

# Strong Alethic Pluralism and the Fragmentation of Logical Validity

Andrea Stollo

University of Trieste

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## Abstract

Truth pluralism has been criticized for its difficulty to vindicate the traditional truth-theoretic account of logical validity. In this paper, we argue that an underestimated version of this problem takes form in the fragmentation of logical validity induced by a plurality of truth properties. We briefly present the difficulty differentiating it from the similar one connected with mixed inferences, and we argue that joining forces with logical pluralism does not alleviate the burden from the truth pluralist shoulders. Accordingly, we review some solutions that have been proposed for mixed inferences, assessing whether they can also fix the fragmentation. We show that only some strategies are able to solve both problems.

## Keywords

logical validity; mixed inferences; truth pluralism; truth preservation

## 1 Strong truth pluralism and logical validity

Assume a form of truth pluralism along the following lines.<sup>1</sup> There is only one truth *predicate* in the language (synonymic expressions aside); there is only one *concept* of truth associated to it (understood for simplicity as a kind of mental representation in a broad sense); but there

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<sup>1</sup> For an overview of truth pluralism see, for instance, Lynch and Pedersen [2018]. Wright [1992] and Lynch [2009] are two classic works.

are many (more than one) *properties* of truth varying in areas of discourses about different subject matters. If we add the specification that each truth property is confined to certain areas, so that there is no generic property of truth, we have a quite standard template for a strong version of alethic pluralism. Notice that while there is not a general property of truth, the predicate and the concept are general. This means that generic claims such as ‘truth is the goal of inquiry’ or ‘everything god says is true’ are possible and legitimate, but they go with a multiplicity of area-specific truth properties, rather than with a generic property of truth. In a sense, the truth predicate and the truth concept, which are unique and generic, are just convenient shortcuts to gather together a gerrymandered collection of properties. Those who feel uneasy with this discrepancy between the unitary concept/predicate versus the plurality of properties could also postulate a merely *insubstantial* property of generic truth (something like the property of falling under the concept of general truth). In Lewis’ jargon, such a property would be abundant, and thus insubstantial, for it would correspond to a non joint carving predicate.<sup>2</sup> Such an insubstantial, abundant generic truth property is the best a strong pluralist can afford. A helpful way to look at this is combining the usual deflationary stance about truth at the level of predicate and concept with a pluralist and inflationary view about the truth properties of specific areas of discourse.

As is well known, strong truth pluralism has difficulties in accounting for discourses that involve different domains, since in such cases a general property for truth would come in handy. In particular, strong pluralism is hard to square with the venerable definition of logical validity as necessary truth preservation.<sup>3</sup> The problem is immediately clear if we consider a valid inference in which premises and conclusion mix different areas of discourse. If there is not a single property of truth shared by all premises and conclusion, what property of truth is preserved? Strong pluralism is in trouble. This is the problem of mixed inferences. Although strong pluralists have implicitly accepted this formulation, it should be noted that the problem is theoretical and it does not depend on what notion of validity truth pluralists have actually endorsed.<sup>4</sup> The problem emerges from the apparent incompatibility between strong truth pluralism and a traditional account of logical validity.

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<sup>2</sup> Lewis [1983]. For an application of the abundant/sparse distinction to truth see Asay [2014], Stollo [2014], Edwards [2016].

<sup>3</sup> Williamson [1994], Tappolet [1997].

<sup>4</sup> See for example Cotnoir [2013] or Gamester [2019].

The point is not what truth pluralists accept, but what they are arguably forced to exclude.<sup>5</sup>

Mixed inferences have been addressed in different ways and a few, potentially successful solutions have been provided.<sup>6</sup> However, the problem is not the only obstacle posed by logical validity, since there is a different and possibly deeper issue for strong truth pluralism in its vicinity. This is the problem of avoiding that the property of validity is fragmented in a vast plurality of disparate properties as a consequence of the various truth properties that can be preserved.<sup>7</sup> Although this fragmentation also has to do with the challenge of combining strong truth pluralism with the traditional account of logical validity as necessary truth preservation, it does not arise only in the context of mixed inferences. This fragmentation problem has been recently put forward and shown to affect various theories of truth, such as deflationism and strong pluralism, in different ways. In this paper we expand that discussion in two ways. On the one hand, we relate the fragmentation problem to logical pluralism, assessing whether and in what measure fragmentation poses a genuine threat. On the other hand, we discuss the prospects of adapting the solutions proposed for mixed inferences to also fix the fragmentation of validity. Since the latter issue can also emerge in the case of *non*-mixed inferences, it is far from obvious that such an extension is possible. The final upshot will be that only some strategies are able to solve both problems. The paper follows this just outlined plan.

