New Fourier Transform Approach to the Synthesis of Shaped Patterns by Linear Antenna Arrays

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***Abstract*—A new Fourier Transform (NFT) approach is developed for the synthesis of shaped patterns radiated by linear antenna arrays. The proposed method exploits in an innovative way the FT relation between the source distribution and the radiated pattern. Precisely, the finite dimension of real sources is firstly taken into account by using the sampling theorem to approximate the desired pattern as a band-limited function. It is this step that allows one to obtain an important performance improvement. Then, a continuous source is evaluated from the approximate desired pattern to finally obtain the element excitations. Numerical examples validate the method.**

I. Introduction

The synthesis of shaped patterns by linear antenna arrays (LAAs) has been a canonical problem since many decades and still arises in many modern applications [1]–[3]. A great variety of synthesis algorithms have been developed in the literature, deterministic [4]–[6], stochastic [7], [8] or hybrid [9], [10]. Among the deterministic algorithms, the Fourier Transform (FT) approach [11] has received great consideration. This method is based on the FT relation between the excitation distribution $a(z)$ of a linear source and the radiated far-field space factor $SF(u)$:

$$SF(u) = \int_{-\infty}^{\infty} a(z) \exp(jzu)\ dz, \tag{1}$$

$$a(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} SF(u) \exp(-jzu)\ du, \tag{2}$$

where $u = k \sin\theta$, being $k = 2\pi/\lambda$ the wavenumber (λ is the wavelength), θ the angle from broadside, and $j = \sqrt{-1}$. In the above relations, (1) can be viewed as an analysis equation, whereas (2) can be regarded as a synthesis equation. Precisely, given a desired shaped pattern $SF_{\mathrm{d}}(u)$, (2) provides the excitation distribution $a(z)$ which exactly yields the desired pattern. Of course, the relation is exact only for infinite sources, while real cases only deal with distributions of finite dimensions. So, the excitation provided by (2) is truncated:

$$a(z) = 0 \text{ if } |z| \geq L/2, \tag{3}$$

where L is the source length. Truncation (3) yields an approximate space factor, which indeed is proved to have the minimum least-mean-square error from the desired space factor over the entire u–domain [11] (this is not true when only the visible region is considered, i.e., $[-k, k]$).

The new FT (NFT) method proposed in this paper still exploits the synthesis equation (2) for the shaped beam synthesis of LAAs. However, importantly, the finite dimension of the source is firstly taken into account, as it is described in details in the next section (Section II), with special attention on its novel aspects. Then, Section III proposes two numerical examples that show how the NFT method improves the results of the original FT approach. Finally, Section IV summarizes the main conclusions and proposes possible future developments.

II. Problem and New Fourier Transform Approach

In a Cartesian system $O(x, y, z)$, consider a LAA composed by N elements arranged on the z–axis at the positions $z_n, n = 1, \ldots, N$. Let $\mathbf{z} = [z_1, \ldots, z_N]^T$ and $\mathbf{a} = [a_1, \ldots, a_N]^T$ be the column vector of the element positions and of the complex excitations, respectively (superscript T denotes transpose operator). The array pattern can be expressed as:

$$F(\mathbf{a}; \mathbf{z}; u) = \sum_{n=1}^{N} a_n \ \exp(jz_n u), \tag{4}$$

which can be viewed as the discretized form of a space factor in (1), when:

$$a(z) = \sum_{n=1}^{N} a_n \delta(z - z_n), \tag{5}$$

where δ is the Dirac delta function. Importantly, the distribution in (5) satisfies condition (3), thus its space factor is band-limited and, by the sampling theorem it can be expressed in terms of its samples f_m, and approximated as:

$$F_{\mathrm{a}}(u) = \sum_{m=-M}^{M} f_m \ \mathrm{sinc}\left[\frac{\omega_{\mathrm{s}}}{2}\left(u - \frac{2\pi m}{\omega_{\mathrm{s}}}\right)\right], \tag{6}$$

being $\omega_{\mathrm{s}} (> L)$ the sampling rate. Starting by this consideration, the NFT approach firstly solves the following problem: *given a desired (complex) radiation pattern $F_{\mathrm{d}}(u)$, defined and non-zero in the visible region $[-k, k]$, the $2M+1$ coefficients f_m of the approximate pattern $F_{\mathrm{a}}(u)$ of the form (6) are searched, in such a way as to minimize the mean-square-error:*

$$\int_{-k}^{k} \left| F_{\mathrm{d}}(u) - \sum_{m=-M}^{M} f_m \mathrm{sinc}\left[\frac{\omega_{\mathrm{s}}}{2}\left(u - \frac{2\pi m}{\omega_{\mathrm{s}}}\right)\right]\right|^2 du. \tag{7}$$

