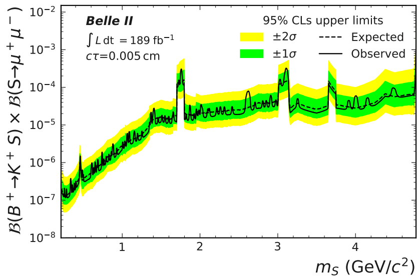
Figure X 10: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to e^+e^-)$ for lifetimes $5 < c\tau < 10000$ cm.



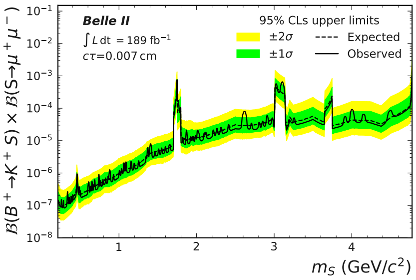
(a) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.001$ cm.



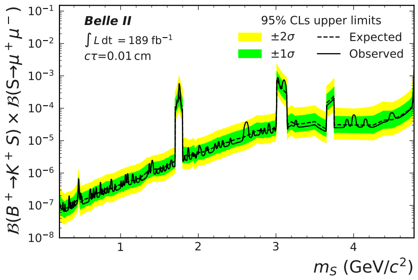
(b) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.003$ cm.



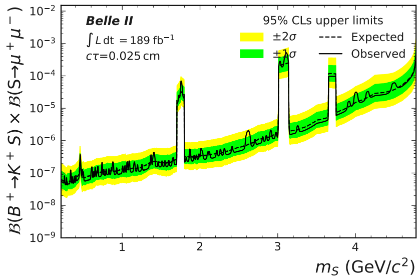
(c) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.005$ cm.



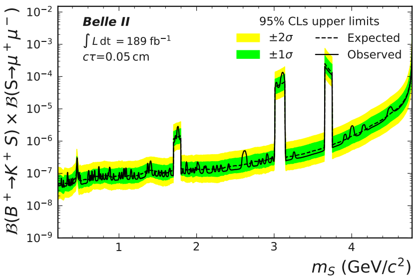
(d) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.007$ cm.



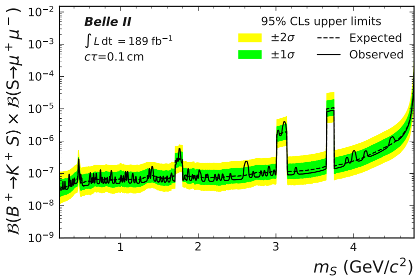
(e) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.01$ cm.



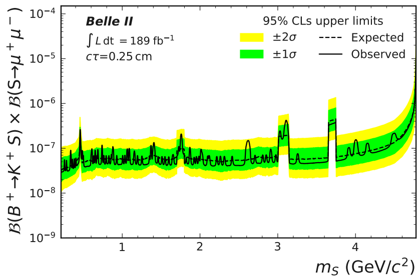
(f) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.025$ cm.



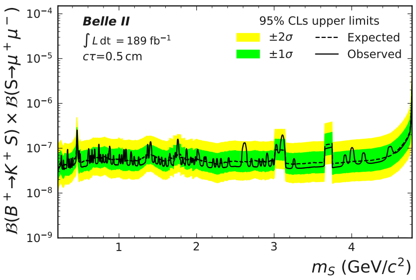
(g) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.05$ cm.



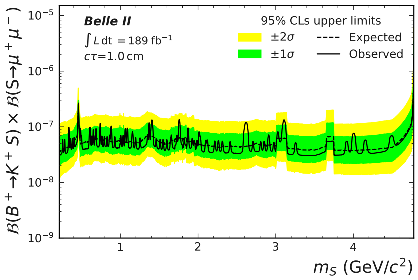
(h) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.100$ cm.



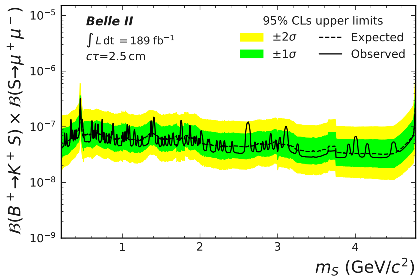
(i) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.25$ cm.



(j) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.5$ cm.

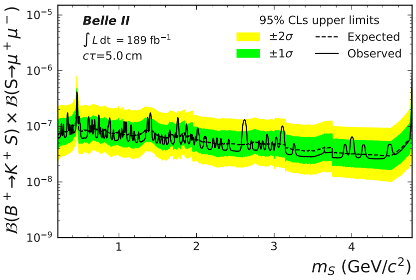


(k) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 1$ cm.

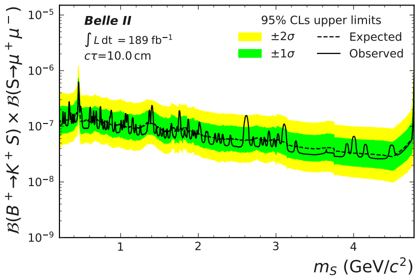


(l) $B^+ \to K^+S, S \to \mu^+\mu^-$, lifetime of $c\tau = 2.5$ cm.

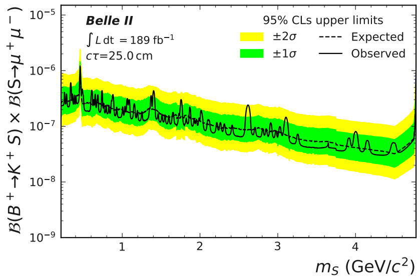
Figure X 11: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to K^+S) \times \mathcal{B}(S \to \mu^+\mu^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm.



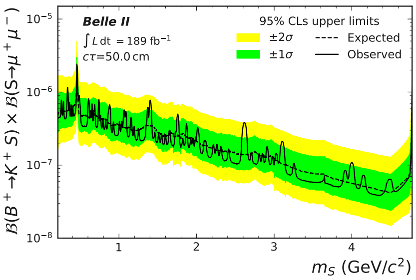
(a)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 5$ cm.