## 2 Non-mixed inferences and the fragmentation of logical validity

How can inferences involving different truth properties be valid in the same way? This is the gist of the fragmentation problem. According to it, valid inferences whose components belong to different areas of discourse cannot have the same property of logical validity because they preserve different truth properties. Consider the following example for illustration. Suppose that from premise A, belonging to area<sub>1</sub> and apt to be true<sub>1</sub>, the conclusion B, also belonging to area<sub>1</sub> and apt to be true<sub>1</sub>, can be validly inferred. Call this

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<sup>5</sup> This is the standard reconstruction of the nature of the debate, explicitly given, for example, in Cotnoir [2013].

<sup>6</sup> Attempted strategies include, for example, Pedersen [2006] and Cotnoir [2013], beside the more recent proposals discussed below. Stollo [2016] offers a survey and a critical discussion of some of these accounts.

<sup>7</sup> For an extensive presentation see Stollo [2024].

inference<sub>1</sub>. What does the validity of inference<sub>1</sub> consist of for strong pluralism? The answer is straightforward: it consists in necessary preservation of the property of truth<sub>1</sub>. In any case in which A is true<sub>1</sub>, also B is true<sub>1</sub>. No problem here. Consider then a non-mixed inference taking place in area<sub>2</sub>. Suppose that from premise C, belonging to area<sub>2</sub> and apt to bear truth<sub>2</sub>, the conclusion D, also belonging to area<sub>2</sub> and apt to be true<sub>2</sub>, can be validly inferred. Call this inference<sub>2</sub>. Again, what does the logical validity of inference<sub>2</sub> consist of for the strong pluralist? Analogously to the previous case, it consists in necessary truth<sub>2</sub> preservation. So far so good. A puzzle, however, immediately arises once we ask the strong pluralist whether inference<sub>1</sub> and inference<sub>2</sub> are valid in the same way, namely whether they instantiate the same property of validity. The answer must be ‘no’. The validity of inference<sub>1</sub> consists in truth<sub>1</sub> preservation—call it validity<sub>1</sub>—, whereas the validity of inference<sub>2</sub> consists in truth<sub>2</sub> preservation,—call it validity<sub>2</sub>. Since, given truth pluralism, truth<sub>1</sub> and truth<sub>2</sub> are different, for the strong pluralist the various truth properties are not versions of a generic way of being true, so that truth<sub>1</sub> and truth<sub>2</sub> are indeed apples and oranges. This is what makes the pluralist strong rather than weak.<sup>8</sup> It then follows that also validity<sub>1</sub> and validity<sub>2</sub> are not forms of a more general property of logical validity, but they are apples and oranges too. For the strong truth pluralist, logical validity is as fragmented and gerrymandered as truth is. Moreover, although the fragmentation of the unity of validity strikingly emerges in some non-mixed inferences, it generalizes. The properties of validity proliferate when mixed inferences are also considered. Indeed, since in any case in which different combinations of truth properties are involved a further property of validity is yielded, the situation escalates quickly.

One might try to fix this obtained fragmentation by resorting to the concept of generic truth, speaking of preservation of the properties falling under the concept of generic truth. This would certainly provide a generic *concept* of validity as necessary truth preservation, under which both validity<sub>1</sub> and validity<sub>2</sub> fall. However,—and this is the crucial point—from the perspective of strong pluralists, a concept is not enough to secure a uniform property. Concepts do not automatically carve the underlying reality at joints, otherwise a substantial generic truth property—anathema to strong pluralists—would be automatically postulated by a generic truth concept and strong truth pluralism would be just incoherent. By contrast, concepts and their associated platitudes can be somehow superficial, possibly motivated

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<sup>8</sup> Whereas weak pluralists admit a generic property of truth whose area-specific properties are different manifestations. See Lynch [2009].

by cognitive or practical purposes, while lacking a metaphysical import. For cognitive or practical businesses, putting apples and oranges together may be a good choice, but one should refrain from hastily reading metaphysics out of this. This means that even if a generic concept of validity is obtained from the generic concept of truth, it only produces a gerrymandered collection of properties of validity impotent to fix the fragmentation. Generic validity is abundant and fragmented as the property of generic truth is.

### 3 Logical pluralism

According to the previous remarks, strong truth pluralism implies an exacerbated form of strong validity pluralism, imposing a multiplicity of different ways of being valid. A first reaction to this result may be that of deeming it as unproblematic. Strong pluralists should just acknowledge the fragmentation and incorporate validity pluralism, no matter how inflated it would be, into strong truth pluralism.

Before arguing that such a dismissive attitude is costly, we should emphasize that, even if this was the outcome, standard presentations and discussions of strong truth pluralism should register and explore its consequences. The debate on truth pluralism, instead, has been largely conducted under the impression that the only problem with logical validity was a problem of preservation, typically displayed by mixed inferences, not a problem of unity, exemplified also by non-mixed inferences. Moreover, despite some occasional discussions and overlaps, it has been mostly held that pluralism about truth and pluralism about logic are fundamentally independent enterprises.<sup>9</sup> Acknowledging the fragmentation of validity imposed by strong truth pluralism would imply that the connection is actually stronger and perhaps unavoidable, at least from the side of truth pluralism. Having said that, we can move on to the problems of fragmentation, showing that a combination with logical pluralism is not as helpful as it may appear.

