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Radiation Pressure Sensor

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***Abstract* - Mechanical elements with dimensions in the nanometer range, at least in one direction, have been successfully employed as sensors in various devices. Their mechanical properties must be known with maximum precision in order to quantify the sensor response to external excitation. This often poses a significant challenge due to the mechanical fragility of the sensor elements. Here we present a measurement of the mechanical response of a 100 nm thick silicon nitride membrane. The external excitation force is provided by a laser beam modulated in amplitude, while the displacement of the membrane is measured by a Michelson interferometer with a homodyne readout.**

Keywords – radiation pressure, sensor, mechanical resonator, mechanical impedance

I. INTRODUCTION

The development of production methods and technologies has led to a realization of smaller and ever more sensitive sensors. One example are the thin high-stress membranes made of silicon nitride produced by the Canadian company Norcada. The membranes are basically a thin (~ 50 nm) silicon nitride window stretched over a 200 µm thick silicon frame. Their high mechanical quality factor ($Q \sim 10^5$) and high resonance frequency (~ 10^5 Hz) has made them a valuable tool in various types of experiments. Their use ranges from quantum optics experiments, where light is controlled by light [1] or, more recently, as a force sensor in an experiment searching for the dark energy [2].

Dark energy, which makes up to 70% of the Universe, was introduced in order to provide an explanation for the accelerated expansion of the Universe. One of the most popular models includes a scalar field with a screening "chameleon" mechanism, where the mass of the field depends on the local matter density [3]. This allows for the accelerated expansion, while evading conflict with laboratory experimental results [4]. Since the mass of the field depends on the local matter density, the associated particle must reflect off a density boundary when its energy before the boundary is lower than after, thereby transferring momentum in this process. The transferred momentum can be detected as a displacement of, or force applied to, the boundary. This makes the detection of the chameleon field possible with a suitable sensor which must be calibrated. A calibration scheme where a known force is applied on the membrane has been proposed [5]. This scheme, where the force is provided by the radiation pressure of a laser light beam is used also in gravitational wave detectors where it simulates the expected signal [6].

In this paper we present a study of the mechanical response of a membrane to a sinusoidal varying force. The frequency of modulation was varied in a range from 10 kHz to 100 kHz and the response of the sensor was studied.

II. THEORETICAL BACKGROUND

The membranes are two-dimensional mechanical resonators whose motion is governed by the partial differential equation [7]

$$\frac{\partial^2\psi}{\partial t^2} = v^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right), \tag{1}$$

where v is the speed of wave propagation. This equation has a solution depending on the indices m, n and is given by

$$\psi_{mn} = [A_{mn}\cos(\omega_{mn}t) + B_{mn}\sin(\omega_{mn}t)]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right). \tag{2}$$

The indices m and n are integer numbers, and a and b represent membrane dimensions in the x and y direction, respectively. The coefficients A_{mn} and B_{mn} depend on the initial conditions and can be calculated, but they are not interesting for this work. The frequencies of the membrane oscillation modes are given by

$$\omega_{mn} = \pi v\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}. \tag{3}$$

Eq. (3) is usually used to identify the oscillation modes in the displacement spectrum of the membrane. An example of such spectrum is shown in Figure 1.

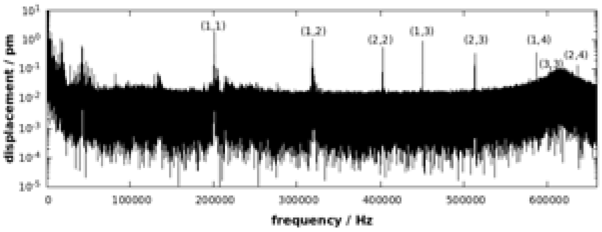

Figure 1. Calibrated frequency spectrum of the oscillations of 2x2 mm^2, 50 nm thick, membrane. The mode indices *m,n* are shown in parenthesis. The frequencies of higher order modes are related to the frequency of the fundamental mode by Eq. (3).

