A Quantitative Extension of Interval Temporal Logic over Infinite Words

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Abstract

Model checking (MC) for Halpern and Shoham's interval temporal logic HS has been recently investigated in a systematic way, and it is known to be decidable under three distinct semantics (state-based, trace-based and tree-based semantics), all of them assuming *homogeneity* in the propositional valuation. Here, we focus on the *trace-based semantics*, where the main semantic entities are the infinite execution paths (traces) of the given Kripke structure. We introduce a quantitative extension of HS over traces, called *Difference* HS (DHS), allowing one to express timing constraints on the difference among interval lengths (*durations*). We show that MC and satisfiability of full DHS are in general undecidable, so, we investigate the decidability border for these problems by considering natural syntactical fragments of DHS. In particular, we identify a maximal decidable fragment DHS_{simple} of DHS proving in addition that the considered problems for this fragment are at least 2EXPSPACE-hard. Moreover, by exploiting new results on linear-time hybrid logics, we show that for an equally expressive fragment of DHS_{simple}, the problems are EXPSPACE-complete. Finally, we provide a characterization of HS over traces by means of the one-variable fragment of a novel hybrid logic.

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1 Introduction

Interval Temporal Logics (ITLs, see [17, 29, 34]) provide an alternative setting for reasoning about time with respect to the more popular *Point-based* Temporal Logics (PTLs) whose most known representatives are the linear-time temporal logic LTL [30] and the branching-time temporal logics CTL and CTL* [15]. ITLs assume intervals, instead of points, as their primitive temporal entities allowing one to specify temporal properties that involve, e.g., actions with duration, accomplishments, and temporal aggregations, which are inherently "interval-based", and thus cannot be naturally expressed by PTLs. The most prominent example of ITLs is *Halpern and Shoham's modal logic of time intervals* (HS) [17] featuring modalities for any Allen's relation [1]. The satisfiability problem for HS turns out to be highly undecidable for all interesting (classes of) linear orders [17] both for the full logic and most of its fragments [13, 22, 26].

Model checking (MC) of (finite) Kripke structures against HS has been investigated in recent papers [23, 24, 25, 27, 5, 6, 7, 28, 3, 8] which provide more encouraging results. In the model checking setting, each finite path of a Kripke structure is an *interval* having a labelling derived from the labelling of the component states: a proposition letter holds over an interval if and only if it holds over each component state (*homogeneity assumption* [31]). Most of the results have been obtained by adopting the so-called *state-based semantics* [27]:

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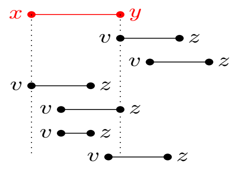
intervals/paths are "forgetful" of the history leading to their starting state, and time branches both in the future and in the past. In this setting, MC of full HS is decidable: the problem is at least Expspace-hard [4], while the only known upper bound is non-elementary [27]. The known complexity bounds for full HS coincide with those for the linear-time fragment BE of HS which features modalities ⟨B⟩ and ⟨E⟩ for prefixes and suffixes. Whether or not model checking for BE can be solved elementarily is a difficult open question. On the other hand, in the state-based setting, the exact complexity of MC for many meaningful (linear-time or branching-time) syntactic fragments of HS, which ranges from **coNP** to $\mathbf{P^{NP}}$, Pspace, and beyond, has been determined in a series of papers [5, 7, 9, 11, 8].
The expressiveness of HS with the state-based semantics has been studied in [6], together with other two decidable variants: the *computation-tree-based semantics* and the *traces-based* one. For the first variant, past is linear: each interval may have several possible future, but only a unique past. Moreover, past is finite and cumulative, and is never forgotten. The trace-based approach instead relies on a linear-time setting, where the infinite paths (traces) of the given Kripke structure are the main semantic entities. It is known that the computation-tree-based variant of HS is expressively equivalent to finitary CTL^* (the variant of CTL^* with quantification over finite paths), while the trace-based variant is equivalent to LTL [6]. The state-based variant is more expressive than the computation-tree-based variant and expressively incomparable with both LTL and CTL^* [6].
In this paper, we introduce a quantitative extension of the interval temporal logic HS under the *trace-based semantics*, called *Difference* HS (DHS). The extension is obtained by means of equality and inequality constraints on the temporal modalities which allow to specify integer bounds on the difference between the durations (lengths) of the current interval and the interval selected by the modality. The logic DHS can also encode in a succinct way constraints on the duration of the current interval. Thus, the considered framework non-trivially generalizes well-known discrete-time quantitative extensions of standard LTL, such as Metric Temporal Logic (MTL) [20], where one can essentially express integer bounds on the duration of the interval having as endpoints the current position and the one selected by the temporal modality.
We prove that MC and satisfiability of full DHS are in general undecidable. Thus, we investigate the decidability border of these problems by considering the syntactical fragments of DHS obtained by restricting the set of allowed constrained modalities. In particular, we prove that for the syntactical fragment, namely DHS_{simple}, whose constrained temporal modalities are associated with the Allen's relations subsuming the subset relation or its inverse, the problems are decidable, though at least 2Expspace-hard. On the other hand, we show that any constrained modality not supported by DHS_{simple} is inherently problematic, since the addition to HS of such a modality leads to undecidability. These results are a little surprising since it is well-known that under the adopted strict semantics admitting singleton intervals, all temporal modalities in HS can be expressed in terms of the ones associated with the Allen's relations subsuming the subset relation or its inverse. Additionally, we identify an expressively complete fragment of DHS_{simple} for which satisfiability and model checking are shown to be Expspace-complete. The upper bound in Expspace is obtained by an elegant automaton-theoretic approach which exploits as a preliminary step a linear-time translation of the fragment of DHS_{simple} into a quantitative extension of the one-variable fragment of *linear-time hybrid logic* HL [16, 32, 2]. Finally, we provide a characterization of HS over traces in terms of a novel hybrid logic, namely SHL_1, which lies between the one-variable and the two-variable fragment of HL. We prove that there are linear-time translations from HS formulas into equivalent formulas of SHL_1, and vice versa.

Table 1 Allen's relations and corresponding HS modalities.

Allen relation	HS	Definition w.r.t. interval structures
MEETS	$\langle A \rangle$	$[x,y]\, \mathcal{R}_A \,[v,z] \iff y = v$
BEFORE	$\langle L \rangle$	$[x,y]\, \mathcal{R}_L \,[v,z] \iff y < v$
STARTED-BY	$\langle B \rangle$	$[x,y]\, \mathcal{R}_B \,[v,z] \iff x = v \wedge z < y$
FINISHED-BY	$\langle E \rangle$	$[x,y]\, \mathcal{R}_E \,[v,z] \iff y = z \wedge x < v$
CONTAINS	$\langle D \rangle$	$[x,y]\, \mathcal{R}_D \,[v,z] \iff x < v \wedge z < y$
OVERLAPS	$\langle O \rangle$	$[x,y]\, \mathcal{R}_O \,[v,z] \iff x < v < y < z$

Example



2 Preliminaries

We fix the following notation. Let $\mathbb{Z}$ be the set of integers, $\mathbb{N}$ the set of natural numbers, and $\mathbb{N}_+ \stackrel{\text{def}}{=} \mathbb{N} \setminus \{0\}$. For a finite or infinite word w over some alphabet, $|w|$ denotes the length of w ($|w| = \infty$ if w is infinite) and for all $0 \leq i < |w|$, $w(i)$ is the $(i+1)$-th letter of w.

We fix a finite set $\mathcal{AP}$ of atomic propositions. A *trace* is an infinite word over $2^{\mathcal{AP}}$.

Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two logics interpreted over traces. For a formula $\varphi \in \mathfrak{F}$, $\mathcal{L}(\varphi)$ denotes the set of traces satisfying φ. Given $\varphi \in \mathfrak{F}$ and $\varphi' \in \mathfrak{F}'$, φ and φ' are *equivalent* if $\mathcal{L}(\varphi) = \mathcal{L}(\varphi')$. The satisfiability problem for $\mathfrak{F}$ is checking for a given formula $\varphi \in \mathfrak{F}$, whether $\mathcal{L}(\varphi) \neq \emptyset$.

Kripke Structures. A *(finite) Kripke structure* over $\mathcal{AP}$ is a tuple $\mathcal{K} = (\mathcal{AP}, S, E, \mathit{Lab}, s_0)$, where S is a finite set of states, $E \subseteq S \times S$ is a left-total transition relation, $\mathit{Lab} : S \to 2^{\mathcal{AP}}$ is a labelling function assigning to each state s the set of propositions that hold over it, and $s_0 \in S$ is the initial state. An infinite path π of $\mathcal{K}$ is an infinite word over S such that $\pi(0) = s_0$ and $(\pi(i), \pi(i+1)) \in E$ for all $i \geq 0$. A finite path of $\mathcal{K}$ is a non-empty infix of some infinite path of $\mathcal{K}$. An infinite path π induces the trace given by $\mathit{Lab}(\pi(0))\mathit{Lab}(\pi(1))\ldots$. We denote by $\mathcal{L}(\mathcal{K})$ the set of traces associated with the infinite paths of $\mathcal{K}$. For a logic $\mathfrak{F}$ interpreted over traces, the *model checking* (MC) *problem against* $\mathfrak{F}$ is checking for a given Kripke structure $\mathcal{K}$ and a formula $\varphi \in \mathfrak{F}$, whether $\mathcal{L}(\mathcal{K}) \subseteq \mathcal{L}(\varphi)$.

