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ABSTRACT

Cyclic loads represent a typical demand on ship structures, and all the stress concentration zones (hot spots) are potential sources of fatigue cracking.

Experience shows which are the most critical structural details that need to be regularly inspected. Periodic surveys, which are normally carried out to assess the serviceability of ship structures throughout their lifetime, should obviously be especially directed to the inspection of such details.

Once a crack has been detected, Fracture Mechanics is a suitable tool for assessing the residual fatigue life of the structure in question, so that sound decisions can be taken about repairing or replacing it.

This paper deals with the problem of setting up a procedure for predicting the growth of fatigue cracks, so that the time remaining before failure may be properly estimated.

The proposed procedure is focused on the assessment, by means of finite element models, of the stress intensity factors that play the main role in determining the crack growth rate.

The whole procedure is illustrated with reference to a welded joint that is typical in ship structures, and which also has been the subject of full-scale tests. This example can help in drawing general guidelines for other practical applications.

INTRODUCTION

In the design stage the fatigue strength of ship structures can be evaluated by applying Palmgren-Miner's linear cumulative damage theory. In short, this procedure requires a reliable estimate of load history, in terms of stress ranges ($\Delta\sigma$ or S) and corresponding number of cycles N, and the availability of a proper S-N curve.

On the other hand, when a crack has already started and has been detected on a certain structural detail, a suitable approach for assessing its residual fatigue life is based on Fracture Mechanics theory. Whereas the above-mentioned design approach based on the use of S-N curves is aimed at avoiding the development of unstable cracks, the analysis based on Fracture Mechanics starts from the assumption that a crack already exists, and is aimed at evaluating the crack growth in order to establish if and when it will become unstable. Therefore, Fracture Mechanics can play an important role in the program of periodic surveys normally carried out on ship structures. Indeed, when in the course of a structural survey a crack is detected, its seriousness in relation to the interval of time remaining until the next survey needs to be determined.

For this purpose, a proper tool can be based on the results (in terms of crack growth) of systematic analyses carried out on the structural detail in question for various initial crack lengths and with reference to an expected load history.

In the next section a procedure to establish the residual fatigue life of a cracked structural detail will be outlined.

BASIC CONCEPTS OF FRACTURE MECHANICS THEORY

Ship structures are built up through various welded connections. All welding procedures may entail, for different reasons, the creation of microcracks that may grow as a consequence of the presence of cyclic loads.

A crack can extend in different modes: the opening or tensile mode (Mode I), the sliding or in-plane shear mode (Mode II), and the tearing or antiplane shear mode (Mode III).

The Mode I loading, in which the two faces of the crack undergo displacements normal to the faces themselves, is the most common encountered in the actual engineering situations involving fatigue mechanisms, while the other loading modes are less frequent and are usually less important in most practical cases.

The two faces of a crack meet in a sharp tip, where stresses and strains can be critical. In fact, in the tip region the material is subjected to stresses that may lead to a splitting phenomenon initiating a crack propagation.

The stress field in the neighbourhood of the tip has a typical pattern capable of being expressed in analytical form. The magnitude of the stresses at a given point is dependent on a stress field parameter K usually called the *stress intensity factor*. More specifically, K is evaluated according to the above-mentioned displacement modes of the crack faces, and correspondingly denoted K_I, K_{II}, and K_{III}.

In general, the stress intensity factor K, for a certain structural body under a given loading condition, can be expressed by:

$$K = \beta\, s_o\, \sqrt{\pi\, a} \tag{1}$$

where a is the length of the crack, $\beta = \beta(a)$ is a shape function that depends on the crack geometry (especially on its length, which is the only concern in two-dimensional cases), while s_o is the "nominal" stress on the point of the crack tip determined as if the crack did not exist.

To provide a better understanding of the terms a and s_o used above, it is useful to be more specific: the term a in the case of an edge crack is simply the crack length, while for center cracks (i.e., two-tip cracks) it is half of the total crack length, in other words a is the distance covered by the tip of the crack from its initiation; the term s_o for Mode I is to be understood as the stress σ_o in the direction perpendicular to the crack faces, while for Mode II it is the tangential stress τ_o acting in the direction parallel to the faces of the crack. In what follows attention will be focused on Mode I only (for simplicity, therefore, K_I will be written as K, omitting the subscript).

Linear Elastic Fracture Mechanics theory (LEFM-theory), with reference to an infinite plate with a central crack under remote biaxial loading, gives the stress distribution in the region near the crack tip. The stress functions were first obtained by Westergaard for equibiaxial loading, and were later modified by Sih to cover the case of biaxial loading with uniform tensile stresses on the remote boundaries equal to σ_∞ and $\kappa\,\sigma_\infty$ in the directions perpendicular and parallel to the crack faces, respectively (Fig. 1). κ is a constant that is equal to 1 for the equibiaxial loading condition and is equal to 0 for uniaxial loading.

In Fig. 1 an arbitrary stressed element is shown with its cylindrical coordinates (r, θ, z) taking their origin at the crack tip (z direction is perpendicular to the x-y plane).

