SUPPLEMENTAL MATERIAL
FOR
BOUNDS ON LORENTZ INVARIANCE VIOLATION FROM MAGIC OBSERVATION OF GRB 190114C

The MAGIC Collaboration

A. Analysis sensitivity

When building the likelihood function, we have assumed certain spectral and temporal distributions of the signal events. These were determined using our data, however, only up to a certain precision and under several theoretical assumptions [17, 19]. Therefore, we cannot determine in an unbiased way their level of accuracy. In particular, we know that the minimal model of the intrinsic light curve (Eq. (8) in the main text) does not correctly describe the temporal distribution of the signal events. Because of this, we cannot presume that the PDF for L is a χ^2 with one degree of freedom, or that the estimator of η distributes as a Gaussian around the true value η_{true}. Thus, to evaluate the PDF of the η estimator we apply the maximum likelihood method to 1000 mock data sets. Each of them is generated starting from the measured data set, first "reshuffling" the event arrival times, and then applying once the bootstrapping resampling technique. Reshuffling consists of reassigning randomly the measured arrival times to the different observed events. In this way, we remove any energy-time correlation present in the data (in particular, any LIV effect), without altering the overall spectral and temporal distributions of the signal. Bootstrapping creates samples of the same size by randomly selecting events (repetition is allowed) from the reshuffled data set, and therefore allows the measured spectral and temporal distributions to vary within their natural statistical uncertainties.

We maximize the likelihood for each of the reshuffled-bootstrapped samples, and make the histogram of the resulting best fits. This gives us the PDF of our estimator, shown in Fig. 1. Since the reshuffling procedure was supposed to remove any energy-time correlation present in the data, the expected mean of the distribution is 0. The apparent deviation from 0 we interpret as the bias η_{bias} of our analysis. From the PDF we determine the p-value of the null hypothesis, i.e., the significance of the detection of a LIV effect, as its integral above $\eta_{\mathrm{uncal.}}$ and below $2\eta_{\mathrm{bias}} - \eta_{\mathrm{uncal.}}$. Our results for the theoretical LC, $p_{\eta_1} = 0.78$ and $p_{\eta_2} = 0.59$, are consistent with the null hypothesis.

Note that this procedure is not applicable to the minimal LC model. Since the likelihood profile has no minimum, the bias is not well-defined. Furthermore, the minimal model is by construction valid only to obtain robust model-independent upper limits on η.

B. Confidence interval calibration

Since the PDF of L is not a χ^2 distribution, we cannot rely on the standard technique of finding the values of η for which L reaches the 3.84 threshold. Instead, we build a PDF of the values of η corresponding to an arbitrary value of the L threshold and calculate the quantiles of the PDF below (above) η_{bias}. We repeat this procedure for different values of the L threshold until the quantiles are 2.5% (see Fig. 2). The value of the L threshold obtained in this way we use to determine the "uncalibrated" upper (lower) limit $\eta^{\mathrm{UL}}_{\mathrm{uncal.}}$ ($\eta^{\mathrm{LL}}_{\mathrm{uncal.}}$). Finally, we compute the fully calibrated upper (lower) limits by subtracting η_{bias} from uncalibrated upper (lower) limits

$$\eta^{\mathrm{UL}} = \eta^{\mathrm{UL}}_{\mathrm{uncal.}} - \eta_{\mathrm{bias}} \tag{1}$$

$$\eta^{\mathrm{LL}} = \eta^{\mathrm{LL}}_{\mathrm{uncal.}} - \eta_{\mathrm{bias}}. \tag{2}$$

This procedure differs again from the standard Neyman construction of CIs, which is not feasible because it requires Monte Carlo simulations. However, it should produce equal results provided the PDF for $\eta - \eta_{\mathrm{true}}$ is symmetric with respect to its mean, and does not depend on η_{true}.