2.1 Allen's relations and Interval Temporal Logic HS

An interval algebra to reason about intervals and their relative orders was proposed by Allen [1], while a systematic logical study of interval representation and reasoning was done a few years later by Halpern and Shoham, who introduced the interval temporal logic HS featuring one modality for each Allen relation [17].

Let $\mathbb{U} = (U, <)$ be a linear order over the nonempty set $U \neq \emptyset$, and $\leq$ be the reflexive closure of $<$. Given two elements $x, y \in U$ such that $x \leq y$, we denote by $[x, y]$ the (non-empty closed) *interval* over U given by the set of elements $z \in U$ such that $x \leq z$ and $z \leq y$. We denote the set of all intervals over $\mathbb{U}$ by $\mathbb{I}(\mathbb{U})$. Table 1 gives a graphical representation of the Allen's relations $\mathcal{R}_A$, $\mathcal{R}_L$, $\mathcal{R}_B$, $\mathcal{R}_E$, $\mathcal{R}_D$, and $\mathcal{R}_O$ for the given linear order together with the corresponding HS (existential) modalities. For each $X \in \{A, L, B, E, D, O\}$, the Allen's relation $\mathcal{R}_{\overline{X}}$ is defined as the inverse of relation $\mathcal{R}_X$, i.e. $[x, y]\, \mathcal{R}_{\overline{X}}\, [v, z]$ if $[v, z] \mathcal{R}_X [x, y]$.

HS formulas φ over $\mathcal{AP}$ are defined as follows:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle X \rangle\, \varphi$$

where $p \in \mathcal{AP}$ and $\langle X \rangle$ is the existential temporal modality for the (non-trivial) Allen's relation $\mathcal{R}_X$, where $X \in \{A, L, B, E, D, O, \overline{A}, \overline{L}, \overline{B}, \overline{E}, \overline{D}, \overline{O}\}$. The size $|\varphi|$ of a formula φ is the number of distinct subformulas of φ. We also exploit the standard logical connectives $\vee$

and $\rightarrow$ as abbreviations, and for any temporal modality $\langle X \rangle$, the dual universal modality $[X]$ defined as: $[X]\,\psi \stackrel{\text{def}}{=} \neg \langle X \rangle \neg\psi$. Given any subset of Allen's relations $\{\mathcal{R}_{X_1}, .., \mathcal{R}_{X_n}\}$, we denote by $\mathsf{X_1 \cdots X_n}$ the HS fragment featuring temporal modalities for $\mathcal{R}_{X_1}, .., \mathcal{R}_{X_n}$ only.

The logic HS is interpreted on *interval structures* $\mathcal{S} = (\mathcal{AP}, \mathbb{U}, Lab)$, which are linear orders $\mathbb{U}$ equipped with a labelling function $Lab : \mathbb{I}(\mathbb{U}) \rightarrow 2^{\mathcal{AP}}$ assigning to each interval the set of propositions that hold over it. Given an HS formula φ and an interval $I \in \mathbb{I}(\mathbb{U})$, the satisfaction relation $I \models_{\mathcal{S}} \varphi$, meaning that φ holds at the interval I of $\mathcal{S}$, is inductively defined as follows (we omit the semantics of the Boolean connectives which is standard):

$$I \models_{\mathcal{S}} p \quad \Leftrightarrow p \in Lab(I)$$
$$I \models_{\mathcal{S}} \langle X \rangle\, \varphi \quad \Leftrightarrow \text{there is an interval } J \in \mathbb{I}(\mathbb{U}) \text{ such that } I\, \mathcal{R}_X\, J \text{ and } J \models_{\mathcal{S}} \varphi$$

It is worth noting that we assume the *non-strict semantics of HS*, which admits intervals consisting of a single point. Under such an assumption, all HS-temporal modalities can be expressed in terms of $\langle B \rangle, \langle E \rangle, \langle \overline{B} \rangle$, and $\langle \overline{E} \rangle$ [34]. As an example, $\langle D \rangle\, \varphi$ can be expressed in terms of $\langle B \rangle$ and $\langle E \rangle$ as $\langle B \rangle \langle E \rangle\, \varphi$, while $\langle A \rangle\, \varphi$ can be expressed in terms of $\langle E \rangle$ and $\langle \overline{B} \rangle$ as $([E]\, \neg\top \wedge (\varphi \vee \langle \overline{B} \rangle\, \varphi)) \vee \langle E \rangle([E]\, \neg\top \wedge (\varphi \vee \langle \overline{B} \rangle\, \varphi))$.

Interpretation of HS over traces. In this paper, we focus on interval structures $\mathcal{S} = (\mathcal{AP}, (\mathbb{N}, <), Lab)$ over the standard linear order on $\mathbb{N}$ ($\mathbb{N}$-interval structures for short) satisfying the *homogeneity principle*: a proposition holds over an interval if and only if it holds over all its subintervals. Formally, $\mathcal{S}$ is *homogeneous* if for every interval $[i, j]$ over $\mathbb{N}$ and every $p \in \mathcal{AP}$, it holds that $p \in Lab([i, j])$ if and only if $p \in Lab([h, h])$ for every $h \in [i, j]$. Note that homogeneous $\mathbb{N}$-interval structures over $\mathcal{AP}$ correspond to traces where, intuitively, each interval is mapped to an infix of the trace. Formally, each trace w induces the homogeneous $\mathbb{N}$-interval structure $\mathcal{S}(w)$ whose labeling function Lab_w is defined as follows: for all $i, j \in \mathbb{N}$ with $i \leq j$ and $p \in \mathcal{AP}$, $p \in Lab_w([i, j])$ if and only if $p \in w(h)$ for all $h \in [i, j]$. This mapping from traces to homogeneous $\mathbb{N}$-interval structures over $\mathcal{AP}$ is evidently a bijection. For a trace w, an interval I over $\mathbb{N}$, and an HS formula φ, we write $I \models_w \varphi$ to mean that $I \models_{\mathcal{S}(w)} \varphi$. The trace w satisfies φ, written $w \models \varphi$, if $[0, 0] \models_w \varphi$. For an interval $I = [i, j]$ over $\mathbb{N}$, we denote by $|I|$ the length of I, given by $j - i + 1$.

It is known that HS over traces has the same expressiveness as standard LTL [6], where the latter is expressively complete for standard first-order logic FO over traces [19]. In particular, the fragment AB of HS is sufficient for capturing full LTL [6]: given an LTL formula, one can construct in linear-time an equivalent AB formula [6]. Note that when interpreted on infinite words w, modality $\langle B \rangle$ allows to select proper non-empty prefixes of the current infix subword of w, while modality $\langle A \rangle$ allows to select subwords whose first position coincides with the last position of the current interval. For each $k \geq 1$, we denote by len_k the B formula capturing the intervals of length k: $\mathsf{len}_k \stackrel{\text{def}}{=} (\underbrace{\langle B \rangle \ldots \langle B \rangle}_{k-1 \text{ times}} \top) \wedge (\underbrace{[B] \ldots [B]}_{k \text{ times}} \neg\top)$.

3 Difference Interval Temporal Logic

In this section, we introduce a quantitative extension of the logic HS under the trace-based semantics, we call *Difference HS* (DHS for short). The extension is obtained by means of equality and inequality constraints on the temporal modalities of HS which allow to compare the difference between the length of the interval selected by the temporal modality and the length of the current interval with an integer constant.

The set of DHS formulas φ over $\mathcal{AP}$ is inductively defined as follows:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle X \rangle\, \varphi \mid \langle X \rangle_{\Delta\sim c}\, \varphi$$

where $p \in \mathcal{AP}$, $\sim \in \{<, \leq, =, >, \geq\}$, $c \in \mathbb{Z}$, and $\langle X \rangle_{\Delta\sim c}$ is the existential *constrained* temporal modality for the Allen's relation $\mathcal{R}_X$ where $X \in \{A, L, B, E, D, O, \overline{A}, \overline{L}, \overline{B}, \overline{E}, \overline{D}, \overline{O}\}$. We exploit the symbol Δ in $\langle X \rangle_{\Delta\sim c}$ to emphasize that the constraint $\sim c$ refer to the difference between the lengths of two intervals, the one selected by the modality $\langle X \rangle$ and the current one. For any constrained modality $\langle X \rangle_{\Delta\sim c}$, the dual universal modality $[X]_{\Delta\sim c}$ is an abbreviation for $\neg \langle X \rangle_{\Delta\sim c} \neg\varphi$. We assume that the constants c in the difference constraints are encoded in binary. Thus, the size $|\varphi|$ of a DHS formula φ is defined as the number of distinct subformulas of φ multiplied the number of bits for encoding the maximal constant occurring in φ. The semantics of the constrained modalities is as follows:

- $I \models_w \langle X \rangle_{\Delta\sim c}\, \varphi \Leftrightarrow$ for some interval J such that $I\, \mathcal{R}_X\, J$ and $|J| - |I| \sim c$, $J \models_w \varphi$.