To see the damaging consequences of the fragmentation of validity, it is useful to consider the impact on logic as a discipline. Logic, at least with respect to its so called canonical application and not merely as an area of pure mathematics, presents itself as a theoretical enterprise whose core aim is characterizing, by mathematical means, the inferences that are logically valid. Implicit in the legitimacy of pursuing this goal is the idea that valid inferences form a natural collection, exhibiting a common feature whose investigation is

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<sup>9</sup> For some exceptions and overlaps see Wyatt and Pedersen [2018], Lynch and Shapiro [2019].

a sensible task. In other words, it is a presupposition of logic that the subject matter of logic consists in a certain uniform phenomenon. There is a property of logical validity, exemplified by some arguments, and logic studies its nature and properties. This quick but reasonable description is immediately undermined by strong truth pluralism, since, given the argument above, it implies that the assumption that there is a uniform subject matter is illusory. According to truth pluralism, the properties of validities are many, and they have no objective similarity keeping them together. Hence, logical inquiry proceeds under a mere illusion that there is a somehow uniform phenomenon corresponding to validity, although the various validities are not similar to one another. Such a mistake can even be explained by truth pluralists, since logicians can be regarded to have been apparently misled by the conceptual superficialities stressed above. By relying on a generic concept of truth, they yielded a generic concept of validity and then isolated the presumed corresponding subject matter. However, as already emphasized, since the concept of truth does not carve the truth properties at any joints, also the resulting concept of validity is an umbrella term covering a patchwork of different properties, to which a unique label, 'validity', is forcefully applied. Logicians put together apples and oranges and studied them naively thinking that they have a homogeneous basket of fruits.<sup>10</sup> By contrast, given the fragmentation of validity, logicians should also fragment their discipline in many disparate inquiries, each one addressing a specific property of validity. A discipline studying preservation of correspondence, a discipline studying preservation of coherence, a discipline studying an inference with a premise apt to correspond and the conclusion apt to cohere, and so on and so forth for any possible combination, because any combination presents a radically different property of validity. The illusion that there is a common nature or shared similarities among these properties, vindicating their study in a single discipline, should be abandoned. People working on logic should be promptly alerted by this discovery and update their practice accordingly.

Given the revisionary power of the obtained fragmentation, one may think that Logical Anti-Exceptionalism, or AEL for brevity, may be relevant here.<sup>11</sup> However, the relevance of AEL is questionable. First, as the recent debate has shown, it is not clear how AEL should

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<sup>10</sup> Literally, this metaphor is wrong, because apples and oranges do share objective similarities: they are all fruits. Not so in the case of validities and truth properties.

<sup>11</sup> See Hjortland [2017], Hjortland and Martin [2022, 2024, 2025].

be understood and if it really provides a useful theoretical category.<sup>12</sup> Second, even putting such worries aside, according to the most typical account, AEL holds that the methodology of logic should be continuous with that of other sciences, namely abductive, and large (if not unique) weight should be given to empirical evidence. This however is unrelated to the thesis that validity is uniform or fragmented, which apparently is made neither easier nor harder to defend in the context of AEL. In other words, the fragmentation of validity and AEL are orthogonal. It may be objected that if AEL is articulated in terms of rejection of at least one traditional idea,<sup>13</sup> and uniformity is taken to be a traditional thesis, then we do obtain a form of AEL. Accordingly, the argument showing that strong pluralism leads to fragmentation would also show that strong pluralism is committed to AEL. If AEL is considered a legitimate position, the fragmentation would be a non problematic consequence. However, this line of thought is flawed. On the one hand, when understood as tradition rejection, the tenability of AEL depends on what traditional feature is rejected and why. Tradition rejection is not attractive per se. On the other hand, what is rejected here is the uniformity of validity in favour of a precise form of fragmentation. No significant work has been done in the context of AEL in this respect. At most, the issue may be connected to logical pluralism, and through logical pluralism only indirectly related to AEL. Hence, it is better to just put AEL aside and consider logical pluralism directly, as done next.

We emphatically stressed the apparent problematicity of accepting a plurality of properties of validity and the ensuing forced fragmentation of logical inquiries. However, a possible reaction to my remarks may be that this is an over exaggeration, given that pluralism in logic is an actual position that does not seem to have the disruptive consequences we allude to. On the contrary, logical pluralism can even do better justice to the various practices of logicians, meeting increasing sympathy and perhaps replacing logical monism as the dominant or default position. If so, interesting as it may be, the plurality of logical validities implied by truth pluralism is not a problem for logic.

