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Feedback control of social distancing for COVID-19 via elementary formulae

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Abstract.– Social distancing has been enacted in order to mitigate the spread of COVID-19. Like many authors, we adopt the classic epidemic SIR model, where the infection rate is the control variable. Its differential flatness property yields elementary closed-form formulae for open-loop social distancing scenarios, where, for instance, the increase of the number of uninfected people may be taken into account. Those formulae might therefore be useful to decision makers. A feedback loop stemming from model-free control leads to a remarkable robustness with respect to severe uncertainties and mismatches. Although an identification procedure is presented, a good knowledge of the recovery rate is not necessary for our control strategy.

Keywords: Biomedical control, COVID-19, social distancing, SIR model, flatness-based control, model-free control, robustness, identifiability, algebraic differentiator

1. INTRODUCTION

In two years an abundant mathematically oriented literature has been devoted to the worldwide COVID-19 pandemic. Some of the corresponding calculations had even a significant political impact (see, *e.g.*, Adam (2020); Quintana *et al.* (2021)). Note that in the field of mathematical epidemiology of infectious diseases the role of modeling human behavior became increasingly important in the last 15 years. It gave birth to a novel research field named *behavioral epidemiology* of infectious diseases: see, *e.g.*, Manfredi & d'Onofrio (2013); Wang *et al.* (2016).

A novel control technique for improving the social distancing is presented here. This fundamental topic has already been tackled by many authors: see, *e.g.*, Al-Radhawi *et al.* (2022); Ames *et al.* (2020); Angulo *et al.* (2021); Berger (2022); Bisiacco & Pillonetto (2021); Bliman & Duprez (2021); Bliman *et al.* (2021); Bonnans & Gianatti (2020); Borri *et al.* (2021); Charpentier *et al.* (2020); Di Lauro *et al.* (2021a,b); Dias *et al.* (2022); Efimov & Ushirobira (2021); Gevertz *et al.* (2021); Godera *et al.* (2021); Greene & Sontag (2021); Ianni & Rossi (2021); Jing *et al.* (2021); Köhler *et al.* (2021); McQuade *et al.* (2021); Morato *et al.* (2020a,b); Morgan *et al.* (2021); Morris *et al.* (2021); O'Sullivan *et al.* (2020); Péni *et al.* (2020); Pillonetto *et al.* (2021); Sadeghi *et al.* (2021); Sontag (2021); Stella *et al.* (2022); Tsay *et al.* (2020). Most of those papers are based on the famous *SIR* (*Susceptible-Infected-Recovered/Removed*) model, which goes back to 1927 (Kermack & McKendrick (1927)), or on some modifications of its *compartments*. This communication is also using the SIR model:

- When, like in several papers, the *infection rate* is the control variable, the SIR model is *(differentially) flat* (Fliess *et al.* (1995)). Remember that flatness-based control is one of the most popular model-based control setting, especially with respect to concrete applications: see, *e.g.*, Beltran-Carbajal *et al.* (2021); Bonnabel & Clayes (2020); Diwold *et al.* (2022); Kogler *et al.* (2022); Li *et al.* (2021); Lorenz-Meyer *et al.* (2020); Miunske (2020); Richter *et al.* (2021); Sahoo & Chiddarwar (2020); Sanchez *et al.* (2020); Schörghuber *et al.* (2020); Steckler *et al.* (2021); Sekiguchi *et al.* (2021); Tal & Karaman (2021); Thounthong *et al.* (2021); Tognon & Franchi (2021); Zauner *et al.* (2021) for some recent publications. Note that flatness has already been utilized by Hametner *et al.* (2021) for studying COVID-19 but with other purposes.
- There are severe uncertainties: model mismatch, poorly known initial conditions, ... We therefore close the loop around the reference trajectory via *model-free* control, or *MFC*, in the sense of Fliess & Join (2013, 2021a). MFC, which is easy to implement, has already been illustrated in a number of practical situations. Some new contributions are listed here: Gu *et al.* (2021); Ismail *et al.* (2021); Jin *et al.* (2021); Kuruganti *et al.* (2021); Lv *et al.* (2022); Manzoni & Rampazzo (2021); Mao *et al.* (2021); Michel *et al.* (2022); Mousavi *et al.* (2021); das Neves & Angélico

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(2021); Sancak *et al.* (2021); Sehili & Boukhezzar (2022); Srour *et al.* (2021); Sun *et al.* (2021); Xu *et al.* (2020, 2021); Wang *et al.* (2022, 2020a); Wang & Wang (2020b); Zhang *et al.* (2020, 2021); Zhou *et al.* (2021). Let us single out here the excellent work by Truong *et al.* (2021) on ventilators, which are related to COVID-19.