Note that the constrained modalities of the form $\langle X \rangle_{\Delta\prec c}$ where $\prec \in \{<, \leq\}$ are *upward-monotone* in the sense that for all constants c and c' such that $c' \geq c$, $I \models_w \langle X \rangle_{\Delta\prec c}\, \varphi$ entails that $I \models_w \langle X \rangle_{\Delta\prec c'}\, \varphi$. On the other hand, the constrained modalities of the form $\langle X \rangle_{\Delta\succ c}$ where $\succ \in \{>, \geq\}$ are *downward-monotone*, i.e., for all constants c and c' such that $c' \leq c$, $I \models_w \langle X \rangle_{\Delta\succ c}\, \varphi$ entails that $I \models_w \langle X \rangle_{\Delta\succ c'}\, \varphi$. Thus, we say that a formula φ is *monotonic* if it does not use equality constraints $\Delta = c$ as subscripts of the temporal modalities.
We consider the following fragments of DHS and their monotonic versions:

- The fragment DHS_{simple} which disallows constrained modalities for the Allen's relations $\mathcal{R}_A$, $\mathcal{R}_L$, $\mathcal{R}_O$, and their inverses, and for any Allen's relation $\mathcal{R}_X$, the fragment DHS_X allowing constrained modalities for the Allen's relation $\mathcal{R}_X$ only.
- For any subset of Allen's relations $\{\mathcal{R}_{X_1}, .., \mathcal{R}_{X_n}\}$, the fragment $\mathsf{D}(\mathsf{X_1} \ldots \mathsf{X_n})$ featuring temporal modalities for $\mathcal{R}_{X_1}, .., \mathcal{R}_{X_n}$ only, and the common fragment of $\mathsf{D}(\mathsf{X_1} \ldots \mathsf{X_n})$ and DHS_{simple}, denoted by $\mathsf{D}_{simple}(\mathsf{X_1} \ldots \mathsf{X_n})$.

Expressiveness issues. As mentioned in Section 2, all the temporal modalities of HS can be expressed in terms of $\langle B \rangle, \langle E \rangle, \langle \overline{B} \rangle$, and $\langle \overline{E} \rangle$. In the considered quantitative setting, these interdefinability results cannot be generalized to the constrained versions of the temporal modalities. In particular, we will show in Section 5 that the fragment DHS_{simple} of DHS, featuring constrained modalities only for the Allen's relations $\mathcal{R}_B$, $\mathcal{R}_D$, $\mathcal{R}_E$, and their inverses, is not more expressive than HS. On the other hand, we will establish in Section 4 that the fragments DHS_X, where $X \in \{A, L, O, \overline{A}, \overline{L}, \overline{O}\}$, are highly undecidable.

Unlike HS, in DHS_{simple} we can succinctly express that an arbitrary HS property φ holds in the *maximal proper sub-intervals* of the current non-singleton interval by the formula $(\langle E \rangle_{\Delta\geq -1}\, \varphi) \wedge (\langle B \rangle_{\Delta\geq -1}\, \varphi)$. Moreover, we can succinctly encode constraints on the length of the current interval. For an integer $n > 0$, the DHS_{simple} formula $\langle B \rangle_{\Delta\leq -n+1} \top$ (resp., $\neg \langle B \rangle_{\Delta\leq -n} \top$) characterizes the intervals of length at least (resp., at most) n.

▶ **Example 1.** We consider the behaviour of a scheduler serving N processes which continuously request the use of a common resource. The behaviour of each process P_i, with $1 \leq i \leq N$, is represented by the Kripke structure $\mathcal{K}_{P_i}$, depicted in Figure 1, whose atomic propositions p_I^i, p_R^i, and p_U^i label the states where the process is idling, requests the resource, and uses the resource, respectively. The behaviour of the scheduler H is modeled by the Kripke structure $\mathcal{K}_H$ in Figure 1 whose propositions q_I, $q_{U_1}, \ldots q_{U_N}$ label the states where H is idling or assigns the resource to the i-th process (proposition q_{U_i}). The considered Kripke structure $\mathcal{K}_{Sched}$, depicted in Figure 1 for $N = 2$, is the Cartesian product of the Kripke

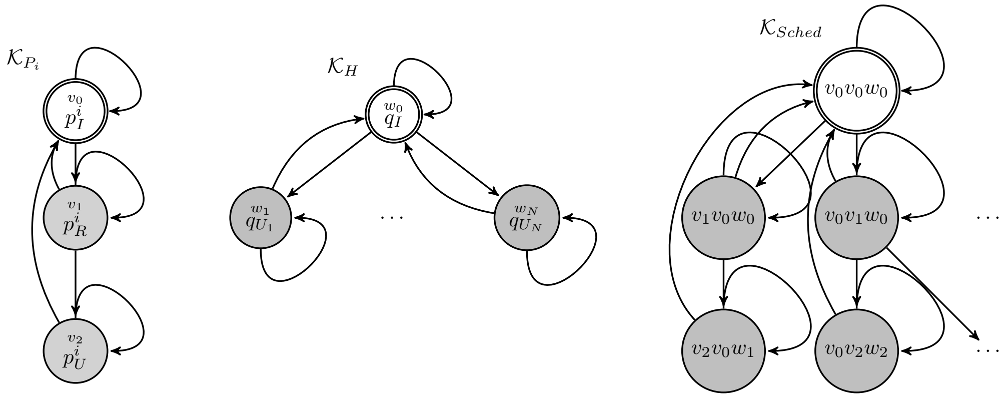


Figure 1 The Kripke structure $\mathcal{K}_{Sched}$ for two processes.

structures $\mathcal{K}_{P_1}, \dots, \mathcal{K}_{P_N}, \mathcal{K}_H$ with the additional requirement that the scheduler is in state w_i *iff* the i-th process is in state v_2. The set of atomic propositions labelling each compound state is the union of the sets of propositions labelling the component states.

As an example of specification, we consider the requirement that the i-th process can unsuccessfully iterate a request (i.e., without finally having the resource granted) for an interval of at least m and at most M time units. This can be expressed in DHS_{simple} as:

$$[\mathrm{A}]\,[\mathrm{A}][(Max_{p_R^i} \wedge \langle\overline{\mathrm{B}}\rangle_{\Delta\le 1}\,\langle\mathrm{E}\rangle\, p_I^i) \rightarrow (\langle\mathrm{B}\rangle_{\Delta\le -m+1}\top \wedge \neg\,\langle\mathrm{B}\rangle_{\Delta\le -M}\top)]$$

where for a proposition p, $Max_p \stackrel{\text{def}}{=} p \wedge (\neg\,\langle\overline{\mathrm{B}}\rangle\, p) \wedge (\neg\,\langle\overline{\mathrm{E}}\rangle\, p)$ captures the maximal length intervals where p homogeneously holds. Note that $\langle\overline{\mathrm{B}}\rangle_{\Delta\le 1}\,\langle\mathrm{E}\rangle\, p_I^i$ ensures that the maximal homogeneous interval where p_R^i holds is followed by a p_I^i-state.

4 Undecidability of DHS

In this section, we establish that model checking and satisfiability for the novel logic DHS are highly undecidable even for the fragments DHS_X, where $X \in \{A, L, O, \overline{A}, \overline{L}, \overline{O}\}$.

▶ **Theorem 2.** *Model checking and satisfiability for the fragment* DHS_X *of DHS, where* $X \in \{A, L, O, \overline{A}, \overline{L}, \overline{O}\}$, *are* Σ_1^1*-hard even if the unique constant used in the constraints is* 0, *and in case* $X \in \{A, O, \overline{A}, \overline{O}\}$ *even if the unique exploited constraint is* ≥ 0 *(or, dually,* ≤ 0*).*

We prove Theorem 2 for the part concerning the satisfiability problem for the fragments DHS_A, DHS_L, and DHS_O (the parts for the model checking problem and for the fragments $\mathsf{DHS}_{\overline{A}}$, $\mathsf{DHS}_{\overline{L}}$, and $\mathsf{DHS}_{\overline{O}}$ being similar). We provide polynomial-time reductions from the *recurrence problem* of *non-deterministic Minsky* 2*-counter machines* [18]. Fix such a machine which is a tuple $M = (Q, \Delta, \delta_{init}, \delta_{rec})$, where Q is a finite set of (control) locations, $\Delta \subseteq Q \times L \times Q$ is a transition relation over the instruction set $L = \{\mathsf{inc}, \mathsf{dec}, \mathsf{if\_zero}\} \times \{1, 2\}$, and $\delta_{init} \in \Delta$ and $\delta_{rec} \in \Delta$ are two designated transitions, the initial and the recurrent one. For each counter $c \in \{1, 2\}$, let $Inc(c)$, $Dec(c)$, and $Zero(c)$ be the sets of transitions $\delta \in \Delta$ whose instruction is (inc, c), (dec, c), and $(\mathsf{if\_zero}, c)$, respectively.

