The Interplay Between Voluntary Vaccination and Reduction of Risky Behavior: A General Behavior-Implicit SIR Model for Vaccine Preventable Infections

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Abstract The onset in the last 15 years of behavioral epidemiology has opened many new avenues for epidemiological modelers. In this manuscript we first review two classes of behavioral epidemiology models for vaccine preventable diseases, namely behaviour-implicit SIR models with prevalence-dependent vaccination (at birth and among older individuals), and prevalence-dependent contact rate.

Subsequently, we briefly propose a general framework of behavior–dependent nonlinear and linear Forces of Infection (FoI) valid for a vast family of infectious diseases, and including delays and 'epidemic memory' effects.

Finally and mainly, we develop a new general behavioral SIR model. This model combines the two aforementioned types of behavioral phenomena, previously considered only separately, into a single unified model for behavioral responses. The resulting model allows to develop a general phenomenological theory of the effects of behavioral responses within SIR models for endemic infections. In particular, the model allows to complete the picture about the complicate interplay between different behavioral responses acting on different epidemiological parameters in triggering sustained oscillations of vaccine coverage, risky behavior, and infection prevalence.

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1 Introduction

The birth of mathematical and computational epidemiology dates back to a century ago about, when a few mathematical pioneers developed the main cornerstone ideas and models of the new discipline [21, 29]. Their ground-breaking idea lied in the description of the key process namely, infection transmission from an infected to a susceptible individual, by the law of mass action imported from Statistical Mechanics. Accordingly, contagion is abstracted as a chemical reaction that can or cannot occur (with a certain probability) upon the random encounter of two individuals. Social contacts between individuals are in their turn abstracted: individuals contact each other at random, as the particles of a perfect gas colliding in a box [7].

In particular, still owing to the Statistical Mechanics paradigm, the two key parameters, namely the *contact rate* per individual, and the *transmission rate* per contact, are taken as natural constants of human behavior, possibly mirroring the social characteristics of a given community or setting, at a certain time moment.

Building on extensions of this simple idea, more recent pioneering contributions aiming to better integrate models with data [2, 19, 20], have allowed mathematical models of infectious diseases to leave their traditional, abstract, bio-mathematical environment, to become central supporting tools for public health decisions. Main instances are for example the determination of the duration of school closure during a pandemic outbreak, or the fraction of new-born children to be immunized for a vaccine-preventable infection, as is the case of measles and pertussis. This critical role has conferred to mathematical epidemiology a prominent role in policy making, by allowing a substantial advance of public health as a scientific discipline.

Some of contemporary models are highly sophisticated in both their mathematical/computational structure, and in their data requirements [26]. In these sophisticated models, the patterns of social contacts with which individuals contact each other, classified according to a range of characteristics (e.g. age, level of social/sexual activity, etc.) are the key determinants of the transmission of both close-contact infections, as influenza or measles, and of sexually transmitted infections (STIs), such as HIV/AIDS.

Nonetheless, as it was pointed out [17], even in such highly sophisticated models there remained a key missing layer: the humans' behavior. Indeed, even these realistic models continue to treat contact and transmission rates as natural constants, exactly in the same way as the simple SIR model. This means that individuals' social behavior is totally unaffected by the state of the disease. Briefly, this means for example that during an epidemic outbreak individuals will continue to contact each other at the same rate regardless of how low or high is the perceived risk of acquiring infection or even of dying from it. As contact patterns are usually measured from normal situations [27], the resulting models are therefore unlikely to apply under the complicate and stressed social conditions that might result during a dangerous epidemic or a during a period of panic raised by a pandemic threat [17].

Similarly, the models used in the current public health practice to evaluate the impact of childhood immunization programs most often treat vaccine uptake as a constant [2]. This implies to postulate that the vaccination coverage prevailing in the community is totally unaffected by individuals' risk perceptions about the disease and the vaccine. Such an hypothesis is at odd with the fact it is the degree of adhesion of the public that will ultimately determine the success of the program, especially when the program is voluntary or when laws for mandatory vaccines are not carefully enforced.

Clearly, this static human behavior, which is the ultimate legacy of the Statistical Mechanics paradigm, is an unrealistic abstraction, which at best can apply in some particular situations (e.g. an epidemics of a non-threatening and non-costly infection). Indeed, by their very nature, humans are neither static nor passive. For example, they can decide to spontaneously change their social behavior in response to a pandemic threat, can redirect their sexual activity towards partners perceived as less-at-risk in response to news about a dangerous STI known to circulate in the population, or can decide not to vaccinate their children after having compared perceived costs and benefits of a vaccination program, thereby threatening its success. A central role in these decisions is played by the way communication technologies affect the shape and the speed of spread of the relevant information.