To assess such a reaction we need to articulate logical pluralism more, and compare it with the version obtained by truth pluralism. Although logical pluralism comes in many versions, at its core there is the thesis that there is more than one correct logic. For example, classical logic may be correct, but so can be a non classical logic, like intuitionistic logic or a three valued logic. It is a task of logical pluralism to develop this idea and investigate

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<sup>12</sup> See Rosenberg and Shapiro [2021] for criticisms.

<sup>13</sup> As in Hjortland and Martin [2022].

the various forms this can or cannot take. Such forms include treating logics as models, introducing a contextual dimension, and so on.<sup>14</sup> Two aspects are relevant for our present issue. The usual versions of logical pluralism accept a plurality of *general* logics, since the various admitted logics are not confined to limited areas of discourse. This is a first disanalogy with truth pluralism. Secondly, all correct logics need not give form to different *properties of validity*. On the contrary, validity may be one and the same thing (like necessary truth preservation) captured by different logics. Logical pluralism need not be, and often is not, a pluralism of properties of validities. This is another difference with the kind validity pluralism we obtained by truth pluralism. Indeed, our argument had no direct impact on what and how many logics are correct, but only on how many properties of validity there are. It is also possible that there are many properties of validity, but only one correct logic. Thus, our argument from truth pluralism implies a form of validity pluralism that is different from the dominant views of logical pluralism. For other versions and an overview, see Russell [2014].

At this point, one might note that recognizing a plurality of logics, as opposed to a plurality of properties of validities, is actually enough to secure the practice of logicians. Logicians often do not care about the metaphysical question about the nature of validity, for they typically pay attention only to the extensional adequacy of the set of valid inferences captured by certain logics. Thus the uniformity of the property of validity or its alleged fragmentation has no direct impact on the actual practice of logicians. The disruptive effect we were dramatizing may thus be considered exaggerated. However, on the one hand, the issue of the nature of validity and the logic instantiating it are not unrelated. Hence, some impact would arguably be had on at least some practice. On the other hand, and in any case, regardless of the impact on logical practice, the philosophically neglected point would remain at least at the theoretical level: strong pluralism implies a plurality of properties of validity.

An opponent may then insist that there actually are prominent forms of logical pluralism that admits a plurality of properties of validities along a plurality of logics. According to one of the chief versions of logical pluralism, described by Beall and Restall,<sup>15</sup> for example, the characterization of validity as truth preservation in all cases hinges on an underdetermined notion of case, for which we have the following schematic characterization called *Generalised*

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<sup>14</sup> These correspond to Cook [2010] and Caret [2017].

<sup>15</sup> Beall and Restall [2005].

*Tarski Thesis (GTT):*

**GTT.** *An argument is valid<sub>x</sub> if and only if in every case<sub>x</sub> in which the premises are true, also the conclusion is true.*

According to the authors, this underdetermined notion of case can be specified in different ways, as signaled by the index<sub>x</sub>. For example, a case can be a possible world, or a model defined in the usual classic first order way, or it can be considered as a stage of inquiry, or as an incomplete or even inconsistent situation. Depending on what a 'case' is taken to be, the resulting precisifications of GTT give rise to different analyses of logical validity (valid<sub>x</sub>). These various accounts of validity, in turn, may be realized by different logics, such as classic, intuitionistic and relevant logic. We thus have a version of logical pluralism that makes room also for a plurality of properties of validity. Logical pluralism can then show that the obtained fragmentation need not be problematic.

The analogy between this kind of logical pluralism and the validity pluralism induced by strong truth pluralism should not be overestimated, though. Two points of departure are worth noting. First, the various analyses of validities obtained by GTT are the result of the underdetermination and an ensuing plurality of the notion of case, not of truth. Moreover, even if truth is relativized to the kind of case under consideration, we have a form of truth relativism not of truth pluralism. Second, at least as presented, nothing prevents this case-based logical pluralism from admitting a generic notion of case (corresponding to the undetermined 'case<sub>x</sub>'), which can be specified in different ways. Once such a generic notion of case is available, however, the alleged similarity evaporates. By relying on a generic notion of case, in fact, unity among the various case-based validities can be secured. There is a straightforward sense in which all valid inferences are valid in the same way, provided by the general notion of case. This is the same unity we would obtain if, besides a generic concept of truth, also a generic property of truth were available. Case-based logical pluralism, however, has no reason to adopt such a strong formulation. On the contrary, given the fragmentation that would ensue, it has good reasons to reject it. Hence, Beall and Restall's pluralism does not seem to lead to a form of strong fragmentation of validity, let alone to the same kind of fragmentation induced by truth pluralism.