An M-configuration is a pair (δ, ν) consisting of a transition $\delta \in \Delta$ and a counter valuation $\nu : \{1, 2\} \rightarrow \mathbb{N}$. A computation of M is an *infinite* sequence of configurations of the form $((q_0, (op_0, c_0), q_1), \nu_0), ((q_1, (op_1, c_1), q_2), \nu_1), \dots$ such that for each $i \geq 0$: (i) $\nu_{i+1}(3 - c_i) = \nu_i(3 - c_i)$; (ii) $\nu_{i+1}(c_i) = \nu_i(c_i) + 1$ if $op_i = \mathsf{inc}$; (iii) $\nu_{i+1}(c_i) = \nu_i(c_i) - 1$

if $op_i = \mathsf{dec}$; and (iv) $\nu_{i+1}(c_i) = \nu_i(c_i) = 0$ if $op_i = \mathsf{if\_zero}$. A *recurrent computation* is a computation starting at the initial configuration (δ_{init}, ν_0), where $\nu_0(c) = 0$ for each $c \in \{1, 2\}$, which visits the transition δ_{rec} infinitely often. The *recurrence problem* is to decide whether for the given machine M, there is a recurrent computation. This problem is known to be Σ_1^1-complete [18].

For each $X \in \{A, L, O\}$, we construct a DHS_X formula $\varphi_{M,X}$ such that M has a recurrent computation *iff* φ_M is satisfiable. The reduction for the fragment DHS_L, given in the following, is quite different from the ones for the fragments DHS_A and DHS_O, which are given in [12]. Indeed, while the quantitative versions of modalities $\langle \mathrm{A} \rangle$ and $\langle \mathrm{O} \rangle$ allow to impose quantitative constraints on adjacent encodings of M-configurations, this is not possible for the quantitative version of modality $\langle \mathrm{L} \rangle$ whose semantics is not “local”, and for this modality, a different encoding of the computations of M is required.

We exploit some auxiliary DHS formulas. Let ψ be an arbitrary DHS formula. Formulas *left*(ψ) and *right*(ψ) assert that ψ holds at the singular intervals corresponding to the left and right endpoints, respectively, of the current interval.

$$\mathit{left}(\psi) \stackrel{\text{def}}{=} (\mathsf{len}_1 \wedge \psi) \vee \langle \mathrm{B} \rangle (\mathsf{len}_1 \wedge \psi) \qquad \mathit{right}(\psi) \stackrel{\text{def}}{=} \langle \mathrm{A} \rangle (\mathsf{len}_1 \wedge \psi)$$

For the current interval $[i, j]$, *right_next*(ψ) (resp., *left_next*(ψ)) asserts that ψ holds at the singleton interval $[j+1, j+1]$ (resp., $[i+1, i+1]$), while *Int*(ψ) requires that there is an internal position $i < h < j$ such that ψ holds at the singleton interval $[h, h]$.

$$\mathit{right\_next}(\psi) \stackrel{\text{def}}{=} \langle \mathrm{A} \rangle (\mathsf{len}_2 \wedge \langle \mathrm{A} \rangle (\mathsf{len}_1 \wedge \psi)) \qquad \mathit{left\_next}(\psi) \stackrel{\text{def}}{=} \mathit{left}(\mathit{right\_next}(\psi))$$

$$\mathit{Int}(\psi) \stackrel{\text{def}}{=} \langle \mathrm{B} \rangle (\neg \mathsf{len}_1 \wedge \mathit{right}(\psi))$$

Reduction from the recurrence problem for DHS_L. Some ideas in the proposed reduction for the logic DHS_L are taken from [14], where it is shown that model checking one-counter automata against LTL with registers in undecidable.

We first provide a characterization of the recurrent computations of M. Let $\xi = \delta_0, \delta_1, \ldots$ be an infinite sequence of M-transitions. We say that ξ satisfy the *consecution requirement* if (i) $\delta_0 = \delta_{init}$, (ii) for all $i \geq 0$, δ_i is of the form (q_i, op_i, q_{i+1}), and (iii) for infinitely many $j \geq 0$, it holds that $\delta_j = \delta_{rec}$. In order to characterize the sequences ξ for which there exists a corresponding computation of M, we associate a positive natural number (called *value*) to each transition δ_i along ξ. For each counter $c \in \{1, 2\}$, we require that the value associated to a transition δ_i of ξ which increments counter c is obtained by incrementing the natural number associated to the previous incrementation of counter c, if any, along ξ. A similar requirement is imposed on the transitions along ξ decrementing counter c except that the values associated to c-decrementations must not exceed the values associated to previous c-incrementations. Intuitively, this ensures that at each position $i \geq 0$ along ξ, the value of counter c is never negative. In order to simulate the zero-test, we require that for each transition δ_i associated to a zero-test for c, the previous values associated to c-incrementations correspond to previous values associated to c-decrementions.

Formally, a *flat configuration* is a pair (δ, n) consisting of a transition $\delta \in \Delta$ and a positive natural number $n > 0$ such that $n = 1$ if $\delta \in \mathit{Zero}(c)$ for some counter c. We say that n is the value of (δ, n). A *well-formed M-sequence* is an infinite sequence $\rho = (\delta_0, n_0), (\delta_1, n_1), \ldots$ of flat configurations satisfying the following requirements:

- The infinite sequence of transitions $\delta_0, \delta_1, \ldots$ satisfies the consecution requirement.
- *Increment progression* (resp., *Decrement progression*): for each counter $c \in \{1,2\}$, let $\xi = (\delta_{i_0}, n_{i_0}), (\delta_{i_1}, n_{i_1}), \ldots$ be the (possibly empty) ordered sub-sequence of the flat configurations in ρ associated with incrementation (resp., decrementation) of counter c. Then, $n_{i_0} = 1$ and $n_{i_h} = n_{i_{h-1}} + 1$ for all $0 < h < |\xi|$.
- *Increment domination*: for each $c \in \{1,2\}$ and $j \geq 0$ such that $\delta_j \in \mathit{Dec}(c)$, there is $0 \leq h < j$ such that $\delta_h \in \mathit{Inc}(c)$ and $n_h \geq n_j$.
- *Zero-test checking*: let $c \in \{1,2\}$ and $j \geq 0$ such that $\delta_j \in \mathit{Zero}(c)$ and there are $h < j$ such that δ_h is a c-incrementation or c-decrementation. Then, the greatest $h_{\max}$ of such h is associated to a c-decrementation and for each $h < h_{\max}$ such that δ_h is a c-incrementation, it holds that $n_h \leq n_{h_{\max}}$.

▶ **Lemma 3.** *There is a recurrent computation of M* iff *there is a well-formed M-sequence.*

Construction of the DHS_L formula $\varphi_{L,M}$. Let $\mathcal{AP} \overset{\text{def}}{=} \Delta \cup \{1, \#\}$. A flat configuration (δ, n) is encoded by the finite word $\{\delta\} \cdot \{1\}^n \cdot \{\#\}$. A well-formed M-sequence $\rho = (\delta_0, n_0), (\delta_1, n_1), \ldots$ is encoded by the trace obtained by concatenating the codes of the flat configurations visited by ρ starting from the first one.

We construct a DHS_L formula $\varphi_{L,M}$ characterizing the well-formed M-sequences.

$$\varphi_{L,M} \overset{\text{def}}{=} \varphi_{\mathsf{con}} \wedge \varphi_{\mathsf{inc}} \wedge \varphi_{\mathsf{dec}} \wedge \varphi_{\mathsf{if\_zero}} \wedge \varphi_{\mathsf{dom}}$$

The conjunct φ_{con} is a formula in the AB-fragment of DHS_L capturing the traces which are concatenations of codes of flat configurations and satisfy the consecution requirement. The construction of φ_{con} is an easy task and we omit the details here. The conjunct φ_{inc} (resp., φ_{dec}) ensures the increment (resp., decrement) progression requirement. We focus on the formula φ_{inc} (the definition of φ_{dec} being similar) which requires that (i) the value associated to the first c-incrementation, if any, is 1, and (ii) if a c-incrementation $\mathcal{I}$ with value n_1 is followed by a c-incrementation with value n_2, then $n_2 > n_1$ and there is also a c-incrementation following $\mathcal{I}$ with value $n_1 + 1$. The first requirement can be easily expressed by an AB formula. The second requirement is captured by the following DHS_L formula.

$$\bigwedge_{c\in\{1,2\}} \bigwedge_{\delta\in \mathit{Inc}(c)} [\mathrm{A}]\,[\mathrm{A}]\Big((\mathit{left}(\delta) \wedge \mathit{right}(\#) \wedge \neg \mathit{Int}(\#)) \rightarrow \Big[\neg \bigvee_{\delta'\in \mathit{Inc}(c)} \langle \mathrm{L}\rangle_{\Delta\leq 0}(\mathit{left}(\delta') \wedge \mathit{right}(\#))$$
$$\wedge \Big(\bigvee_{\delta'\in \mathit{Inc}(c)} \langle \mathrm{A}\rangle\, \mathit{right}(\delta') \rightarrow \bigvee_{\delta'\in \mathit{Inc}(c)} \langle \mathrm{L}\rangle_{\Delta=0}(\mathit{left}(\delta') \wedge \neg \mathit{Int}(\#) \wedge \mathit{right\_next}(\#))\Big)\Big]\Big)$$