The need to seriously account for human behavior has led in the last 10 years to a deep rethinking of the mathematical modeling of infectious diseases. This has in turn led to the birth of a new branch of mathematical epidemiology, which we termed the *behavioral epidemiology* (BE) of infectious diseases [24]. As argued in [4, 24], BE has an intrinsically multi-disciplinary core, aiming to combine classical epidemiological modeling [2, 3, 7] and behavioral sciences, namely sociology, psychology, economics, anthropology etc, to improve our understanding of the complex interplay between infection dynamics and the related underlying human behavior.

Behavioural epidemiology has now grown rapidly and a summary of its many different facets can be found in a number of reviews appeared on the subject [18, 24, 32] to which the interested reader can refer.

This work has a twofold goal. First, we aim to introduce some of the basic ideas of behavioural epidemiology and their mathematical and public health implications for the dynamics and control of vaccine preventable infectious diseases such as e.g., measles and pertussis. In particular we do this by presenting (1) some basic SIR models with voluntary vaccination where individuals change their propensity to vaccinate their children depending on the perceived changes in the risks of infection and disease, and (2) some basic SIR models where instead individuals change their social contact patterns—modulating their behaviour at risk—still based on their perceived risks of infection and disease. In relation to this we rely on work from [9, 11, 24] where individuals' responses are modulated from awareness of the current and past trend of infection summarised by suitable phenomenological information indexes. Second, we aim at combining the previous two issues—which have typically been considered separately in the available behavioural epidemiology literature—by proposing a general model where awareness of risks can affect both

the propensity to vaccinate as well as the contact rate. This double feedback, though not well documented for current vaccine preventable endemic infections, is likely to occur in many circumstances. Surely a well documented example has been represented by the alert arising from the doubling in the number of deaths from invasive meningococcal disease observed in the Tuscany region of Italy between 2015 and 2016 [8, 28, 31]. As meningitis is perceived as a very serious disease, the scaring public news appearing at the time jointly with the offer of free vaccination from the local public authorities, were able to dramatically increase the vaccination coverage in all population age groups i.e., the newborn (those typically targeted by the Italian public health system for free vaccination against meningococci), the young, the adolescents as well as the adults [28]. On the other hand it is known that the concurring—worried—public health communications recommending avoidance of possible risky behavior especially among adolescents (including e.g., avoidance of exchanges of cigarettes, glasses, etc.) were also taken very seriously, resulting also in a reduction in the contact and transmission rates relevant for meninogocci transmission. However, we expect such effects to arise in many other circumstances. For example an ongoing measles epidemics might—at least at the local level—stimulate an upward pressure on vaccine coverage as well as protective behaviours such as e.g., not to send to schools non–vaccinated children. Motivated by these considerations, we propose, as a first step, a general SIR model for vaccine preventable endemic infections where individuals can respond to changes in their perceptions of risks by modulating both their propensity to vaccinate among newborn but also, and mainly, among older individuals, as well as their contact rate.

This article is organised as follows. In Sect. 2 we review the behavior of SIR models of endemic infections for mandatory vs voluntary—behaviour dependent—vaccination. In Sect. 3 we discuss SIR models with behavior—dependent contact rates. In Sect. 4 we present and investigate our general SIR model with behavior responses in both vaccination propensity and contact patterns. Concluding remarks follow.

2 The SIR Model: Mandatory vs. Voluntary Vaccination

2.1 The Case of Constant Vaccination Coverage

The basic SIR model for the control of endemic infections assumes vaccination at birth at constant coverage p, which is reminiscent of a situation where a mandatory immunization program exists. The resulting model is as follows:

$$S' = \mu (1 - p) - \mu S - \beta(t) SI \tag{1}$$

$$I' = I(\beta (t) S - (\mu + \nu)) \tag{2}$$

$$R' = \mu p + \nu I - \mu R \tag{3}$$

where we denoted by S, I, R ($S + I + R = 1$, allowing to omit the third equation) the fractions of individuals who are, respectively, *susceptible* to acquiring infection, *infective*, i.e. able to retransmit infection to others, and *removed* because of e.g. immunity acquired after recovery. The infective fraction I is also called the *infection prevalence*. The function $\beta(t)$ denotes the transmission rate which is typically time-dependent. The other demo-epidemiological parameters are: $\mu > 0$ which denotes both the birth and death rates, assumed identical to ensure that the population is stationary over time, and $\nu > 0$ which is the rate of recovery from infection.

