Editors' Suggestion

Taming Mass Gaps with Anti–de Sitter Space

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Anti–de Sitter space acts as an infrared cutoff for asymptotically free theories, allowing interpolation between a weakly coupled small-sized regime and a strongly coupled flat-space regime. We scrutinize the interpolation for theories in two dimensions from the perspective of boundary conformal theories. We show that the appearance of a singlet marginal operator destabilizes a gapless phase existing at a small size, triggering a boundary renormalization group flow to a gapped phase that smoothly connects to flat space. We conjecture a similar mechanism for confinement in gauge theories.

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Introduction—A striking aspect of quantum field theory (QFT) is the possibility that a classically marginal coupling becomes dimensionful at the quantum level, generating an energy scale Λ from "nothing." This induces drastic changes in the low energy spectrum, particularly evident in examples like Yang-Mills (YM) theory in four dimensions. Despite classical expectations of massless excitations, the quantum theory exhibits a mass gap, a phenomenon yet to be rigorously proven for YM [1,2].

Studying QFT in the anti–de Sitter space (AdS) [3] offers a new window into the dynamics through boundary conformal correlators [4–15]. The AdS radius (L) acts as a tunable parameter, enabling interpolation between a weakly coupled regime ($\Lambda L \ll 1$) and a regime where the flat-space physics is recovered ($\Lambda L \gg 1$) [4,5]. Previous work [16] on YM in AdS_4 discussed an important consequence of the mass gap: Dirichlet boundary conditions (BC) at $\Lambda L \ll 1$ result in a family of boundary conformal theories, with a global symmetry mirroring the bulk gauge group and massless gluons in the bulk spectrum. These BCs cannot persist at $\Lambda L \gg 1$ due to the mass gap in flat space, hinting at a confinement transition, the precise mechanism of which remains elusive. Rephrasing the transition in the language of CFT could provide a clear target for the conformal bootstrap [17–19] to prove the mass gap.

Motivated by these ideas, we study two-dimensional models in AdS_2 with a dynamically generated gap; the $O(N)$ nonlinear σ model and the $O(N)$ Gross-Neveu (GN) model, both of which have been studied since the discovery of asymptotic freedom as toy models for four-dimensional gauge theories.

As in YM in four dimensions, we find families of BCs that exist only for small values of ΛL and disappear at $\Lambda L \gg 1$.

In contrast to YM, in which the boundary theory at $\Lambda L \ll 1$ is characterized by the presence of a global symmetry, these examples are characterized by the presence of exactly marginal operators and an associated conformal manifold. This is the consequence of a BC that breaks a continuous global symmetry of the bulk theory, i.e., the AdS analog of the spontaneous symmetry breaking (SSB). These BCs must disappear at $\Lambda L \sim 1$ since the symmetry is unbroken in the flat-space vacuum.

The transitions in these examples can be quantitatively analyzed using the large N solvability of the theories. In both cases the transition is due to a boundary operator, singlet under global symmetries of the boundary theory. It is irrelevant at weak coupling but becomes marginal at the transition point, triggering a RG instability.

We start by explaining general properties of these transitions based on symmetry arguments and anomalies. We then move to explain the results of the large N analysis.

Tilt operators, boundary conformal manifolds and transitions—Consider a QFT in AdS_{d+1} with a continuous symmetry G and a boundary condition $|B\rangle\rangle$. We let $\mathfrak{g}$ be the corresponding Lie algebra and $\mathbf{t}^a$ its generators. The symmetry acts on $|B\rangle\rangle$ through the topological defects,

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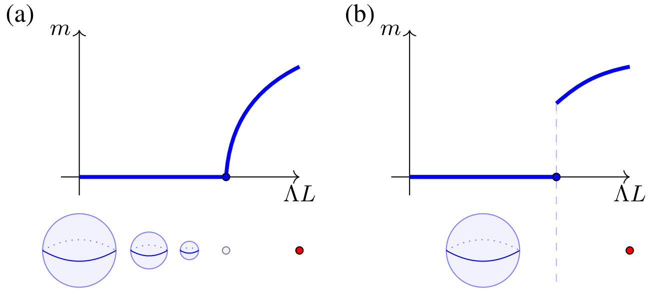