The conjunct $\varphi_{\mathsf{if\_zero}}$ expresses the zero-test checking requirement. It ensures that for each counter c, (i) there is no c-incrementation $\mathcal{I}$ s.t. the first c-operation following $\mathcal{I}$ is a zero-test, and (ii) there is no c-incrementation followed by a c-decrementation $\mathcal{D}$ with a smaller value such that the first c-operation following $\mathcal{D}$ is a zero-test. The first requirement can be easily expressed by an AB formula. The second requirement is captured in DHS_L as follows.

$$\neg \bigvee_{c\in\{1,2\}} \bigvee_{\delta_i\in \mathit{Inc}(c)} \bigvee_{\delta_d\in \mathit{Dec}(c)} \bigvee_{\delta_0\in \mathit{Zero}(c)} \langle \mathrm{A}\rangle\, \langle \mathrm{A}\rangle \Big((\mathit{left}(\delta_i) \wedge \mathit{right}(\#) \wedge \neg \mathit{Int}(\#)) \wedge$$
$$\langle \mathrm{L}\rangle_{\Delta<0}[\mathit{left}(\delta_d) \wedge \mathit{right}(\#) \wedge \langle \mathrm{A}\rangle(\mathit{right}(\delta_0) \wedge \bigwedge_{\delta\in \mathit{Inc}(c)\cup \mathit{Dec}(c)\cup \mathit{Zero}(c)} \neg \mathit{Int}(\delta))]\Big)$$

Finally, the conjunct φ_{dom} characterizes the increment domination requirement. One can easily check that the following conditions capture increment domination.

- If there is some c-decrementation, then there is some c-incrementation.
- c-incrementations have values greater than previous c-decrementations.
- If a c-incrementation $\mathcal{I}$ with value n is not followed by other c-incrementations, then each c-decrementation following $\mathcal{I}$ has a value smaller or equal to n.

We focus on the third requirement which can be expressed in DHS_L as follows (the specification of the first and second requirements are simpler):

$$\bigwedge_{c\in\{1,2\}} \bigwedge_{\delta_i \in \mathit{Inc}(c)} [\mathrm{A}]\,[\mathrm{A}]\Big(\big[\mathit{left}(\delta_i) \wedge \mathit{right}(\#) \wedge \neg \mathit{Int}(\#) \wedge [\mathrm{A}] \bigwedge_{\delta \in \mathit{Inc}(c)} \neg \mathit{right}(\delta)\big] \longrightarrow$$
$$\neg \langle \mathrm{L} \rangle_{\Delta > 0} \bigvee_{\delta_d \in \mathit{Dec}(c)} [\mathit{left}(\delta_d) \wedge \mathit{right}(\#) \wedge \neg \mathit{Int}(\#)]\Big)$$

Note that the unique constant used in the constraints of $\varphi_{L,M}$ is 0. By construction, the DHS_L formula $\varphi_{L,M}$ captures the traces encoding the well-formed M-sequences. Thus, by Lemma 3, $\varphi_{L,M}$ is satisfiable iff M has a recurrent computation.

5 Decidable fragments of DHS

In this section, we show that model checking and satisfiability of DHS_{simple} are decidable though 2EXPSPACE-hard. Moreover, by exploiting new results on the *linear-time hybrid logic* HL [16, 32, 2], we show that for the fragment of DHS_{simple} given by monotonic $\mathsf{D}_{simple}(\mathsf{AB\overline{B}})$, the considered problems are exactly EXPSPACE-complete. Note that DHS_{simple} represents the maximal fragment of DHS which is not covered by the undecidability results of Section 4, while $\mathsf{D}_{simple}(\mathsf{AB\overline{B}})$ corresponds to the extension of $\mathsf{AB\overline{B}}$ with the constrained versions of the modalities for the Allen's relations $\mathcal{R}_B$ and $\mathcal{R}_{\overline{B}}$. Additionally, we provide a characterization of HS in terms of a novel hybrid logic which lies between the one-variable and the two-variable fragment of HL. We establish that there are linear time translations from HS formulas into equivalent formulas of the novel logic, and vice versa. This result is of independent interest since while for the one-variable fragment of HL, model checking and satisfiability are EXPSPACE-complete [32, 2], for the two-variable fragments of HL, these problems are already non-elementarily decidable [32, 2].

Constrained HL. HL [16, 32, 2] extends standard LTL + past by first-order concepts. Here, we consider a constrained version of HL (CHL) where the temporal modalities are equipped with timing constraints. Formally, CHL formulas φ over $\mathcal{AP}$ and a set X of (position) variables are defined by the following syntax:

$$\varphi \stackrel{\text{def}}{=} \top \mid p \mid x \mid \neg\,\varphi \mid \varphi \wedge \varphi \mid \mathsf{F}_{\sim c}\varphi \mid \mathsf{P}_{\sim c}\varphi \mid {\downarrow} x.\varphi$$

where $p \in \mathcal{AP}$, $x \in X$, $\sim \in \{<, \leq, =, >, \geq\}$, $c \in \mathbb{Z}$, $\mathsf{F}_{\sim c}$ is the *constrained strict eventually* modality and $\mathsf{P}_{\sim c}$ is its past counterpart, and ${\downarrow}x$ is the *downarrow binder* operator which assigns the variable name x to the current position. A formula is *monotonic* if it does not use equality constraints $= c$. We also exploit the constrained modalities $\mathsf{G}_{\sim c}$ (*always*) and $\mathsf{H}_{\sim c}$ (*past always*) as abbreviations for $\neg\mathsf{F}_{\sim c}\neg\varphi$ and $\neg\mathsf{P}_{\sim c}\neg\varphi$, respectively. The standard strict eventually (resp., always) modality F (resp., G) corresponds to $\mathsf{F}_{>0}$ (resp., $\mathsf{G}_{>0}$), and its past counterpart P (resp., H) corresponds to $\mathsf{P}_{>0}$ (resp., $\mathsf{H}_{>0}$). The logic HL [16, 32, 2] corresponds to the CHL fragment using only the temporal modalities F and P. We denote by CHL_1 and CHL_2 (resp., HL_1 and HL_2) the one-variable and two-variable fragments of CHL (resp., HL). A CHL sentence is a formula where each variable x is not free (i.e., occurs in the scope of modality $\downarrow x$). The size $|\varphi|$ of a CHL formula φ is the number of distinct subformulas of φ multiplied the number of bits for encoding the maximal constant occurring in φ.

CHL formulas φ are interpreted over traces w. For a position $i \geq 0$ and a *valuation* g assigning to each variable a position, the satisfaction relation $(w, i, g) \models \varphi$ is defined as follows (we omit the semantics of propositions and Boolean connectives which is standard):

$$
\begin{array}{ll}
(w,i,g) \models x & \Leftrightarrow i = g(x) \\
(w,i,g) \models \mathsf{F}_{\sim c}\,\varphi & \Leftrightarrow \text{there is } j > i \text{ such that } j - i \sim c \text{ and } (w,j,g) \models \varphi \\
(w,i,g) \models \mathsf{P}_{\sim c}\,\varphi & \Leftrightarrow \text{there is } j < i \text{ such that } i - j \sim c \text{ and } (w,j,g) \models \varphi \\
(w,i,g) \models \downarrow x.\varphi & \Leftrightarrow (w,i,g[x \mapsto i]) \models \varphi
\end{array}
$$

where $g[x \mapsto i](x) = i$ and $g[x \mapsto i](y) = g(y)$ for $y \neq x$. We write $(w, i) \models \varphi$ to mean that $(w, i, g_0) \models \varphi$, where g_0 maps each variable to position 0, and $w \models \varphi$ to mean that $(w, 0) \models \varphi$.

From $\mathsf{D}_{simple}(\mathsf{AB}\overline{\mathsf{B}})$ to CHL_1. We show that (monotonic) $\mathsf{D}_{simple}(\mathsf{AB}\overline{\mathsf{B}})$ formulas can be translated in linear time into equivalent (monotonic) CHL_1 sentences. For a constraint $\sim c$, we write $(\sim c)^{-1}$ for $\sim' -c$, where $\sim'$ is the inverse of $\sim$. For example, $<$ is the inverse of $>$, while $\leq$ is the inverse of $\geq$.

