Shallow water resistance estimation for semi-displacement slender ships

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Abstract. Estimating shallow water resistance for a vessel is an issue of primary importance for inland water navigation, not only for determining the total power absorbed by the propulsors but also for predicting the far-field wave pattern necessary to assess the wake-washing effect on the shores. Once it is not possible performing model tests for a specific water depth or it is not possible to execute complex CFD simulations, it is necessary to estimate the shallow water effect and add it to the deepwater results. Traditional empirical methods usually refer to the subcritical regime for shallow water; however, for several applications, also critical and supercritical regimes require attention. The present work analyses the shallow water effect adopting a linearised theory for wave resistance prediction using the thin ship approximation and an empirical formulation for the viscous resistance. Such a methodology allows for determining a procedure for estimating shallow water effects for slender semi-displacement ships operating also in critical and supercritical regimes. The novel resulting process is tested on a reference semi-displacement hull designed for navigation in lagoons. The results are compared with conventional empirical methods and RANS calculations in shallow water, highlighting the agreement of the proposed method with the CFD calculations for the critical and supercritical regimes.

Keywords. Shallow water, semi-displacement ships, Slender body, CFD, supercritical regime

1. Introduction

Estimating performances for a new vessel implies the ability to evaluate, since the initial stages of design, peculiar characteristics like resistance. Depending on the operative profile of the ship, resistance should not be estimated only for calm deepwater conditions but also for additional cases. As an example, ships should operate in open waters subject to the action of wind and waves, which means evaluating the added resistance of the environmental loads [1]. Likewise, vessels designed for inland waters or coastal areas like lagoons should operate in areas characterised by shallow waters or restricted water (i.e. narrow channels) [2]. Therefore, methods for a reliable estimation of these particular cases should be available for the different stages of the design, considering the amount of information at disposal and the relative costs of the methodologies.

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The present study concerns the estimation of a methodology for a fast estimate of shallow water resistance for the specific case of semi-displacement slender hulls. These vessels are commonly used for passenger transportation inside lagoons, where the environment presents a combination of narrow and large channels and purely shallow water pathways [3]. The speed constraints in the narrow channels usually limitate the navigation to the subcritical regime; however, in large channels or wide shallow water areas, the navigation reaches the critical and supercritical regimes.

Such conditions impact the wave pattern generated by the vessel and, consequently, the total resistance. Therefore, suitable methods should be employed to predict the ship's behaviour from the early design stages. In the open literature, the available methodologies for a fast estimate of shallow water resistance consider mainly the subcritical regime [4,5]. The estimation of the critical and supercritical situations can be investigated through model tests or numerical simulations [6]. Model tests cannot be easily performed in an early design stage as they require the availability of an almost definitive hull form and a budget that is not always available for the project of small units. Concerning the simulations, different approaches and methods can be employed. The most accurate solution is given by CFD viscous flow calculations [7]; however, also this approach require a computational effort not compatible with the necessities of an early design stage. Another technique is the adoption of potential flow calculations, faster than viscous flow ones, and sufficiently accurate for the prediction of the wave pattern of the vessel. As semi-displacement vessels are generally slender, the slender-body approximation [8,10] is also an option that can be pursued in the early design stage as it provides fast results and could be applied also to hull form variations without too much computational effort.

In any case, the potential flow computation do not consider the viscous part of the total resistance. To this end, it is here proposed to establish a procedure adopting a hybrid approach, which is evaluating the wave resistance in shallow water with the slender body approximation and the viscous resistance through an empirical formulation. Such a combination allows for considering also the critical and supercritical regimes in the evaluation of the shallow water resistance of a semi-displacement slender vessel.

In the present work, a reference hull form representative of a slender semi-displacement vessel designed for the navigation in lagoons [11] is employed to test the hybrid process. The developed procedure is then compared with standard literature methodologies for shallow water resistance estimation and with RANS simulations. The paper firstly introduce the general methods for shallow water resistance estimation. Afterwards, the novel hybrid process is described, providing at the end the worked example on the reference vessel, highlighting the compliance of the obtained results with RANS simulations.