The oscillations are thermally excited, i.e. by Brownian stochastic force with zero mean value, and they are damped by a viscous force with damping rate *D* [8]. The equation of motion of a viscously damped mechanical oscillator with a sinusoidally driving force is

$$m\ddot{x} + D\dot{x} + kx = F\ \sin(\omega t) \quad (4)$$

Where *m* is the mass, *k* is the spring constant of the undriven harmonic oscillator without damping and *D* is the damping coefficient. The constants *k* and *m* are related to the natural frequency of the harmonic oscillator by the usual relation $\omega_0 = \sqrt{\frac{k}{m}}$.

The equation of motion has a general solution which includes the transient solution and steady state response. We are interested only in the latter since we assume that the system has reached equilibrium. The steady state solution becomes

$$x(t) = \frac{F(\sin \omega t - \varphi)}{\sqrt{m^2(\omega^2-\omega_0^2)^2+(D\omega)^2}} \quad (5)$$

where φ is the phase between the driving force and displacement.

The amplitude of the steady state is the value of interest since it is directly measured and is given by

$$A = \frac{F}{\sqrt{m^2(\omega^2-\omega_0^2)^2+(D\omega)^2}} \quad (6)$$

The spectrum from Fig. (1) could be used to obtain the mechanical impedance, however due to the feeble signal the procedure is not straightforward. To verify the response of the sensor to a driving force *F* in (4), a calibration was performed using the radiation pressure of a laser beam with sinusoidally modulated intensity.

III. Experimental Setup

The measurements were carried out with an experimental setup designed and constructed in the Laboratory for Quantum and Nonlinear Optics at Department of Physics and Centre for micro- and nanosciences and technologies (NANORI) at University of Rijeka. The heart of the setup is a Michelson interferometer where the membrane is placed at the end of the so-called signal (SIG) arm, while in the other arm of the interferometer, called local oscillator (LO), a piezo-actuated mirror is used to maintain a constant phase relation between the two arms. The beams propagating in the two arms are also labeled SIG and LO. The experimental scheme can be seen in Fig 2.

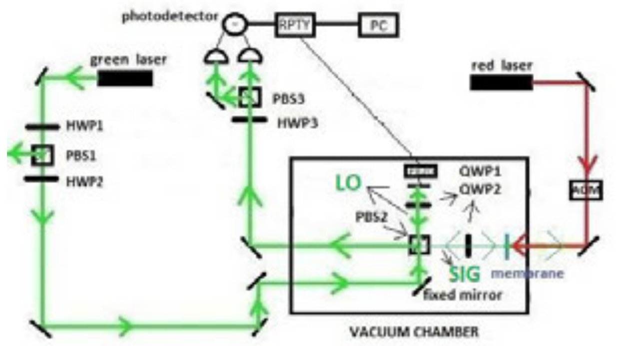

Figure 2. Experimental setup for mechanical impedance measurements. The Michelson interferometer is placed in a vacuum chamber. The green laser is used for the measurement while the red laser is used for excitation. More details are given in the text.

The information on the phase between the SIG and LO beams is read out by a balanced homodyne detector [9] and passed to a single board computer, called Red Pitaya (Rptya) [10] used for control and data acquisition. A custom software has been developed for this purpose.