In order to be more specific consider a flat system with a single input u and a single output y. Assume that y is a flat output. Our strategy (see also Villagra & Herrero-Pérez (2012); Fliess *et al.* (2021b)) may be summarized as follows:

(1) To any output reference trajectory $y^\star$ corresponds at once thanks to flatness an open-loop control $u^\star$.
(2) Let z be some measured output. Write $z^\star$ the corresponding reference trajectory. Set $u = u^\star + \Delta u$, where Δu is the control of an *ultra-local* local model (Fliess & Join (2013)). Its output $\Delta z = z - z^\star$ is the tracking error. Closing the loop via an *intelligent controller* (Fliess & Join (2013)) permits to ensure local stability around $z^\star$ in spite of severe mismatches and disturbances.

Our paper is organized as follows:

- Section 2 shows that the SIR model, where the infection rate is the control variable, is flat and the population of recovered/removed individuals is a flat output; the recovery rate is identifiable in the sense of Fliess *et al.* (2008).
- Section 3 is devoted to a flatness-based control strategy, *i.e.*, to a feedforward approach. Elementary closed-form of the control and state variables are easily derived. Various scenarios, where for instance the number of uninfected persons is increased, may thus be easily suggested to decision makers.
- Closing the loop via an intelligent proportional regulator, stemming from model-free control, is the subject of Section 4. Computer simulations confirm an excellent robustness with respect to severe uncertainties.
- A time-varying recovery rate is estimated in Section 5 via *algebraic estimation* methods (Fliess *et al.* (2008)). Techniques from Section 4 show however good performances if this rate is wrongly assumed to be constant.
- Some suggestions for future investigations and someconcluding remarks may be found in Section 6.

2. MODELING ISSUES

2.1 The SIR model

The SIR model (see, *e.g.*, Weiss (2013) for a nice introduction) reads:

$$\begin{cases} \dot{S} = -\beta I S \\ \dot{I} = \beta I S - \gamma I \\ \dot{R} = \gamma I \end{cases} \tag{1}$$

S, I and R, which are non-negative quantities, correspond respectively to the fractions of susceptible, infected and recovered/removed individuals in the population. We may set therefore

$$S + I + R = 1 \tag{2}$$

β, $0 < \underline{\beta} \leq \beta \leq \overline{\beta}$, which is here the control variable,[1] and the parameter $\gamma > 0$ are respectively the infection and recovery rates.

2.2 Flatness

Equations (1)-(2) show that System (1) is flat and that R is a flat output (Fliess *et al.* (1995)). The other system variables may be expressed as *differential rational functions* of R, *i.e.*, as rational functions of R and its derivatives up to some finite order:

$$I = \frac{\dot{R}}{\gamma} \tag{3}$$

$$S = 1 - R - \frac{\dot{R}}{\gamma} \tag{4}$$

$$\beta = -\frac{\dot{S}}{IS} = \frac{1}{S}\left(\frac{\dot{I}}{I} + \gamma\right) \tag{5}$$

Remark 1. If γ is not constant, but a differentiable function of time, Equations (3)-(4)-(5) remain valid: System (1) is still flat and R is still a flat output. Equation (5) shows however that $\dot{\gamma}$ is needed.

2.3 An addendum on the SEIR model

The *SEIR* model (see, *e.g.*, Brauer & Castillo-Chavez (2012)) is a rather popular extension of the SIR model:

$$\begin{cases} \dot{S} = -\beta I S \\ \dot{E} = \beta I S - \alpha E \\ \dot{I} = \alpha E - \gamma I \\ \dot{R} = \gamma I \end{cases} \tag{6}$$

where $\alpha > 0$ is an additional parameter. Equation (2) becomes

$$S + E + I + R = 1. \tag{7}$$

Equations (6)-(7) show that the SEIR model is also flat and that R is a flat output:

$$\begin{cases} I = \dfrac{\dot{R}}{\gamma} \\ E = \dfrac{\dot{I} + \gamma I}{\alpha} = \dfrac{\ddot{R} + \gamma \dot{R}}{\gamma \alpha} \\ S = 1 - R - I - E = 1 - R - \dfrac{\dot{R}}{\gamma} - \dfrac{\ddot{R} + \gamma \dot{R}}{\gamma \alpha} \\ \beta = -\dfrac{\dot{S}}{IS} \end{cases}$$

2.4 Identifiability of the recovery rate

Equation (5) yields

$$\gamma = \beta S - \frac{\dot{I}}{I}$$

γ is a differential rational function .of R and β: It is thus *rationally identifiable* (Fliess *et al.* (2008)).

