A Regression Based Approach for Valuing Longevity Measures

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Abstract. This paper addresses the ever-prominent issue of how to evaluate and forecast future longevity dynamics. Indeed, studying the evolution of mortality and/or the cost of longevity risk is a major task for both demographers and actuaries. In contrast to the usual period-based evaluation, we consider the problem of approximating the distribution of future life expectancy with a cohort-based perspective. In particular, we suggest an application of the Least-Squares Monte Carlo approach, which allows to overcome the straightforward nested simulations method. The method is applied to the family of CBDX models, and results and comparisons between different models, males and females, and period and cohort approaches, are presented.

Keywords: LSMC · Longevity risk · Stochastic mortality

1 Introduction

The analysis of mortality, and consequently of the evolution of various longevity indices, is always under study by demographers and actuaries. Indeed, policy makers need to quantify and manage the risks deriving from unexpected changes in mortality, which would have major implications for the financial stability and solvability of insurance companies and pension providers.

In contrast to the usual period-based approach, this paper addresses the problem of approximating the distribution of future life expectancy, and provides a simulation scheme with a cohort-based perspective that depends on the future evolution of mortality obtained by relying on extrapolative methods. In this regard, one contribution can be found in [7], where the so-called SCOPE approach to forecast future life expectancy levels, i.e., by conditioning on specific future mortality scenarios, is introduced. Indeed, forecasting longevity indices

with a cohort-based method requires the computation of conditional expectations for which explicit solutions often do not exist. A simple way to solve this problem would be to rely on a nested simulations approach, which unfortunately becomes readily unmanageable and computationally intensive, especially when life expectancy estimates are needed for different cohorts and when stochastic mortality models with multiple factors are considered. To overcome this drawback, [3] proposes a Taylor-series approximation of the involved conditional expectations.

This work, instead, suggests an application of the well-known Least-Squares Monte Carlo (LSMC) approach firstly introduced in the financial field (e.g., see [6]) and then extensively adopted in the actuarial one. The main idea is to approximate conditional expectations by linear combinations of some basis functions depending on the relevant factors that affect the quantity of interest. Among the most important advantages of this method, we can mention its generality and flexibility; indeed, it can be used with any mortality model, regardless of its complexity. Essentially, the methodology proposed in this paper is based on that described in [1], where the problem of evaluating future life annuities is addressed. Even if here we focus on just life expectancy, this methodology may be adopted also for approximating other longevity measures at future dates for which cohort-based estimations are often replaced by period ones for computational simplicity.

The remainder of the paper is structured as follows: Sect. 2 states the problem and briefly explains the proposed methodology, Sect. 3 illustrates some numerical results and finally, in Sect. 4, we draw some conclusions.

2 Life Expectancy and Computational Framework

The objective of this paper is to analyse the evolution of future life expectancy levels. Indeed, even if previous studies have broadly addressed this problem, the majority of them exploited a period approach, therefore neglecting future mortality improvements. To fill this gap, we propose a methodology that allows to adopt a cohort based perspective without increasing the computational complexity.

To this end, let $\mu_{x,t}$ be the instantaneous death rate for an individual aged x at time t. Then, following [2], we assume that the force of mortality is constant over each year of age and calendar. Hence, denoting by $m_{x,t}$ the central death rate at age x in year t, and $p_{x,t}$ the 1-year survival probability of an individual aged x at time t, it follows that $m_{x,t} = \mu_{x,t}$ and $p_{x,t} = e^{-\mu_{x,t}} = e^{-m_{x,t}}$.

Now, we are interested in estimating the residual lifespan of an individual aged x at a future time $T > 0$. We define the *period life expectancy* measure as follows:

$$e^{p}_{x,T} = \frac{1}{2} + \sum_{i=1}^{\omega-x} {}_{i}p_{x,T}, \tag{1}$$

where ${}_ip_{x,T} = e^{-\sum_{k=0}^{i-1} m_{x+k,T}}$ represents the i-th years survival probability for an individual aged x at time T, computed by considering the age-specific mortality rates at time T, and ω is the ultimate age. It is clear from Eq. (1) that further mortality improvements after time T are ignored.