▶ **Proposition 4.** *Given a (monotonic) $\mathsf{D}_{simple}(\mathsf{AB}\overline{\mathsf{B}})$ formula φ, one can construct in linear-time an equivalent (monotonic) CHL_1 sentence.*

Proof. Fix a variable x. In the translation, x and the current position refer to the left endpoint and right endpoint of the current interval in $\mathbb{N}$, respectively. We can assume that the modalities for the Allen's relations $\mathcal{R}_B$ and $\mathcal{R}_{\overline{B}}$ occur only in a constrained form (for example, $\langle \mathrm{B} \rangle$ corresponds to $\langle \mathrm{B} \rangle_{<0}$). Formally, the translation $f : \mathsf{D}_{simple}(\mathsf{AB}\overline{\mathsf{B}}) \mapsto \mathsf{CHL}_1$ is homomorphic w.r.t. the Boolean connectives (i.e., preserves the Boolean connectives) and is inductively defined as follows:

$$
\begin{array}{rclrcl}
f(p) & \stackrel{\text{def}}{=} & p \wedge \neg \mathsf{P}(\neg p \wedge (x \vee \mathsf{P}x)) & f(\langle \mathrm{A} \rangle\,\varphi) & \stackrel{\text{def}}{=} & \downarrow x.\,(f(\varphi) \vee \mathsf{F} f(\varphi)) \\
f(\langle \mathrm{B} \rangle_{\sim c}\,\varphi) & \stackrel{\text{def}}{=} & \mathsf{P}_{(\sim c)^{-1}}(f(\varphi) \wedge (x \vee \mathsf{P}x)) & f(\langle \overline{\mathrm{B}} \rangle_{\sim c}\,\varphi) & \stackrel{\text{def}}{=} & \mathsf{F}_{\sim c} f(\varphi)
\end{array}
$$

By a straightforward induction on φ, we obtain that given a trace w, an interval $[i, j]$, a valuation g such that $g(x) = i$, it holds that $[i, j] \models_w \varphi$ if and only if $(w, j, g) \models f(\varphi)$. The desired CHL_1 sentence φ' equivalent to φ is then defined as follows: $\varphi' \stackrel{\text{def}}{=} \downarrow x.\, f(\varphi)$. ◀

In Section 5.1, we show that model checking and satisfiability of monotonic CHL_1 are EXPSPACE-complete. By [10], for the logic AB over traces, the considered problems are already EXPSPACE-hard. Thus, by Proposition 4 we obtain the following result.

▶ **Theorem 5.** *MC and satisfiability of monotonic $\mathsf{D}_{simple}(\mathsf{AB}\overline{\mathsf{B}})$ are* EXPSPACE*-complete.*

Decidability of DHS_{simple}. We first introduce a variant of CHL, we call *swap CHL* (SCHL). SCHL formulas φ are defined as follows: $\varphi \stackrel{\text{def}}{=} \top \mid p \mid x \mid \neg\,\varphi \mid \varphi \wedge \varphi \mid \mathsf{F}_{\sim c}\,\varphi \mid \mathsf{P}_{\sim c}\,\varphi \mid \mathsf{swap}_x.\varphi$. The novel modality swap_x simultaneously assigns to x the value of the current position and updates the current position to the value previously referenced by x. Formally, its semantics is defined as follows: $(w, i, g) \models \mathsf{swap}_x.\varphi \Leftrightarrow (w, g(x), g[x \mapsto i]) \models \varphi$.

We are interested in the one-variable fragment SCHL_1 of SCHL, and in the unconstrained version SHL_1 of SCHL_1 where the unique temporal modalities are F and P. From a succinctness point of view, the fragment SCHL_1 lies between CHL_1 and CHL_2.

▶ **Proposition 6.** *Given a* CHL_1 *(resp.,* HL_1*) sentence, one can construct in linear time an equivalent* SCHL_1 *(resp.,* SHL_1*) sentence. Moreover, given a* SCHL_1 *(resp.,* SHL_1*) sentence, one can construct in linear time an equivalent* CHL_2 *(resp.,* HL_2*) sentence.*

Proof. The translation function $f : \mathsf{CHL}_1 \mapsto \mathsf{SCHL}_1$ from CHL_1 formulas to SCHL_1 formulas is homomorphic w.r.t. proposition, variables, Boolean connectives and temporal modalities. Moreover, for a CHL_1 formula φ using variable x, $f(\downarrow x.\,\varphi)$ is defined as $\mathsf{swap}_x.\,\mathsf{FP}(x \wedge f(\varphi))$.

For the second part of Proposition 6, let φ be a SCHL_1 formula using variable x, and let x_1 and x_2 be two distinct variables. For each $h = 1, 2$, we define a CHL_2 formula $F(\varphi, x_h)$ using only variables x_1 and x_2 and such that only x_h can occur free in $F(\varphi, x_h)$. The mapping F is homomorphic w.r.t. propositions, Boolean connectives and temporal modalities, and is defined as follows for variable x and the swap modality: $F(x, x_h) \stackrel{\text{def}}{=} x_h$ and $F(\mathsf{swap}_x.\,\varphi, x_h) \stackrel{\text{def}}{=} \downarrow x_{3-h}.\,\mathsf{FP}(x_h \wedge F(\varphi, x_{3-h}))$. By a straightforward induction on the structure of the SCHL_1 formula φ, given a trace w, $h = 1, 2$, two positions $i, j \geq 0$, a valuation g s.t. $g(x) = j$, and a valuation g' s.t. $g'(x_h) = j$, it holds that $(w, i, g) \models \varphi$ iff $(w, i, g') \models F(\varphi, x_h)$. Hence, if φ is a SCHL_1 sentence, then $\downarrow x_h.\,F(\varphi, x_h)$ is a CHL_2 sentence equivalent to φ. ◀

We show that DHS_{simple} formulas can be converted in exponential time into equivalent SCHL_1 sentences. Moreover, the logic HS over traces exactly corresponds to SHL_1, i.e., there are linear-time translations from HS formulas into equivalent SHL_1 sentences, and vice versa.

▶ **Proposition 7.**

1. *Given a* DHS_{simple} *formula* φ*, one can construct in singly exponential time an equivalent* SCHL_1 *sentence* ψ*. Moreover, if* φ *is a* $\mathsf{D}(\mathsf{BE}\overline{\mathsf{BE}})$ *formula, then* ψ *can be constructed in linear time, and* $\psi \in \mathsf{SHL}_1$ *if* $\varphi \in \mathsf{HS}$*.*
2. *Given a* SHL_1 *sentence* φ*, one can construct in linear time an equivalent* HS *formula* ψ*.*

Proof. We focus on the proof of statement 1 in Proposition 7. A proof of statement 2 can be found in [12]. First note that DHS_{simple} corresponds to $\mathsf{D}(\mathsf{BED}\overline{\mathsf{BDE}})$ since the unconstrained modalities for the Allen's relations $\mathcal{R}_A$, $\mathcal{R}_L$, and $\mathcal{R}_O$ and their inverses can be expressed in linear time into $\mathsf{BE}\overline{\mathsf{BE}}$. Moreover, the constrained versions of the modalities $\langle \mathrm{D} \rangle$ and $\langle \overline{\mathrm{D}} \rangle$ can be easily expressed in $\mathsf{D}(\mathsf{BE}\overline{\mathsf{BE}})$ though with a singly exponential blow-up. For example, for $n > 0$, $\langle \overline{\mathrm{D}} \rangle_{\geq n}\, \varphi$ is equivalent to $\displaystyle\bigvee_{n_1 \geq 0, n_2 \geq 0 : n_1 + n_2 = n} \langle \overline{\mathrm{B}} \rangle_{\geq n_1} \langle \overline{\mathrm{E}} \rangle_{\geq n_2}\, \varphi$.

Thus, it suffices to show that a $\mathsf{D}(\mathsf{BE}\overline{\mathsf{BE}})$ formula can be converted in linear time into an equivalent SCHL_1 sentence. Let x be a position variable. We use the expression $x < cur$ to indicate that the position referenced by variable x is smaller than the current position. The meaning of the expression $x > cur$ (resp., $x = cur$) is similar. Given a $\mathsf{D}(\mathsf{BE}\overline{\mathsf{BE}})$ formula φ and $\tau \in \{x < cur, x > cur, x = cur\}$, we inductively define an SCHL_1 formula $f(\varphi, \tau)$ using variable x. Intuitively, the position referenced by x and the current position represent the endpoints of the interval on which φ is currently evaluated. We can assume that φ contains only constrained temporal modalities. Indeed, the modalities in $\mathsf{BE}\overline{\mathsf{BE}}$ can be trivially converted into equivalent constrained versions. The mapping f is homomorphic w.r.t. the Boolean connectives and is inductively defined as follows:

- $f(p, x < cur) \stackrel{\text{def}}{=} p \wedge \neg\mathsf{P}(\neg p \wedge (x \vee \mathsf{P}x))$.
- $f(\langle \mathrm{B} \rangle_{\Delta\sim c}\, \varphi, x < cur) \stackrel{\text{def}}{=} \mathsf{P}_{(\sim c)^{-1}}[(f(\varphi, x = cur) \wedge x) \vee (f(\varphi, x < cur) \wedge \mathsf{P}x)]$.
- $f(\langle \overline{\mathrm{B}} \rangle_{\Delta\sim c}\, \varphi, x < cur) \stackrel{\text{def}}{=} \mathsf{F}_{\sim c}\, f(\varphi, x < cur)$.
- $f(\langle \mathrm{E} \rangle_{\Delta\sim c}\, \varphi, x < cur) \stackrel{\text{def}}{=} \mathsf{swap}_x.\,\mathsf{F}_{(\sim c)^{-1}}[(f(\varphi, x = cur) \wedge x) \vee (f(\varphi, x > cur) \wedge \mathsf{F}x)]$.