As a rejoinder, an opponent may point out that there are forms of logical pluralism beside the one devised by Beall and Restall that are not given in terms of GTT. In particular, some versions of logical pluralism are derived directly from the variance of truth properties

admitted by truth pluralism.<sup>16</sup> According to these versions, an area-relative form of logical pluralism that parallels truth pluralism is in fact obtained. Since in those versions everything aligns, it might seem that the validity pluralism that is implied by truth pluralism is not generally problematic. However, also in this case, the devil is in the details. First, in these proposals the fragmentation of the property of validity is not explicitly noted, let alone discussed. Its potential drawbacks are not evaluated and its implications are not accommodated. The relevance of the present work is then secured. Second, those proposals are not only quite exceptional but also entirely devised in the field of truth pluralism, rather than in the debate on logical pluralism. They get their support from considerations about truth, rather than from considerations about logic. Teaming with logical pluralism to reinforce the position of strong truth pluralism, showing that the obtained fragmentation of validity is not worrisome, is then not particularly effective. In order to be effective, logical pluralism should provide independent reasons for such a fragmentation, but this is not the case. Finally, as a confirmation of the limited role of forms of logical pluralism based on truth, we should note that they are not even mentioned among the main versions of logical pluralism reported in prominent surveys.<sup>17</sup> One of the reasons may be that variance in different areas of discourse, typical of truth-based versions of logical pluralism, is particularly at odds with the ambition to generality and formality standardly taken as defining traits of logic.

A final move to use logical pluralism as a way to tame the fragmentation of validity may be tempting. One may point out that there are logics that are unapt to receive a treatment in terms of GTT or even truth preservation. Examples abound. Non monotonic and substructural logics without transitivity, for example, do not have features that characterize a Trakonian consequence relation. Moreover, several logics, such as fuzzy logics, can be given algebraic semantics where truth preservation is not the key notion. These logics would show that validity is indeed already fragmented, so that the argued impact of truth pluralism is, in any case, redundant. This objection is tempting, but it misses the target by a large margin. First, that some logics may be given a semantics not in terms of truth preservation is irrelevant. The existence of alternative semantics was never an issue. What would be problematic, at most, would be logics that cannot be given a truth preservationist semantics,

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<sup>16</sup> Pedersen [2014].

<sup>17</sup> Cook [2010], Russell [2014].

or a close enough one.<sup>18</sup> Second, even if such logics existed, their status as authentic logics could be questioned exactly because they do not exhibit truth preservation in any sense. They may be formal systems that may be mathematically interesting but merely superficially resembling logics. Third, all the alleged logics that are not characterizable as necessary truth preserving lay beyond the scope of this debate. If they did not, also the original problem of mixed inferences would be dissolved. If it is not crucial to make sense of validity as truth preservation, because some logics escape this account, then the problem of what truth property is preserved in mixed inferences evaporates. Strong pluralists, like those alternative logicians, would be under no pressure to indicate how they can vindicate an account of validity in terms of truth preservation. Last, the same observations above apply again: the sort of validity pluralism obtained from strong truth pluralism is not found in those alternative logics mentioned above. Thus, they can hardly undermine the significance of the new and radical fragmentation induced from truth pluralism.

Let us take stock before proceeding. We argued that, once a strong form of truth pluralism is adopted and validity is understood as necessary truth preservation, a plurality of properties of validities is produced. We showed that such properties are quite disparate and hardly alike. A gerrymandered collection of properties is yielded and taken under the umbrella term ‘logical validity’ despite being so different from one another. To stress the importance of this result and its potential problematicity, we enlightened its implication for the foundation and practice of logic. It is at this point that logical pluralism seemed relevant. Given that logical pluralism is a widely shared thesis, perhaps having a plurality of properties of logical validity is harmless. The result may be just a reason to combine strong truth pluralism with logical pluralism. Once we pay enough attention to the field of logical pluralism, however, we see that the actual situation is more complicated. On the one hand, most forms of logical pluralism concern the number of logics rather than the number of properties of validities. Even when a plurality of properties of validity is accepted, we arguably do not have the same fragmentation because moderate forms of logical pluralism, rather than strong versions rejecting general properties of validities, are at stake. We might find a suitable form of logical pluralism, able to comfort the strong truth pluralist, only if we turn to truth pluralism itself. In the field of truth pluralism, some suitable logical pluralism can be found. However, the original strategy can hardly be implemented in this

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<sup>18</sup> Also a semantics relying on the notion of at least *as true as*, customary in some many-valued logic, would be unproblematic.

way, since the resulting logical pluralism is no longer independently motivated, so that it is no longer able to tame the various negative effects of the fragmentation. Indeed, this truth-based logical pluralism has a very marginal role in the discussion on logical pluralism. Finally, one may point out that there exist a good number of logics that may have a plurality of alternative properties of validity. However, even if the relevance of such logics were accepted, the fragmentation of validity induced by strong truth pluralism is of a completely different sort. Hence, putting all of this together, we can conclude that an appeal to logical pluralism is unable to neutralize or alleviate the problem of the fragmentation of validity implied by alethic pluralism. The obtained fragmentation remains problematic and calls for a solution.