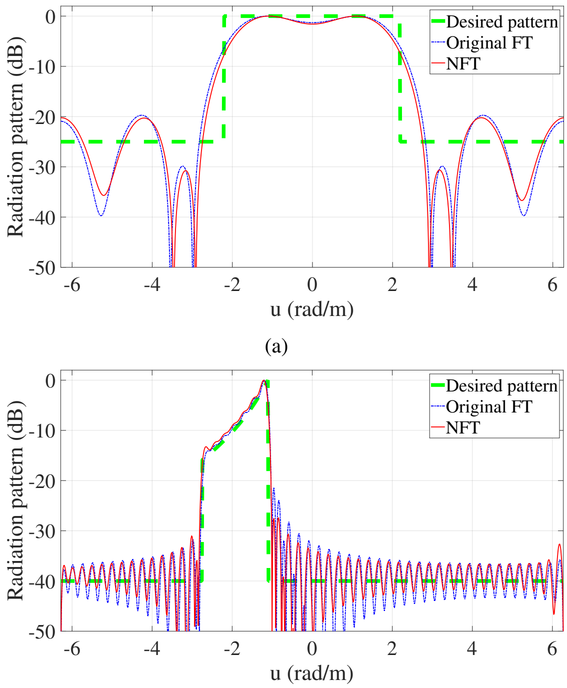
Manipulating (7), after some algebra, the vector $\mathbf{f}$ of $2M+1$ coefficients $f_m, m = -M, \ldots, M$ is obtained as:

$$\mathbf{f} = S^{\dagger}\mathbf{s}, \tag{8}$$

where the superscript $\dagger$ denotes the pseudo-inverse, S is the square matrix whose elements are:

$$S_{mp} = \int_{-k}^{k} v_m(u)\ v_p(u)\ du, \quad m, p = -M, \ldots, M \tag{9}$$

$$v_m(u) = \mathrm{sinc}\left[\frac{\omega_{\mathrm{s}}}{2}\left(u - \frac{2\pi m}{\omega_{\mathrm{s}}}\right)\right], \tag{10}$$



(a)

(b)

Fig. 1. (a): Example 1: flat-top pattern, $N = 13, \omega_{\mathrm{s}} = 18.9\lambda$. (b): Example 2: squared cosecant pattern, $N = 101, \omega_{\mathrm{s}} = 58.0\lambda$. ($M = 400$ is assumed for the approximate patterns of both examples.)

and $\mathbf{s}$ is the vector whose elements are:

$$s_m = \int_{-k}^{k} v_m(u)\ F_{\mathrm{d}}(u)\ du, \quad m = -M, \ldots, M. \tag{11}$$

Once the coefficients f_m are evaluated using (8), the band-limited approximate desired pattern is obtained by (6). This latter pattern is used in the synthesis equation (2) to obtain the continuous source distribution $a(z)$, which gives by (5) the element excitations, a_n. In the next section two numerical examples are provided.

III. Numerical Examples

The first example deals with a LAA of $N = 13$ equally spaced elements at the positions $z_n = [-(N+1)/2+n]d, n = 1, \ldots, N$, with $d = \lambda/2$. The desired pattern is the flat-top shown in Fig. 1(a). The NFT approach is applied with $\omega_{\mathrm{s}} = 18.9\lambda$, providing the excitations in Table I and the pattern in Fig. 1(a). For comparison purpose, the same problem is also solved with the original FT approach. As it can be seen, the NFT provides slightly lower maximum sidelobe level (–20.2 dB vs –19.7 dB) and narrower beam.

TABLE I
Example 1: normalized element excitations (a_n/a_7).

n	{6,8}	{5,9}	{4,10}	{3,11}	{2,12}	{1,13}
original FT	0.6927	0.3148	-0.0399	-0.1848	-0.1103	0.0393
NFT	0.6937	0.3413	-0.0190	-0.1810	-0.1373	0.0190

In the second example, the desired pattern is a squared cosecant (see Fig. 1(b)), and a larger LAA is used, which is composed by $N = 101$ equally spaced elements with $d = \lambda/2$. The patterns obtained by the original FT approach and the NFT (with $\omega_{\mathrm{s}} = 58.0\lambda$) are shown in Fig. 1(b). As it can be seen, the desired pattern is satisfactorily approximated, and the NFT provides a considerably lower maximum sidelobe level (–27.5 dB vs –21.4 dB, the element excitations a_n are not listed for space reasons.) Both numerical examples are obtained using Matlab R2019b on a commercial laptop and require only 0.1 s and 0.8 s, respectively.

IV. Conclusion

The canonical problem of synthesis of shaped beams by LAAs is solved by a novel FT approach. First, the proposed method exploits the finite dimension of the source to approximate the desired pattern as a band-limited function. Then, the approximate version of the desired pattern is used in the synthesis equation to evaluate a continuous source distribution, which is finally sampled to obtain the element excitations. Current efforts aim to extend the method, which is computationally simple and provides improved results with respect to the original FT approach, by considering planar array geometries and the geometrical synthesis by a density tapering approach [12].

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