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## Implementing the use of Energy Bar Charts in the framework of an Early Physics approach

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## Implementing the use of Energy Bar Charts in the framework of an Early Physics approach

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**Abstract.** Introductory college courses use the Multiple Representations (MR) method for teaching/learning energy processes. It helps students understand concepts which are challenging to learn, like energy, and to solve related problems. Although this method is well-recognised in the context of Physics Education and researchers, it is less known by high school teachers because of its limited use in Physics textbooks. We report a recent experience where we accompanied teachers in their Pedagogical Content Knowledge (PCK) revision and in the building of an innovative way of teaching using conceptual fragmentation. The assessment confirmed the teaching efficiency of using Multiple Representations tools such as Energy Bar Charts.

### 1. Introduction: Multiple Representations and Disciplinary Languages

During the last twenty years, Physics Education Research has suggested using Energy Bar Charts as an efficient description of energy processes [1, 2, 3]. Experts often apply these qualitative representations to understand problems [1], whereas novices prefer resolving them with a mathematical approach [4]. While a qualitative representation is a tool for reasoning [2], a purely mathematical approach could be affected by errors caused by the procedural application.

Involving qualitative representation in problem-solving strategies is now a starting point for teachers to integrate their instructions [3]. Passing from qualitative to quantitative representations is one of the goals of the Multiple Representations (MR) approach [5, 6, 7]. Its use is relevant for problem-solving, representing a physical process differently and in a proper form for the content-building knowledge process [3, 5].

The standard tools of physical world representation are words, sketches or pictures, diagrams, graphs, and equations [1, 3]. Each is more than a simple representation [5] since they are related to a specific disciplinary language used for describing Physics situations [7]. For instance, representing in words means using Verbal Language (or “natural” because it refers to the Spoken Language) [8], representing through mathematical equations means using Maths Language [9], and so on for the other representations.



Each representation is relevant since having its Language with specific semantics and grammar activates specific cognition abilities and grants a stricter control of cognition strategy for learning. Students can achieve a deeper comprehension of the physical world by passing from one representation to another, developing the ability to translate observations, estimates, concepts, and descriptions of the phenomena from one disciplinary Language into another. The MR approach, promoting a combined usage of different representations, empowers such ability [6]. More generally, the MR approach makes learning more accessible through the combination of different representations and their complementary role and is beneficial for students in their learning process and conceptualisation [10].

Representations work on two different levels: external and internal [10]. The external representations are those that are communicated to other people, i.e., pictures, text narrations, graphs, and symbols [6]; the internal ones concern mental models, “structural analogies of situations or processes” [6], sometimes depicted also by metaphors [11]. According to the process of building knowledge, external representations could direct to “knowledge and structure in the environment, as physical symbols, objects, or dimensions” [12]; internal ones guide the knowledge and structure in memory, like propositions, productions, schemas, neural networks, or other forms [12]. The mutual exchange between external and internal representations enhances abstraction, extension, and relation.

In teaching practices, MR are commonly used mainly in problem-solving activities [14, 15]; what is less common is their blending into content-building explanations.

As a reasoning tool, MR could enact a deep conceptual understanding of physical phenomena. This has already been studied and found to happen, for instance, in force, work and energy [1, 2, 3, 15, 16] explanation concepts in introductory Physics courses.

For students at the University who have been exposed for many years (typically thirteen) to mathematics and science courses, the usage of MR and the ability to use different disciplinary languages can be somehow already acquired and the conceptual effectiveness can be achieved [3, 16]. However, this is only partly true: many university students, mainly those in their first years, struggle to understand multiple representations of concepts [35]. This could be reconducted to the limited introduction of its usage during secondary high school instruction.

At an early stage of learning a scientific discipline, linguistic difficulties augment those referred to as conceptual ones.

