

Appendix S1

Synthetic table with all mathematical symbols and equations used in the main text

Variables and indices	Name	Definitions	Details	References
Basic variables				
N	Species richness	Number of species in a sample site		
p_i		Relative abundance of species i in the sample site	i ranges from 1 to N with $0 < p_i \leq 1$ and $\sum_{i=1}^N p_i = 1$.	
d_{ij}		Functional dissimilarity between species i and j	For any i and j , we impose $d_{ij} = d_{ji}$ and $d_{ii} = 0$.	
s_{ij}		Functional similarity between species i and j	If d_{ij} is in the range $[0, 1]$, we have $s_{ij} = 1 - d_{ij}$.	
Indices				
$S = 1 - \sum_{i=1}^N p_i^2$	Simpson diversity or Gini-Simpson diversity	S is the probability that two individuals drawn at random with replacement from the sampling site belong to different species.		Gini, 1912 Simpson, 1949
$D = \sum_{i=1}^N p_i^2$	Simpson dominance	D is the probability that two individuals drawn at random with replacement from the sampling site belong to different species.	$D = 1 - S$	Simpson, 1949
$Q = \sum_{i,j=1}^N p_i p_j d_{ij}$	Quadratic diversity	Q is the mean functional dissimilarity between two individuals drawn at random with replacement from the sample site.		Rao, 1982

$1 - Q = \sum_{i,j=1}^N p_i p_j s_{ij}$	Functional homogeneity	Mean functional similarity between two individuals drawn at random with replacement from the sample site.		Ricotta et al., 2016
$R = \sum_{i \neq j}^N p_i p_j s_{ij}$	Functional redundancy	Mean functional similarity between two randomly selected individuals of different species.	$R = S - Q$ $D + R + Q = 1$	Ricotta et al., 2016
$1 - R = 1 - \sum_{i \neq j}^N p_i p_j s_{ij}$	Functional uniqueness	A measure of the lack of insurance against the loss of ecosystem processes due to local species extinctions	$1 - R = D + Q$	Ricotta et al., 2016 This paper
Species' contributions to indices				
$q_i = \left(\sum_{j=1}^N p_j d_{ij} \right)$		Unweighted contribution of species i to functional diversity Q	$Q = \sum_{i=1}^N p_i q_i$	This paper
$r_i = \left(\sum_{j=1, j \neq i}^N p_j s_{ij} \right)$		Unweighted contribution of species i to functional redundancy R	$R = \sum_{i=1}^N p_i r_i$ $p_i + r_i + q_i = 1$	This paper

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