Working Memory and Mathematical Learning

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Introduction

An increasing number of students show severe mathematical difficulties. Between 5% and 10% of children and adolescents experience a substantial learning deficit in at least one area of mathematics (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005). The identification of these mathematical difficulties is fundamental if we consider the negative widespread drawbacks determined by math difficulties. Basic mathematical skills are regularly used in everyday life, and their deficiency affects both employment opportunities and socio-emotional well-being. In addition, results of recent studies show how mathematical abilities predict financial and educational success, particularly for women (Geary, Hoard, Nugent, & Bailey, 2013). It is therefore crucial to promote an early identification of children at risk for mathematical learning difficulties at preschool level and develop effective evidence-based mathematics curricula considering all the cognitive processes involved in the development of mathematical skills.

In the last decades, various studies investigated the cognitive factors, defined as *precursors*, that underlie the development of mathematical abilities. The identification of these cognitive markers of mathematical learning plays a key role in the early identification of children that may develop math difficulties and disabilities. Competencies that specifically predict mathematical abilities, such as digit recognition, magnitude understanding, and counting, may be considered domain-specific precursors. General cognitive abilities, such as working memory, processing speed, and intelligence, which may predict performance not only in mathematics but also

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in other school subjects, can be considered as domain-general precursors (Passolunghi & Lanfranchi, 2012).

In this chapter, we will not discuss in detail the domain-specific precursors of mathematical learning, but we will focus on a key general precursor, the working memory, and its influence on the mathematical learning processes.

Working Memory (WM): A Domain-General Precursor of Mathematical Learning

Domain-general cognitive abilities such as memory, attention, or processing speed are important precursors of school learning. Of all these general cognitive skills, several studies demonstrated that working memory is a key predictor of mathematical competence. The term "working memory" (WM) refers to a temporary memory system that plays an important role in supporting learning during the childhood years because its key feature is the capacity to both store and manipulate information. Various models of the structure and function of working memory exist, but in the present chapter we will refer to the multicomponent model of working memory proposed by Baddeley and Hitch in 1974 and revised in succeeding years (Baddeley, 2000). Baddeley's models consist of three main parts. The two "slave" systems of working memory (i.e., the phonological loop and visuospatial sketchpad) are specialized to process language-based and visuospatial information, respectively. The central executive, which is not modality-specific, coordinates the two slave systems and is responsible for a range of functions, such as the attentional control of actions. The distinction between the central executive system and specific memory storage systems (i.e., the phonological loop and visuospatial sketchpad) in some way parallels the distinction between the working memory, involving storage, processing, and effortful mental activity, and the short-term memory, typically involving situations in which the individual passively holds small amounts of information (Swanson & Beebe-Frankenberger, 2004).

In this multicomponent model, the central executive is responsible for control and regulation of cognitive processes in which executive functions are involved. Miyake et al. (2000) identified three main executive functions in working memory: inhibition, updating, and shifting. Inhibition involves the ability to suppress dominant responses, shifting involves the ability to shift strategies when attending to multiple tasks or mental processes, and updating involves the ability to replace outdated and irrelevant information by maintaining only a restricted set of elements in working memory.

More recently, Baddeley (2000) added a fourth component to his model, the episodic buffer, which is a limited-capacity system that both integrates and provides temporary storage of information from the two subsystems and long-term memory. Developmental research related to this fourth component is very limited, so in this chapter we will focus on the first three components of Baddeley's working memory model.

Verbal short-term memory is traditionally assessed using tasks that require the participant to recall a sequence of words (e.g., word span task forward) or numbers (e.g., digit span task forward). On the other hand, tasks such as the visual pattern test are designed to assess visuospatial short-term memory. In the visual pattern test, participants are presented with matrix patterns of black and white squares and are required to memorize patterns of increasing complexity. All these tasks designed to assess short-term memory skills require individuals to recall a sequence of verbal or visual information in the same format of presentation. Differently, working memory capacity is reliably assessed by dual tasks in which the individual is required to store and, at the same time, process increasing amounts of information. An example of verbal working memory tasks is the listening span task (Daneman & Carpenter, 1980). Participants are presented with an increasing number of sentences, they are required to judge whether the sentences are true or false, and then at the end of each set, they have to recall the last word of each of the sentences of the set.