In the most well-known case of constant transmission rate $\beta(t) = \beta$, the previous SIR model, which always admits a disease-free equilibrium point

$$DFE = (1 - p, 0, p),$$

has a simple threshold behavior depending on the interplay between the basic reproduction number $\Re_0 = \beta/(\mu + \nu)$ and the vaccine uptake p. Assuming that the basic reproduction number $\Re_0$ exceeds one (ensuring a globally stable endemic equilibrium in absence of immunization) then, if the vaccine-reduced reproduction number $\Re_0(1 - p) > 1$ then the DFE is unstable and the infection continues to persist endemically about its endemic equilibrium, while if the vaccine coverage is large enough to ensure

$$\Re_0(1 - p) \leq 1$$

then the infection will be eliminated i.e., the DFE is globally attractive. The condition $\Re_0(1 - p) < 1$ can be rewritten as:

$$p > p_c,$$

where

$$p_c = 1 - \frac{1}{\Re_0}$$

is the so called *critical immunization coverage*, which we also term the May-Anderson threshold [2].

Finally, if one also takes into account the presence of vaccination at ages older than birth

$$S' = \mu(1 - p) - \mu S - \mu \phi S - \beta(t) S I,$$

where $\mu \phi S$ is the vaccination rate of adults. the parameter ϕ is non–dimensionalised and, given the average lifespan $L = 1/\mu$ the average age at adult vaccination is L/ϕ. One can easily find the following disease elimination condition:

$$\Re_0 \frac{1 - p}{1 + \phi} < 1,$$

which also reads

$$\frac{p + \phi}{1 + \phi} > p_c.$$

2.2 Voluntary Vaccination: A Phenomenological Model

The hypothesis of constant p is clearly an approximation which roughly mirrored mandatory immunization systems enacted by many countries in the past, but it is no more valid under many recent scenarios. Consider for simplicity a voluntary vaccination system where parents take their decisions on whether to immunize or not their children based primarily on perceived costs and benefits of that immunization. These perceived costs and benefits in turn depend on the available information about the state of the infection—and related serious disease—and about the risks that are perceived to be connected with vaccination, i.e., suffering serious side effects from immunization. In such circumstances available information might feedback on the current vaccine uptake thereby affecting infection dynamics. For example, during epoch of low infection incidence individuals might perceive a quite high relative cost from vaccine side effects, therefore reducing their propensity to vaccinate, and the opposite during epidemic phases. Our *behavior-implicit* framework for information related immunization [10–13] considers the following SIR model for a non-fatal childhood infectious disease in a stationary homogeneously mixing population (we omit the R equation since $R = 1 - S - I$):

$$S' = \mu \left(1 - p(M)\right) - \mu S - \phi(M) - \beta(t)SI \tag{4}$$

$$I' = I\left(\beta\left(t\right)S - (\mu + \nu)\right) \tag{5}$$

where the transmission rate $\beta(t) > 0$ is taken either constant or periodically varying with period θ equal to 1 year [2], while functions $p(M) \geq 0$ and $\phi(M) \geq 0$ denote the vaccination coverage at birth and the coverage at subsequent immunizations respectively, both which are taken here to be increasing functions of a suitable *information index M* [11]. The index M summarizes the information on benefits and costs of immunization used by parents to take their vaccination decisions. Thus, M might be any function of the *current*, or *past*, infection prevalence (or incidence), taken as measures of the perceived cost of suffering infection or its serious sequelae, or of the prevalence (or incidence) of vaccine adverse events (VAE), taken as measures of the perceived cost of suffering VAEs. Here we focus on the perceived risk of infection (and related disease) as the driving force of immunization decisions, on the simplifying assumption that the perceived risk of vaccine adverse events is coarsely constant over time.

The forms actually adopted for the vaccine uptake functions $p(M)$ and $\phi(M)$ are such that, first of all

$$p_0 = p(0) > 0$$

and

$$\phi_0 = \phi(0) > 0$$

where the fixed components p_0 and ϕ_0 mirror the presence of a sub-population vaccinating independently of the state of information on infection and disease. Moreover, we assume that $p_1(M) = p(M) - p_0$ and $\phi_1 = \phi(M) - \phi_0$ are increasing functions mirroring, respectively, parents' and adults' reaction to increasing perceived risk from the disease. For example, taking M to be the current infection prevalence I, previous formulation amounts to state that when infection prevalence increases, people in the group influenced by information react by increasing their children and/or their own vaccine uptake, and vice-versa. Of course, for very *large* levels of M we assume p_1 to saturate to some level $p_1^{\text{sat}} \leq 1 - p_0$. A saturating level is not required for $\phi_1(M)$, although it is reasonable. The functions p_1 and ϕ_1 are continuous and differentiable, except at a finite number of points.

2.3   Modeling the Information Index $M(t)$

The index M can be taken to represent a measure of the perceived risk following infection (including serious sequelae), which summarizes the way information on infection and its serious sequelae, and ensuing perceptions on benefits and costs (of measures to be adopted to reduce risks), affect perceptions about risks. We can assume that M is given by a continuous function $\omega(S, I)$ with $\partial_I \omega(S, I) > 0$.