2. Shallow water resistance estimation

Predicting vessel performances in shallow water conditions consists, traditionally, of estimating the resistance variations to the infinite depth conditions. Different approaches allow for assessing the resistance, following diverse techniques and levels of accuracy. Firstly, model tests permit the measurement of the resistance components, measuring, in model scale, a condition comparable with the reality case. Another way consists of performing dedicated CFD computations employing, for example, a RANSE solver. Also, this case, with proper discretisation, allows for reproducing a state comparable to the full-scale operational conditions of the vessel.

Both methods capture the different behaviours of the wave system through the regimes characterising the shallow water environment. It is possible to define the shallow water regimes by defining a Froude number as a function of the water depth h. The Froude number is as follows:

$$Fn_h = \frac{V}{gh} \tag{1}$$

where V is the vessel speed and g is the acceleration of gravity. The Fn_h number values define the following shallow water regimes:

1. *Subcritical*: This condition refers to $0.4 < Fn_h < 1.0$. The wave system is composed of both transversal and divergent wave components, with the divergent wave angle θ increasing with Fn_h.
2. *Critical*: This condition refers to $Fn_h = 1$. The wave propagation angle θ approaches 90 deg, which means the wave system is composed of a single crest advancing in the same direction as the vessel motion.
3. *Supercritical*: This condition refers to $Fn_h > 1$. The wave system is composed of the divergent components only, with the propagation angle θ decreasing with the Fn_h increase according to $\theta = \arcsin V/\sqrt{gh}$.

The above cathegorisation is theoretical, as vessels can reach a critical condition also for Fn_h values lower than 1 (between 0.88 and 0.96) according to the fullness of the ship.

However, it is not always possible to perform complex calculations or model tests in an early design stage. Therefore, the literature offers some correction methods to derive shallow water results from the deep water resistance. As mentioned, the purpose of this study is to develop a methodology providing a sufficiently accurate estimation for the early design stage in all hydrodynamic regimes. For such a reason, the following sections give an overview of the standard methods for shallow water resistance to identify which one can be the starting point for developing the new procedure.

2.1. Schlichtling-Landweber method

One of the most known empirical methods for resistance evaluation in shallow water is the Schlichtling-Landweber one. The first studies have been carried out by Schlichtling [4] and have been extended by the work of Landweber [5]. The resulting final method covers the subcritical regime, so it is valid for $Fn_h < 1$.

The method divides the total resistance into a frictional and a residual component. When a vessel is sailing in restricted water with depth h and speed V_h, the water has a higher relative speed V_I due to the section restriction under the hull. This speed increment affects the frictional resistance R_F, which is consequently increasing. The speed change depends on the ratio A_x/h, where A_x is the maximum sectional area of the hull.

For the correction on residual resistance R_R, the method considers that the wave generated by the vessel is longer than the wave generated by the same ship in deep water conditions. Therefore, the ratio between the speed V_I and the fictitious speed at infinite depth V_∞ is as follows:

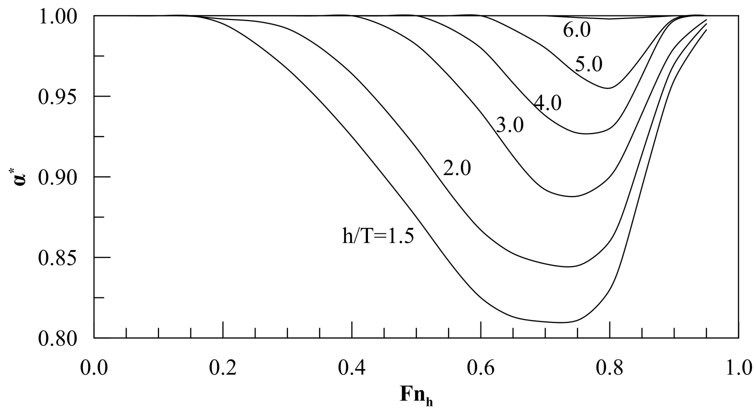

Figure 1. Karpov diagram for the frictional resistance correction.

$$\frac{V_I}{V_\infty} = \sqrt{\tanh Fn_h^{-2}} \tag{2}$$

where the Fn_h refers to the speed V_∞. It follows that a vessel travelling in shallow water with depth h has the following total resistance derived from the infinite depth one:

$$R_T\left(V_h\right) = R_F\left(V_I\right) + R_R\left(V_\infty\right) \tag{3}$$

The method is simple to apply once the resistance curve for infinite depth is known. For such a reason, this method or equivalent ones, like Schilichtling-Lackemby [12] formulation, are commonly used for shallow water estimation. However, the prediction is limited to the subcritical regime.