## 4 Solving the problem

The problem of mixed inferences, namely the challenge for strong alethic pluralism to determine what area-specific truth property is preserved in valid inferences, has been subjected to some debate in the last few years. Different, more or less effective solutions have been offered.<sup>19</sup> One could then wonder whether these solutions to the problem of mixed inferences could also provide a solution to the problem of fragmentation of logical validity. Although the two problems are different, it would be theoretically convenient if they could be solved in one blow. In the rest of the paper I discuss to what extent this strategy works. In particular, I analyze three solutions to mixed inferences that are currently among the best ones and evaluate whether they can also solve the problem of fragmentation. I argue that one fails, whereas the other two succeed to various degrees. Before doing that, however, a cautionary remark is worth doing. The solutions to mixed inferences discussed below have been already extensively defended by the respective authors in their published articles. Given the absence in the literature of explicit objections to such proposals, here we assume that they all somehow work with respect to the problem of mixed inferences and represent the best options currently on the market.<sup>20</sup> Thus, although some complications for each one are briefly noted for the sake of debate, for present purposes what only matters is how the selected strategies perform with respect to the problem of fragmentation. New

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<sup>19</sup> See Stollo [2016] for a partial critical survey.

<sup>20</sup> Other solutions, such as Cotnoir [2013], have been instead criticized, for example in Keefe [2018] and Stollo [2018]. A critical review of some proposals is also given in Stollo [2016].

objections to their ability to solve the problem of mixed inferences and evaluation in that respect are thus beyond the scope of the present work.

#### 4.1 Validity through truth dependencies

The first solution to the problem of mixed inferences is what we call the structural solution.<sup>21</sup> It is not really a solution but a dissolution of the difficulty, since it aims at showing that it is not even an authentic problem. The strategy can be presented in two steps. First, we should stress that talking of “truth preservation” is metaphorical talk. What is actually meant is a conditional link stating that, in a valid inference, if the premises are true, the conclusion is also true. Secondly, this claim can be carefully formulated in a way that remains neutral on how many truth properties there are. Accordingly, in a valid inference we have necessary truth dependencies between premises and conclusion. Namely, an inference is valid if in any case in which the premises have their specific truth property, also the conclusion has its specific truth property. This structural covariance is all that is needed. In particular, there is no reason, with respect to validity, to require that the truth properties of premises and conclusion are the same, as long as they vary in the right way. Once it is acknowledged that such a reconstruction is adequate, it is easy to note that no problem of mixed inferences arises. The problem was illusory and it has been dissolved. Nonetheless, this approach is impotent with respect to the problem of fragmentation. Indeed, it helps make the problem emerge in a clearer form.

To evaluate the structural account with respect to the fragmentation problem, it can be useful to lay down the provided characterization of logical validity, mentioned above. According to this understanding we have the following formal rendering of truth dependencies:

##### *Structural Validity (SV)*<sup>22</sup>

An argument from premises  $\{A_1, \dots, A_n\}$  to conclusion B, such that  $F_1$  falls under the concept of truth and is area-specific for  $A_1$ ,

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$F_n$  falls under the concept of truth and is area-specific for  $A_n$ ,

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<sup>21</sup> Sketched in Lynch and Pedersen [2018] and fully developed by Gamester [2019].

<sup>22</sup> For the sake of readability, my formulation is slightly different from the one in Gamester [2019].

G falls under the concept of truth and is area-specific for B, is , iff: necessarily, if ( $A_1$  is  $F_1, \dots$ , and  $A_n$  is  $F_n$  ), then B is G.

It is important to note that SV is fully available to a strong truth pluralist, since it allows for a multiplicity of area-specific truth properties and, crucially, it does not rely on any general property of truth, but just on a concept of truth.<sup>23</sup> Such a concept, however, cannot keep the properties it denotes objectively together because they do not have a common nature nor exhibit objective resemblance. It gathers them together just for extrinsic reasons. We thus have disparate relations of dependencies treated as they formed a uniform phenomenon, by forcing them under the cognitive umbrella of the generic concept of truth. Once this aspect is stressed, it becomes apparent that the fragmentation of the property of validity is implied. So framed, our examples of inferences involving different truth properties and thus yielding different properties of validity are straightforwardly produced. The moral is that SV may solve the problem of mixed inferences but, rather than solving the fragmentation, it makes the issue arise in full clarity.