In particular, M might be any function of the *current*, or *past*, infection prevalence or incidence, e.g. of the form $M = h(\beta(t)S(t)I(t))$ (e.g. $M = k_* \beta(t)S(t)I(t)$ with $k_* > 0$), or $M = g(I)$ (e.g. $M = kI$ with $k > 0$). For example, if $M = kI$ might define the perceived risk of serious disease as the product of the perceived risk of infection, given by some linear function of prevalence I, times the perceived risk of serious disease given infection. For the sake of simplicity since now on we shall deal with functions $M = g(I)$. Following the review in [18], this hypothesis amounts to build a *prevalence-based* model, as opposite to belief-based, where the use of information is *global*, i.e. homogeneously available to everyone, as opposite to local, as is the case for spatially structured models.

More realistically M also depends on past values of state variables, as for many infections information typically becomes available only with a delay, due to a number of procedures (such as laboratory confirmations, reporting to public health

authorities, and diffusion by the available channels), and moreover awareness in the population requires time. In this case M will take the more general form:

$$M(t) = \int_{-\infty}^{t} g(I(\tau))K(t - \tau)d\tau \tag{6}$$

where K is a probability density function called the *delaying kernel* [23].

As for function $g(I)$, we assume that $g(0) = 0$ and $g'(I) > 0$.

Here, besides the trivial kernel $K(t) = \delta(t)$, where δ is the Dirac function, yielding the unlagged case

$$M(t) = g(I(t)),$$

we consider two main types of delaying kernels, i.e. the well-known *exponentially fading memory* kernel $K(t) = a \exp(-at)$, with expectation $< t >$ given by the fading time scale $T = 1/a$ [23], and the kernel:

$$K(t) = \frac{1}{T_1 - T_2} \left(e^{-t/T_1} - e^{-t/T_2} \right). \tag{7}$$

The latter kernel, introduced in [16], represents a parsimonious way to model the effects of information handling by individuals, as it accounts for two sub-processes possibly occurring independently and at different time-scales: (1) formation and acquisition of information, with time-scale T_1, and (2) memory fading of acquired information, with time-scale T_2. Often the first process is much faster than the second. Note that if $T_1 \approx 0$ then $K(t) \approx (1/T_2)e^{-t/T_2}$, i.e. $K(t)$ collapses into an exponentially fading memory with time scale T_2. This kernel has expectation $< t >= T_1 + T_2$ and $Var(t) = T_1^2 + T_2^2$. Compared to the exponentially fading memory, which assigns maximum weight to current information—usually unavailable—this kernel satisfies $K(0) = 0$, mirroring negligible use of current information, as in the commonly used Erlang kernels of higher order [23]. However, unlike the latter kernels, which consider sub-processes having the same time scale, the kernel (7) is much more flexible. Note indeed that the first order Erlang kernel $K(t) = a^2 t \exp(-at)$, corresponds to (7) in the case where $T_1 = T_2$. We term (7) the *acquisition-fading* kernel. As for Erlang kernels, also (7) is reducible to ordinary differential equations (ODEs).

Under the exponentially fading memory the Eq. (6) reduces to the single ODE:

$$M' = a(g(I) - M) \tag{8}$$

Finally, under the *acquisition-fading* kernel (7) Eq. (6) reduces to the following pair of ODEs:

$$M_1' = a_1 \left(g(I) - M_1 \right) \tag{9}$$

$$M_2' = a_2 \left(M_1 - M_2 \right), \tag{10}$$

where $a_1 = 1/T_1$, $a_2 = 1/T_2$, $M(t) = M_2(t)$.

The Erlang first order kernel i.e., the exponentially fading memory, corresponds to the particular case $a_1 = a_2$.

3   Behavior–Modulated Contact Rate

In epidemic model, a key concept is the *infection incidence*, which represents the absolute number of new infection cases per unit of time. Although intuitive from the epidemiological point of view, the *infection incidence* constitutes a modelling challenge [5, 25]. For a generalized family of SIR model with one class of susceptible and one of infectious, denoting as X the total number of susceptible, Y the total number of infectious and N the total population size, and with J the incidence, generalizing Begon and coworkers [5] one has:

$$J = X \times C \times \pi_1 \times \pi_2 \tag{11}$$

where: C is the average number of general contacts per time unit, π_1 the probability that a contact is with an infectious subject and π_2 is the probability that a contact with an infectious subject induces the infection of the susceptible subject.

Clearly, π_1 is a function of the state variables [5]:

$$\pi_1 = \pi_1(X, Y, N)$$

whereas, in the classical epidemiology view C and π_2 were considered constant. The product

$$FoI = C\pi_2\pi_1(X, Y, N) \tag{12}$$

is the *force of infection* (FoI), and represents the *per capita* rate at which susceptible individuals acquire infection per unit of time.