2.2. Karpov diagrams

Another empirical method for shallow water resistance determination is the Karpov diagrams [6]. Like Shliechtling-Landweber, this method starts from the knowledge of the infinite depth resistance curve for the analysed vessel, dividing the resistance into frictional and residual components. However, the diagrams cover a broader range of conditions.

The process considers a starting deep water resistance in the following form:

$$R_{T_\infty} = \frac{1}{2}\rho S\left[\left(C_F + \Delta C_F\right)V_\infty^2 + C_R V_\infty^2\right] \tag{4}$$

where C_F is the frictional resistance coefficient, ΔC_F is the correction to the frictional resistance, C_R is the residual resistance coefficient, ρ is the water density and S is the ship wetted surface. From equation (4), the shallow water resistance at a depth h assumes the following form:

$$R_{T_h} = \frac{1}{2}\rho S\left[\left(C_F + \Delta C_F\right)V_1^2 + C_R V_2^2\right] \tag{5}$$

where the two velocities are $V_1 = V_\infty/\alpha^*$ and $V_2 = V_\infty/\alpha^{**}$.

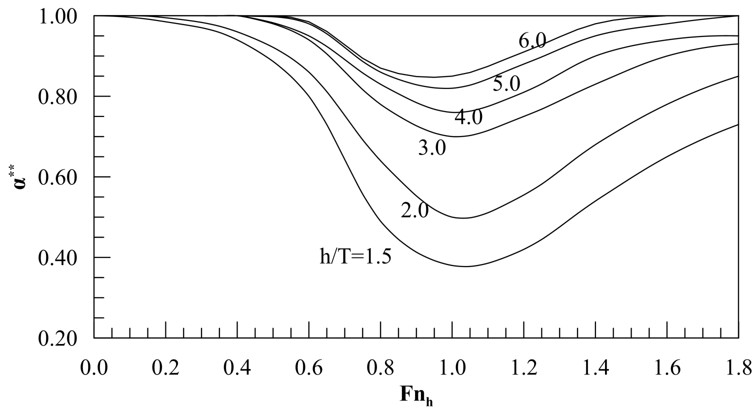

Figure 2. Karpov diagram for the residual resistance correction.

The parameters α^{*} and α^{**} derive from the so-called Karpov diagrams (see Figures 1 and 2) expressed as a function of Fn_h and h/T ratio, with T the mean vessel draugth. From the diagrams it can be observed that the method covers all the three hydrodynamics regimes, being the Fn_h values defined up to 1.8. However, the main limitations of the methodology relate to the h/T ratio. In fact, the method is valid for $1.5 \leq h/T \leq 10.0$; therefore, for the so-called ultra-shallow water cases the method is no more applicable.

2.3. RANSE Numerical methods

Besides the empirical correction methods, which imply the knowledge of the infinite depth resistance, numerical methods allow for directly estimating the vessel resistance in shallow water conditions. The most used methodology for CFD calculations in shallow water conditions adopts the discrete Reynolds-Averaged Navier-Stokes equations (RANSE) resolution. With a proper domain discretisation and physical setup, RANSE solvers can provide a reliable estimate of the vessel resistance in shallow water together with the dynamic behaviour of the ship.

However, the wave pattern and the presence of the bottom relatively closer to the water-air interface imply the adoption of a finer calculation grid than needed for infinite-depth calculations. Furthermore, a larger domain than the standard infinite-depth simulations is necessary. Therefore, the simulations in shallow water conditions require more calculation effort than infinite-depth ones, considering also the higher physical time needed to reach a stable solution.