Also with respect to the problem of mixed inferences, however, the strategy may be not so good. A major problem that affects this strategy is that it risks being self undermining. The reason is that, if a generic notion of truth is dispensed in favor of truth dependencies, then the same attitude could be adopted to dispense with truth dependencies as well. Accordingly, the solution gives rise to a slippery slope making not just general truth but also area-specific truth disappear. SV could be replaced with a purely structural account according to which an argument from premises  $\{A_1, \dots, A_n\}$  to conclusion B is valid, iff necessarily, if ( $A_1$  is  $F_1, \dots$ , and  $A_n$  is  $F_n$ ), then B is G. We thus have that the final outcome of the structural strategy is that of replacing a truth theoretic account of validity with a deflationary one.<sup>24</sup> The initial request of keeping the standard truth theoretic account fixed is thus missing. To this objection, however, it may be replied that the strong pluralist might propose independent reasons to admit the area-specific truth properties and thus block the slippery slope.

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<sup>23</sup> This passage is not explicit enough in Gamester [2019] formulation, thus possibly giving the impression that no fragmentation would follow.

<sup>24</sup> As in Shapiro [2011, 2022].

## 4.2 Validity through logical truths

Let us consider a second solution to mixed inferences. This approach aims at giving an account of validity as truth preservation by selecting a single truth property among the various area specific ones accepted by strong pluralists.<sup>25</sup> Such a goal may look unattainable, since it may seem incompatible with the very nature of mixed inferences, which involve different areas and different local truth properties. The only way to rely on a single truth property, seemingly, would be that of treating it as a generic property of truth, something that, as we know, is anathema to the strong pluralist. There is a way to circumvent this obstacle, though. It can be presented in three steps. First, by relying on the deduction theorem, we can note that any valid inference can be converted into a corresponding logical truth. Second, we should note that all logical truths are compounded sentences. No atomic sentence is a logical truth.<sup>26</sup> Third, compounded sentences can be assigned a specific property of truth not shared by atomic sentences. The last step is connected to another main issue of strong truth pluralism, namely how to treat true compounds with parts from different areas of discourse. If a compound has compounding parts from different areas, what area dictates the truth property of the whole compound? This problem, called the problem of mixed compounds, has received various treatments. According to a prominent one, relevant for the present proposal, compounds have their specific truth property, unrelated to those of its atomic parts.<sup>27</sup> Such a specific truth property can be thought of as a kind of logical or compositional property of truth. If the specific truth property of a true compound is this compositional truth property, giving a proper account of validity is straightforward. Once the elements mentioned above are all combined, the validity of an inference can be taken to consist in the necessary compositional truth of the sentence obtained by suitable applications of the deduction theorem. In this way we have a truth theoretic account that only relies on an area-specific property of truth and the problem of mixed inferences is solved.

Against such a solution to mixed inferences, one may object that there are many logics

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<sup>25</sup> The proposal has been fully articulated and defended in Stollo [2016]. Here I briefly summarized the main points.

<sup>26</sup> Here implicit reference is made to regimentation in standard propositional language, as usual in logical practice. This assumption, in any case, is common in the truth pluralism debate and in the problem of mixed compounds in particular, as demonstrated, for example, in Edwards [2009] or Gamester [2019].

<sup>27</sup> Edwards [2009].

which do not validate a deduction theorem, so that this strategy has a limited application. However, various replies are available.<sup>28</sup> First, the strategy still works for the main and more traditional logics. The force of the objection is thus limited. Second, in many cases those logics can be extended in such a way that a deduction theorem becomes provable. In those logics for which this is not possible, for example because the deduction theorem is blocked by lack of compactness, one may even dispute their status as authentic logics.<sup>29</sup> Third, for all relevant cases, one may also move at the meta-level, considering the meta-level conditional represented by the turnstile, rather than transforming the inferences into an object level implication.<sup>30</sup> Accordingly, an argument is valid not because a certain truth property is necessarily preserved from premises to conclusion, but because the meta-level entailment is compositionally true. Namely, an inference from  $A_1, \dots, A_n$  to  $B$  is valid iff it is compositionally true that  $A_1, \dots, A_n$  entails  $B$ .<sup>31</sup>

What about the issue of fragmentation of the property of validity? As it should be already clear, such a problem does not even arise in this context. Since the account is grounded on a single area-specific truth property, there is no fragmentation at all. Indeed, even if there were fragmentation at some level, it would be fixed by the appeal to a single property of truth providing the glue keeping all valid inferences together. We thus have a first example of a solution taking care of both problems at the same time. In this case, however, some fragmentation may still emerge for those logics recalcitrant to a deduction theorem.

### 4.3 Validity through designation

Soon after the problem of mixed inferences had been noted, an apparently simple solution was proposed.<sup>32</sup> The problem of what truth property is preserved, in a context in which there is a plurality of such properties, is very close to an issue inherent to the study of many-valued logic, where it receives a common and apparently unproblematic solution. Many-valued logicians gather the suitable values together into a certain collection, identifying some

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<sup>28</sup> For a discussion see Stollo [2016].

<sup>29</sup> This is the case, for example, for second order logic.

<sup>30</sup> This may still not work for non compact logics, to which, however, the previous considerations apply.