- $f(\langle \overline{\mathrm{E}} \rangle_{\Delta\sim c}\, \varphi, x < cur) \stackrel{\text{def}}{=} \mathsf{swap}_x.\, \mathsf{P}_{\sim c}\, f(\varphi, x > cur)$.
- $f(p, x > cur) \stackrel{\text{def}}{=} p \wedge \neg \mathsf{F}(\neg p \wedge (x \vee \mathsf{F}x))$.
- $f(\langle \mathrm{B} \rangle_{\Delta\sim c}\, \varphi, x > cur) \stackrel{\text{def}}{=} \mathsf{swap}_x.\, \mathsf{P}_{(\sim c)^{-1}}[(f(\varphi, x = cur) \wedge x) \vee (f(\varphi, x < cur) \wedge \mathsf{P}x)]$.
- $f(\langle \overline{\mathrm{B}} \rangle_{\Delta\sim c}\, \varphi, x > cur) \stackrel{\text{def}}{=} \mathsf{swap}_x.\, \mathsf{F}_{\sim c}\, f(\varphi, x < cur)$.
- $f(\langle \mathrm{E} \rangle_{\Delta\sim c}\, \varphi, x > cur) \stackrel{\text{def}}{=} \mathsf{F}_{(\sim c)^{-1}}[(f(\varphi, x = cur) \wedge x) \vee (f(\varphi, x > cur) \wedge \mathsf{F}x)]$.
- $f(\langle \overline{\mathrm{E}} \rangle_{\Delta\sim c}\, \varphi, x > cur) \stackrel{\text{def}}{=} \mathsf{P}_{\sim c}\, f(\varphi, x > cur)$.
- $f(p, x = cur) \stackrel{\text{def}}{=} p$.
- $f(\langle \mathrm{X} \rangle_{\Delta\sim c}\, \varphi, x = cur) \stackrel{\text{def}}{=} \neg\top$ for each $X \in \{B, E\}$.
- $f(\langle \overline{\mathrm{B}} \rangle_{\Delta\sim c}\, \varphi, x = cur) \stackrel{\text{def}}{=} \mathsf{F}_{\sim c}\, f(\varphi, x < cur)$.
- $f(\langle \overline{\mathrm{E}} \rangle_{\Delta\sim c}\, \varphi, x = cur) \stackrel{\text{def}}{=} \mathsf{P}_{\sim c}\, f(\varphi, x > cur)$.

By a straightforward induction on the structure of φ, we obtain that given a trace w, a position $j \geq 0$, and a valuation g such that $g(x) = j$, the following holds:

- $[j, j] \models_w \varphi$ iff $(w, j, g) \models f(\varphi, x = cur)$.
- For each $i > j$, $[j, i] \models_w \varphi$ iff $(w, i, g) \models f(\varphi, x < cur)$.
- For each $i < j$, $[i, j] \models_w \varphi$ iff $(w, i, g) \models f(\varphi, x > cur)$.

It follows that the SCHL_1 sentence $\mathsf{swap}_x.\, f(\varphi, x = cur)$ is equivalent to φ. Note that the previous sentence is in SHL_1 if $\varphi \in \mathsf{HS}$. ◀

By [32, 2], model checking and satisfiability of CHL_2 are decidable though with a non-elementary complexity. We can show that for the logic DHS_{simple}, the considered problems are at least 2EXPSPACE-hard even for the fragment given by monotonic $\mathsf{D}_{simple}(\mathsf{ABE})$ (for a proof, see [12]). Moreover, note that CHL formulas can be trivially translated into equivalent formulas of first-order logic FO over traces. Thus, by the first-order expressiveness completeness of the fragment AB of HS [6] (under the considered trace-based semantics), and Propositions 6–7, we obtain the following result.

▶ **Theorem 8.** *Model checking and satisfiability of DHS_{simple} are decidable and* 2EXPSPACE-*hard even for the fragment given by monotonic $\mathsf{D}_{simple}(\mathsf{ABE})$. Moreover, DHS_{simple}, monotonic $\mathsf{D}_{simple}(\mathsf{AB\overline{B}})$, and HS have the same expressiveness.*

5.1 EXPSPACE-completeness of monotonic CHL_1

In this section, we describe an asymptotically optimal automata-theoretic approach to solve satisfiability and model checking of monotonic CHL_1 (MCHL_1 for short), which is based on a direct translation of MCHL_1 sentences into *two-way alternating finite-state word automata* (2AWA) equipped with standard generalized Büchi acceptance conditions.

A MCHL_1 formula is in *monotonic normal form* (*MNF*) if negation is applied only to atomic propositions and variables, and the constrained temporal modalities are of the form $\mathsf{O}_{\leq c}$ with $c \geq 1$ and $\mathsf{O} \in \{\mathsf{F}, \mathsf{G}, \mathsf{P}, \mathsf{H}\}$. A MCHL_1 formula φ can be easily converted in linear-time into an equivalent MCHL_1 formula in *MNF* φ_M, called the *MNF* of φ (for details, see [12]). The *dual* $\widetilde{\varphi_M}$ *of* φ_M is the *MNF* of $\neg\varphi_M$.

Characterization of the satisfaction relation. We fix a monotonic CHL_1 formula φ with variable x, where x may occur free. W.l.o.g. we assume that φ is in *MNF*, and $\mathcal{AP}$ is the set of atomic propositions occurring in φ. First, we give an operational characterization of the satisfaction relation $w \models \varphi$ which non-trivially generalizes the classical notion of Hintikka-sequence of LTL. Essentially, for each trace w and valuation g of variable x, we associate to w and g infinite sequences $\rho = A_0, A_1, \ldots$ of sets, where for each $i \geq 0$, A_i is an *atom* and intuitively describes a maximal set of subformulas of φ which can hold at position i along w w.r.t. the valuation g. As for LTL, the notion of atom syntactically captures the semantics of Boolean connectives. The fixpoint characterization of the unconstrained temporal modalities and the semantics of the constrained temporal modalities are locally captured by requiring that consecutive pairs A_i, A_{i+1} along the sequence ρ satisfy certain syntactical constraints. Finally, the sequence ρ has to satisfy additional non-local conditions reflecting the liveness requirements ψ in the eventually subformulas $\mathsf{F}\psi$ of φ, and the semantics of the binder modality $\downarrow x$. Now, we give the technical details.

A formula ψ is a *first-level subformula* of φ if there is an occurrence of ψ in φ which is not in the scope of modality $\downarrow x$. The *closure* $cl(\varphi)$ of φ is the smallest set containing (i) x, $\top$, the propositions in $\mathcal{AP}$, formula $\mathsf{P}_{\leq 1}\top$, and (ii) all the first-level subformulas ψ of φ together with $\mathsf{F}_{\leq 1}\psi$ and $\mathsf{P}_{\leq 1}\psi$, and (iii) the duals of the formulas in the points (i) and (ii). Note that $\varphi \in cl(\varphi)$ and $|cl(\varphi)| = O(|\varphi|)$. Moreover, the set $obl(\varphi)$ of *φ-obligations* is the set of pairs of the form $(\mathsf{O}_{\leq c}\psi, d)$ such that $\mathsf{O} \in \{\mathsf{F}, \mathsf{P}, \mathsf{G}, \mathsf{H}\}$, $\mathsf{O}_{\leq c}\psi \in cl(\varphi)$, $c > 1$, and $1 \leq d \leq c - 1$. Intuitively, the obligations are exploited for capturing in a succinct way the semantics of the constrained temporal modalities. In particular, an obligation of the form $(\mathsf{F}_{\leq c}\psi, d)$ asserted at a position i means that there is $j > i$ such that ψ holds at position j, and $i + d$ *is the smallest of such* j. Note that two distinct obligations associated to the same formula $\mathsf{F}_{\leq c}\psi$ cannot hold simultaneously at the same position. Dually, an obligation of the form $(\mathsf{G}_{\leq c}\psi, d)$ asserted at a position i means that there is $j > i$ such that ψ holds at all positions in $[i+1, j]$, and $i + d$ *is the greatest of such* j. The meaning of the obligations associated to the past constrained modalities is similar. Evidently, $|obl(\varphi)| = 2^{O(|\varphi|)}$. A *$\varphi$-atom* A is a subset of $cl(\varphi) \cup obl(\varphi)$ such that $\top \in A$ and the following holds, where $c > 1$ and $\mathsf{O} \in \{\mathsf{F}, \mathsf{P}, \mathsf{G}, \mathsf{H}\}$:

- for each $\psi \in cl(\varphi)$, $\psi \in A$ iff $\widetilde{\psi} \notin A$;
- for each $\psi_1 \wedge \psi_2 \in cl(\varphi)$, $\psi_1 \wedge \psi_2 \in A$ iff $\{\psi_1, \psi_2\} \subseteq A$;
- for each $\psi_1 \vee \psi_2 \in cl(\varphi)$, $\psi_1 \vee \psi_2 \in A$ iff $\{\psi_1, \psi_2\} \cap A \neq \emptyset$;
- for each $\mathsf{O}_{\leq c}\psi \in cl(\varphi)$, *there is at most one obligation* of the form $(\mathsf{O}_{\leq c}\psi, d)$ in A.

It is worth noting that the set $\mathsf{Atoms}(\varphi)$ of φ-atoms has a cardinality which is at most singly exponential in $|\varphi|$, i.e. $|\mathsf{Atoms}(\varphi)| = 2^{O(|\varphi|)}$. A φ-atom A is *initial* if A does not contain formulas of the form $\mathsf{P}\psi$ or $\mathsf{P}_{\leq c}\psi$ and obligations of the form $(\mathsf{O}_{\leq c}\psi, d)$ with $\mathsf{O} \in \{\mathsf{P}, \mathsf{H}\}$.