Computational methods based on RANSE calculation have different resolution methodologies and setups. The present study adopts the STAR-CCM+ solver to solve RANS equations according to the following assumptions. The RANS equations have been solved with a segregated approach on algebraic multi-grids [13], using Rhie-Chow interpolation for pressure-velocity coupling [14]. The control over the total solution is obtained through the SIMPLE algorithm [15]. The effect of the turbulent fluctuations on the mean flow is approximated through the realisable $k-\varepsilon$ turbulence model [16], which solves transport equations for the kinetic energy k and dissipation rate ε. The free surface is captured using a Volume of Fluid (VOF) approach with High Resolution Interface

Capturing Scheme (HRIC) based on the Compressive Interface Capturing Scheme for Arbitrary Meshes (CISCAM). The adoption of a segregated approach implies the selection of an implicit time step for the resolution, selected in the present study according to ITTC recommendations for CFD computations.

Besides, for the specific case of shallow water, different modelling is needed for the vessel motions compared to standard resistance calculations in STAR-CCM+ solver. The commonly used techniques rotates and translates the calculation domain according to the calculated trim and sinkage. Such an assumption is valid for small movements, especially for the rotation. However, in shallow water, small rotations may generate a sloshing effect in the calculation domain, due to the presence of the bottom. Therefore, an overset mesh approach needs to be used, rotating and translating only a small volume around the vessel according to the calculated rigid body motions.

Thanks to these settings it is possible to evaluate both unrestricted and restricted water resistence for the vessel and compare it with other methods. However, the computational effort of this methodology is too high for a fast application in the early design stage, thus alternative approaches should be preferred for a fast evaluation of shallow water resistance. Furthermore, a proper estimation of resistance through CFD simulations require the adoption of calculation grids and physical setups validated with experiments, in order to achieve an adequate reliability for the results.

2.4. Slender body theory

The slender body assumption is one of the first numerical methods adopted for the evaluation of wave resistance for a vessel. The first formulations, like the one given by Mitchell, were valid for hulls supposed to be defined by mathematical formulations and for the case of deep water. Afterwards, the method was extended to hulls defined by a set of source distributions and for shallow water.

In the specific case of shallow water condition the Laplace equation $\nabla\phi^2 = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$ needs to be solved in the range $-h < z < 0$ subject to the following boundaries conditions:

- $\frac{\partial\phi}{\partial z} = 0$ for $z = -h$ (impermeability of the bottom).
- $\frac{\partial^2\phi}{\partial x^2} + k_0\frac{\partial\phi}{\partial z} = 0$ for $z = 0$ (linearised Kelvin boundary condition on the free surface).
- $\frac{\partial\phi}{\partial y} = \pm VY_x(x,z)$ for $y = 0$ (impermeability of the hull).

Besides these equations, an additional radiation condition needs to be added for avoiding the propagation of waves in front of the vessel. Another assumption concerns the hull, which is supposed to be symmetric to the y-axis with offsets $y = Y_x(x,z)$ and $k_0 = g/V^2$. Under these assumptions, it can be stated that the wave making can be generated by distributions of Havelock sources, which automatically satisfies the Kelvin free surface boundary condition.

Adopting this strategy it is possible to evaluate the total energy in the far-field wave pattern through a Mitchell integral as follows:

$$R_W = \frac{\pi}{2}\rho V_\infty \int_{-\pi/2}^{pi/2} |A(\theta)|^2 cos^3\theta \mathrm{d}\theta \tag{6}$$

where $A(\theta)$ is the complex amplitude of the linearised ship wave integral also known as free-wave spectrum or Kochin function.

Equation (6) is valid for both the deep water and shallow water case, what is changing is the formulation of $A(\theta)$. For the deep water case, the formulation is quite simple, while for shallow water it is more complicated [8]. The Kochin function for shallow water conditions is valid for all the hydrodynamic regimes.

Adopting this approach, it is possible to estimate the wave resistance of the vessel in shallow water when the geometry is known. Several studies by Tuck [8,9] show that such an approach reproduces quite well the wave pattern of slender vessels in shallow water as well as in unrestricted waters.