FIG. 1. Two scenarios for the phase transition. (a) Continuous transition. (b) Discontinuous transition.

$$U_\alpha = e^{i\alpha_g \int \star j}, \qquad j = \sum_a j_a \mathfrak{t}^a, \tag{1}$$

mapping $|B\rangle\rangle$ to $|B_g\rangle\rangle \equiv U_g|B\rangle\rangle$. Generically, $|B\rangle\rangle$ is invariant only under a subgroup $G_\perp \subset G$, whose algebra will be denoted by $\mathfrak{g}_\perp$. The symmetry predicts the universal bulk-to-boundary OPE of the bulk conserved current,

$$\star j_a(x,y) \sim \tau_a(x) + \cdots \qquad \mathfrak{t}^a \in \mathfrak{g}/\mathfrak{g}_\perp, \tag{2}$$

where x and y parametrize the boundary and the radial directions of AdS, respectively, and τ_a's are called the *boundary tilt operators*. Generalizing the arguments of [20], we can show, using AdS isometries, that τ_a's are exactly marginal. See the Supplemental Material [21]. Turning them on, one obtains a boundary conformal manifold $\mathcal{M}_{G/G_\perp}$ encoding the SSB of G to $G_\perp$ in the bulk. Unlike usual conformal manifolds encountered in CFTs, all points in $\mathcal{M}_{G/G_\perp}$ are physically equivalent, as they are related by the action of the G symmetry [30,31].

One might expect $\mathcal{M}_{G/G_\perp}$ to exist at any ΛL as the symmetry G does. This turns out to be incorrect. We found that a marginal $G_\perp$-singlet operator appears in the boundary spectrum at $\Lambda L \sim 1$. One can show under rather general assumptions that this leads to the disappearance of the BC and triggers the RG flow [32,33]. (A similar mechanism for the screening of conformal line defects was recently discussed in a series of works [34–36].) This boundary transition generates bulk mass gap, for which we found two scenarios (see Fig. 1): (a) Continuous ("second-order") transition: $\mathcal{M}_{G/G_\perp}$ shrinks to zero size, merging into a G-preserving BC. The BCFT data, as well as the bulk mass gap, change continuously across the transition point. This happens in the O(N) σ model. (b) Discontinuous ("first-order") transition: At *finite* size, $\mathcal{M}_{G/G_\perp}$ becomes unstable and undergoes a transition. In this case, the BCFT data jump discontinuously. It is sometimes possible to argue for a discontinuous transition based on mixed anomaly, which forbids the merger of BCs with different symmetries. This happens in the GN model.

Large N examples—We now study two large N examples with a boundary phase transition and a dynamical bulk mass generation. The examples cover both scenarios described above.

$O(N)$ sigma model: Consider the 2d O(N) σ model, whose effective action at large N is given by

$$\mathcal{S} = \int \frac{1}{2}(\partial\phi)^2 + \sigma\left(\phi^2 - \frac{N}{g^2}\right) + N \operatorname{tr}\log\left[-\Box + 2\sigma\right]. \tag{3}$$

In flat space, the vacuum preserves O(N) symmetry and ϕ^i's acquire mass $M \sim \Lambda = \mu e^{-(2\pi/g^2)}$. By contrast, in AdS$_2$ the model has a weakly coupled SSB phase with $N-1$ massless Goldstone fields [5] and a VEV $\langle\phi^i\rangle = \sqrt{N}\Phi^i$, with $(\Phi)^2 = -(1/2\pi)\log(\Lambda L)$. This does not contradict the Coleman-Mermin-Wagner theorem [37,38] as AdS space comes with a natural IR regulator.

