

# CONTEMPORARY MATHEMATICS

789

## The Diverse World of PDEs Algebraic and Cohomological Aspects

Alexandre Vinogradov Memorial Conference  
Diffieties, Cohomological Physics, and Other Animals  
December 13–17, 2021  
Independent University of Moscow  
and Moscow State University, Moscow, Russia

I. S. Krasil'shchik  
A. B. Sossinsky  
A. M. Verbovetsky  
Editors

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2020 *Mathematics Subject Classification*. Primary 13-XX, 14-XX, 34-XX, 35-XX, 46-XX, 53-XX, 55-XX, 58-XX, 68-XX, 81-XX.

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### Library of Congress Cataloging-in-Publication Data

Names: Alexandre Vinogradov Memorial Conference on Diffieties, Cohomological Physics, and Other Animals (2021 : Moscow, Russia) | Krasil'shchik, I. S. (Iosif Semenovich), editor. | Sossinsky, A. B. (Aleksei Bronislavovich), editor. | Verbovetsky, Alexander, editor. | Vinogradov, A. M. (Aleksandr Mikhaïlovich), honoree.

Title: The diverse world of PDEs : algebraic and cohomological aspects / I.S. Krasil'shchik, A.B. Sossinsky, A.M. Verbovetsky, editors.

Description: Providence, Rhode Island : American Mathematical Society, [2023] | Series: Contemporary mathematics, 0271-4132 ; volume 789 | "Alexandre Vinogradov Memorial Conference on Diffieties, Cohomological Physics, and Other Animals, December 13–17, 2021, Independent University of Moscow and Moscow State University, Moscow, Russia"—title page. | Includes bibliographical references.

Identifiers: LCCN 2023006316 | ISBN 9781470473556 (paperback) | ISBN 9781470474096 (ebook)

Subjects: LCSH: Differential equations, Partial—Congresses. | Differential equations, Nonlinear—Congresses. | Geometry, Differential—Congresses. | Homology theory—Congresses. | AMS: Commutative algebra. | Algebraic geometry. | Ordinary differential equations. | Partial differential equations. | Functional analysis. | Differential geometry. | Algebraic topology. | Global analysis, analysis on manifolds. | Computer science. | Quantum theory.

Classification: LCC QA377 .A555 2023 | DDC 515/.353—dc23/eng20230626

LC record available at <https://lcn.loc.gov/2023006316>

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To the memory of Alexandre Vinogradov (1938-2019)



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## The Editors Preface

A remarkable Soviet/Russian/Italian mathematician Alexandre M. Vinogradov passed away on September 20, 2019 (see the obituary in *Russian Math. Surveys* **75** (2020), no. 2, 369–375). Independent University of Moscow in collaboration with the Faculty of Mechanics and Mathematics of Moscow Lomonosov State University organized an international conference *Diffieties, Cohomological Physics, and Other Animals* in memory of Vinogradov ([https://gdeq.org/amv\\_conf](https://gdeq.org/amv_conf)). The Conference took place in the period December 13–17, 2021 and comprised a very wide spectrum of topics that more or less reflect the wideness of Vinogradov’s mathematical interests. The current volume of *Contemporary Mathematics* contains the articles that deal with interplay between nonlinear PDEs and algebra and cohomological theories.

I. Krasil’shchik, A. Sossinsky, A. Verbovetsky  
Independent University of Moscow



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This volume contains the proceedings of the Alexandre Vinogradov Memorial Conference on Diffieties, Cohomological Physics, and Other Animals, held from December 13–17, 2021, at Independent University of Moscow and Moscow State University, Moscow, Russia.

The papers reflect the modern interplay between partial differential equations and various aspects of algebra and computer science. The topics discussed are: relations between integrability and differential rings, supermanifolds, differential calculus over graded algebras, noncommutative generalizations of PDEs, quantum vector fields, generalized Nijenhuis torsion, cohomological approach to the geometry of differential equations, the argument shift method, Frölicher structures in the formal Kadomtsev–Petviashvili hierarchy, and computer-based determination of optimal systems of Lie subalgebras.

The companion volume (Contemporary Mathematics, Volume 788) is devoted to Geometry and Mathematical Physics.



ISBN 978-1-4704-7355-6



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CONM/789