3. Hybrid prediction method

The present study aims to develop a procedure for a fast evaluation of shallow water resistance for semi-displacement slender ships for all hydrodynamic regimes. Conventional empirical methods give an approximated resolution not capable of adequately modelling cases like the supercritical regime, where resistance can be lower than in the deep water case. The purpose is to develop a correction method for correcting a resistance evaluated for deep water conditions through empirical methods or other sources like model tests or numerical calculations. Therefore, the definition of the total resistance is important to ensure consistency between the infinite depth prediction and the new one in shallow water. Here, the division is made considering the viscous resistance R_V and the wave resistance R_W and considering the effect of shallow water separately on the two components.

3.1. Wave resistance

The slender-body assumption is a suitable method for estimating wave resistance correction, being the procedure applicable to slender semi-displacement vessels. The slender-body theory is a simplified formulation that, for the shallow water case, needs further simplification by considering the far-field wave approximation.

However, despite the approximation, the slender-body theory allows for predicting quite well the wave patterns of a slender vessel. Therefore, the method can compare deep and shallow water wave generation. Consequently, the theory can estimate relative differences between the wave resistance variations between the analysed cases. Then, the process needs two calculations to evaluate the wave resistance in shallow water with a depth of h. The first calculation determines the wave resistance in infinite depth through the slender-body assumption through a selected speed range. The second calculation covers the same speed range at depth h. Afterwards, it is possible to determine the non-dimensional increase/decrease of the wave resistance in shallow water as follows:

$$\Delta R_W = 1 - \frac{R^*_{W_h}}{R^*_W} \tag{7}$$

where $R^*_{W_h}$ and R^*_W are the wave resistance in shallow water and infinite depth evaluated with the slender-body approximation.

Consequently, the wave resistance in shallow water derives from the following formulation:

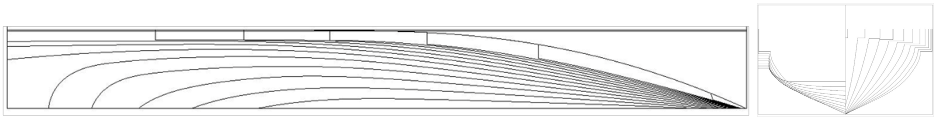

Figure 3. Waterlines (*left*) and transversal sections (*right*) of the reference vessel.

$$R_{W_h} = R_{W_\infty} \Delta R_W \tag{8}$$

where R_{W_∞} is the wave resistance in infinite depth obtained by statistical methods, CFD calculations or model tests.

3.2. Viscous resistance

The presence of the sea bottom influences the viscous resistance of the vessel. The changes in the pressure distribution along the hull, and a different boundary layer growth, increase the viscous forces on the vessel surface. Here, the Millward formulation [17] determines the viscous resistance increase in shallow water. Previous studies highlight the correspondence between this formulation and double-body viscous flow simulations. Such correlation is valid when the h/T ratio is above 0.8-1.0. When this constraint is satisfied, the formulation is as follows:

$$R_{V_h} = R_{V_\infty} \left[1 + 0.644 \left(\frac{T}{h} \right)^{1.72} \right] \tag{9}$$

where T is the mean draught of the vessel, and R_{V_∞} is the viscous resistance for infinite depth conditions.

This formulation is valid for subcritical and transcritical regimes. Once the vessel travels in supercritical conditions, it is reasonable to suppose that the viscous resistance remains equal to the infinite depth case. This assumption is coherent with the Karpov diagram for α^* (see Figure 1), where no correction is needed for the frictional resistance above $Fn_h = 1$.

Knowing wave resistance through equation (8) and viscous resistance through equation (9), the final total resistance R_{T_h} in shallow water is the sum of the two contributions.

4. Applied example

The described correction procedure is here tested on a reference semi-displacement hull designed for navigation in lagoons. The results of the proposed method are then compared with those coming from standard correction methods and RANSE calculation previously described.

4.1. The reference vessel

The reference hull is a semi-displacement slender vessel having the main characteristics reported in Table 1 and the hull form shown in Figure 3. The vessel is designed for the navigation in lagoons, more precisely for passenger transportation in the Venice lagoon

Table 1. Reference hull main characteristics.