Boundary conditions. Imposing the Dirichlet BC $\phi^i|_{\partial AdS} = \Phi^i$ in the SSB phase, one obtains a boundary theory with a conformal manifold $\mathcal{M}_{O(N)/O(N-1)} \equiv \mathcal{M}_{S^{N-1}}$. Goldstone modes can be seen explicitly by expanding ϕ^i around the VEV $\Phi^i = \Phi n^i$:

$$\phi^i = (\sqrt{N}\Phi + \rho)n^i + \pi^i, \qquad \pi \cdot n = 0. \tag{4}$$

The tilt operators $\tau^i = \partial_y \pi^i$ parametrize the tangent space TS^{N-1} and arise from the pullback of the bulk O(N) current. [Concretely the O(N) current is $j^{ij}_\mu = \phi^{[i}\partial_\mu\phi^{j]}$, using the small y expansion $\phi^i = \Phi^i + y\partial_y\phi^i$ we find that $n_i j^{ij}_y = \Phi\partial_y\pi^j$.] The presence of $\mathcal{M}_{S^{N-1}}$ ensures that the bulk Goldstone modes remain massless.

By contrast the O(N)-singlet BC $\Phi^i = 0$ corresponds to the standard symmetry-preserving phase, with the mass gap Σ being the VEV of the σ field. It exists for $\Lambda L \geq \frac{1}{4}$ and all the way to the flat space limit $\Lambda L \gg 1$. See Fig. 2 for a graphical representation.

These two BCs merge at $\Lambda L = 1$ where the radius of $\mathcal{M}_{S^{N-1}}$ shrinks to zero. The merger is signaled by the appearance of a marginal boundary operator $\hat{\sigma}$ on $\mathcal{M}_{S^{N-1}}$, the lightest scalar in the σ bulk-to-boundary OPE. In addition to these two BCs, the model has Neumann boundary conditions, which exist only at small ΛL. Their physics is described in the Supplemental Material [21].

Details. We now provide some details. We begin by studying the bulk phase structures following [5]. Minimizing the effective potential gives the gap equations

$$\Sigma\Phi^i = 0, \qquad \Phi^2 - \frac{1}{g^2} + \operatorname{tr}\left[\frac{1}{-\Box + 2\Sigma}\right] = 0. \tag{5}$$

A SSB solution requires $\Sigma = 0$. Using the regulated AdS$_2$ propagator [5] we absorb the divergence of the trace in the definition of the regulated coupling $1/g^2_{\rm reg}$. Introducing the dynamically generated scale $\Lambda = \mu e^{-2\pi/g^2_{\rm reg}}$ we find

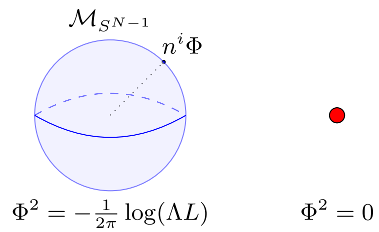

FIG. 2. The boundary conformal manifolds for O(N)-breaking and O(N)-preserving boundary conditions in the O(N) sigma model.

$$\Phi^2 = -\frac{1}{2\pi}\log(\Lambda L), \tag{6}$$

Φ^2 should be interpreted as the radius of $\mathcal{M}_{S^{N-1}}$. The spectrum of the σ bulk-to-boundary OPE is extracted from the poles of the AdS propagator

$$\langle\sigma(\mathbf{x}_1)\sigma(\mathbf{x}_2)\rangle = -\frac{1}{2}\int_{-\infty}^{\infty} d\nu \frac{1}{B(\nu) + \frac{2\Phi^2}{\nu^2+\frac{1}{4}}}\Omega_\nu(\mathbf{x}_1,\mathbf{x}_2), \tag{7}$$

where $B(\nu) = (1/2\pi)[(\psi(i\nu/2+3/4)+\psi(-i\nu/2+3/4)+\log(4)+2\gamma)/(\nu^2+\frac{1}{4})]$ is the 2D bubble function, and γ is Euler's constant. See Ref. [5] or the Supplemental Material [21]. The dimensions of the lightest ($\hat{\sigma}$) and the second lightest ($\hat{\sigma}_2$) operator are shown in Fig. 3. At $\Lambda L = 1$, $\hat{\sigma}$ hits marginality and decouples from the spectrum. The decoupling can be seen in the bulk-to-boundary OPE coefficient $b^2_{\hat{\sigma}\sigma}$, given by the residue of the pole of (7). Near the transition point ($\Phi = 0$) it reads

$$b^2_{\hat{\sigma}\sigma} = \frac{144}{\pi^2}\Phi^2 + O(\Phi^4). \tag{8}$$

After the transition $b_{\hat{\sigma}\sigma}$ becomes imaginary, showing that the SSB BC becomes a complex BCFT [39].