As far as the function $\pi_1(X, Y, N)$ is concerned, for the SIR model the two most popular choices are: (1) the classical mass action law $\pi_1 = qY$ leading to the following force of infection;

$$FoI = \beta Y$$

(2) the frequency dependent mass action law $\pi_1 = hY/N$. leading to the following force of infection

$$FoI = \beta\frac{Y}{N} = \beta I$$

In both cases the term β, called the transmission rate, is taken as a natural constant of human behavior.

In the mathematical epidemiology literature, the first step beyond the Statistical mechanics paradigm yielding the first epidemiological model with behavioral change was the *behavior-implicit*, *prevalence-dependent* SIR epidemic model proposed by Capasso and Serio in the seventies [6]. In [6] the contact rate β, until then taken as constant, is allowed to be a decreasing function of infection prevalence I. This implies that the Force of Infection (FoI), in the case of 'frequency dependent mass action law' takes the following non-linear form [6]:

$$FoI(I) = \beta(I)I \tag{13}$$

with: $\beta'(I) < 0$. The authors pointed out that, unlike standard mass action formulations, this could make the FoI to become a non-monotone function of the prevalence (e.g. if $\beta(I) = \beta_0(1 + hI^2)^{-1}$).

The authors motivated their formulation with the possibility of behavioral changes in response to the changing epidemiological conditions that appear as the epidemic out in the population. For example, during epochs in which disease prevalence is perceived to be high, also the risk of infection might be perceived as high, thereby inducing changes in individuals' contact behavior to reduce risks, thereby ultimately affecting also the actual risk of getting infected. Today Capasso and Serio's formulation would be classified as a *behavior-implicit* [4] formulation, to mirror the fact that behavior is embodied into the mathematical model in an *implicit* manner, i.e. via a nonlinear specification of the FoI possibly mirroring individuals responses to changing epidemiological conditions, rather than incorporating rules explicitly describing the agents' behavior. Since [6], several other works have investigated epidemic models with a non-linear FoI [1, 22, 30]. With reference to the expression of the incidence rate (11) and of the generalized force of infection (12), we propose here the following form of FoI:

$$FoI = C(M)\pi_2(M)\pi_1(X, Y, N) = \widehat{\beta}(M)\pi_1(X, Y, N) \tag{14}$$

where $C'(M) < 0$, $\pi_2'(M) < 0$, $\widehat{\beta}(M) = C(M)\pi_2(M)$ and, as a consequence, $\widehat{\beta}'(M) < 0$. In defining the generalized behaviour-dependent FoI of (14) we have taken into the account that: (1) behaviour can modify (with different patterns and intensity, of course) both the average number of contacts per time unit and the probability of getting the infection when in contact with an infectious; (2) behaviour is not only based on the knowledge of current stage of spread of the disease but also on the memory of past epidemic history.

Finally, note that (11) and (14) can be easily extended to more complex models where multiple epidemic state variables and more complex patterns of transmission are considered.

3.1 Extending the Capasso–Serio Behavioral Model

By noting that behavior changes in turn require changes in the individuals' information endowment, in [9] we also attempted to generalise previous *behavior-*

implicit models of endemic infections (such as e.g., measles), by representing the contact rate β along the same notion of *information-dependent* behavior we developed in [11] for the vaccination coverage.

This led us to consider simple SIR dynamic models of recurrent endemic infections where the contact rate is a phenomenological function of an *information index M* sharing the above described characteristics, yielding the following FoI:

$$FoI(M) = \beta(M)I, \tag{15}$$

where $\beta'(M) < 0$. This assumption yields the following SIR model with behaviour-dependent contact rate [9]:

$$S' = \mu(1 - S) - \beta(M)IS \tag{16}$$

$$I' = \beta(M)IS - (\mu + \nu)I \tag{17}$$

completed by Eq. (6), governing the dynamics of M, and by the balance equation of the removed fraction $R(t)$: $R(t) = 1 - S(t) - I(t)$.