Remark 2. The above equation does not work for an identifiability purpose if γ is time-varying: $\dot{\gamma}$ is sitting

[1] Softening social distancing implies increasing $\beta(t)$.

on its right hand-side. If we assume that I and S are measured, Equation (4) yields

$$\gamma = \frac{\dot{I} - \beta IS}{I} \tag{8}$$

γ is still rationally identifiable with respect to I, S, β. It will be useful in Section 5.

3. FLATNESS-BASED CONTROL

3.1 Preparatory calculations

Set

$$\boxed{I_{\text{reference}}(t) = I_0 e^{-\lambda t}}$$

where $t \geq 0$, $0 \leq I_0 \leq 1$, and $\lambda \geq 0$ is some constant parameter. If we set $R(0) = 0$, it yields

$$R_{\text{reference}}(t) = \frac{\gamma I_0}{\lambda}(1 - e^{-\lambda t})$$

$$S_{\text{reference}}(t) = 1 - \frac{\gamma I_0}{\lambda}\left(1 - e^{-\lambda t}\right) - I_0 e^{-\lambda t}$$

and the open-loop control

$$\boxed{\beta_{\text{flat}}(t) = \frac{\gamma - \lambda}{1 - \frac{\gamma I_0}{\lambda}\left(1 - e^{-\lambda t}\right) - I_0 e^{-\lambda t}}}$$

Thus

$$\lim_{t \to +\infty} \beta_{\text{flat}}(t) = \frac{\lambda(\gamma - \lambda)}{\lambda - \gamma I_0} \tag{9}$$

The following inequalities are staightforward:

$$\gamma I_0 < \lambda < \gamma \tag{10}$$

$\lambda < \gamma$ follows from $\beta > 0$; $\gamma I_0 < \lambda$ follows from

$$\lim_{t \to +\infty} S(t) = 1 - \frac{\gamma I_0}{\lambda} = S(\infty) > 0 \tag{11}$$

Introduce the more or less precise quantity β_{accept}, where $\underline{\beta} < \beta_{\text{accept}} < \overline{\beta}$. It stands for the "harshest" social distancing protocols which are "acceptable" in the long run. Equation (9) yields therefore

$$\frac{\lambda(\gamma - \lambda)}{\lambda - \gamma I_0} = \beta_{\text{accept}}$$

The positive root of the corresponding quadratic algebraic equation $\lambda^2 + (\beta_{\text{accept}} - \gamma)\lambda - \gamma I_0 \beta_{\text{accept}} = 0$ is

$$\boxed{\lambda_{\text{accept}} = \frac{\gamma - \beta_{\text{accept}} + \sqrt{\Delta_{\text{accept}}}}{2}}$$

where $\Delta_{\text{accept}} = (\gamma - \beta_{\text{accept}})^2 + 4\gamma I_0 \beta_{\text{accept}} \geq 0$. The fundamental inequality

$$\gamma I_0 < \lambda_{\text{accept}} < \gamma$$

follows from

$$\lim_{\lambda \downarrow \gamma I_0} \frac{\lambda(\gamma - \lambda)}{\lambda - \gamma I_0} = +\infty, \quad \lim_{\lambda \uparrow \gamma} \frac{\lambda(\gamma - \lambda)}{\lambda - \gamma I_0} = 0$$

Equation (11) leads to the notation

$$S_{\text{accept}}(\infty) = 1 - \frac{\gamma I_0}{\lambda_{\text{accept}}}$$

The inequality

$$S(\infty) < S_{\text{accept}}(\infty) \quad \text{if} \quad \lambda < \lambda_{\text{accept}}$$

demonstrates that the proportion of uninfected people decreases if the social distancing obligations are relaxed.

3.2 Two computer experiments

Set $\gamma = 0.1$, $\beta_{\text{accept}} = 0.22$. Figure 1 displays the open-loop evolutions of β, I, S when $\lambda = \lambda_{\text{accept}}$. Those behaviors are quite satisfactory.

