AMODAL COMPLETION OF COINCIDENTALLY OCCLUDED ANGLES

Appendices to:

"Amodal completion of coincidentally occluded angles:

A matter of visual approximation"

Andrea Dissegna, Walter Gerbino* and Carlo Fantoni

Department of Life Sciences, University of Trieste, Italy

*CORRESPONDENCE: Walter Gerbino, gerbino@units.it

Appendix 1

Figure A1.1 illustrates how the assumption of collinearity constrains the relation between two pairs of sequential adjustments referred to the same side. Consider the case in which a sequence of adjustments for a given edge includes a tilt error τ larger than an extrapolation error ε (red probes). Under the assumption that observers adjust both dot and line (black) probes of the opposite edge of the same side along the straight line defined by the line probe relative to the first edge, the extrapolation error of the (black) dot probe should be $\varepsilon' = 2 \tau - \varepsilon$ [Equation 1] and the expected tilt error of the (black) line probe $\tau' = \tau$ [Equation 2]. Equations 1 and 2 subsume the special case in which $\tau = \varepsilon$. When extrapolation and tilt errors for one edge are equal, then $\tau = \varepsilon =$ $\tau' = \varepsilon'$.



Figure A1.1. When extrapolation and tilt errors at one edge differ (red probes), a collinear solution for the opposite edge of the same side (black probes) implies an extrapolation error $\varepsilon' = 2 \tau - \varepsilon$ and the same tilt error $\tau' = \tau$.

Appendix 2

Extrapolation errors

We analyzed extrapolation errors using a MAX*lmer* model (see the main text for details), including Display, Angle, and Edge as fixed effects.

The pattern of extrapolation errors is depicted in Figure A2.1. Means, standard errors and associated *p*-values for their difference from 0 are reported in Table A2.1. The model accounted for a substantial amount of variance in the data ($R^2 = 0.87$). The main effect of Edge was significant (F(1, 22.55) = 59.92, p < 0.001). Specifically, extrapolation errors for the near edge were overall compatible with a CW rotation of about -2.45° (s.e.m. = 0.64°), while extrapolation errors for the far edge were overall compatible with a CCW rotation of about 2.66° (s.e.m. = 0.65°).

The main effect of Angle was significant (F(1, 16.33) = 41.14, p < 0.001). Extrapolation errors for the target angle were overall compatible with a CCW rotation of about 1.39° (s.e.m. = 0.55°), while extrapolation errors for the control angle were overall compatible with a CW rotation of about -1.16° (s.e.m. = 0.85°).

The Angle × Edge interaction was significant (F(1, 28.61) = 7.22, p = 0.012). Specifically, a significant difference was found between control and target angle conditions for the near edge ($\chi^2(1) = 22.38, p < 0.001$), but not for the far edge ($\chi^2(1) = 0.80, p = 0.372$). Notice that this interaction could be reasonably expected given that, in the control angle condition, far and near edges are associated with two distinct angles, one for each side of the target angle (see Figure 4 in the main text). Other effects did not reach statistical significance.



Figure A2.1. Columns represent the mean extrapolation errors in different conditions, with error bars showing the respective standard errors.

Table A2.1. Extrapolation errors in different conditions. The table reports the adjusted means of the fitted MAX*lmer* model, with the respective standard error and *p*-value for a *t* test against 0.

Display	Angle	Edge	Adjusted mean of extrapolation error	s.e.m.	р
AC	Control	Far	1.06	0.41	0.019*
AC	Control	Near	-3.02	0.83	0.008*
AC	Target	Far	2.56	0.91	0.016*
AC	Target	Near	-0.37	0.46	0.445
No completion	Control	Far	2.37	0.92	0.019*
No completion	Control	Near	-5.17	1.04	< 0.001*
No completion	Target	Far	4.61	0.79	< 0.001*
No completion	Target	Near	-0.24	0.34	0.495
Mosaic	Control	Far	3.31	1.44	0.032*
Mosaic	Control	Near	-5.70	1.58	< 0.001*
Mosaic	Target	Far	1.36	1.12	0.241
Mosaic	Target	Near	0.12	0.86	0.887

* Asterisks next to p-values indicate the statistical significance of the intercept in the MAXImer model in different conditions of the Display \times Angle \times Edge design.