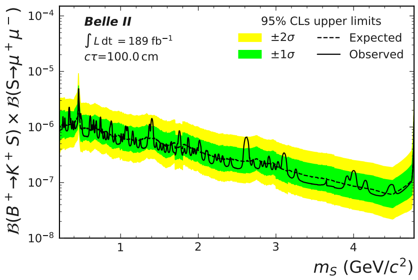
(b)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 10$ cm.



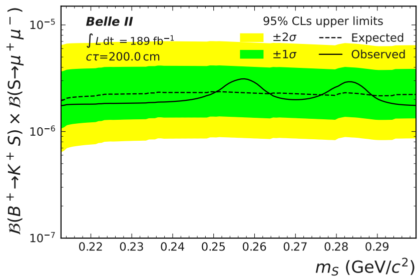
(c)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 25$ cm.



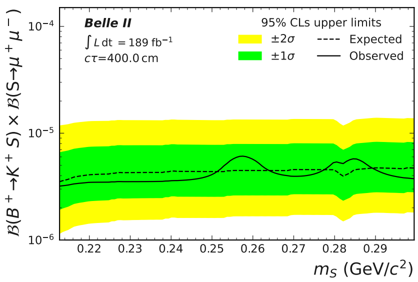
(d)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 50$ cm.



(e)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 100$ cm.

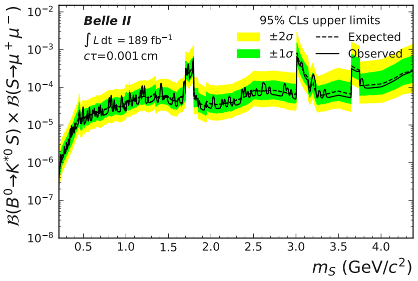


(f)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 200$ cm.

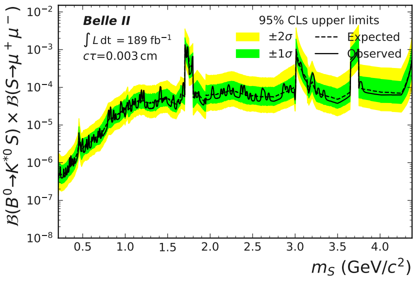


(g)$B^+ \to K^+S, S \to \mu^+\mu^-$,
lifetime of $c\tau = 400$ cm.

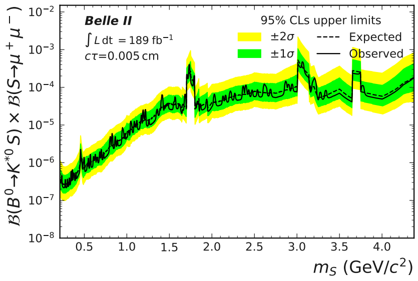
Figure X 12: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to K^+S) \times \mathcal{B}(S \to \mu^+\mu^-)$ for lifetimes $5 < c\tau < 400$ cm.



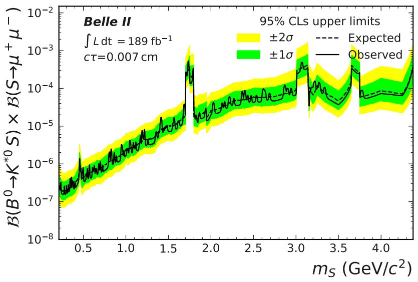
(a) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.001$ cm.



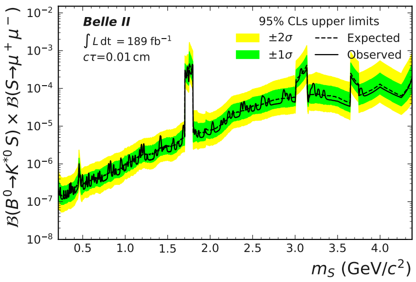
(b) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.003$ cm.



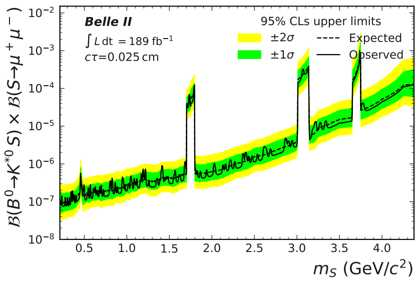
(c) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.005$ cm.



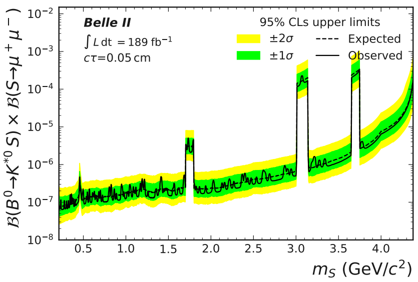
(d) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.007$ cm.



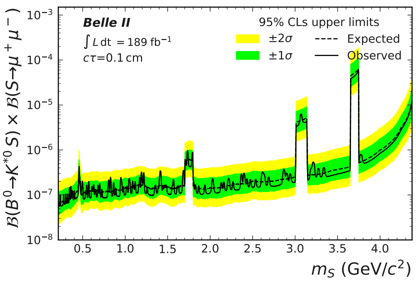
(e) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.01$ cm.



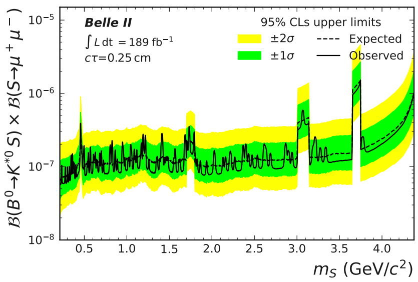
(f) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.025$ cm.



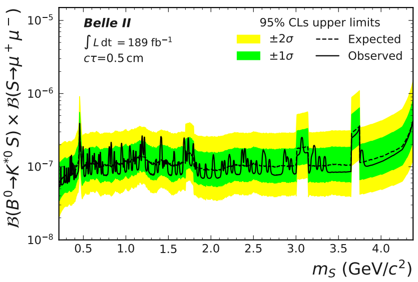
(g) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.05$ cm.



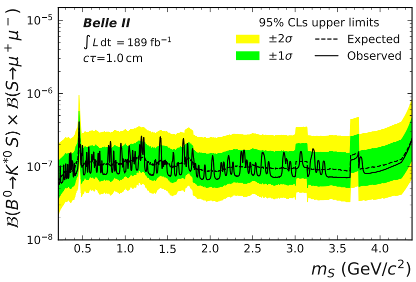
(h) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.100$ cm.