So, the level of secondary education is crucial for two main reasons. Firstly, students develop their internal cognitive functions such as abstraction [17] at that level. For this reason, they may still have difficulties in generalisation, formalisation and modelling. These abilities are performed through well-designed content building and straightforward disciplinary linguistic usage. Students usually cannot acquire these abilities on their own. They improve their skills by teachers’ instructions and methodologies. MR tools are a way of supporting this cognitive process [17] for the so-called internal representations. Secondly, most external representations are familiar with the disciplinary Languages of Mathematics studies [18]. Mathematical tools are essential to understand physical description [19, 20] deeply. However, since students in secondary schools are still learning them while studying physics, they may face significant difficulties integrating one discipline's instructions with the other. Therefore, teachers have to build the Phys-Maths interplay to better activate linguistic representations in graphs, symbols, equations, etc. [19, 20]. MR conceptualise external representations through Mathematical Languages descriptions. These two factors may affect the performance of effective use of MR in secondary education. Teachers must consider these cognitive features in developing and using MR. Their teaching strategies might differ from those needed for the same content at a higher level of instruction, as assessed by monitoring their Content Knowledge for Teaching [2]. Consequently, two actions concerning the teaching/learning process must be taken into account: 1) to tailor teaching to meet the development of students’ content knowledge building and 2) empower learning effectiveness by using reasoning tools.

## 2. Development: Training for the Integration of MR use in Physics Teaching

To adopt MR in instructional strategies, we devoted our efforts to teachers' professional in-service training. In Italy, the Initial Education of Secondary School Teachers has been traditionally limited to acquiring the subject matter content knowledge, with limited access to the scientific debate about the students' conceptual change [36]. Notwithstanding a process of school reform, the implicit teaching model is still mastery learning in many school realities. Consequently, teachers rely on in-service teacher education activities, as opportunities to reflect on their established teaching practices and to introduce some elements of change.

We met some teachers during their lessons, observing and monitoring their actions [21, 22, 23]. We recognised their footprints by referring to their Pedagogical Content Knowledge (PCK) for Physics teaching [24]. We involved teachers in a profound revision of their PCK, keeping in mind the five points featured by Etkina [24]. The revision also strengthened the teachers' typical Phys-Maths interplay [22, 25] in their PCK. Lehavi and his colleagues identified four prevalent patterns in Phys-Maths interplay [25]: exploration, construction, broadening and application. They stressed the evidence between expert and master teachers based on the patterns used. They revealed that "the practice of employing different patterns of the Phys-Maths interplay can distinguish master teachers from other expert teachers" [25]. Our recent monitoring provided evidence of the prevalent choice in Italian teachers' practising [21]; we noted that teachers prefer an application pattern with some weak integration with the construction one. This preference is not dependent on teachers' professional experience or their academic graduation (if they have a degree in Maths or Physics studies).

Furthermore, it is not dependent on whether they teach in high schools for scientific studies or technical-professional ones. Mathematics controls the process in both cases, even in an early Physics learning experience [23] (in the first years of secondary school).

In Physics lessons, teachers use mathematics to demonstrate physics laws and in their classroom discourses. Then, they assess students' learning with many mathematics exercises and problem-solving applied to physics phenomena [26]. This restricts the building knowledge process by excluding three different points of view: epistemological, linguistic/procedural, and phenomenological. These limits also affect students' learning outcomes on argumentation skills [23]. Students are trained in applying formulas to explain physical phenomena; they are not involved in exploring the limits of validity, verifying the model used or testing hypotheses and ideas through conceptual experiments.

We encouraged teachers to adopt all patterns, revise their PCK and devote particular attention to developing argumentation skills [27]. We gave them a research perspective: starting from the experience they provided to test new instructional strategies. In this way, they better-understood students' difficulties; they changed their orientation towards physics teaching and curriculum and scaffolded how to use MR reasoning tools to develop students' argumentation skills in a broad integration of Phys-Maths interplay patterns. To achieve this complex revision of their PCK, we organised the training following three targets:

- ongoing exchange of experiences (interfacing teachers with researchers and vice versa);
- classes case study practices as an active instructional laboratory;
- dedicated content knowledge for teaching workshops (one for teachers at middle secondary school and one for high secondary school).