A long-standing body of research suggests that there is a direct influence of working memory on mathematical skills (De Smedt et al., 2009; Passolunghi, Mammarella, & Altoè, 2008; Passolunghi, Vercelloni, & Schadee, 2007). Longitudinal studies show that working memory performance assessed in preschool years predicts mathematical achievements several years after kindergarten (Gathercole, Brown, & Pickering, 2003; Mazzocco & Thompson, 2005). These results support the hypothesis that working memory is a distinct and significant correlate of early numerical abilities. However, the same cannot be said of either verbal or visuospatial short-term memory (Passolunghi, Lanfranchi, Altoè, & Sollazzo, 2015). Indeed, there is substantial evidence for separating the involvement of short-term and working memory as correlates of mathematical learning (Shah & Miyake, 2005; Swanson, 2006), with active working memory skills having an essential influence on early numerical abilities and later mathematical performance. Indeed, even the simplest mathematics calculations require WM processes: temporary storage of problem information, retrieval of relevant procedures, and processing operations to convert the information into numerical output. These same processes are needed even for simple number comparison tasks: the child needs to map the different number symbols onto the corresponding quantities, store them into memory, and then integrate this with the incoming information to performing the task.

Despite the growing evidence that WM plays a fundamental role in the development of mathematical abilities, there is still an absence of shared consensus about the relative extent of the involvement of domain-specific and domain-general precursors in the development of mathematical abilities. Some authors, for example, highlight the importance of domain-specific precursors such as the approximate number system (ANS) in the development of mathematical learning (Halberda, Mazzocco, & Feigenson, 2008). The ANS is an innate system, which allows the manipulation of quantities and magnitudes in an approximate way. A typical example of ability underlying ANS consists in approximately estimating computation results or in comparing two or more sets of elements to identify, without counting, which could be the most numerous. The involvement of ANS in mathematical learning is nevertheless very much debated. Indeed, while some studies account for its significant role, many others do not. Moreover, while some authors report deficits associated with ANS in children with or at risk for mathematical learning disability, others highlighted impairments in making comparisons between quantities, but only when quantities are represented by symbols and not when using nonsymbolic, approximate numerosities. In order to further investigate the relation between domain-specific and domain-general precursors of mathematical development, we conduct a wide assessment of memory components and domain-specific factors, such as the ANS (Passolunghi, Cargnelutti, & Pastore, 2014). A large sample of first grade typically developing children was tested at both beginning and end of their Grade 1. Both general (working memory and intelligence) and specific (ANS) precursors were evaluated by a wide battery of tests and put in relation to concurrent and subsequent mathematical skills. Results demonstrated that working memory and intelligence were the strongest precursors in both assessment times. ANS had instead a milder role, which lost significance by the end of the school year. Some authors argue that the relationship between ANS performance and mathematics achievement may in fact be an artefact of the WM (inhibitory control) demands of some trials of the numerosity comparison task (e.g., Gilmore et al., 2013; Soltész, Szűcs, & Szűcs, 2010).

Contribution of WM Components to Mathematical Learning

With regard to the contribution of the three core components of working memory to the development of mathematical skills, many studies showed a direct association between executive function and children's early emergence and development of mathematical abilities across a wide age range. For example, dual-task studies suggest that central executive resources are implicated in children's arithmetic performance (e.g., Imbo & Vandierendonck, 2007), and longitudinal data found that inhibitory control predicted later mathematical outcomes (Blair & Razza, 2007; Mazzocco & Kover, 2007). On the other hand, children who are poor in mathematics also have a poor performance in central executive tasks, especially in tasks that require the inhibition of irrelevant information and updating (Passolunghi, Cornoldi, & De Liberto, 1999; Passolunghi & Siegel, 2001, 2004; St Clair-Thompson & Gathercole, 2006).

Spatial skills and visuospatial working memory were also found to be related to children's early counting ability and general mathematical competence (e.g., Passolunghi & Mammarella, 2012). Indeed, the visuospatial sketchpad appears to support the representation of numbers in counting, arithmetic calculations, and especially mental calculation (McKenzie, Bull, & Gray, 2003). This component is also fundamental in the process of problem-solving, because it allows the individual to build a visual mental representation of the problem (Holmes & Adams, 2006). Moreover, visuospatial WM abilities assessed in the preschool years predict complex arithmetic, number sequencing, and graphical representation of data in primary school (Bull, Espy, & Wiebe, 2008).