4. MODEL-FREE CONTROL

4.1 Ultra-local model

Set $\Delta I(t) = I(t) - I_{\text{reference}}(t)$, $\beta(t) = \beta_{\text{flat}}(t) + \Delta\beta(t)$. In order to take into account the various uncertainties, introduce the *ultra-local* model (Fliess & Join (2013))

$$\frac{d}{dt}\Delta I = F + \mathfrak{a}\Delta\beta \tag{12}$$

- The function F, which is data-driven, subsumes the poorly known structures and disturbances.
- The parameter $\mathfrak{a}$, which does not need to be precisely determined, is chosen such that the three terms in Equation (12) are of the same magnitude.
- $F_{\text{est}} = -\frac{6}{\tau^3}\int_{t-\tau}^{t}\left((t - 2\sigma)\Delta I(\sigma) + \mathfrak{a}\sigma(\tau - \sigma)\Delta\beta(\sigma)\right)d\sigma$, where $\tau > 0$ is "small", gives a real-time estimate, which in practice is implemented via a digital filter.

4.2 Intelligent proportional controller

Introduce (Fliess & Join (2013)) the *intelligent proportional controller*, or *iP*,

$$\boxed{\Delta\beta = -\frac{F_{\text{est}} + K_P \Delta I}{\mathfrak{a}}} \tag{13}$$

where K_P is a tuning gain. Equations (12) and (13) yield

$$\frac{d}{dt}\Delta I + K_P \Delta I = F - F_{\text{est}}$$

Set $K_P > 0$. Then $\lim_{t \to +\infty} \Delta I(t) \approx 0$ if the estimate F_{est} is "good," *i.e.*, if $F - F_{\text{est}}$ is "small." Local stability is ensured.

Remark 3. When compared to classic PIs and PIDs (see, *e.g.*, Åström & Murray (2008)), the gain tuning of the iP is straightforward.

4.3 Computer experiments

The sampling time interval is 2 hours. In Equations (12) and (13), $\mathfrak{a} = 0.1$, $K_P = 1$. Figure 2 displays excellent results in spite of errors on initial conditions and of the fuzzy character of any measurement of the social distancing. This fuzziness is expressed here by an additive corrupting white Gaussian noise $\mathcal{N}(0, 5.10^{-3})$ on β.

5. ON THE RECOVERY RATE γ

Assume now that γ is a differentiable time function. Equation (8) yields the algebraic estimator

$$\gamma_{\text{est}} = \frac{[\dot{I}]_{\text{est}} - \beta IS}{I} \tag{14}$$

where $[\dot{I}]_{\text{est}}$ is an estimate of $\dot{I}$ obtained along the lines developed by Mboup *et al.* (2009) and Othmane *et al.* (2021) for *algebraic differentiators*. Figure 3-c displays excellent results. The flatness-based computer experiments is achieved as in Section 3.2, *i.e.*, $\gamma = 0.1$ is assumed to be constant. Lack of space prevents us from examining more realistic situations. Closing the loop via model-free control yields as demonstrated in Figures 3-a-b a satisfactory behavior. Is the exact knowledge of the recovery rate unimportant?

6. CONCLUSION

Casella (2021) questions the relevance and usefulness of such control-theoretic considerations for non-pharmaceutical mitigation policies against COVID-19. We certainly do not claim to set aside those objections in this preliminary short study. The combination however of flatness-based and model-free controls presents nevertheless some major advantages as demonstrated here and by Villagra & Herrero-Pérez (2012) and Fliess *et al.* (2021b).

An extra theoretical effort must be made in order to design control strategy as close as possible to the real epidemic control enacted by Public Health authorities. Summarizing, we consider this results proposed in this work as a theoretical ideal framework, to be filled with a more realistic picture: an implementable non-pharmaceutical control strategy. Preliminary results, which we recently obtained, indicate that the methodology here proposed is in the right direction (see Join *et al.* (2022)).