We now define the function Succ_φ which maps each atom $A \in \mathsf{Atoms}(\varphi)$ to a subset of $\mathsf{Atoms}(\varphi)$. Intuitively, if A is the atom associated with a position i of the given trace w, then $\mathsf{Succ}_\varphi(A)$ contains the set of atoms associable to the next position $i + 1$ (w.r.t. a given valuation of variable x). Formally, $A' \in \mathsf{Succ}_\varphi(A)$ iff A' is not initial and the following holds:

- *F-requirements*: for all $\mathsf{F}\psi \in cl(\varphi)$, $\mathsf{F}\psi \in A \Leftrightarrow \{\mathsf{F}\psi, \psi\} \cap A' \neq \emptyset$.
- *P-requirements*: for all $\mathsf{P}\psi \in cl(\varphi)$, $\mathsf{P}\psi \in A' \Leftrightarrow \{\mathsf{P}\psi, \psi\} \cap A \neq \emptyset$.
- *G-Requirements*: for all $\mathsf{G}\psi \in cl(\varphi)$, $\mathsf{G}\psi \in A \Leftrightarrow \{\mathsf{G}\psi, \psi\} \subseteq A'$.
- *H-requirements*: for all $\mathsf{H}\psi \in cl(\varphi)$, $\mathsf{H}\psi \in A' \Leftrightarrow \{\mathsf{H}\psi, \psi\} \subseteq A$.
- *$\mathsf{F}_{\leq c}$-requirements*: for all $\mathsf{F}_{\leq c}\psi \in cl(\varphi)$,
  - $\mathsf{F}_{\leq c}\psi \in A \Leftrightarrow$ *either* $\psi \in A'$ *or* $c > 1$ and $(\mathsf{F}_{\leq c}\psi, d) \in A'$ for some $1 \leq d < c$;
  - for each $1 \leq d < c$, $(\mathsf{F}_{\leq c}\psi, d) \in A \Leftrightarrow$ *either* $d = 1$ and $\psi \in A'$, *or* $d > 1$, $\psi \notin A'$, and $(\mathsf{F}_{\leq c}\psi, d - 1) \in A'$.

- $P_{\leq c}$*-requirements*: for all $\mathsf{P}_{\leq c}\psi \in cl(\varphi)$,
  - $\mathsf{P}_{\leq c}\psi \in A' \Leftrightarrow$ *either* $\psi \in A$ *or* $c > 1$ and $(\mathsf{P}_{\leq c}\psi, d) \in A$ for some $1 \leq d < c$;
  - for each $1 \leq d < c$, $(\mathsf{P}_{\leq c}\psi, d) \in A' \Leftrightarrow$ *either* $d = 1$ and $\psi \in A$, *or* $d > 1$, $\psi \notin A$, and $(\mathsf{P}_{\leq c}\psi, d-1) \in A$.
- $G_{\leq c}$*-requirements*: for all $\mathsf{G}_{\leq c}\psi \in cl(\varphi)$,
  - $\mathsf{G}_{\leq c}\psi \in A \Leftrightarrow \psi \in A'$ and, in case $c > 1$, either $\mathsf{G}_{\leq c}\psi \in A'$ or $(\mathsf{G}_{\leq c}\psi, c-1) \in A'$;
  - for each $1 \leq d < c$, $(\mathsf{G}_{\leq c}\psi, d) \in A \Leftrightarrow$ *either* $d = 1$, $\psi \in A'$, and $\mathsf{F}_{\leq 1}\psi \notin A'$, *or* $d > 1$, $\psi \in A'$, and $(\mathsf{G}_{\leq c}\psi, d-1) \in A'$.
- $H_{\leq c}$*-requirements*: for all $\mathsf{H}_{\leq c}\psi \in cl(\varphi)$,
  - $\mathsf{H}_{\leq c}\psi \in A' \Leftrightarrow \psi \in A$ and, in case $c > 1$, either $\mathsf{H}_{\leq c}\psi \in A$ or $(\mathsf{H}_{\leq c}\psi, c-1) \in A$;
  - for each $1 \leq d < c$, $(\mathsf{H}_{\leq c}\psi, d) \in A' \Leftrightarrow$ *either* $d = 1$, $\psi \in A$, and $\mathsf{P}_{\leq 1}\psi \notin A$, *or* $d > 1$, $\psi \in A$, and $(\mathsf{H}_{\leq c}\psi, d-1) \in A$.

Note that Succ_φ captures the semantics of the constrained modalities in accordance to the intended meaning of the associated obligations. Let w be a trace and $\ell \geq 0$. A *φ-sequence over the pointed trace* (w, ℓ) is an infinite sequence $\rho = A_0, A_1, \ldots$ of φ-atoms such that:

- A_0 is initial, $x \in A_\ell$, and $x \notin A_i$ for each $i \neq \ell$;
- for each $i \geq 0$, $A_i \cap \mathcal{AP} = w(i)$ (*propositional consistency*), and $A_{i+1} \in \mathsf{Succ}_\varphi(A_i)$;
- *Fairness*: for each $\mathsf{F}\psi \in cl(\varphi)$ and for infinitely many $i \geq 0$, either $\psi \in A_i$ or $\mathsf{F}\psi \notin A_i$.

The standard fairness requirement ensures that the requirements ψ in the first-level subformulas $\mathsf{F}\psi$ of φ are eventually satisfied. In order to capture the semantics of the modality $\downarrow x$, we now give the notion of *fulfilling φ-sequence* by induction on the nesting depth of $\downarrow x$. Formally, a φ-sequence $\rho = A_0, A_1, \ldots$ over (w, ℓ) is *fulfilling* if for all $i \geq 0$ and $\downarrow x.\, \psi \in A_i$, there is a fulfilling ψ-sequence $\rho' = A'_0, A'_1, \ldots$ over the pointed trace (w, i) such that $\psi \in A'_i$.

The notion of fulfilling φ-sequence over a pointed trace (w, ℓ) provides a characterization of the satisfaction relation $(w, i, g) \models \varphi$ with $g(x) = \ell$ (a proof is in [12]).

▶ **Theorem 9.** *Let ϕ be a MCHL$_1$ sentence in MNF. Then, $w \in \mathcal{L}(\phi)$ if and only if there exists a fulfilling ϕ-sequence $\rho = A_0, A_1, \ldots$ over $(w, 0)$ such that $\phi \in A_0$.*

Automata-theoretic approach for MCHL$_1$. By Theorem 9, given a MCHL$_1$ sentence φ in *MNF*, it is not a difficult task to construct in singly exponential time a generalized Büchi 2AWA $\mathcal{A}_\varphi$ accepting $\mathcal{L}(\varphi)$. Given an input trace w, $\mathcal{A}_\varphi$ guesses a φ-sequence $\rho = A_0, A_1, \ldots$ over $(w, 0)$ by simulating it in forward mode along the "main" path of the run-tree. At the i^{th}-node of such a path, $\mathcal{A}_\varphi$ keeps track in its state of the φ-atom A_i. Moreover, in order to check that ρ is fulfilling, for each binder formula $\downarrow x.\, \psi \in A_i$, $\mathcal{A}_\varphi$ recursively checks the existence of a fulfilling ψ-sequence $\rho' = A'_0, A'_1, \ldots$ over (w, i) by guessing the ψ-atom A'_i, with $\{x, \psi\} \subseteq A'_i$, and by activating two secondaries copies: the first one moves in backward mode by guessing the finite sequence $A'_{i-1}, \ldots, A'_0$, and the second one moves in forward mode by guessing the infinite sequence $A'_{i+1}, A'_{i+2}, \ldots$. Details about the construction of $\mathcal{A}_\varphi$ can be found in [12]. By [33, 21], generalized Büchi 2AWA can be converted on the fly and in singly exponential time into equivalent Büchi nondeterministic finite-state automata (Büchi NWA). Recall that non-emptiness of Büchi NWA is NLogspace-complete, and the standard model checking algorithm consists in checking emptiness of the Büchi NWA given by the synchronous product of the given Kripke structure with the Büchi NWA associated with the negation of the given formula. Thus, we obtain algorithms for satisfiability and model-checking of monotonic CHL$_1$ which run in non-deterministic single exponential space. Therefore, since Expspace = NExpspace, and for the logic HL$_1$, the considered problems are already Expspace-hard [32], we obtain the following result.

▶ **Corollary 10.** *Model checking and satisfiability of monotonic CHL$_1$ are* Expspace*-complete.*

6 Conclusion

We have investigated decidability and complexity issues for satisfiability and model checking of a quantitative extension of HS, namely DHS, under the trace-based semantics. The novel logic provides constrained versions of the HS temporal modalities which can express bounds on the difference between the durations of the current interval and the interval selected by the modality. A different and natural choice would have been to consider constraints on the sum of the durations. In this setting, one can show that the logic HS extended with sum constraints is decidable under the trace-based semantics by means of an exponential-time translation of formulas into equivalent HL_2 sentences.

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