Therefore, to describe the actual life course of an individual aged x at time $T > 0$, let us introduce the concept of *cohort life expectancy* defined as

$$e_x^c(T) = \frac{1}{2} + \sum_{i=1}^{\omega - x} {}_ip_x(T), \tag{2}$$

where ${}_ip_x(T) = \mathbb{E}_T\left[e^{-\sum_{k=0}^{i-1} m_{x+k,T+k}}\right]$ represents the (conditional) i-th years survival probability for an individual aged x at time T, and $\mathbb{E}_T[\cdot]$ is the conditional expectation given the information available at the future date T. As already mentioned, cohort life expectancy is not as commonly evaluated, unlike its period counterpart, since it requires the calculation of a conditional expectation. Note that both Eqs. (1) and (2) are the discrete versions of period and cohort life expectancy measures given, for instance, in [5].

Forecasting life expectancy at future times requires projections of mortality onto the future. For this reason, we introduce the computational framework on which we build some numerical results. In particular, we make use of stochastic mortality models in order to capture the possible time evolution of mortality, and in this regard we consider the recently introduced CBDX family (see [4]). Hence, let $D_{x,t}$ denote the number of deaths at age x and calendar year t, which is assumed to be Poisson distributed with parameter $E_{x,t}m_{x,t}$, where $E_{x,t}$ denotes the central exposure. Then, according to [4], the central death rate at age x and calendar year t can be modelled as

$$\log m_{x,t} = \alpha_x + \sum_{i=1}^{N} f^{(i)}(x)\kappa_t^{(i)} + \gamma_{t-x},$$

where α_x is a static age parameter, $\boldsymbol{\kappa}_t = \left(\kappa_t^{(1)}, \ldots, \kappa_t^{(N)}\right)$ is the time index, γ_{t-x} incorporates the cohort effects, and $f^{(i)}(x)$ is a known age-modulating function. In particular, [4] considers the case of $N \in \{1, 2, 3\}$ (named CBDX1, CBDX2 and CBDX3, respectively), and proposes as modulating functions $f^{(1)}(x) = 1, f^{(2)}(x) = (x - \bar{x})$ and $f^{(3)}(x) = \left[(x - \bar{x}) - \sigma_x^2\right]$, where $\bar{x}$ and σ_x^2 represent the mean and variance of the ages in the data. To project mortality into the future, the time indices are assumed to follow a multivariate random walk with drift, while the cohort effect is modelled as a univariate ARIMA model.

2.1 Valuation Procedure

Computing the quantity in Eq. (2) is not a trivial task since explicit expressions do not always exist. In particular, this is the case of the valuation framework previously introduced. For this reason, a straightforward solution would be a

nested simulations scheme. The latter is computationally challenging since it requires a huge number of simulations. An alternative methodology has been proposed by [3], which consists in approximating conditional expectations by Taylor-series expansions. However, also this approach would be time-demanding since multiple simulations sets are needed in order to estimate the involved coefficients. For this reason, on the basis of [1] we adopt a very flexible tool for approximating conditional expectations, i.e. the LSMC method. Indeed, to the best of our knowledge, this methodology has been extensively used in many fields but it has not yet been proposed in the demographic context. The main idea is to express conditional expectations through linear combinations of some basis functions (e.g. simple or orthogonal polynomials) depending on the relevant risk factors that affect the evolution of mortality (in our case, the time indices $\boldsymbol{\kappa}_t$ and γ_{t-x}), and use regression across simulations against those factors. Hence, we will evaluate Eq. (2) by regression. Moreover, we refer the readers to [1] for more details.

3 Numerical Results

In this Section we provide some numerical results based on the previously introduced framework. In particular, we analyse the evolution of life expectancy with both cohort and period life tables. The analysis considers males and females in England and Wales population. The models have been calibrated on the mortality data over the period 1965–2018 and range of ages 60–89, obtained from the Human Mortality Database, excluding the first and last 5 cohorts to avoid overfitting. We assume that year 2018 is time 0, and that life tables are closed using a log-linear procedure up to the ultimate age $\omega = 120$. Finally, all computations are based on $n = 20000$ trajectories, and the LSMC algorithm exploits as basis functions simple polynomials of order $p = 2$. Under this setting, we analyse the evolution of life expectancy of both males and females aged $x = 65$ at different future times $T = 2019, \ldots, 2053$ (35 years).