The last target guided our efforts toward exploring the meaning of building Content Knowledge for Teaching [2]. Table 1 summarises the working groups for the three targets and the number of teachers and classes involved.

**Table 1.** Teachers’ training and classroom activities during the 2020-21 school year.

Target Description	Features
Ongoing exchange of experiences (discussions, meetings, lessons planning)	About 9 teachers in different secondary schools (almost 30 hours per teacher)
Classes case study practices	From 20 to 50 hours/class – in 10 different classes
Content knowledge for teaching workshops	About 40 teachers involved in workshops activities

During the training, we noticed that the teachers were sincerely motivated and deeply involved to revise their PCK. They embraced with interest the adoption of an approach integrated with MR for a better Phys-Maths interplay structuring adequate disciplinary Languages. In the proposed approach, we suggested using MR in the context of conceptual knowledge fragmentations through the so-called Knowledge Segments [28].

Conceptual knowledge fragmentation consists of content management, simplifying concept building and emphasising disciplinary languages. These two elements (conceptual knowledge fragmentation and use of MR) should characterise every lesson step. Teachers explain all those aspects that become unclear through purely mathematical formalism, where they are usually constrained to stay during conceptual building, partly losing physical meaning. What is clear for teachers in mathematical language description and conceptualisation [19, 20] is opaque to most students [29]. For this reason, they need more representations in different disciplinary languages to activate the process of building physical knowledge [30]. The other representations offer teachers a “conceptual place” where they can develop students’ reasoning and start to improve argumentations [27]. It conducts the learning process to build epistemological knowledge [23, 31, 32].

Here we report an example of classroom practice realised with two teachers and their three classes (involving about 70 students). We choose the adoption of Energy Bar Charts in Content Knowledge for Teaching Energy [2], as these are well-defined and studied in Physics Education Research [1, 2, 3, 16, 37]. The Energy topics are present in every Physics introductory course in Italian high schools, even if they have different curriculum studies and different levels of disciplinary treatment. So, it fulfils the curricular requirements. The following example documented an activity developed on the related content about Mechanical Energy Conservation. This specific topic has an introductory depiction through the Energy Bar Charts description. We could consider it as the first step in conceptualising more complex Energy processes. We devoted particular attention to Disciplinary Languages by representing and translating them to each other in the physical context we were exploring. The Energy Bar Chart is one of the possible representations that students have to manipulate to understand energy exchanges. Teachers took part in projecting through discussions, comparisons between textbooks, researching of problems and exercises to realise the activities. In learning by doing between peers (the teachers and the researchers), we implemented innovation in teaching practices.

### 3. Implementation: Instructional Activities for Energy Content Learning

Firstly, we planned a detailed activity (12 hours per class) to respond to curricular goals, students’ backgrounds and teachers’ needs. We prepared all the materials with teachers, reviewed times and steps, and tried to integrate the use of multiple representation tools into the process of content building.

Various online applications help implement MR activities during physics classes. We selected Desmos (<https://www.desmos.com>) and Phet Simulations (<https://phet.colorado.edu/>). Desmos is an online free web platform, available also as an applet, used mainly for Mathematics

Education, but it has great potential also in Physics Education. It requires free registration for teachers. It is a graphing and teaching tool that allows everyone to personally build online teaching activities or use something else already prepared by others. Therefore, Desmos has high flexibility in content management, and it has been a key tool for managing online activities during the period affected by COVID-19 restrictions. There are two further advantages to using Desmos: it promotes real interactivity among students. It allows teachers to check the progress of students' solutions, thanks to the simultaneous control panel, as seen in Figure 1.