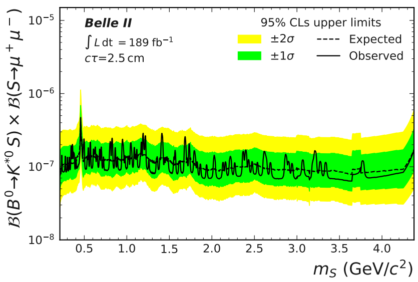
(i) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.25$ cm.



(j) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 0.5$ cm.

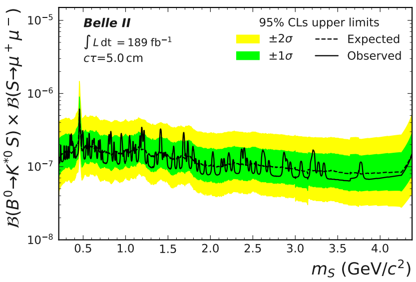


(k) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 1$ cm.

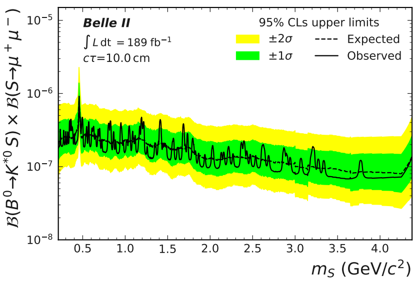


(l) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 2.5$ cm.

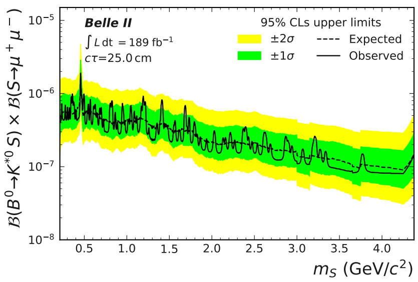
Figure X 13: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to \mu^+\mu^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm.



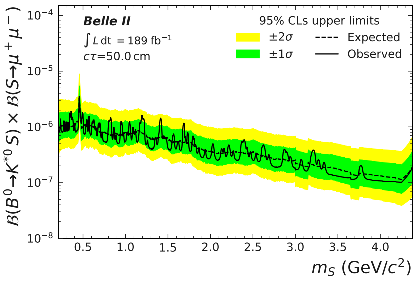
(a)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 5\,\mathrm{cm}$.



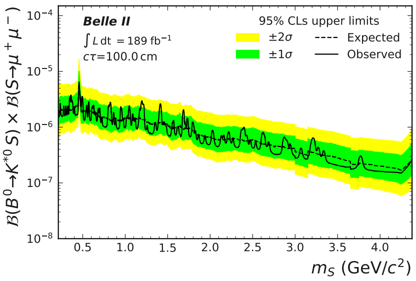
(b)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 10\,\mathrm{cm}$.



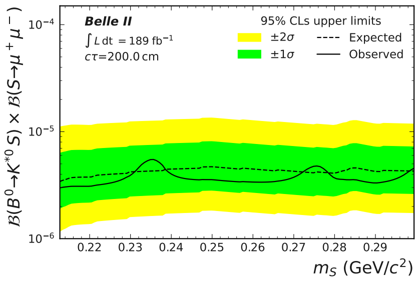
(c)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 25\,\mathrm{cm}$.



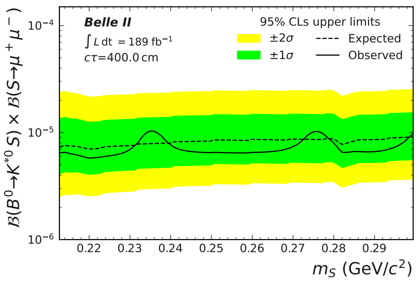
(d)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 50\,\mathrm{cm}$.



(e)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 100\,\mathrm{cm}$.

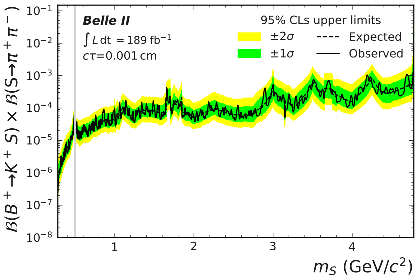


(f)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 200\,\mathrm{cm}$.

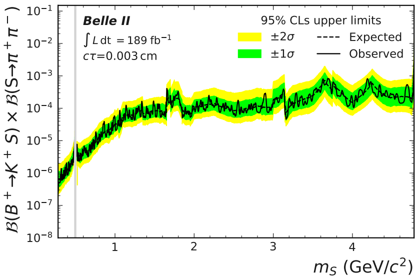


(g)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \mu^+\mu^-$, lifetime of $c\tau = 400\,\mathrm{cm}$.

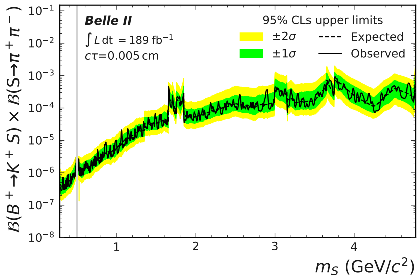
Figure X 14: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to \mu^+\mu^-)$ for lifetimes $5 < c\tau < 400$ cm.



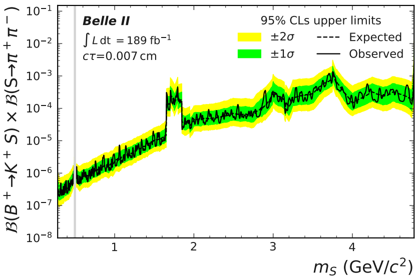
(a) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.001$ cm.



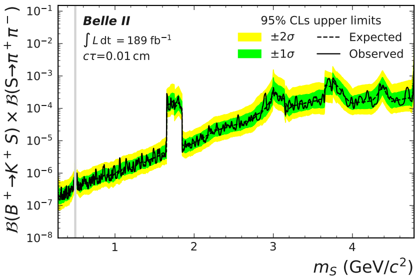
(b) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.003$ cm.