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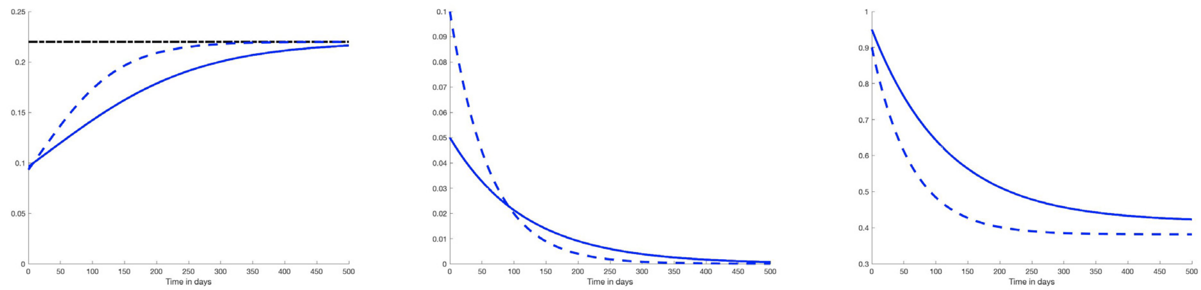
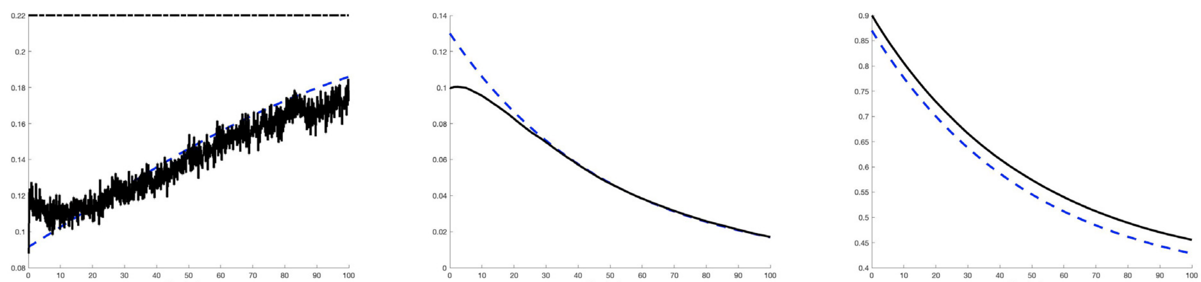


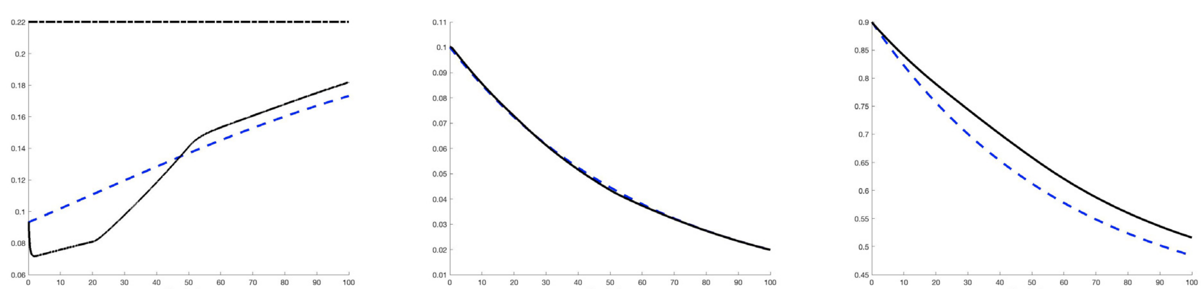
(a) β (b) I (c) S

Figure 1. Open loop control strategy. Trajectories corresponding to two distinct initial conditions for the infectius $I_0 = 0.05$ (single-dashed curves: -) and $I_0 = 0.1$ (double-dashed curves: - -). Left panel: plot of the transition rate $\beta(t)$; central panel: plot of the infectious fraction $I(t)$; right panel: plot of the fraction of susceptible subjects $S(t)$.

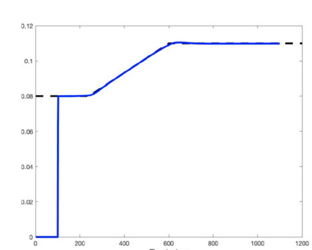


(a) β (b) I (c) S

Figure 2. Effect of both errors on initial conditions and of the fuzziness of measurements of social distancing. In all panels, dashed blue line represent the reference trajectories. Left panel: plot of the transition rate $\beta(t)$; central panel: plot of the infectious fraction $I(t)$; right panel: plot of the fraction of susceptible subjects $S(t)$.



(a) β – (blue - -): reference trajectory (b) I – (blue - -): reference trajectory (c) S – (blue - -): reference trajectory



(d) γ (- -) and γ_{est} (blue –)

Figure 3. Impact of the estimation of the time-varying recovery rate γ.