The O(N)-singlet BC ($\Phi^i = 0$) can be analyzed similarly. The Breitenlohner-Freedman (BF) bound in two dimensions is $m^2L^2 \geq -\frac{1}{4}$. Since the mass of ϕ^i is 2Σ, this BC exists only for $2\Sigma \geq -\frac{1}{4}$ that is $\Lambda L \geq \frac{1}{4}$.

At $\Lambda L = \frac{1}{4}$, the solution merges with the Neumann BC discussed in the Supplemental Material [21], which continues to $\Lambda L \ll 1$, see Fig. 3. If we instead increase ΛL, the dimension of $\hat{\sigma}$ intersects with that of $\hat{\sigma}_2$ of the SSB BC at $\Lambda L = 1$, leading to a continuous interpolation of the two BCs. Continuing further, it reproduces the known flat space results at $\Lambda L \gg 1$. As $\hat{\sigma}$ is always irrelevant, the O(N)-singlet BC is stable all the way to the strong coupling regime.

Gross-Neveu model: The second example is the Gross-Neveu model, with the large N effective action

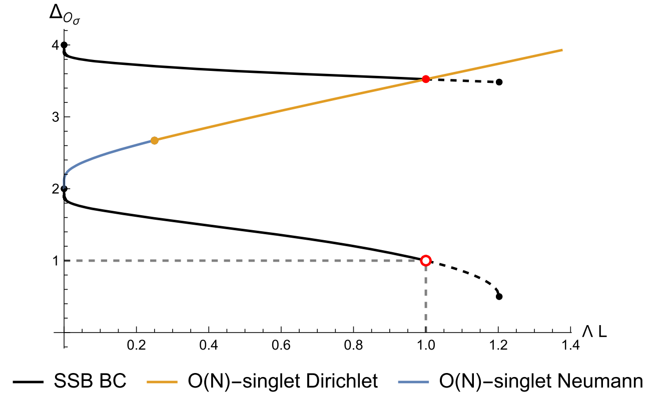

FIG. 3. Dimensions of operators in the boundary spectrum of the bulk σ. Black lines are the lightest ($\hat{\sigma}$) and the second lightest ($\hat{\sigma}_2$) operators in the SSB BC and the orange line is the lightest operator ($\hat{\sigma}$) in the O(N)-singlet one. The blue line is for the Neumann BC.

$$\mathcal{S} = \int \bar{\psi}_i \nabla\!\!\!/ \psi^i + \sigma\bar{\psi}_i\psi^i - \frac{N}{2g}\sigma^2 - N\mathrm{tr}\log[\nabla\!\!\!/ + \sigma], \tag{9}$$

ψ^i ($i = 1, \ldots, N$) being N Dirac fermions. At finite g, in addition to $(-1)^F$, the model has a symmetry:

$$G = O(2N)_V \times \mathbb{Z}_2^{F_L}. \tag{10}$$

$\mathbb{Z}_2^{F_L}$ is left-moving fermion parity acting by $\psi^i \to \gamma_3\psi^i$ and $\sigma \to -\sigma. G$ contains a $\mathbb{Z}_4^A$ axial symmetry, generated by $\mathbb{Z}_2^{F_L}$ together with a discrete O$(2N)_V$ rotation sending $\psi \to i\psi$. The axial symmetry thus acts as

$$\psi^i \to i\gamma_3\psi^i, \qquad \sigma \to -\sigma. \tag{11}$$

In flat space, $\mathbb{Z}_4^A$ is spontaneously broken to $\mathbb{Z}_2^F$ by a σ condensate, which gives a mass to the Dirac fermions [40].