4 A General SIR Model Embedding Behavioral Feedbacks on Both Vaccination Propensity and the Contact Rate

As discussed in the Introduction, in the available behavioural epidemiology literature dealing with endemic, vaccine preventable, infectious diseases, the feedback that the awareness of changes in trends of infection prevalence—as modulated by information—might yield on behavior towards the disease, has been investigated separately i.e., either for its effects on vaccination coverage or for those on the contact rate. However, it is reasonable to expect that in many circumstances behavioral changes might involve both the propensity to vaccinate as well as the contact rate. Consistently, in this section we propose a new general SIR model to investigate the synergy between these two feedbacks, on the assumption that the information background on which decisions to switch to a different behaviour are taken is summarised by the same information index M. In particular we include in the model both prevalence-dependent vaccination of newborn as well of older individuals to allow the possibility, for older individuals who avoided vaccination at birth, to consider later vaccination during epochs of increasing perceived risks. The dependence of the behaviours on the prevalence is mediated by the information index M. This yields to the following model

$$S' = \mu(1 - p(M) - S) - \mu\phi(M)S - \beta(M; t)IS \tag{18}$$

$$I' = \beta(M)IS - (\mu + \nu)I \tag{19}$$

to be complemented by a model linking the information index to the spread of the diseases and by $R = 1 - S - I$. We assume there that β is both a decreasing function of M and a constant or periodic function of the time. In the general case, thus, the integro-differential system (18)–(19)–(6) forms a *family of models*. By specific choices of the delay kernel $K(\tau)$, a range of models can be derived from the general family of models (18)–(19)–(6).

The results we previously obtained in [11] and [9] will thus become particular subcases of the more general results we now derive for model (18)–(19)–(6).

In relation to the proposed new model there are two main substantive questions, i.e. (1) how perceptions of risks related to the disease might affect behavior-dependent vaccination as well as contact behaviour, and how this in turn affects infection control, and (2) how behavior might affect the dynamical pattern of infection, e.g. by triggering oscillations.

4.1 Modelling Human Behavior and Its Implications for Infection Control

We recall here the first question: *'How perceptions of risks related to the disease might affect behavior-dependent vaccination as well as contact behaviour? And how this in turn affects infection control?'* We start noticing that the family of models (18)–(19)–(6) always admits the disease-free equilibrium (DFE):

$$DFE = \left(\frac{1 - p_0}{1 + \phi(0)}, 0, 0 \right) \tag{20}$$

The stability properties of the DFE are provided by the following theorem which holds regardless the actual form of the information index M

Theorem 1 *Under θ-periodic $\beta(0, t)$, the DFE (20) of (18)–(19)–(6) is globally asymptotically stable (GAS) if:*

$$Q = \frac{1 - p_0}{1 + \phi(0)} \frac{1}{\mu + \nu} \frac{1}{\theta} \int_0^\theta \beta(0, u)du < 1. \tag{21}$$

If instead $Q > 1$, then the DFE is unstable.

The proof of Theorem 1 is based on the fact that the differential inequality

$$S' \leq \mu(1 - p(0) - (1 + \phi(0))S)$$

implies that asymptotically

$$S(t) \leq S_\infty = \frac{1 - p_0}{1 + \phi(0)}.$$

Thus, asymptotically

$$I' \leq I(\beta(0, t)S_\infty - (\mu + v)).$$

As a consequence if $Q < 1$ then $I(t) \to 0^+$. Moreover, the linearization equation for $I(t)$ by setting $I = 0 + i + O(i^2)$ is

$$i' = i(\beta(0, t)S_\infty - (\mu_v))$$

As a consequence: (1) $Q < 1$ guarantees both the local and global stability of the DFE; (2) $Q > 1$ implies the unstability of the DFE.

Note that if $\beta(0, t)$ is constant then condition (21) becomes the well-known one reported in Sect. 2 i.e.,

$$\Re_0 \frac{1 - p_0}{1 + \phi(0)} \leq 1$$

where $\Re_0 = \beta(0)/(\mu + v)$ is the basic reproduction number of the SIR model for endemic infections [2, 7].

The interpretation of condition (21) follows from the proper understanding of Q. Quantity Q represents indeed the appropriate vaccine reproduction number computed in the correspondence of the baseline vaccine coverage for newborn (p_0) and older individuals ($\phi(0)$) respectively, and in presence of the normal social contact rate ($\beta(0, t)$) which are associated to situations of minimal perceived risk ($M = 0$). In particular, the previous result recall us that elimination turns out to be feasible only if the baseline risks conditions that are perceived under circumstances of minimal infection circulation ($M = 0$) are capable to stimulate an overall vaccination coverage (of both newborn and older individuals) already in excess of the critical threshold p_c. Otherwise, elimination can never be achieved even if the overall uptake $p(t) = p(M(t))$ could *temporarily* reach values as high as 100% during epochs of high prevalence and therefore high perceived risks.