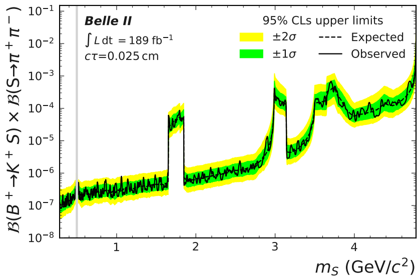
(c) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.005$ cm.



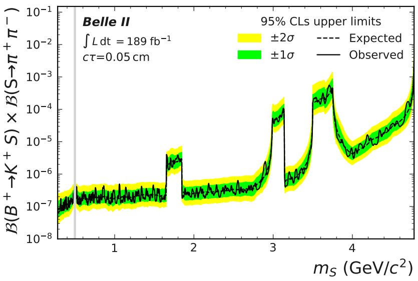
(d) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.007$ cm.



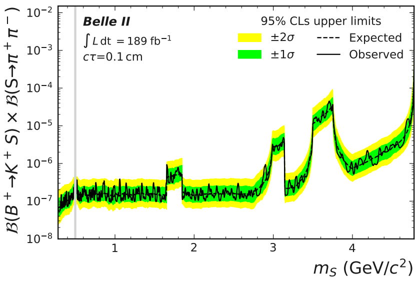
(e) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.01$ cm.



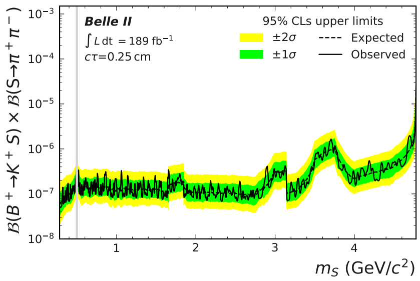
(f) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.025$ cm.



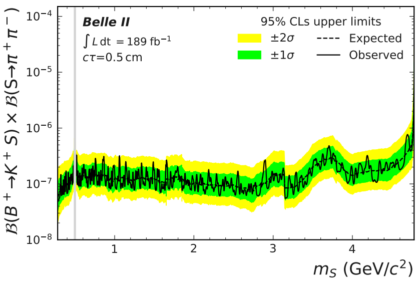
(g) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.05$ cm.



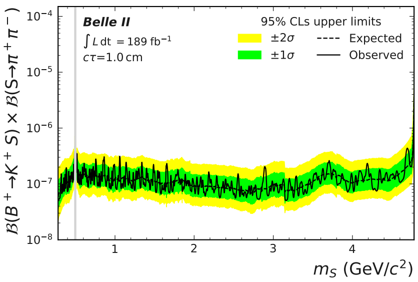
(h) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.100$ cm.



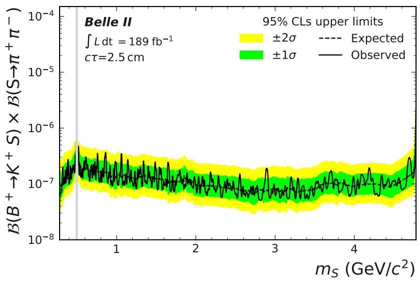
(i) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.25$ cm.



(j) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.5$ cm.

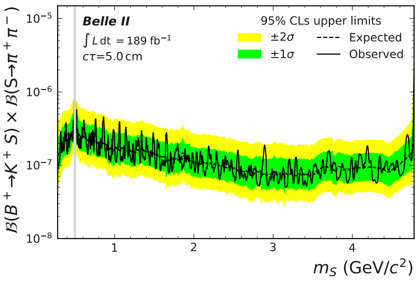


(k) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 1$ cm.

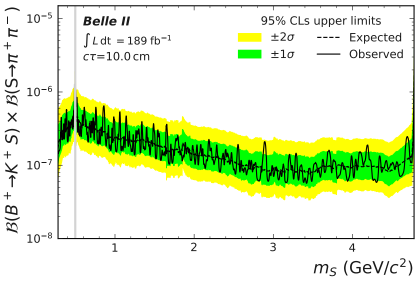


(l) $B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 2.5$ cm.

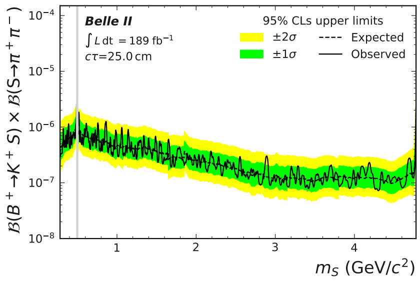
Figure X 15: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to K^+S) \times \mathcal{B}(S \to \pi^+\pi^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm. The region corresponding to the fully-vetoed K^0_S for $S \to \pi^+\pi^-$ is marked in gray.



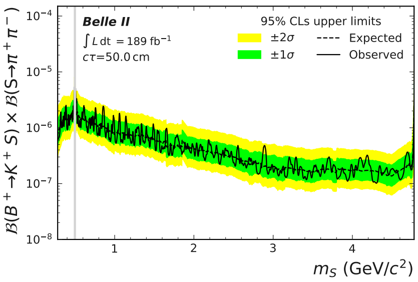
(a)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 5$ cm.



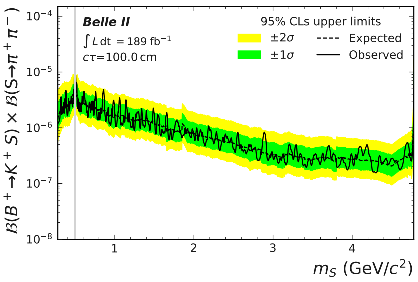
(b)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 10$ cm.



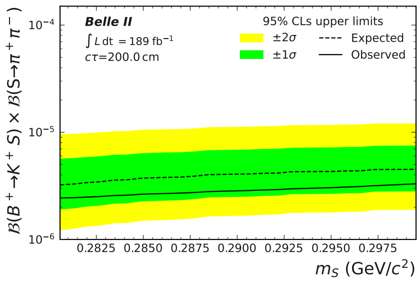
(c)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 25$ cm.