Boundary conditions. This model also has two natural BC's, which preserve either the vector O$(2N)_V$ or the axial $\mathbb{Z}_4^A$ see Fig. 4. The two cannot be preserved at the same time due to the mixed anomaly:

$$I = \pi i \int A_A \cup c_1(F_V). \tag{12}$$

$\mathbb{Z}_4^A$-preserving BCs exist at weak coupling. They read, in Weyl components $[\psi = (\chi_L, \chi_R)^t$ and $\bar{\psi} = (\chi_R^*, -\chi_L^*)]$:

$$|A;\eta\rangle\!\rangle\colon\ (\chi_L^*)_i = e^{i\eta}(\chi_R)^i. \tag{13}$$

These preserve an $[\mathrm{O}(N) \times \mathbb{Z}_4^A]/\mathbb{Z}_2^F$ symmetry. [The O(N) symmetry comes from an $U(N)^*$ symmetry of the free theory, which acts on the Weyl components as $\chi_L \to U\chi_L$ and $\chi_R \to U^*\chi_R$. The intersection of this group with the vector O$(2N)_V$ gives O(N).] The real scalar $\eta \in [0, 2\pi)$ parametrizes an S^1 submanifold of $\mathcal{M}^A_{O(2N)/O(N)}$, whose tilt

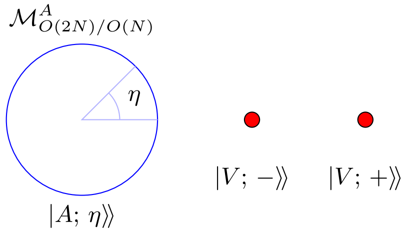

FIG. 4. Maximally symmetric boundary conformal manifolds for the GN model.

operators include the pullback of the $U(1)_V$ current. In this BC, σ is not allowed to condense and the bulk fermions remain massless.

$\mathrm{O}(2N)_V$ vector-preserving BCs are given by

$$|V;\pm\rangle\rangle\colon\ (\chi_L)^i = \pm(\chi_R)^i, \tag{14}$$

These are the BCs associated with SSB of the axial symmetry. For each BC ($\pm$), there are two physically distinct solutions to the gap equation. One of them ("Neumann-like") exists only at small ΛL while the other ("Dirichlet-like") continues to the flat-space limit $\Lambda L \gg 1$. Here we focus on the Dirichlet-like $|V;\pm\rangle\rangle_D$ leaving the other $|V;\pm\rangle\rangle_N$ to the Supplemental Material [21]. The two Dirichlet BC's $|V;\pm\rangle\rangle_D$ are exchanged by the axial symmetry.

The dimensions of the lightest operator $\hat{\sigma}$ in the bulk σ OPE in these BC's are given in Fig. 5. In addition, there is a "shadow" of $\mathbb{Z}_4^A$ preserving BC, denoted by $\widetilde{|A;\eta\rangle\rangle}$. At $\Lambda L \ll 1$, it arises from deforming $|A;\eta\rangle\rangle$ by a double-trace operator $\mathcal{O}_\sigma = \hat{\sigma}^2$. As usual in the double-trace deformation at large N [41–43], the dimensions of $\hat{\sigma}$ in $|A;\eta\rangle\rangle$ and $\widetilde{|A;\eta\rangle\rangle}$ are related by $\Delta_{\hat{\sigma}} \to 1 - \Delta_{\hat{\sigma}}$.

The BCs $|A;\eta\rangle\rangle$ and $\widetilde{|A;\eta\rangle\rangle}$ merge at $\Lambda L = (e^{-\gamma}/4)$. At the merger, the double-trace operator $\mathcal{O}_\sigma$ becomes marginal, triggering a RG flow whose endpoint we conjecture to be the vector-preserving BC, $|V,\pm\rangle\rangle_D$. This transition is discontinuous and the BCFT data jump across the transition, as the axial and vector-preserving BCs cannot merge due to mixed anomaly (12) [44,45].