Tilt errors

We analyzed tilt errors using a MAX*lmer* model including Display, Angle, and Edge as fixed effects ($R^2 = 0.79$). The pattern of tilt errors is depicted in Figure A2.2. Means, standard errors and associated *p*-values for their difference from 0 are reported in Table A2.2. The main effect of Edge was significant (F(1, 28.18) = 63.09, p < 0.001), with tilt errors of the near edge overall compatible with a CW rotation of about -3.18° (s.e.m. = 0.70°) and tilt errors of the far edge overall compatible with a CCW rotation of about 1.90° (s.e.m. = 0.71°).

The main effect of Angle was significant (F(1, 48.48) = 16.50, p < 0.001), with tilt errors of the target angle overall compatible with a CCW rotation of about 0.19° (s.e.m. = 0.65°) and tilt errors of the control angle overall compatible with a CW rotation of about -1.43° (s.e.m. = 0.88°).

The Display × Edge interaction was significant (F(2, 31.75) = 4.64, p = 0.017). However, pairwise comparisons (with Holm's correction) did not reveal any significant difference between conditions for neither the far nor the near edge. Results of comparisons for the far edge were as follows: AC-Mosaic, $\chi^2(1) = 0.0012, p = 0.973$; AC-No completion, $\chi^2(1) = 4.4472, p = 0.143$; Mosaic-No completion: $\chi^2(1) = 4.7883, p = 0.143$. Results of comparisons for the near edge were as follows: AC-Mosaic, $\chi^2(1) = 0.8886, p = 0.692$; AC-No completion, $\chi^2(1) = 4.0393, p = 0.143$; Mosaic-No completion, $\chi^2(1) = 6.234, p = 0.075$.

The Angle × Edge interaction was also significant F(1, 40.15) = 5.23, p = 0.028. Specifically, a significant difference was found between target and control conditions for the near edge ($\chi^2(1) = 13.1938$, p = 0.001), but not for the far edge ($\chi^2(1) = 0.1189$, p = 0.73). Other effects did not reach statistical significance.



Figure A2.2. Columns represent the means of tilt errors for different conditions, with error bars showing the respective standard errors.

Display	Angle	Edge	Adjusted mean of tilt error	s.e.m.	р
AC	Control	Far	0.48	0.52	0.363
AC	Control	Near	-3.47	0.82	< 0.001*
AC	Target	Far	1.71	1.16	0.184
AC	Target	Near	-2.21	0.71	0.008*
No completion	Control	Far	3.86	1.41	0.040*
No completion	Control	Near	-7.28	1.42	< 0.001*
No completion	Target	Far	3.35	0.90	0.002*
No completion	Target	Near	-1.83	0.48	0.001*
Mosaic	Control	Far	1.84	0.63	0.008*
Mosaic	Control	Near	-3.54	0.92	0.001*
Mosaic	Target	Far	0.29	0.84	0.728
Mosaic	Target	Near	-0.74	0.95	0.444

Table A2.2. Tilt errors in different conditions. The table reports the adjusted means of the fitted MAX*lmer* model with the respective standard error and p-value for a t test against 0.

* Asterisks next to p-values indicate the statistical significance of the intercept in the MAXImer model in different conditions of the Display × Angle × Edge design.

Correlation between extrapolation and tilt errors

As evident by comparing Figures A2.1 and A2.2, there was a substantial correlation between extrapolation and tilt errors. A comprehensive analysis of this correlation can be found in subsection 3.1 of the main text. Nonetheless, to provide a comprehensive picture of the previous analysis of extrapolation and tilt errors, we include here the results of the best fitting model, MAXlmer2, which can be found in the main text. This model incorporates the Display × Extrapolation Error × Cluster interaction as a fixed effect ($r_c = 0.88$). The analysis revealed that tilt errors increased proportionally with extrapolation errors, at a rate $\beta = 0.59 \pm 0.02$ (t(706.58) = 20.86, p < 0.001). Notably, the model also demonstrated that the association between extrapolation and tilt errors covaried with the Cluster factor included in the model. In particular, the model revealed a 3-way Display \times Extrapolation Error \times Cluster interaction (F(2, 137.08) = 8.45, p < 0.001, $\eta_p^2 = 0.08$), consistent with a different pattern of adjustments across the three Display types. This suggests that the results of the analysis of the simple means of extrapolation and tilt errors need to be interpreted with caution and that it is crucial to consider the Cluster factor to properly understand the relationship between these variables. A more comprehensive description of the relevance of the Cluster factor for our hypotheses can be found in subsection 2.1. of the main text.