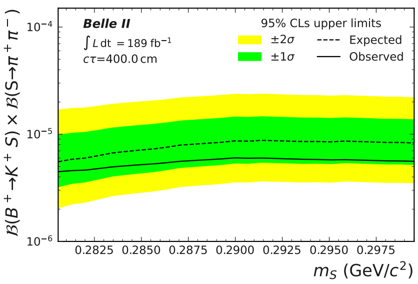
(d)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 50$ cm.



(e)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 100$ cm.

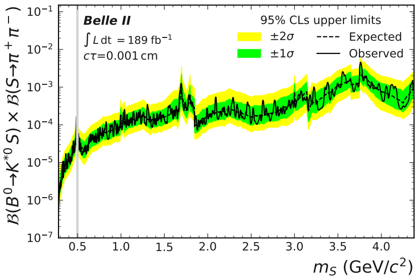


(f)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 200$ cm.

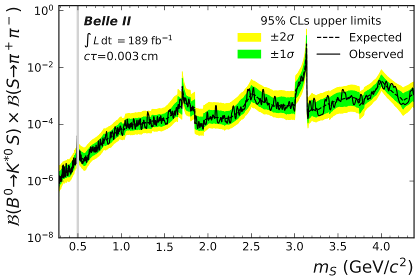


(g)$B^+ \to K^+S, S \to \pi^+\pi^-$, lifetime of $c\tau = 400$ cm.

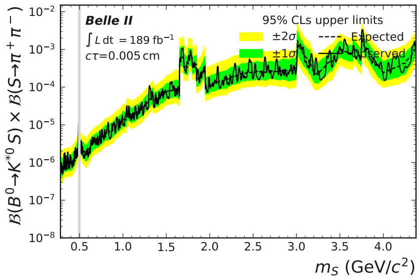
Figure X 16: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to K^+S) \times \mathcal{B}(S \to \pi^+\pi^-)$ for lifetimes $5 < c\tau < 400$ cm. The region corresponding to the fully-vetoed K^0_S for $S \to \pi^+\pi^-$ is marked in gray.



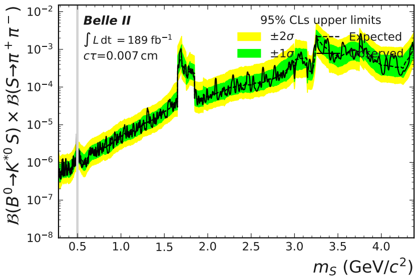
(a)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.001\,\mathrm{cm}$.



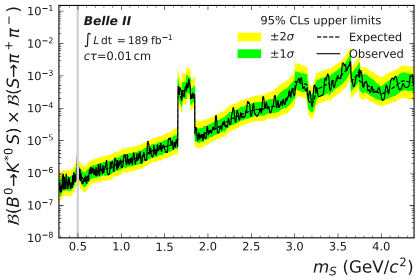
(b)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.003\,\mathrm{cm}$.



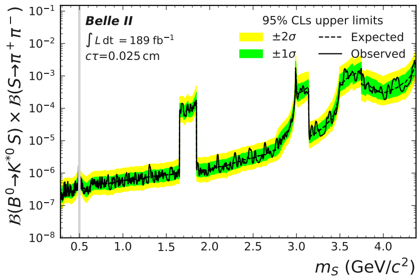
(c)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.005\,\mathrm{cm}$.



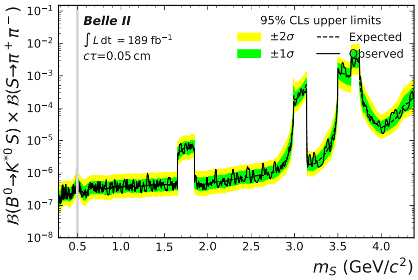
(d)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.007\,\mathrm{cm}$.



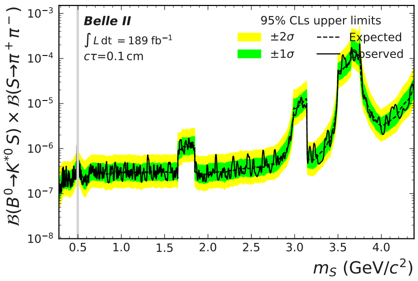
(e)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.01\,\mathrm{cm}$.



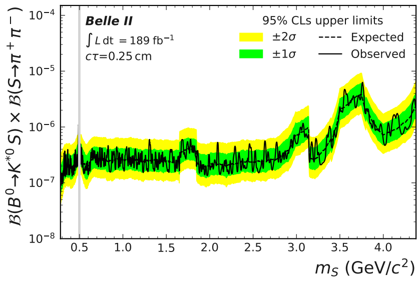
(f)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.025\,\mathrm{cm}$.



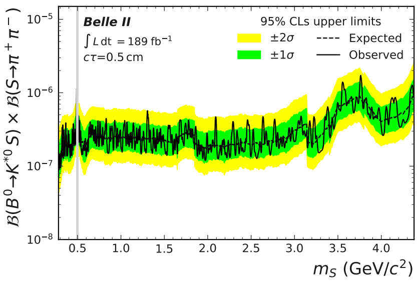
(g)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.05\,\mathrm{cm}$.



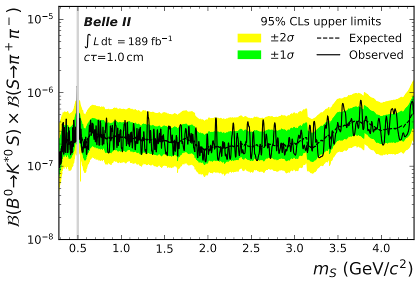
(h)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.100\,\mathrm{cm}$.



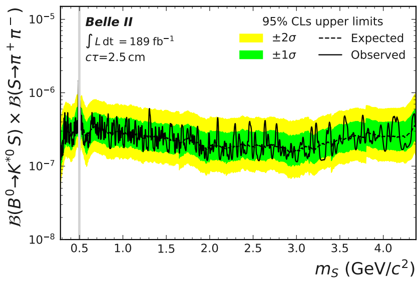
(i)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.25\,\mathrm{cm}$.



(j)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 0.5\,\mathrm{cm}$.