The measurement beam (green) is produced by a laser where an infrared beam is duplicated in frequency. The green light during the propagation encounters a half-wave plate (HWP1) used to determine the intensity of the light used for measurement. This is done together with a polarizing beam splitter (PBS1) where a fraction of light is passing unperturbed while the rest is exiting on the side and does not contribute to the measurement. After the first optical elements the light encounters the HWP2 which together with PBS2 determines the intensity distribution between LO and SIG beams. Most of the intensity circulates in the LO arm where a quarter wave plate (QWP1) is placed. It transforms linearly polarized light to elliptical polarization, and when the light beam is reflected by a mirror going through QWP1 again, its polarization is changed from elliptical to linear but at right angle with respect to the initial one. This results in light coming out of the other port of PBS2 and in separating the ingoing from outgoing light. The same process takes place in the SIG arm, where QWP2 is placed and the light again comes out at the exit port of the PBS2. The two beams with orthogonal linear polarizations propagate to HWP3, which then rotates the polarization of the incoming light by 45 degrees resulting in two effects. In the PBS3 the SIG and LO beams are mixed and can interfere, and the outputs of the PBS3become of equal intensity. The intensities of two beams are read by a balanced photodiode detector, which subtracts the resulting photocurrents, then converts the output to a voltage and amplifies it. This signal is digitized by Rptya and used to maintain the phase between the LO and SIG beam. This is done by applying the voltage provided by a proportional-integral-derivative (PID) loop to a piezo actuator moving the mirror in the LO arm of the interferometer. The measurements were done by locking the phase to the gray fringe of the interferometer. The data digitized by Rptya is also transferred to a personal computer (PC) where it is stored for off-line analysis. The digitized waveforms are analysed in frequency by a Fourier transform, which gives displacement amplitudes as functions of frequency.

The excitation (red) beam is produced by a HeNe laser. It is modulated in intensity and frequency by an acoustooptical modulator (AOM). Here only the modulation in intensity is relevant. A sinusoidal signal at an arbitrary frequency is provided to the intensity modulation input of an AOM driving circuit. The result is a sinusoidally varying intensity at a given frequency. The corresponding radiation pressure that is changing in time provides the driving force.

IV. RESULTS

The measurements were done with a *5x5 mm²*, *100 nm* thick membrane that was also used as sensor at CERN during data taking at the CAST experiment. The request for the sensor calibration comes from the requirement to give a limit to the chameleon radiation pressure force obtained during the measurement campaign reported in [5]. While the sensor is self-calibrating in terms of displacement, as it is sufficient to measure the distance between the fringes during mirror movement in the LO interferometer arm, the force calibration is more involved. To this end, a known external driving force must be provided. Here it was accomplished with the photon radiation pressure force as previously described. The radiation pressure in case of electromagnetic waves is related to the beam intensity *I* by

$P = \frac{2I}{c}$. To obtain the force, the pressure P must be multiplied by an effective area S. Since the intensity I is defined as the ratio of incident power p and the area S, $I = \frac{p}{S}$, the radiation pressure force becomes $F = P \cdot S = 2\frac{p}{c}$.

The force then depends only on the light power p, which can be easily measured by a calibrated optical power meter. The power of the excitation beam used was $p = 1.5$ mW while only a fraction, $p_{ref} = 300$ µW, was reflected from the membrane. The displacement of the membrane at the modulation frequency was measured and the result can be seen in the Fig. 3.

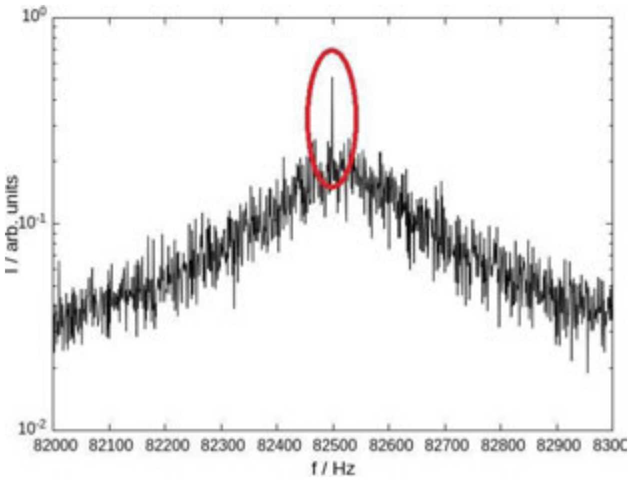

Figure 3. Frequency spectrum of the fundamental mode oscillations of a 5x5 mm², 100 nm thick, membrane. The wide peak is thermally excited, while the narrow peak in the red ellipse is excited by external driving force. The displacement is given in arbitrary units. The calibration constant was separately determined and it is $k = 0.7$ pm/a.u.