The results of studies that considered the role of the phonological loop in children's mathematical processing have been unclear. Dual-task studies showed that 8–9-year-old children (but not younger children) use a verbal approach supplemented by visuospatial resources during online arithmetic performance (McKenzie et al., 2003). In the field of learning disabilities, some studies found no differences in phonological loop abilities between children with and without mathematical difficulties, especially when differences in reading ability were controlled (Passolunghi & Siegel, 2001, 2004). Other authors suggest that the phonological loop is involved in basic fact retrieval (Holmes & Adams, 2006).

The role of each working memory component in mathematical cognition must be considered to vary with expertise and development (Meyer, Salimpoor, Wu, Geary, & Menon, 2010), with an increasing involvement of the phonological loop in mathematical cognition from the age of 7 onward (Rasmussen & Bisanz, 2005).

Working Memory, Word Problems, and Calculation

One of the main goals of mathematical education is to develop students' ability to solve mathematical word problems. This ability is important both for academic success and for problem-solving in everyday life. However, mathematical word problem solution is very demanding and difficult for many students.

In the school setting, mathematical word problems are typically presented as a short story that includes relevant numerical information, the "problem data," and a question (e.g., John bought 4 pizzas with 8 slices each. He and his friends Bruce ate 12 slices of the pizzas. How many slices were left?). The solution of the problem requires the use of arithmetic operations (i.e., addition, subtraction, multiplication, or division) and the execution of several different cognitive processes. Initially, in the understanding phase, children must formulate a cognitive representation of the information drawn from the text of the problem. This initial cognitive representation requires discriminating relevant from irrelevant information. Subsequently, in the solution phase, they need to formulate a plan for solving the problem. Devising a plan involves choosing appropriate sub-goals for the solution and consequently includes the choice of the correct arithmetic operations and algorithms. Finally, they have to correctly perform the calculations.

A more strict focus on word arithmetic problem-solving suggests that working memory can be critically involved even when the written text is still available. Indeed, text comprehension requires incoming information to be integrated with previous information maintained in the working memory system. Furthermore, the complete comprehension of the problem requires that the solvers build up a mental representation of the problem, which involves the capacity of the working memory system. According to Baddeley's three component model, the central executive is probably more specifically and strongly involved in this process than the articulatory loop. In fact, problem-solving does not simply involve the maintenance of given information, but it requires its control, i.e., that this information is examined for relevance, selected or inhibited according to its relevance, integrated, used, and so on. Baddeley (1990) also suggested that reading comprehension involves the central executive more than the articulatory loop. This suggestion seems to apply even more to written word arithmetic problem-solving which requires not only text comprehension but also additional operations on it.

Arithmetic calculation is another important academic skill that children learn when they start formal education. Basic addition skills are fundamental milestones for the development of multiplications skills and increasingly complex arithmetic abilities. The substantial body of research focused on identifying the cognitive processes that underlie arithmetic calculation stresses once again the important role played by working memory. For instance, to perform an addition (e.g., 13 + 9) without being able to use a pen and paper, we must temporary retain the phonological representations of the numbers. The next step would be to employ one or more procedures (e.g., counting) to combine the numbers and produce an answer. Alternatively, employing carrying or regrouping strategies involves maintaining recently processed information while performing other mental operations. First of all, we have to retain the 2 from adding 3 + 9. Next we add the 1 from the tens column of the 13 to the 1 from the tens column of the 12 produced from adding the 3 + 9. Finally, we would need to add the products held in working memory, resulting in the correct solution.

These examples show clearly how the cognitive processes involved in performing arithmetic calculations are embedded within the working memory system. For instance, even the simplest mathematics calculations require the temporary storage of problem information, retrieval of relevant procedures, and processing operations to convert the information into numerical output. Studies have also shown that the different working memory components (e.g., visuospatial sketchpad, phonological loop, and central executive) play specialized and unique roles in arithmetic calculation (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Higher working memory capacity is associated with higher accuracy in solving complex arithmetic problems in adults as well as in children. In particular, children with higher working memory abilities tend to use more sophisticated strategies such as decomposition instead of less sophisticated strategies such as finger counting (Geary, Hoard, Byrd-Craven, & DeSoto, 2004).