Student	9 College	10 Inhelder	11 Inhelder	12 Conflic	13 Taskul	14 Taskul	15 Detern	16 Detern	17 Inheld	18 E se m	19 Analiz	20 Pizpe	21 Taskul	22 Detern
Diana Bousoo G...	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
Kyne Santos	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
Charles Lewis Re...	X	*	*	*	*	*	*	*	*	*	*	*	*	*
Moon Duchin	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
Eugenia Cheng	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
John Sims	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
Aloia Prieto Lang...	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
Hoang Xuan Sinh	✓	*	*	*	*	*	*	*	*	*	*	*	*	*
MC Escher	✓	*	*	*	*	*	*	*	*	*	*	*	*	*

Figure 1. Example of Desmos control panel for teachers.

Combined with Desmos, we used Phet Interactive Simulations. The latter is a project developed by the University of Colorado (Figure 2). Students can simulate interesting everyday situations or laboratory experiments and discover or explore physical phenomena in a game-like environment. Phet Simulations allow students to measure physics quantities and predict behaviours [38].

We have chosen some well-known examples of exercises from the literature in Physics Education research [1]. We used them with the students after the activity on energy conservation using MR. The constructed Desmos activity consists of about 20 short exercises with a progressively increasing difficulty level (Figure 2).

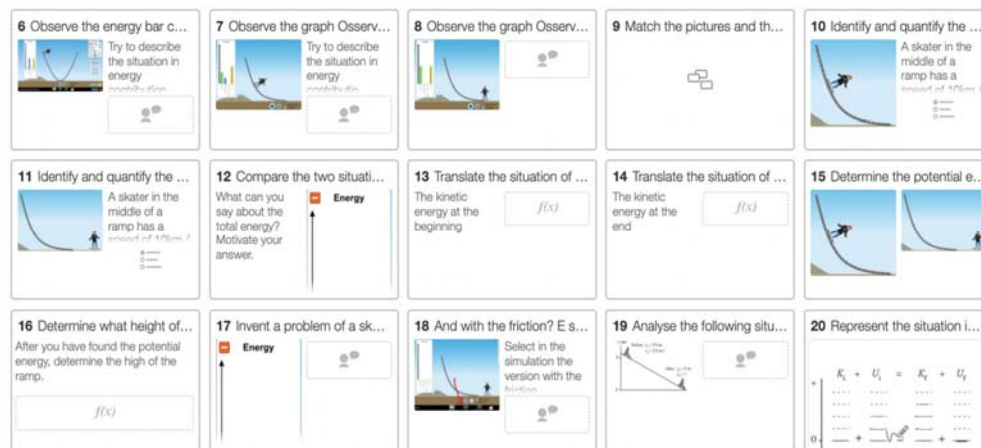


Figure 2. Desmos activity students' screens on energy conservation, including Phet Interactive Simulations and exercises chosen from the literature in Physics Education Research [1]. <https://teacher.desmos.com/activitybuilder/custom/624547fa1dafd8172edb3502/edit?lang=it#step=9b1136b2-3828-480a-8d3b-aba68fe185a5>



We started by asking the students to observe an image representing a physical phenomenon and describe it in the Natural Language. Working with the analogy, we then questioned them about similar phenomena and the relevant physical quantities needed for their description. After introducing the concept of energy and Energy Bar Charts as a possible description of energy conservation, we asked them to work with the simulation and answer some guiding questions, as seen in Figure 3.

An active verb labels all students' instructions: *observe, match, identify, compare, translate, switch, determine, invent, analyse, and represent*. These verbs ensure the active role of students during the activity. They promote student learning by activating their response, which is recorded on the screen and synchronously (or asynchronously according to lesson timing) seen by teachers [39, 40]. This is a crucial point in the activity development. It achieves the active learning goals of challenging students' beliefs through observing real situations made understandable by going deeply into their physical descriptions, in a wide variety of disciplinary languages.