	Symbol	Value	Unit		Symbol	Value	Unit
Length overall	L_{OA}	14.99	m	Design Volume	∇	12.12	m^3
Length on waterline	L_{WL}	14.90	m	Wetted surface	S	38.92	m^2
Breadth overall	B_{max}	3.20	m	Length/Breadth ratio	L_{WL}/B	5.40	-
Breadth on waterline	B	2.86	m	Breadth/Draugth ratio	B/T	3.45	-
Design Draugth	T	0.80	m	Slenderness ratio	$L_{WL}/\nabla^{1/3}$	6.49	-
Design Displacement	Δ	12.44	t	Block coefficient	C_B	0.368	-

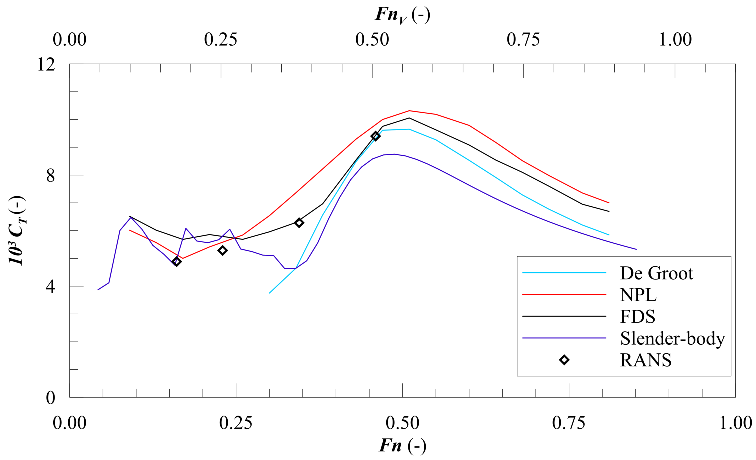

Figure 4. Total resistance of the reference vessel with different methods.

and is subject to all the limitations and constraints imposed by the navigation in that area. For this particular case, the operative profile covers different regimes according to channel size and speed limitation. In particular a consistent part of the travel is in a wide channel (no side effects) with $h = 2$m at a speed of 10.80 knots. This condition corresponds to a supercritical regime with $Fn_h = 1.25$.

As a first step, the resistance in infinite depth should be determined. Here, several methodologies have been applied, providing a comparison between systematic series (NPL and FDS), empirical methods (De Groot) and numerical simulations (RANSE and Slender body). Figure 4 shows the comparison on the total resistance coefficient C_T of the applied methodology. The RANSE calculation have been carried out on grids validated for comparable Froude numbers [18] and therefore could be considered as a reliable prediction for the final resistance level. However, CFD is not suitable for the early design stage but systematic series like the FDS provide a good agreement with numerical data. Therefore, results coming from the FDS systematic series have been considered as the initial infinite depth resistance to apply the hybrid prediction method.

4.2. Hybrid methodology results

Having determined the total resistance in deep water, it is possible to apply the proposed hybrid methodology to the reference vessel. Figure 5 shows the total resistance R_T for the infinite depth and three different water depth h: 5, 3 and 2 metres. It can be observed that,

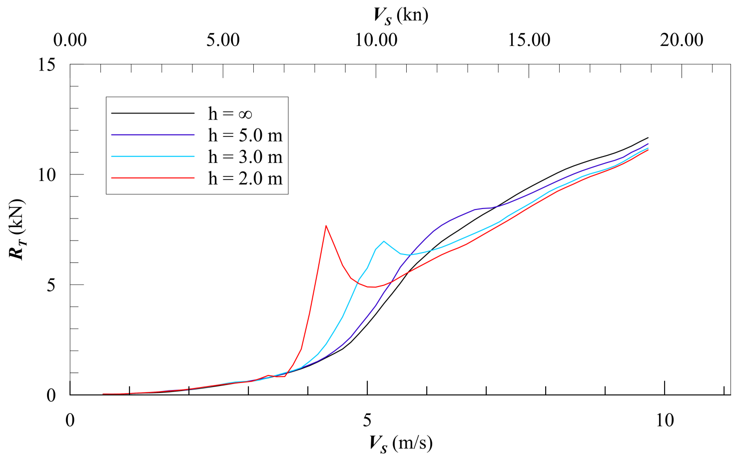

Figure 5. Total resistance according to the proposed hybrid method for different *h* values.