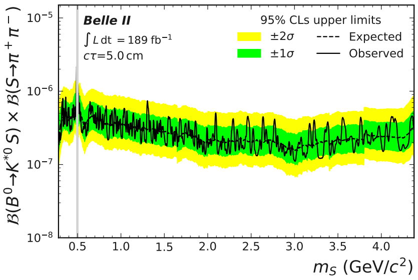


(k)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 1\,\mathrm{cm}$.

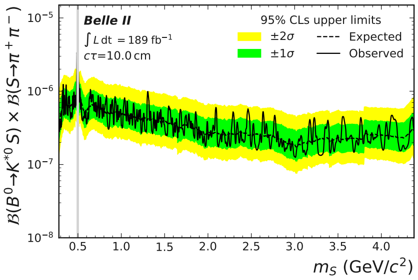


(l)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 2.5\,\mathrm{cm}$.

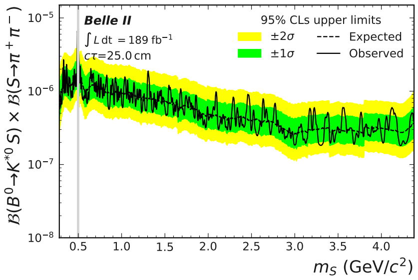
Figure X 17: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to \pi^+\pi^-)$ for
lifetimes $0.001 < c\tau < 2.5$ cm. The region corresponding to the fully-vetoed K^0_S for $S \to \pi^+\pi^-$ is marked in gray.



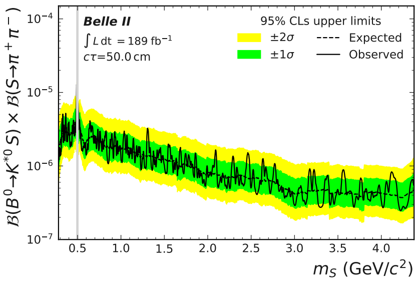
(a) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 5\,\mathrm{cm}$.



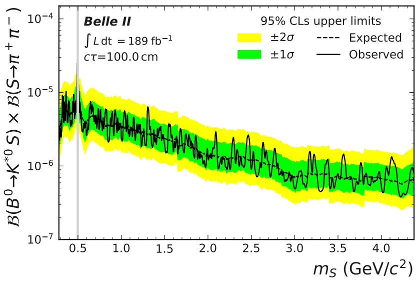
(b) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 10\,\mathrm{cm}$.



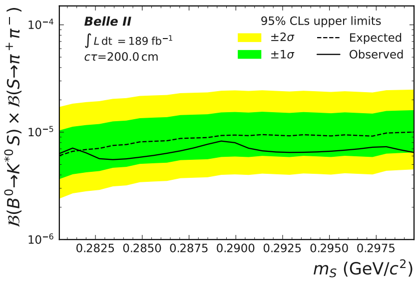
(c) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 25\,\mathrm{cm}$.



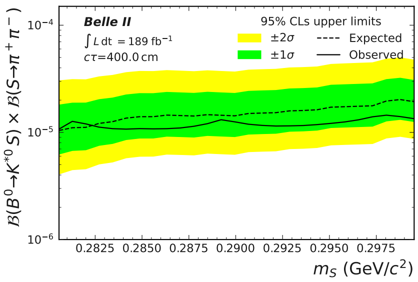
(d) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 50\,\mathrm{cm}$.



(e) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 100\,\mathrm{cm}$.