<sup>31</sup> This treatment aligns, for example, with proposals made by Shapiro [2011, 2022], Williamson [forthcoming] and Iacona [2023].

<sup>32</sup> Beall [2000].

designated values. Validity is then reframed as necessary preservation of designation: necessarily, if the premises have a designated value, also the conclusion has a designated value. Initially, truth pluralists were suggested to adopt the same strategy, by accepting the notion of designation. Despite the appealing comparison with many-valued logic, however, the issues equally soon revealed to be more complicated.<sup>33</sup> The problem is that designation seems to embody the very property of generic truth that strong pluralists reject. In general, resorting to designation must guard against a twofold risk. On the one hand, it must avoid the definition becoming non-standard. If designation is not itself a property of truth, then it seems that validity is not truth preservation as the original definition requires. On the other hand, it must avoid the notion of designation turning out to be a mere property of general truth in disguise. This would make designation unacceptable to strong pluralism. These obstacles seemed to rule out the option and forced the search for alternative strategies like the ones explored above. However, such a dismissal of designation was arguably too hasty. Indeed, it has been shown that the parallel situation of many-valued logic can be explored further, since a similar complication does actually arise also in the logic camp, connected with Suszko's theorem and its interpretation.<sup>34</sup> Indeed, the risks mentioned above, which were initially considered insurmountable, can be circumvented by paying attention to a series of rather technical details that emerge from an in-depth consideration of the foundation of many-valued logic. Although a careful analysis involves subtle issues for which there is no space to do justice here, the relevant upshot for our concerns can be presented in simple terms. First, that designation and truth are different can be demonstrated by showing that they have different properties. Designation and generic truth may thus look the same, and they may even be co-extensional in certain models, but they are not co-extensional in general. Moreover, designation may lack properties (like compositionality) that are arguably essential to semantic notions such as truth and falsity.<sup>35</sup> The accusation of introducing a notion of generic truth in disguise by appealing to designation is thus rejected. Second, *logical* values, such as designation, which are employed to formulate a definition of validity, can be legitimately distinguished from *algebraic* values (such as truth and falsity) that are assigned to sentences in the semantics and

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<sup>33</sup> Tappolet [2000].

<sup>34</sup> Stollo [2021].

<sup>35</sup> This is the case, for example, when Suszko's reduction is applied to Łukasiewicz three valued logic. See Stollo [2021: Appendix] for details.

transmitted in inferences. The algebraic values standing for truth properties are the values that are traded in inferences, despite the fact that designation is the logical notion in terms of which validity is articulated. In this way, also the accusation that the obtained notion of validity is non standard, for it would not be based on truth preservation, is rejected. If treating designation as a mere counterpart of generic truth is wrong and valid inferences can keep preserving truth at the same time, then the many valued-logical solution is indeed available to the strong truth pluralist.

Once designation is made available, both problems related to logical validity are easily solved. The problem of mixed inferences can be solved in the many-valued logic way, by understanding it as preservation of designated values, just stressing the additional remarks that designation is not generic truth and that the general notion of designation does not replace the algebraic values that are designated. The issue of fragmentation, in turn, can be handled by noticing that what keeps together seemingly different validities, in which disparate truth properties are related in premises and conclusion, is the general notion of designation. In particular, the notion of designation can stand for a substantial property, grounding objective similarities among the designated properties.<sup>36</sup> We then have another case of a strategy solving both issues with a single move. Indeed, with respect to the previous option, a solution based on the notion of designation has the advantage that it can be easily generalized to every logic, without relying on the deduction theorem. Moreover, and notably, it aligns truth pluralism back with many-valued logic, as it would be natural to do.

## 5 Conclusion

In this paper I have shown that if logical validity is accounted for in terms of the traditional definition, as necessary truth preservation, strong truth pluralism forces a fragmentation of the property of validity. I have then argued that simply acknowledging this result, treating the ensuing plurality of validities from the perspective of logical pluralism is less theoretically attractive than it may appear. If the loss of unity of the property of validity cannot be simply accommodated, a solution must be provided. I then reviewed three

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<sup>36</sup> Designation as a meta-level property can be related to a second order property akin to what is proposed in the second order functionalist view of truth. See Lynch [2001]. Notably, such a second order view is usually dismissed exactly because the second order property is arguably not an authentic property of truth. See Lynch [2009].

possible options that have been offered in the area of truth pluralism to cope with the close, but different, problem of mixed inferences. I argued that one of the strategies—the structural solution—is impotent with respect to the fragmentation problem, whereas two other strategies (one based on logical truths and one based on designation) succeed at various degrees. Having at disposal accounts that can solve two different issues at the same time is clearly convenient, and makes such solutions superior to the more limited alternative one.

Andrea Stollo  
 University of Trieste  
 Androna Campo Marzio,  
 10 34123 Trieste, Italy  
 andrea.stollo@units.it

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