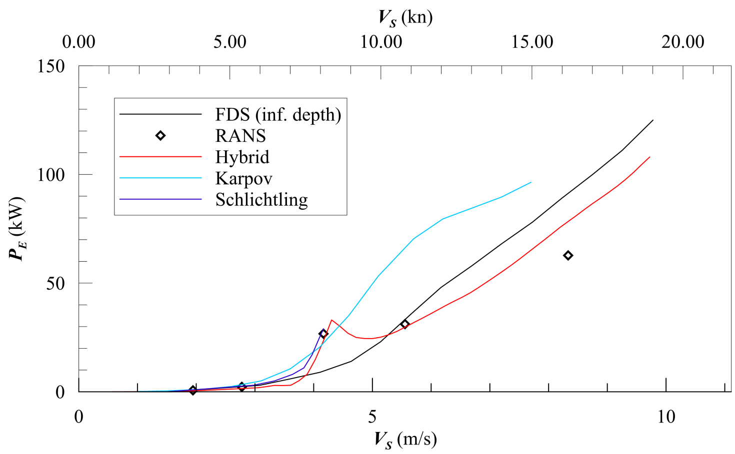

Figure 6. Effective power according to the proposed method, classical methods and RANS calculations for $h = 2$m.

despite Millward formulation gives a viscous resistance increase also for supercritical conditions, the total resistance in shallow water is always lower than the infinite depth for supercritical conditions. For the subcritical case, there is a peak in corrispondence with the critical case and the resistance is higher than the infinite depth one. The methodology capture the different magnitude of the shallow water resistance increase by changing h, highlighting a higher peak amplitude for the lower h values.

4.3. Comparison with RANSE and empirical methods

After the application of the to the reference vessel, the condition referring to $h = 2$ m has been further analysed with the empirical methods of Schlichling-Landwber, Karpov diagrams and with numerical RANSE simulations. Figure 6 shows the comparison of the different methods on the effective power P_E. Analysing the results, it is evident that all methods capture the resistance increase in the subcritical region. For the supercritical region, the Karpov diagrams overestimate the shallow water contribution compared to other methods. This is essentially due to the modelling of R_R in shallow water, which is more consistent for displacement vessel rather then slender hulls. It is worth noticing that the hybrid method show the same trend of advanced RANSE calculations, almost capturing the peak of the critical condition and overestimating of a small amount the resistance in supercritical condition. Therefore, the hybrid methodology present a valuable solution for a fast estimation of shallow water resistance on slender semi-displacement hulls.

5. Conclusions

The work presented a process for evaluating the shallow water resistance of semi-displacement slender ships suitable for the subcritical, critical and supercritical regimes. The methodology determines the contribution of finite depth to the wave and viscous resistance separately. The viscous component is assessed through the empirical formulation given by Millward, while wave resistance change is assessed through the slender body theory. Combining the two corrections allows for determining the total resistance starting from the infinite depth condition, with a process easily applicable in the early design stage. Furthermore, the capability of predicting the resistance in either subcritical, critical or supercritical regimes is a significant enhancement to the traditional methods available in the literature.

The proposed method, tested on a reference vessel, highlights a good agreement with the results provided by applying RANSE calculations, capturing the resistance peak in the proximity of the critical regime. The resistance calculation with the proposed method is still an approximated process. In fact, the slender body approximation is not applicable to all kinds of vessels, like planing crafts or fully displacement ships. Secondly, the method considers the viscous and wave resistance fully separated, even though, in reality, there is a mutual correlation between the two components, something which is captured only with RANSE calculations or model tests.

The last aspect concerns the dynamic trim and sinkage. Shallow water affects the dynamic behaviour of the vessel, something which is not taken into account by the present methodology. Therefore, further improvements can be obtained by considering the trim and sinkage in the correction process through iterative slender body calculations. In any case, the present method is a good starting point for a fast evaluation of shallow water resistance for semi-displacement vessels.

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