Table 1 reports a summary of the distributions of future cohort life expectancy for females at different future times T, obtained by exploiting the different stochastic mortality models. From the table, we can see that each of the proposed models suggests, as expected, an ever increasing life expectancy. In particular, the CBDX1 model provides more optimistic results, while the opposite happens for the CBDX2 model. Moreover, we can appreciate how the uncertainty increases as time passes[1]. All these features can be seen in Fig. 1 that compares the future cohort male and female life expectancy distributions. Figure 1 highlights, first, the increasing uncertainty characterizing the evolution of the longevity metric, and second, gender differences. Indeed, in line with the existing literature, our results depict future life expectancy levels for females constantly above those for males. Finally, in Table 2 we compare cohort and

[1] Similar results were obtained for males, not reported here for space considerations.

period[2] approaches. The table shows how, unsurprisingly, the latter approach persistently under-estimates the desired quantities. What is important to notice is, instead, the magnitude of such under-estimation, which may lead public social systems and life insurance companies to under-estimate the related risks.

Table 1. Summary of the future cohort life expectancy distributions for females aged $x = 65$ at (future) times T. LSMC based on 20000×1 simulations with monomials of degree $p = 2$.

T	Model	Mean	Std. dev.	Skew.	Kurt.	10th Perc.	Median	90th Perc.
2019	CBDX1	23.535	0.284	−0.104	3.058	23.170	23.541	23.900
	CBDX2	21.838	0.273	0.026	3.007	21.489	21.835	22.191
	CBDX3	23.215	0.283	0.085	3.081	22.859	23.211	23.580
2039	CBDX1	25.788	1.154	−0.030	3.031	24.305	25.790	27.256
	CBDX2	23.515	1.194	0.063	3.006	21.999	23.495	25.050
	CBDX3	25.292	1.221	0.026	3.028	23.741	25.286	26.863
2053	CBDX1	27.347	1.453	−0.084	2.981	25.467	27.361	29.202
	CBDX2	24.666	1.571	0.072	3.010	22.668	24.646	26.707
	CBDX3	26.700	1.557	0.031	3.012	24.717	26.690	28.714

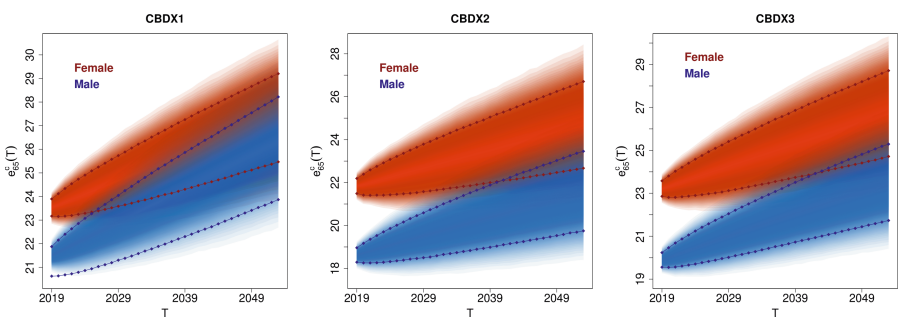

Fig. 1. Distribution of future life expectancy for a cohort of females (red) and males (blue) aged $x = 65$ at (future) times $T \in \{2019, \ldots, 2053\}$. Stochastic mortality models: CBDX1 (left), CBDX2 (centre), CBDX3 (right). LSMC based on 20000×1 trajectories with monomials of degree $p = 2$. Dotted lines represent the 90% prediction intervals.

[2] Period life expectancy estimates have been obtained through a simple Monte Carlo (MC) scheme based on 20000 simulations.

Table 2. Expected future cohort and period life expectancy for females aged $x = 65$ at (future) times T. LSMC (cohort) based on 20000×1 simulations and monomials of order $p = 2$. MC method (period) based on 20000 trajectories.

T	Approach	CBDX1	CBDX2	CBDX3
2019	Period	21.584	21.501	21.530
	Cohort	23.535	21.838	23.215
2039	Period	24.001	22.373	23.680
	Cohort	25.788	23.515	25.292
2053	Period	25.510	23.401	25.084
	Cohort	27.347	24.666	26.700

4 Conclusion

In this paper we addressed the ever-prominent issue of how to evaluate and forecast future longevity dynamics, and in particular we focused on life expectancy. We proposed the LSMC approach that allows to adopt a cohort based perspective, rather than a period one, without increasing the computational complexity. Our results proved to be in line with those already presented in literature. To conclude, we want to strengthen the idea that this methodology can be used to estimate any other longevity measure involving conditional arguments, where cohort measurements are often replaced by period ones for computational simplicity.

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