Research studies in this field emphasize that the central executive plays a greater role in mental calculation compared to the phonological loop (e.g., De Rammelaere, Stuyven, & Vandierendonck, 2001). In particular, the phonological loop plays a major role when calculation involves storing temporary information, whereas carrying operations put a major demand on the central executive processes (Fuerst & Hitch, 2000). Only a limited number of studies examined the role of the visuospatial component of the WM model. These studies showed that visuospatial WM is related to performance in written calculation. In particular, it is important during the initial stages of arithmetic calculation for encoding arithmetic problems presented visually.

Executive Functions of Central Executive Component of WM and Their Role in Mathematics

Within the Baddeley's WM model, the functions of the central executive can be fractionated into at least three separate functions: inhibition, updating, and shifting (Miyake et al., 2000). Executive processes, and in particular inhibition, appear to be particularly important for successful solutions of mathematical word problems (Passolunghi & Siegel, 2001, 2004). Previous research has demonstrated a strong relationship between inhibitory processes and reading comprehension. Specifically, children with reading disabilities perform poorly on working memory tasks that require inhibition of irrelevant information (Chiappe, Hasher, & Siegel, 2000). These findings show how poor comprehenders' performance on working memory tasks is impaired because they are unable to inhibit irrelevant information adequately. The negative consequence of this situation is an overload of their working memory capacity.

The ability to inhibit irrelevant information is also related to the success in problem-solving tasks. Indeed, in both text comprehension and problem-solving, it is necessary to process a great number of information units. Some of these must be rejected in order to maintain only those that are relevant. In particular, in the problem-solving process, the integration of the relevant information into a coherent structure allows a correct and complete mental representation of a text of problem. Passolunghi and Siegel (2001, 2004), for example, found that poor problem-solvers had a deficit in their ability to reduce accessibility of nontarget and irrelevant information (see Passolunghi et al., 1999). These findings are compatible with Engle's (2002) suggestion that individual differences in working memory capacity are not related to how many items can be stored in memory but in the difference in ability of controlling attention and maintaining information in an active, quickly retrievable state. Moreover, he argues that attentional control is related to inhibitory deficits, that is, individuals who have difficulty maintaining attentional focus on the task-relevant information are likely to make intrusion errors.

Another executive function associated with the central executive is the updating of information. Updating is a complex activity that requires attributing different levels of activation to the items presented and maintaining a restricted set of elements activated continuously. A typical measure of updating ability is Morris and Jones' updating task (Morris & Jones, 1990), which requires participants to listen to several lists of letters of varying length (4 to 10). Participants are asked to recall only the final four letters of each list. Since the length of each series is unknown, they are forced to update the information maintained in their WM continuously in order to remember the final four letters only. Updating skills are involved in resolving arithmetic word problems. Indeed, in order to understand word problems, children have to process all information derived from texts. Some information will be inhibited very early because it is not relevant to the solution. Other information will be connected in a coherent model that will be enriched successively by new information. This model will be complete when all the information relevant to solving the question has been integrated. Further information concerning other questions will then be processed and structured in different models. In short, a child who has to update information during a problem-solving task has to select relevant information, to inhibit information already processed but no longer relevant, and to substitute the no longer relevant information with a new one (Passolunghi & Pazzaglia, 2004). Shifting from one model to another requires individuals to update information in working memory, in a fine modulation of the mechanisms of enhancement and inhibition (Passolunghi & Pazzaglia, 2004).

It is widely assumed that updating processes are important also in calculation, in particular during the early development of arithmetical skills. Indeed, arithmetic calculation requires the storage and manipulation of intermediate results, by updating the results of operations such as carrying and borrowing. In line with this view, research shows that children with low updating skills had a poorer performance in solving word problems and calculation, compared to children with higher updating skills (Passolunghi & Pazzaglia, 2004). Recent studies show that updating deficits in children with ADHD may be a further source of their difficulty when solving mathematical problems (Re, Lovero, Cornoldi, & Passolunghi, 2016). In this respect, their impairment in the ability to update information should not differ from the updating difficulties of children with learning difficulties in mathematics showing difficulties in the recall of relevant information and controlled use of problem procedure.