**Figure 3.** An exercise with Energy Bar Charts and their interpretation using Phet Interactive Simulations.

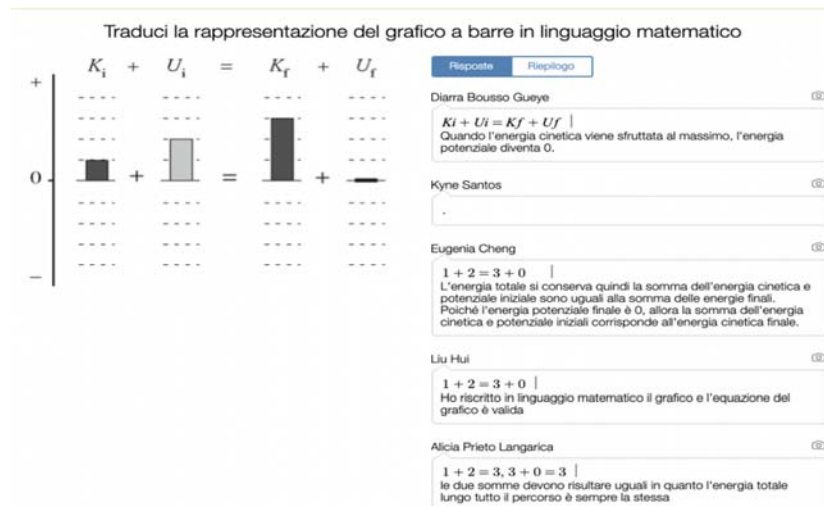
The students must match different descriptions via Energy Bar Charts to the corresponding situations. From the graphs, we finally moved to the mathematical description of the phenomenon. At the end of the activity, we submitted an exercise created by Van Heuleven [1] (Figure 4), asking the students to use Multiple Representations and integrate different Languages.

**Figure 4.** Example of an exercise taken from the literature [1].

#### 4. Discussion and conclusion: towards an Early Physics Approach

Revising the executed activity has suggested some interesting teaching and learning process features. Disassembled content knowledge in all the possible disciplinary languages is the task that teachers tested with success. This strategy allows them to explore and adopt different Phys-Math interplay patterns: on the teacher side, the effect is methodological improvements and on the student side, support for content knowledge building.

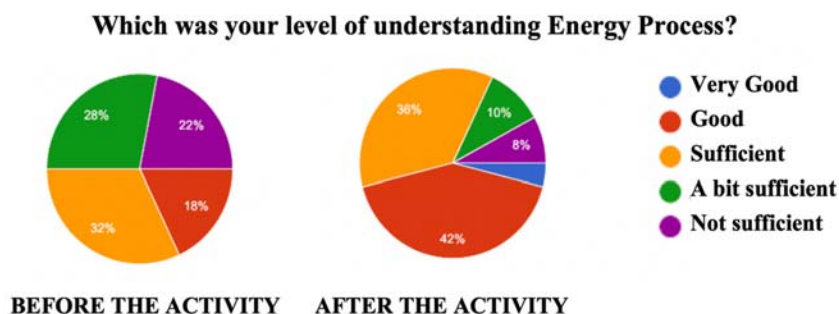
They recognised the conceptual powerfulness of the Energy Bar Charts use and MR tools. At the same time, they improved their awareness of students' difficulties, analysing all their answers in the online activity developed with the Desmos platform (Figure 5). For instance, the example reported in Figure 5 requires filling a gap between reading the graphs, collecting physics information and translating it into a mathematical description. Students need to measure the bars' height instead of seeing their qualitative meaning regarding energy exchange and energy conversion. Teachers were astonished/surprised to discover students' graph reading interpretation.



**Figure 5.** Students' answers collected on Desmos platform activity. In this case, the request was to translate the Energy Bar Chart Representation into Mathematical Language. The students' names are anonymous.

The high level of interactivity created by the platform and the deep insight into students' understanding offered an innovative way to individuate the content process, building knowledge for each student. Before, teachers had never realised how many linguistic gaps could affect students' learning outcomes. Even if teachers had evidenced many difficulties collecting information through students' answers and doubted the efficacy of targeting an adequate concept understanding, students have shown great satisfaction and involvement in the activity done. This can be seen from an overview of their answers to the post-activity survey we submitted (Figure 6).





**Figure 6.** Comparison between how students perceived their level of conceptual understanding pre and post-activity (50 responses collected).