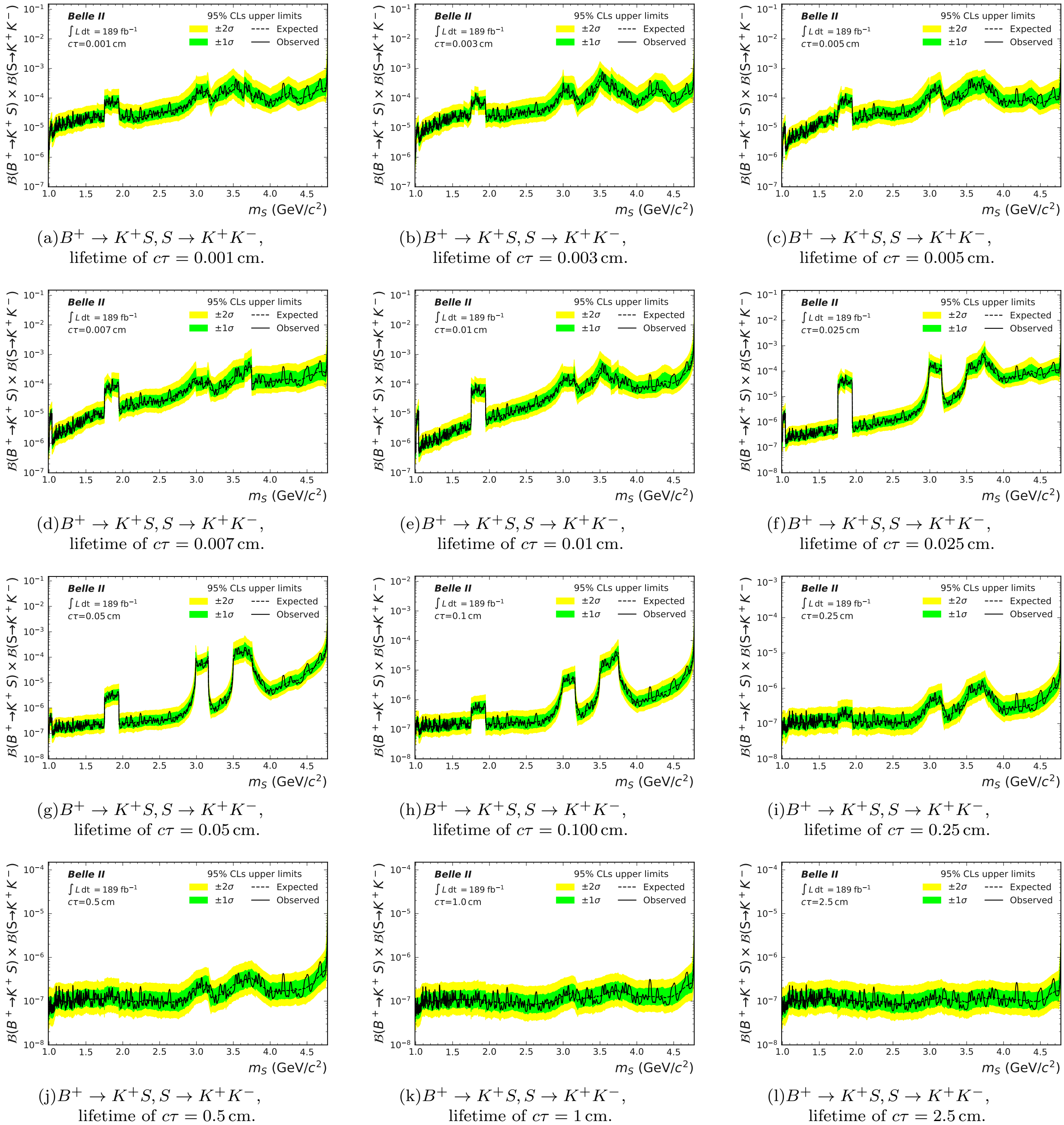


(f) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 200\,\mathrm{cm}$.



(g) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to \pi^+\pi^-$, lifetime of $c\tau = 400\,\mathrm{cm}$.

Figure X 18: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to \pi^+\pi^-)$ for lifetimes $5 < c\tau < 400$ cm. The region corresponding to the fully-vetoed K^0_S for $S \to \pi^+\pi^-$ is marked in gray.



(a) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.001$ cm.

(b) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.003$ cm.

(c) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.005$ cm.

(d) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.007$ cm.

(e) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.01$ cm.

(f) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.025$ cm.

(g) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.05$ cm.

(h) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.100$ cm.

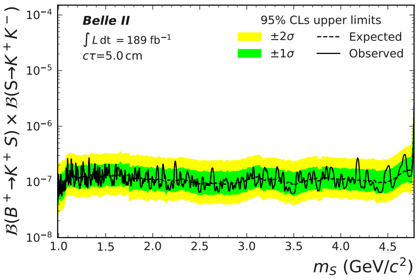
(i) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.25$ cm.

(j) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 0.5$ cm.

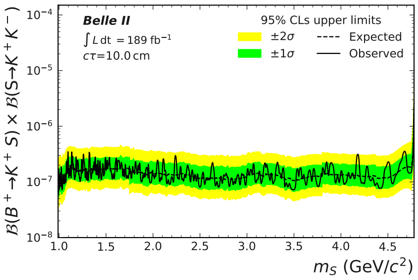
(k) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 1$ cm.

(l) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 2.5$ cm.

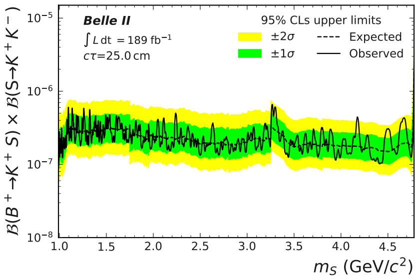
Figure X 19: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to KS) \times \mathcal{B}(S \to K^+K^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm.



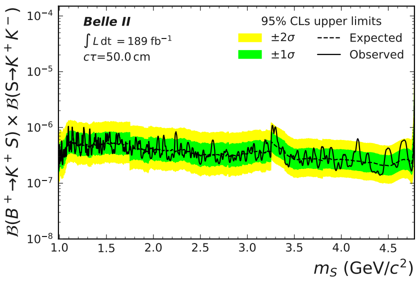
(a) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 5\,$cm.



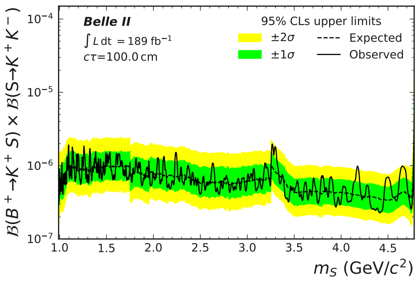
(b) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 10\,$cm.



(c) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 25\,$cm.

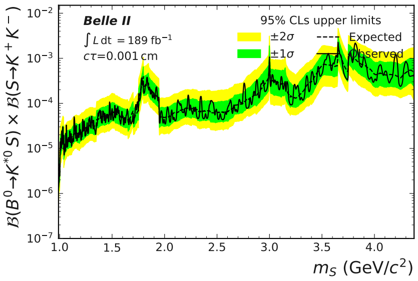


(d) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 50\,$cm.

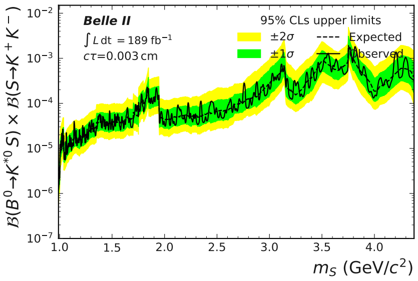


(e) $B^+ \to K^+S, S \to K^+K^-$, lifetime of $c\tau = 100\,$cm.

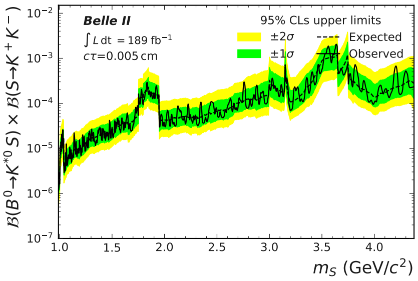
Figure X 20: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^+ \to KS) \times \mathcal{B}(S \to K^+K^-)$ for lifetimes $5 < c\tau < 100$ cm.



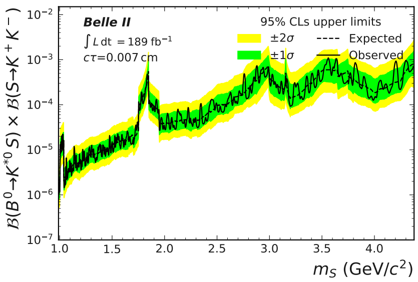
(a)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.001$ cm.



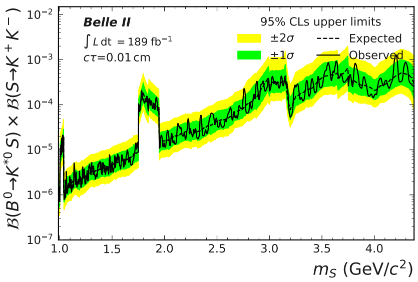
(b)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.003$ cm.