Another executive function is the ability to shift back and forth between multiple tasks, operations, or mental sets. Among the typical complex tests usually used in cognitive and neuropsychological studies to assess executive function, the Wisconsin card sorting task (WCST) involves testing of the shifting processes. The WCST requires matching a series of target cards, presented individually, with any one of four reference cards. The participants are aware that the sorting criterion would change during the task, but they are not explicitly told the exact number of correctly sorted cards to be achieved before the criterion shifts. This test is often conceptualized as a set-shifting task because of its requirement to shift sorting categories after a certain number of successful trials. It is worth noting that some researchers view this task as requiring inhibitory control to suppress the current sorting category before switching to a new one. There is very little research on shifting and mathematical ability, and further research is necessary to clarify this issue. Bull and Sherif (2001) found that the WCST percentage of perseverative responses was negatively correlated with mathematical ability in typically developing primary school children. That is, children with higher mathematics ability made a lower percentage of perseverative responses in this task. These results suggested that the main difficulty for children with mathematical learning difficulties in performing arithmetic tasks is to inhibit a learned strategy and to switch to a new one.

Interestingly, the results of Espy et al. (2004) showed that shifting or mental flexibility did not contribute to mathematical skills in preschool children. They assessed shifting ability by tasks that require rule-based learning and shifting (e.g., spatial reversal task), similar to WCST. It is possible that mental flexibility may contribute more to mathematical abilities in older children, allowing the child to flexibly apply different mathematical procedures in problem-solving and calculation (e.g., borrowing, carrying) to obtain correct mathematical solutions.

Working Memory Training

As extensively described in the previous paragraphs, results from most of the studies conducted up to date suggest working memory abilities influence children's performance in mathematical achievement. Indeed, different mathematical tasks, such as performing mental arithmetic and understanding mathematical word problems, require the storage of information, while it is being processed or integrated with information retrieved from long-term memory. Given the important role played by WM abilities in the development of children's mathematical skills, in the last 15 years, different studies have explored whether mathematical learning problems can be overcome by training specifically designed to enhance working memory. WM was traditionally considered a genetically fixed cognitive ability (Kremen et al., 2007). Therefore, in the past the possibility to enhance WM skills by acting on an individual's environmental experiences and opportunities was not considered. Recently, a growing set of studies with children with typical development and adults has shown that WM skills can be improved through training (e.g., Alloway, Bibile, & Lau, 2013; Kroesbergen, van't Noordende, & Kolkman, 2014; St Clair-Thompson, Stevens, Hunt, & Bolder, 2010).

The debate regarding the effects of WM training is still open: some studies show positive effects of WM training on arithmetic abilities in primary school children using computerized or school-based training procedures (Alloway et al., 2013; Dunning, Holmes, & Gathercole, 2013; Holmes, Gathercole, & Dunning, 2009; St Clair-Thompson et al., 2010). Other authors questioned the effectiveness of WM training concluding that there is no convincing evidence of the generalization of working memory training to other skills (Melby-Lervåg & Hulme, 2013). However, the possibility should be considered that cognitive training applied to younger individuals tends to lead to significantly more widespread transfer of training effects (Wass, Scerif, & Johnson, 2012).

Holmes et al. (2009) provided the first evidence of the efficacy of the computerized "Cogmed" training in overcoming common impairments in working memory and associated learning difficulties in 10-year-old children with low working memory skills. They proposed different training tasks that involve the temporary storage and manipulation of either sequential visuospatial information, verbal information, or both. Children in the training group engaged in the Cogmed program for 35 min a day, for at least 20 days in a period of 5–7 weeks. The majority of the children who completed the program improved on tasks tapping the central executive and the visuospatial sketchpad components of WM. Moreover, a significant increase in mathematics performance assessed with the mathematical reasoning subtest of the Wechsler objective number dimensions (WOND; Wechsler, 1996) was also found, 6 months after the training. St Clair-Thompson et al. (2010) showed the effectiveness of a computerized working memory training ("Memory Booster") in typically developing children aged 5–8 years. The computer program used teaches memory strategies to children, over a period of 6–8 weeks, and resulted in significant improvements in tasks that assess the phonological loop, the central executive, mental arithmetic, and following instructions in the classroom. Enhancing mathematical abilities in 9- to 10-year-old typically developing children is also possible using individual school-based working memory training (Witt, 2011). The WM training program developed by Witt (2011) was carried out over a period of 6 weeks, the children in the intervention group were seen individually, and each training session lasted approximately 15 min. This study suggested that children who underwent working memory training made significantly greater gains in the trained working memory tasks, as well as on an untrained visuospatial working task, compared to a matched control group. Moreover, the training group also made significant improvements in mental arithmetic.