BKT transition.—Abelian bosonization sheds new light on the transition. The bosonized theory is an N-component compact scalar X^i with a potential for the T-dual coordinate $\tilde{X}$ [46,47]:

$$V \sim g\sum_{i\neq j}\cos\left(2\tilde{X}_i\right)\cos\left(2\tilde{X}_j\right). \tag{15}$$

The chiral symmetry is mapped to the $\mathbb{Z}_4^w$ winding symmetry preserved by the potential. Since the fermion bilinear becomes an operator carrying two units of winding charge, its condensation correspond to proliferation of vortices, i.e., it describes an AdS analog of the Berezinskii-Kosterlitz-Thouless (BKT) transition.

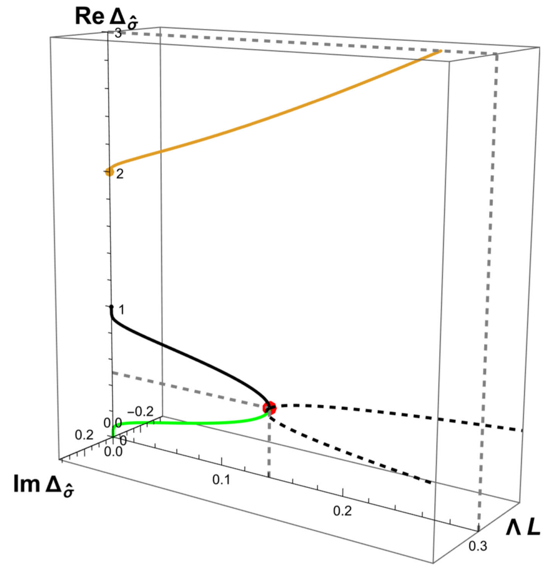

FIG. 5. Dimensions of $\hat{\sigma}$ for $|V;\pm\rangle\rangle_D$ (orange), $|A;\eta\rangle\rangle$ (black), and $\widetilde{|A;\eta\rangle\rangle}$ (green) in the GN model.

Details.—Let us provide some details. The gap equation reads

$$\frac{\Sigma}{g} + \mathrm{tr}\left[\frac{1}{\not{\nabla} + \Sigma}\right] = 0. \tag{16}$$

In the Supplemental Material [21] we show that $\mathrm{tr}[(1/\not{\nabla})] = 0$ with the $|A;\eta\rangle\rangle$ BC, by computing the propagator. As a result the gap equation admits the axial symmetry preserving solution $\Sigma = 0$.

This BC is stable at $\Lambda L \ll 1$ [48] and its boundary spectrum can be read off from the propagator of σ:

$$N\langle\sigma(\mathbf{x}_1)\sigma(\mathbf{x}_2)\rangle = -\int_{-\infty}^{\infty} d\nu \frac{1}{g^{-1} - B_A(\nu)}\Omega_\nu(\mathbf{x}_1,\mathbf{x}_2), \tag{17}$$

where the bubble function $B_A(\nu)$ is computed in the Supplemental Material [21] from the spectral transform of the square of the propagator $\mathrm{tr}[1/\not{\nabla}^2]$. This calculation involves a regularization that requires us to trade g with the scale Λ.

As shown in Fig. 5, the dimension of $\hat{\sigma}$ decreases monotonically from 1 as we increase ΛL. At $\Lambda L = (e^{-\gamma}/4)$, it hits the BF bound $\Delta = \frac{1}{2}$, making $\mathcal{O}_\sigma = \hat{\sigma}^2$ marginal at large N. We conjecture that this triggers the RG flow to $|V;\pm\rangle\rangle_D$. In the proposed scenario, the operator $\hat{\sigma}^2$ which preserves the axial symmetry triggers a flow to a BC which breaks it. This can happen in boundary conformal theories, as is well known in the RG flow from the special to the extraordinary BCFTs of the Ising model [49], triggered by a $\mathbb{Z}_2$ preserving boundary operator.

The bubble function $B_V(\nu)$ for $|V;\pm\rangle\rangle_D$ has been computed in [5], which we review in the Supplemental Material [21]. From this, one can check that $\hat{\sigma}$ in $|V;\pm\rangle\rangle_D$ is always above marginality, ensuring its stability, see Fig. 5.