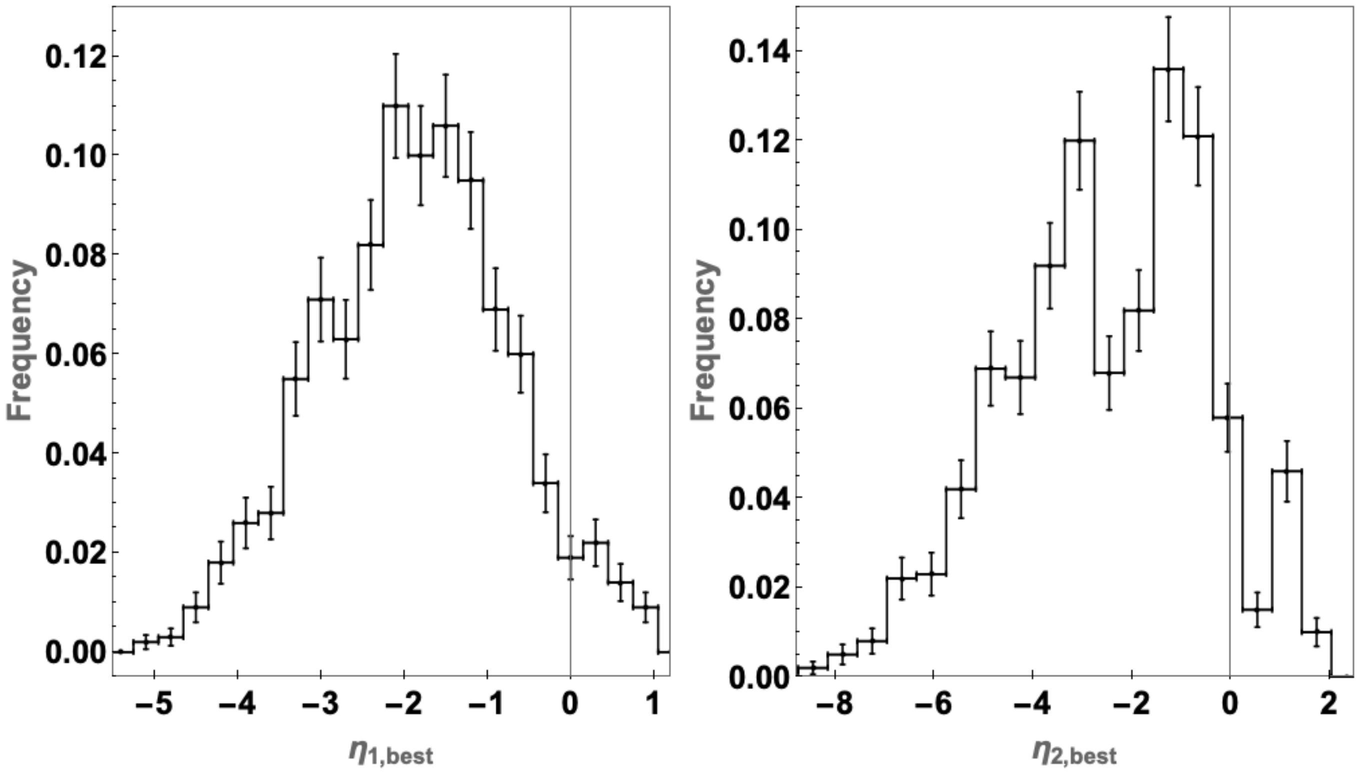

FIG. 1: Distribution of best fits of η_1 (linear case, left) and η_2 (quadratic case, right), obtained from reshuffled-bootstrapped samples and using the theoretical assumption for the intrinsic LC.

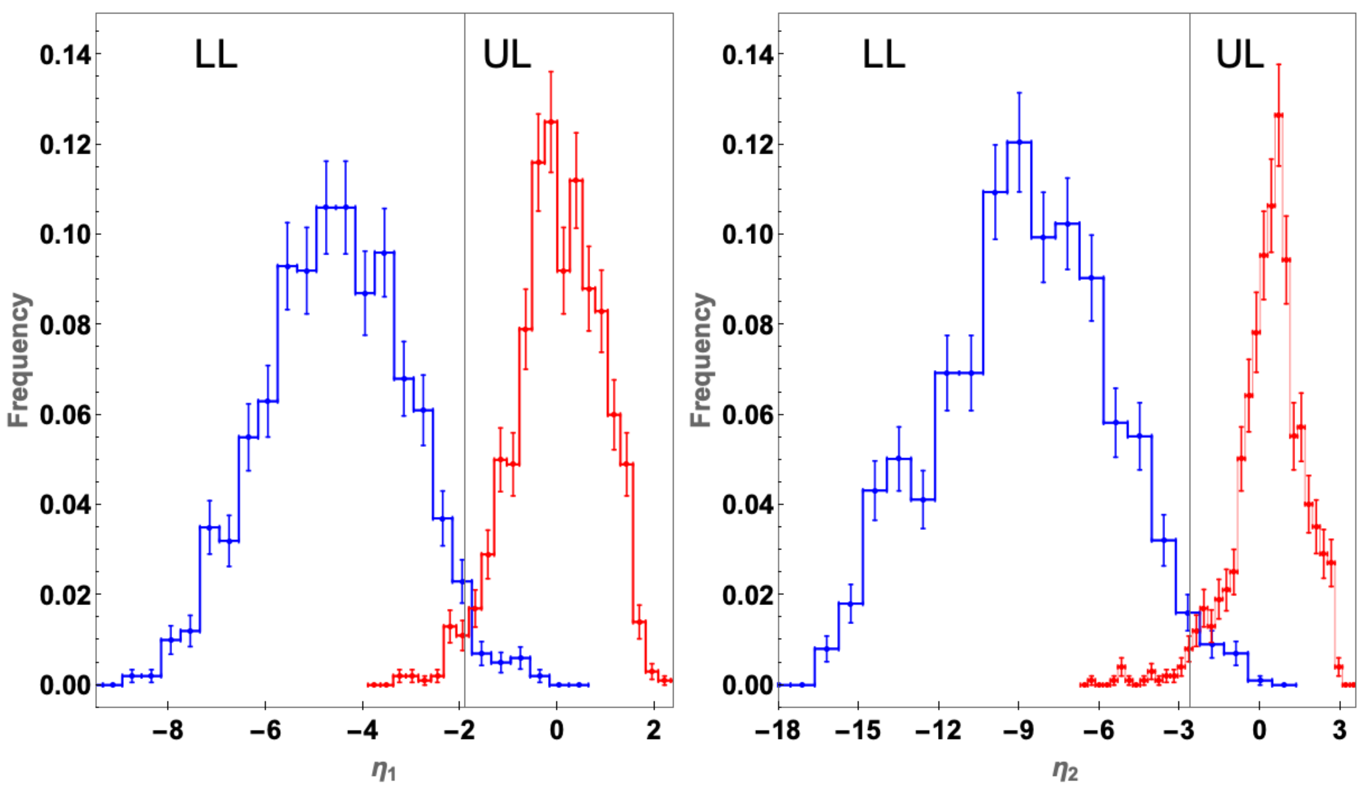

FIG. 2: Distribution of lower (blue) and upper limits (red) for the linear (left) and quadratic case (right), obtained from reshuffled-bootstrapped samples and using the theoretical assumption for the intrinsic LC. The vertical lines indicate respective bias values.