Only a few studies have explored the possibility of enhancing working memory abilities in kindergartners using a specific working memory training. In a recent study (Passolunghi & Costa, 2016), the authors of this section systematically investigated the effects of a training program focused on the enhancement of working memory and a second training program focused on the enhancement of early numeracy. The participants were 48 5-year-old typically developing preschool children. Both the working memory and early numeracy training programs were implemented for 5 weeks, twice weekly, each session lasting 1 h. The working memory training included different paper-and-pencil tasks designed to enhance all three components of Baddeley's working memory model (Baddeley, 1986). On the other hand, the early numeracy training included different paper-and-pencil tasks designed to enhance early numerical abilities such as counting, number-line representation, one-to-one correspondence between quantities and numerals, and quantity comparison. The results of this study showed that the early numeracy intervention specifically improved early numeracy abilities in preschool children. On the other hand, the working memory intervention improved not only verbal and visuospatial working memory abilities but also general early numeracy skills assessed with the early numeracy test (Van Luit, Van de Rijt, & Pennings, 1994). Interestingly, the early numeracy gain obtained in the working memory training group did not differ significantly from the gain obtained in the early numeracy training group. These findings stress the importance of performing activities designed to train working memory abilities, in addition to activities aimed to enhance more specific skills in order to support mathematical development. This kind of activities could be particularly important for those children who are considered to be at risk for developing learning disabilities later on in life. Indeed, WM training seems to be effective in improving math performance also in young children with low early numeracy abilities. The results of a study by Kroesbergen et al. (2014) showed that preschoolers with low numerical skills who participated in a working memory intervention program for 4 weeks significantly improved their working memory and early numeracy skills. The training program consisted of eight 30-minute sessions with hands-on activities, which were implemented in small groups of five children. The positive results of this study suggest that WM training activities can be used with low-performing preschool children, in order to minimize the future learning difficulties that result from WM deficits.

Conclusion

Individual differences in working memory capacity appear to have a strong influence on children's ability to acquire knowledge and new abilities. The great importance of WM in a range of cognitive skills including mathematics has been supported by different studies (see Cowan & Alloway, 2008). Moreover, several researches corroborate a network view of mathematical abilities where domain-general cognitive abilities as working memory sustain the development of mathematical abilities over and above the role of more domain-specific abilities (e.g., Geary, 2011; Passolunghi et al., 2014; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014). In addition, training studies support the same view and suggest that timely action to prevent children from developing early difficulties in mathematical learning should focus both on domain-specific variables, such as number competence, and on more general abilities.

The hypothesis that WM training should improve not only working memory but will also have a transfer effect on early numeracy skills is supported by studies dealing with WM training and transfer effects on math abilities in primary school children and kindergarten (Alloway et al., 2013; Holmes et al., 2009; Kuhn & Holling, 2014; Passolunghi & Costa, 2016; St Clair-Thompson et al., 2010). However, the possibility that the role of working memory training could vary with development should be considered. Most of the studies investigating the effects of WM training focused on school-aged children, while only a few studies have explored the possibility of enhancing working memory (and related early numeracy abilities) in younger children. It is entirely possible that the effects of WM training might be stronger in younger children when the neural system is more malleable to experience (Wass et al., 2012). These results regarding the positive effects of WM training could have interesting implications for classroom practice in preschool and primary school. Performing hands-on activities as well as computerized training tasks designed to boost WM performance may help children to improve cognitive precursors fundamental in future school learning, encouraging the prevention of learning difficulties. Future research should focus on the investigation of the effects of WM training in children who are considered to be at risk for developing learning disabilities. In fact, these kinds of WM training activities could be particularly appropriate for low-performing children in order to minimize the future learning difficulties that result from WM deficits.

One final consideration regards the role played by emotional and motivational aspects in mathematical cognition (Cargnelutti, Tomasetto, & Passolunghi, 2016). Despite the clear importance of domain-general and domain-specific cognitive precursor of mathematical learning, future studies should consider a unitary a complete model which includes also emotional factors such as math anxiety. Even if emotional factors have clearly a potential (negative or positive) impact on a child's development of math skills, very few studies tried to provide a global profile, including both cognitive and emotional factors. This important and topical issue will be further discussed in Chap. 3 of this book.

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