Conjecture for Yang-Mills theory—Based on our findings, we now conjecture a mechanism for confinement of $\mathrm{SU}(N)$ YM in AdS_4. At $\Lambda L \ll 1$, the Dirichlet BC leads to $\mathrm{SU}(N)$ boundary conserved currents J_μ. We first restate three scenarios, proposed in [16], for the disappearance of this BC at $\Lambda L \sim 1$, from the perspective of boundary conformal theories: (1) *Higgs* (continuous): An operator $\mathcal{O}_{\mathrm{adj}}$ in the adjoint representation of $\mathrm{SU}(N)$ hits marginality, leading to a multiplet recombination $\partial_\mu J^\mu \sim \mathcal{O}_{\mathrm{adj}}$ and giving anomalous dimension to J_μ. The leading candidate for $\mathcal{O}_{\mathrm{adj}}$ is $J_\mu J^\mu|_{\mathrm{adj}}$. (2) *Decoupling* (continuous): The two-point function of J_μ vanishes and J_μ's decouple from the spectrum. (3) *Tachyon* (discontinuous): A scalar operator hits the BF bound, destabilizing the BC. Scenario 1 seems improbable as mentioned in [16], due to the difference of numbers of states between multigluons and glueballs. Scenario 2, sharing a decoupling feature with the $\mathrm{O}(N)$ model, is plausible, but the postdecoupling BC remains to be clarified. Scenario 3 faces the issue that the operator becomes marginal (All scalar operators at $\Lambda L \ll 1$ are irrelevant.) before it hits the BF bound, triggering the RG flow earlier [33].

We propose an alternative: (4) *Marginality*: A scalar operator $\mathcal{O}_{\mathrm{sing}}$ singlet under $\mathrm{SU}(N)$ becomes marginal, triggering the RG flow to the Neumann BC. The leading candidate for $\mathcal{O}_{\mathrm{sing}}$ is $\mathrm{tr}(J_\mu J^\mu)$. The transition in this scenario could be either continuous or discontinuous. If continuous, Dirichlet and Neumann BC's smoothly merge, implying the decoupling of J_μ. (This is because there are no counterparts in the Neumann BC at $\Lambda L \sim 1$; e.g., all gluon states are massive, having higher Δ.) This can be seen as a refined version of scenario 2.

However, we think that a discontinuous transition is more likely. This can be argued, for example, if we consider YM with gauge group $SU(4)/\mathbb{Z}_2$ or SO(6). These have $\mathbb{Z}_2^e \times \mathbb{Z}_2^m$ one-form symmetry with mixed anomaly

$$I = \pi i \int B_e \cup \beta(B_m). \tag{18}$$

Similar to the GN example, Dirichlet and Neumann BCs preserve different symmetries ($\mathbb{Z}_2^m$ and $\mathbb{Z}_2^e$) and their merger is forbidden by the 't Hooft anomaly (18).

Discussion—We initiated the study of strong coupling effects in AdS through the lens of boundary conformal theories. The key signature is the appearance of singlet marginal operators, leading to a transition from gapless to gapped phases. There are countless avenues for future explorations: (1) We relied on large N techniques but believe finite N effects do not change the conclusion. It is also interesting to explore alternatives, e.g., the conformal bootstrap, semiclassics, and resurgence [50]. (2) In the O(3) sigma model, the mass generation can be understood through the proliferation of instantons [51]. It is interesting to perform similar analysis in AdS and understand the interplay between semiclassical saddles and the stability of BCs. (3) Also interesting is to test our conjecture on confinement in YM. In particular, if the transition is discontinuous, one can use the current two-point function $\langle JJ\rangle$ to parametrize the AdS radius, and see if solutions to the conformal bootstrap stop existing at nonzero $\langle JJ\rangle$. In practice, more inputs [than the existence of $\mathrm{SU}(N)$ currents] might be needed and a key question is to figure them out.

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$$R\psi^i(x,y) = \gamma^x\psi^i(-x,y), \quad R\bar{\psi}^i(x,y) = -\bar{\psi}^i(-x,y)\gamma^x. \quad (19)$$

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