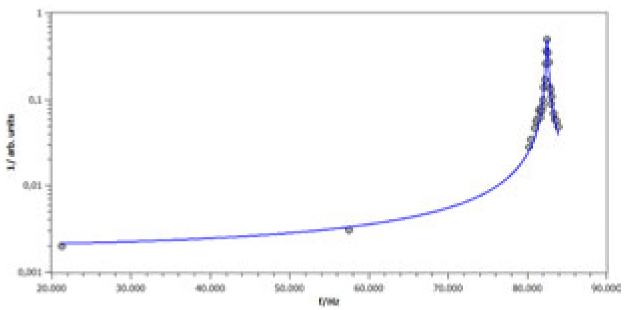

Figure 4. Displacement amplitudes of a membrane for different driving force frequencies. Each gray dot represents one mesurement. It can be seen that the amplitude depends on the driving force frequency. The blue line is result of a least squares fit.

The peak due to the driving force at f = 82500 Hz was observed and the amplitude of the displacement was written down. The procedure was repeated for different values of frequency f of the driving force and the displacement values in arbitrary units were recorded for every case.

The parameters obtained by fitting the Eq. (6) are $\frac{F}{m} = (9.2 \pm 0.3) \cdot 10^6$ in arbitrary units, $\omega_0 = 82495\ \pm 1$ Hz and $D = (7 \pm 1) \cdot 10^{-5}$ a.u.

Since the driving force amplitude $F = 1$ pN is known, the mass m can be extracted from the fit parameter A. With all the parameters known, it is possible to reverse the logic and to obtain a force from measured displacement amplitude. For example, for measured displacement $d = 40 \pm 5$ fm, the force on the membrane is

$$F_m = d\ \sqrt{m^2(\omega^2 - \omega_0^2)^2 + (D\omega)^2} \qquad (7)$$

Which gives a force amplitude of $F = 80 \pm 15$ pN acting on the membrane at 1 kHz frequency.

V. CONCLUSION

We have developed a method for measuring the properties of a membrane mechanical oscillator where the excitation was provided by the electromagnetic radiation force. Displacement peaks due to the external excitation were observed in a range of frequencies around the fundamental mode of the membrane. This method provides a novel way in the determination of properties of mechanical oscillators.

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[1] M. Karuza et al., "Optomechanically induced transparency in membrane-in-the-middle setup at room temperature," Phys. Rev. A 88, 013804 (2013)

[2] S. Arguedas Cuendis et al., " First results on the search for chameleons with the KWISP detector at CAST," Physics of the Dark Universe 26 (2019) 100367

[3] J. Khoury and A. Weltman, “Chameleon cosmology, ” Phys. Rev. D 69, 044026

[4] A. Upadhye, “Dark energy fifth forces in torsion pendulum experiments,” Phys. Rev. D 86, 102003 – Published 7 November 2012

[5] M. Karuza. et al., “KWISP: an ultra-sensitive force sensor for the Dark Energy sector,” Phys. Dark Univ. 12 (2016) 100-104

[6] S. Karki et al., “The Advanced LIGO photon calibrators,” Review of Scientific Instruments 87, 114503 (2016)

[7] E. Kreyszig, Advanced Engineering Mathematics, 10th ed, John Wiley and Sons Inc., pp.575-581

[8] M. Karuza et al., “Optomechanical sideband cooling of a thin membrane within a cavity,” New Journal of Physics 14 (2012) 095015 - Published 17 September 2012

[9] H.A. Bachor and T.C. Ralph, A Guide to Experiments in Quantum Optics, Wiley-VCH, 2004

[10] M. Vretenar, N. Erceg, and M. Karuza, “Energy-resolved coincidence counting using an FPGA for nuclear lifetime experiments,” American Journal of Physics 87 (12):997-1003