Moreover, if $\partial_t \beta(M, t) = 0$ and it holds:

$$\frac{1 - p_0}{1 + \phi(0)} \Re_0 > 1 \tag{22}$$

the system has a unique endemic equilibrium EE $= (S_e, I_e, M_e)$, where $M_e = g(I_e)$,

$$S_e = \frac{1}{\Re_0} K(M_e)$$

and I_e is the unique solution of the equation

$$\frac{\mu}{\mu + v} \left(1 - p(g(I)) - \frac{1}{\Re_0}(1 + \phi(g(I)))K(g(I))\right) = I$$

4.2 Behavioural Responses and Infection Dynamics

We recall here the second main question namely, *How behavior might affect the dynamical pattern of infection (e.g. by triggering oscillations)?*

As we showed in the previous section, the existence and stability of the DFE, as well as the existence and location of the endemic equilibrium hold for the general family of models, independently of the form of the delaying kernel. On the contrary, the stability properties of the endemic state critically depend on $\varrho(\tau)$. Thus, to answer the second substantive question it is necessary to consider specific models of the memory kernel.

For the unlagged case $M = g(I(t))$, and under the assumption that $\partial_t \beta(M, t) = 0$ is constant (meaning that, besides behavioural effects, we rule out other time effects, such as periodicities, on the contact rate) and $p(M)$ and $\phi(M)$ are differentiable, it holds that:

Theorem 2 *Let condition (22) holds. Then the unique endemic state EE of system (18)–(19) in absence of delays is GAS in the positively invariant set:*

$$\Omega^{**} = \left\{ (S, I) \mid S \geq 0, \; I > 0, \; S + I \leq 1, \; S \leq \frac{1 - p_0}{1 + \phi(0)} \right\}. \tag{23}$$

The proof follows by applying the Poincare-Bendixon theorem with weight function $1/I$.

The previous result indicates that inclusion of current information only is not sufficient to trigger oscillations. It is therefore interesting to look at whether the EE can be destabilised when agents base their decisions on past information as well.

In the case of the exponentially fading kernel, we obtain the following three-dimensional family of models:

$$S' = \mu(1 - p(M) - S - \phi(M)) - \beta(M)IS \tag{24}$$

$$I' = \beta(M)IS - (\mu + v)I \tag{25}$$

$$M' = a\,(g(I) - M) \tag{26}$$

Computing the Jacobian matrix at the unique endemic equilibrium, and defining the following quantities:

$$-b_{11} = \partial_S S' = -\mu(1 + \phi(M)) - \beta(M)I < 0$$

$$-b_{12} = \partial_I S' = -\beta(M)S < 0$$

$$\sigma = \partial_M S' = -\mu p'(M) - \mu \phi'(M)S - \beta'(M)IS$$

$$b_{21} = \partial_S I' = \beta(M)I > 0$$

$$\partial_I I' = 0$$

$$-b_{23} = \partial_M I' = \beta'(M)IS < 0$$

$$\partial_S M' = 0$$

$$\partial_I M' = ag'(I) > 0$$

$$\partial_M M' = -a < 0$$

one gets a characteristic equation of the form $c_0 + \sum_1^3 c_i \lambda^i = 0$ where $c_3 = 1$ and:

$$c_2 = a + b_{11} > 0$$

$$c_1 = a(b_{11} + g'(I)b_{23}) + b_{12}b_{21} > 0$$

$$c_0 = a\left(b_{11}g'(I)b_{23} + (b_{12}g'(I) - \sigma)b_{21}\right)$$

All the b_{hk} are positive, and σ has no pre-defined sign in the general case. Thus if

$$\sigma > \sigma^* = b_{12}g'(I) + \frac{b_{11}g'(I)b_{23}}{b_{21}} > 0$$

it can be $c_0 < 0$ and the equilibrium point is unstable. If instead it is $\sigma^* < 0$ then from the Ruth-Hurwitz condition $c_1c_2 - c_0 > 0$ we have the following second-order inequality in a

$$q_2 a^2 + q_1 a + q_0 > 0$$

where:

$$q_2 = b_{11} + g'(I)b_{23} > 0$$

$$q_1 = b_{21}(\sigma + b_{12}(1 - g'(I))) + b_{11}^2$$

$$q_0 = b_{11}b_{12}b_{21} > 0$$

Thus, if

$$\Delta = q_1^2 - 4q_2q_0 \leq 0$$

then the endemic equilibrium is locally asymptotically stable, whereas if $q_1 < 0$ and $\Delta > 0$, i.e. if

$$-q_1 > 2\sqrt{q_2 q_0}$$

i.e.

$$b_{21}(-\sigma - b_{12}(1 - g'(I))) - b_{11}^2 > 2\sqrt{(b_{11} + g'(I)b_{23})\, b_{11}b_{12}b_{21}} \qquad (27)$$

then there exist an interval (a_1, a_2) such that if $a \in (a_1, a_2)$ EE is unstable and Yakubovitch oscillations arise through two Hopf bifurcations at $a = a_1$ and at $a = a_2$; if $a < a_1$ and $a > a_2$ then EE is locally stable.