From a student’s point of view, the conceptual fragmentation supported by the MR is a straightforward simplification in the development of conceptual change. From the survey, we aimed at estimating any improvement obtained by the activity developed. We received positive feedback: students felt involved in the learning process. They also appreciated the applications used as facilitators for following and actively engaging in lesson activities. They accepted the unusual request of giving reasons for their answers. The teacher typically justifies and students assume that the argumentation is the only correct one. But, trying to give a tentative argumentation was, for them, an exciting goal because they were not assessed while doing it. This is crucial for students: they feel free to answer (even if it could be wrong [3]) but continue the activity to understand the content better (engaged in the computer-based tool activity).

We could distinguish two kinds of factors empowering argumentation skills: external and internal. The external factors are included in the activity design targeting and scaffolding students: they are free to argue in time and class context (therefore time and class context are the external factors). In fact, they have the time they need to fill the blank space for answering; with regard to the class context, no one (teacher or students) is directly listening to their answer and evaluating it (it enacts with the same effect of cooperative working groups). From an internal point of view, it regards the way the content knowledge is built; it succeeds a specific need to know, and for this reason, the student tries to investigate the why, how and what of when something happens [8, 36, 41].

Conversely, teachers have to improve a different method for evaluating students’ learning processes and outcomes. It is different from determining the correct answer in a problem-solving test. They stressed this limit in the activities developed, but they recognised how powerful they were in promoting students’ conceptual building.

Among the results, we mainly appreciated the change in teachers’ PCK. Teachers have improved their awareness and the strength of their instructional strategies in terms of efficacy and students’ learning and development. In the meantime, they have recognised how well MR work in the conceptualisation process. Through Desmos, they have acquired a tool for better identification of students’ difficulties. Furthermore, they have tested content knowledge fragmentation as supporting disciplinary Languages integrations.

To summarise, in almost fifty hours of coaching for each teacher, we could observe the start of the process of revision and innovation in PCK [24] and Content Knowledge for Teaching Physics [2]. We are still documenting the effect of this in-service professional training, but some elements already suggest that this can be a strategy that helps to achieve and perform continuously in the future. This goes behind the change of improving an effective scaffolding in teacher practices. It happens if teachers know their teaching and lead their innovation. As well as, students better learn if they are actively participating in their learning process [33]; at the same manner, teachers better teach if they are engaged in their

teaching process investigation [34]. In this process, we offered some teaching tools: for instructional strategies. We suggested using Multiple Representations with particular attention to Disciplinary Languages integration. As a pedagogical approach, we emphasised the preferential role of engaging students in their learning process, offering them the opportunity to argue their answers.

Our research shows that this specific attention guarantees better results for younger students in secondary school.

Furthermore, we suggested that teachers implement content knowledge fragmentation. In disassembling knowledge, they manage contents in a manner that could highlight phenomenological features. Finally, all the new teaching practices and tools suggested have to address the goal of improving students' argumentation skills. And this could be achieved by changing some environmental classroom features, firstly requesting students to justify their knowledge/answers in classroom discourses and activities, such as we tried to highlight in the example reported.

We suggest including all these teaching features in the definition of an *Early Physics* approach, an approach that has to feature during Physics teaching, at its beginning.

There are some limits to overcome: teachers' disposition in guiding students toward an active learning style requires reflection and a revision of the PCK [43]; teachers' knowledge tailored for effective learning [2]; finally teachers' skills in terms of "resources that are activated and drawn upon in the process of habit development and enactment" [43].

But teachers need to make programs, give content, prepare for enrolment tests, and so much more. Traditional teaching fulfils these achievements but does not often develop a positive attitude toward the discipline [33] and sometimes does not orient female students to become scientists, possibly leading to an increase in the gender gap concerning science careers [42].

On our side, we will continue to support teachers' training to help improve teaching in the first years of Physics curricular studies, building a community for in-service teachers that helps the development of strong habits of mind and practice [43].

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