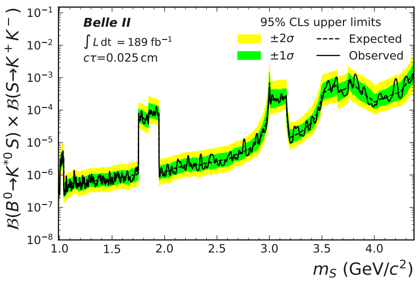
(c)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.005$ cm.



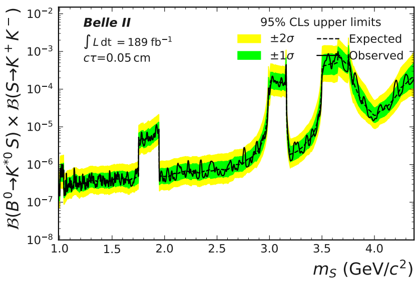
(d)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.007$ cm.



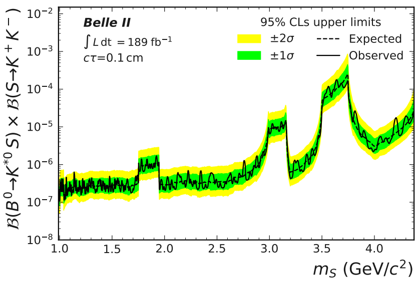
(e)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.01$ cm.



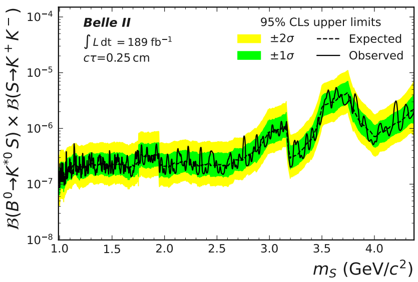
(f)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.025$ cm.



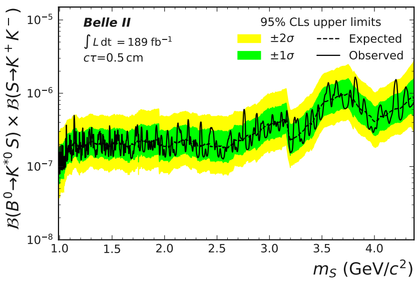
(g)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.05$ cm.



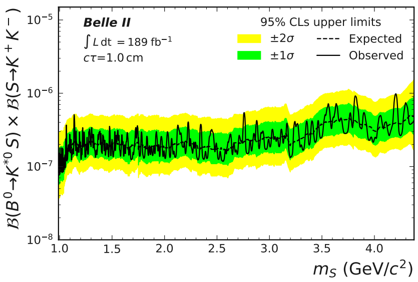
(h)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.100$ cm.



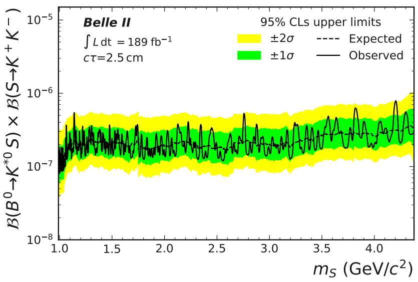
(i)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.25$ cm.



(j)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 0.5$ cm.

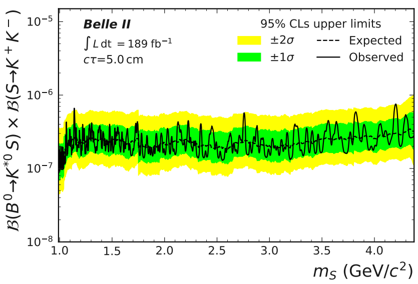


(k)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 1$ cm.

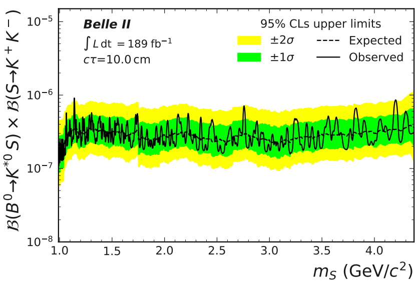


(l)$B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 2.5$ cm.

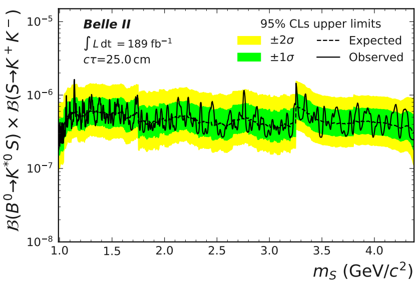
Figure X 21: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to K^+K^-)$ for lifetimes $0.001 < c\tau < 2.5$ cm.



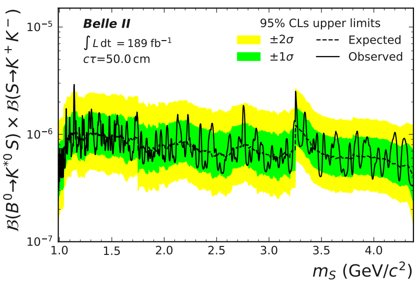
(a) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 5\,$cm.



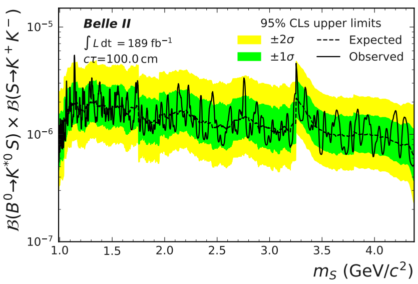
(b) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 10\,$cm.



(c) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 25\,$cm.



(d) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 50\,$cm.



(e) $B^0 \to K^{*0}(\to K^+\pi^-)S, S \to K^+K^-$, lifetime of $c\tau = 100\,$cm.

Figure X 22: Expected and observed limits on the product of branching fractions $\mathcal{B}(B^0 \to K^{*0}(\to K^+\pi^-)S) \times \mathcal{B}(S \to K^+K^-)$ for lifetimes $5 < c\tau < 100$ cm.