Specific subcases where only of the three rates $p(M)$, $\phi(M)$ and $\beta(M)$ was non-null have been investigated in [9–14]. When only vaccination is present, the EE can be destabilized in the presence of an exponentially fading information memory. On the contrary, as shown in [9] for the scenario where human behavior only affects the contact rate β, the EE is locally stable for all $a > 0$ i.e., independently of the magnitude of the average information delay. Thus it is of interest to investigate the role of $\beta'(M)$ in possibly stabilizing the EE. Although inequality (27) is quite complicate, taking into the account that the derivative of the contact rate appears in $b_{23} = -\beta'(M)SI > 0$ and in σ and by rewriting the latter as follows

$$\sigma = -\omega_0 - \beta'(M)SI$$

it yields

$$b_{21}(\omega_0 + \beta'(M)SI - b_{12}(1 - g'(I))) - b_{11}^2 > 2\sqrt{(b_{11} - \beta'(M)SIg'(I))\, b_{11}b_{12}b_{21}} \qquad (28)$$

In other words, the presence of the negative term $\beta'(M)SI$ decreases the l.h.s. of inequality (27) and the presence of the positive term $-\beta'(M)SIg'(I)$ increases its r.h.s. Overall, roughly speaking, this makes less likely the fulfillment of the inequality (27), i.e. less likely the onset of oscillations. In other words the reduction of risky behaviour as a response to perceptions of increasing risk from the disease has beneficial effects, at least as far as the onset of recurrent epidemics is concerned. Note that, focusing instead on the role of $p(M)$ and of $\phi(M)$, one can read in the reversed direction the inequality (27) and conclude that the initiation of voluntary vaccination in a scenario of behavior-dependent contact rate will, on the one hand, contribute to reduce the average infection prevalence I_e (as it is easy to verify) but, on the other hand, may induce recurrent epidemics. As indeed noted in [11, 15] the recurrent oscillations induced by prevalence-dependent vaccination behaviour can be very complicate—even when purely periodical—as they have the potential to generate huge amplitude oscillations with extremely long periodicities, possibly very difficult to predict and handle in a real public health context.

Finally, under the *acquisition-decay* kernel, one yields the following system:

$$S' = \mu(1 - p(M) - S - \phi(M)S) - \beta(M)IS \tag{29}$$

$$I' = \beta(M)IS - (\mu + \nu)I \tag{30}$$

$$M_1' = a_1 (g(I) - M_1) \tag{31}$$

$$M' = a_2 (M_1 - M). \tag{32}$$

Though in principle it is possible to analytically characterise the local stability of the endemic state for the above four-dimensional system, the problem becomes analytically cumbersome also for simple choices of $p(M)$, $\phi(M)$, $g(I)$ and $\beta(M)$. This is also true for the case $a_1 = a_2$.

5 Concluding Remarks

In the first part of this manuscript we have reviewed two main classes of behavioral epidemiology models. The first one is that of SIR models for vaccine preventable endemic infectious diseases with *prevalence-dependent*, *behaviour-implicit*, vaccine coverage at birth and among older individuals in a regime of voluntary vaccination. The second one is that of SIR models for endemic infections with *prevalence-dependent* contact rate. These classes of models [9, 11] replace critical epidemiological parameters typically taken as constant in basic models namely, the vaccine coverage and the contact rate, by general functions of available current and past information on the infection and related serious disease. The underlying idea is that human agents actively respond to changes observed in the (current or past) infection prevalence by adapting their immunization decisions and contact patterns. These models bring a number of relevant novelties compared to classic models [9, 11].

In the second part of the manuscript we developed a new general SIR model combining the two aforemention types of behavioral phenomena, previously considered only separately in the literature, into a single unified model. In this new model individuals respond to changes in the information indexes by continuously adapting (1) the propensity at which they immunize their children at birth, (2) the propensity at which they choose immunization at later ages, and (3) their contact rate, which summarises their behaviour at risk. This combination of behavioral responses, though rarely documented for vaccine preventable infectious diseases of childhood, has certainly occurred for example during the recent alert occurred in Tuscany during 2015–2016, when rates of invasive meningococcal disease and deaths dramatically increased [8, 28, 31]. The alert pushed the Regional public health system towards offering supplementary immunization in a wide range of age groups which was largely accepted by the population, while at the same a number of

possibly risky behaviour surely declined, especially among adolescents and young adults [8, 28].

The resulting model was not designed to specifically describe the meningococcal disease alert in Tuscany, which would have required a more complicate model, but rather to develop a general phenomenological theory of the effects of behavioral responses within SIR models for endemic infections. The model allows to complete the picture offered in previous separate works, in particular suggesting the complicate interplay between different behavioral responses acting on different epidemiological parameters in triggering sustained oscillations of vaccine coverage, risky behavior, and infection prevalence.

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