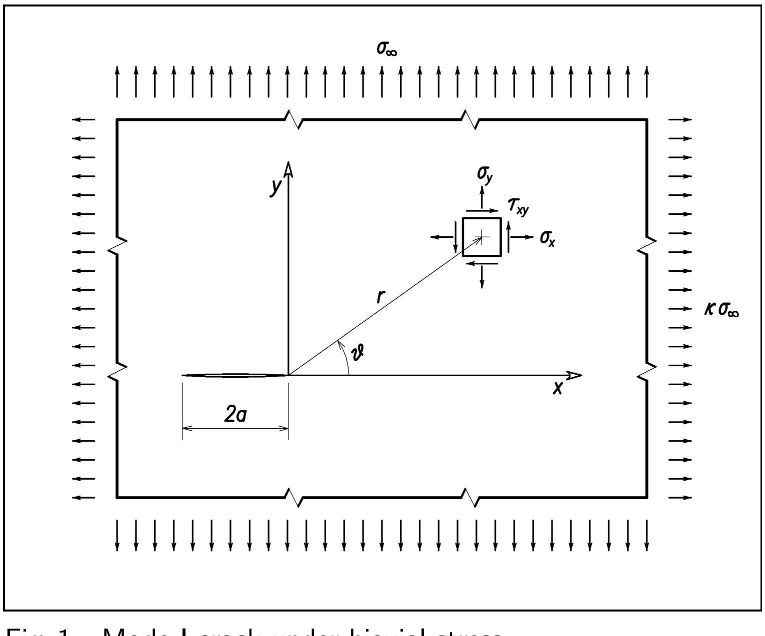

Fig. 1 – Mode I crack under biaxial stress.

The stress field for the opening mode loading (Mode I) in an infinite cracked plate is described by the following generalized expressions:

$$
\begin{aligned}
\sigma_x &= \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\,\sin 3\frac{\theta}{2}\right)-(1-\kappa)\,\sigma_\infty \\
\sigma_y &= \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\,\sin 3\frac{\theta}{2}\right) \\
\tau_{xy} &= \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\,\sin\frac{\theta}{2}\,\cos 3\frac{\theta}{2} \\
\tau_{xz} &= \tau_{yz} = 0 \\
\sigma_z &= \begin{cases} 0 & \text{for plane stress} \\ \nu\,(\sigma_x+\sigma_y) & \text{for plane strain} \end{cases}
\end{aligned}
\tag{2}
$$

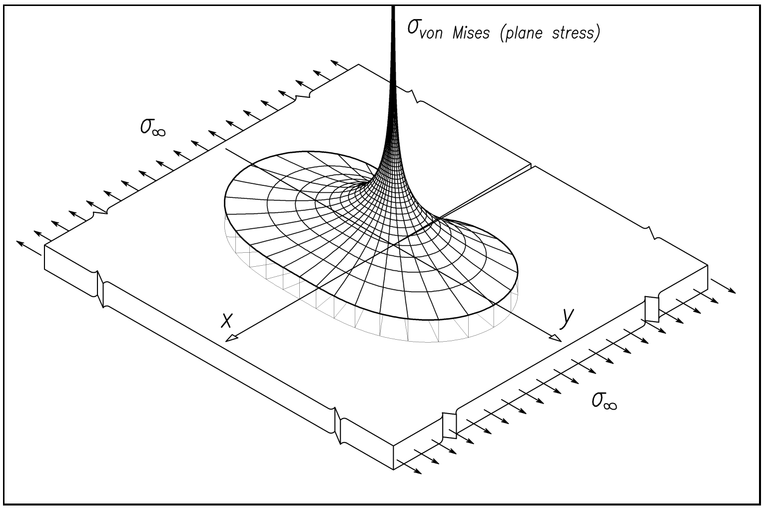

Fig. 2 – von Mises theoretical stresses (plane stress state).

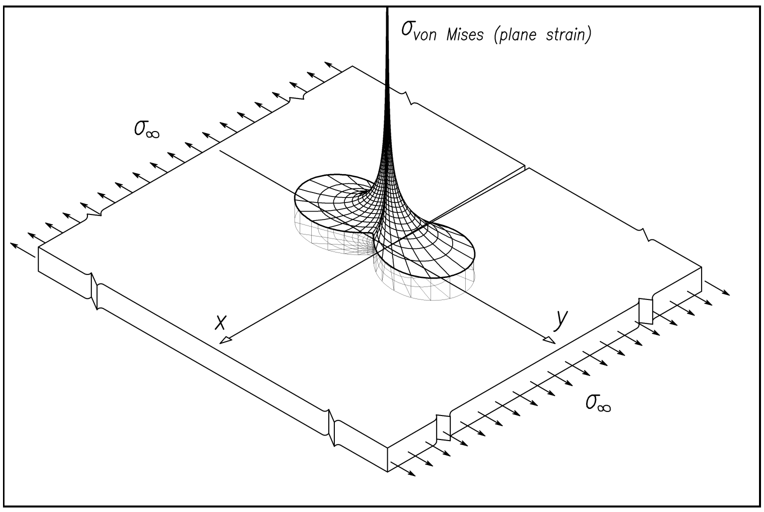

Fig. 3 – von Mises theoretical stresses (plane strain state).

where ν is the Poisson's ratio and $K = \sigma_\infty \sqrt{\pi a}$ is the stress intensity factor (in this case the "nominal" stress σ_o is σ_∞). Clearly, the crack tip is a singularity point for this theoretical stress field.

With reference to expressions (2), it is possible to evaluate the uniaxial equivalent stresses $\sigma_{von\ Mises}$ according to von Mises criterion. Fig. 2 and Fig. 3 show the shape of the $\sigma_{von\ Mises}$ field in the vicinity of the crack tip for the plane stress and the plane strain states, respectively. In both cases, to allow a prompt comparison, the same stress intensity factor K has been assumed.

Obviously, too high stress values (beyond the yield point) have no physical meaning: actually a plastic zone spreads around the crack tip. A proper correction needs then to be introduced to account for this phenomenon.

In Fig. 4 and Fig. 5 the corrected situations for the two above-mentioned cases are shown. The adjustment has been done in accordance with the well-known *Irwin plastic zone correction*: it is worth pointing out that this correction leads to a significant redistribution of the stress levels in the surrounding elastic region as well.

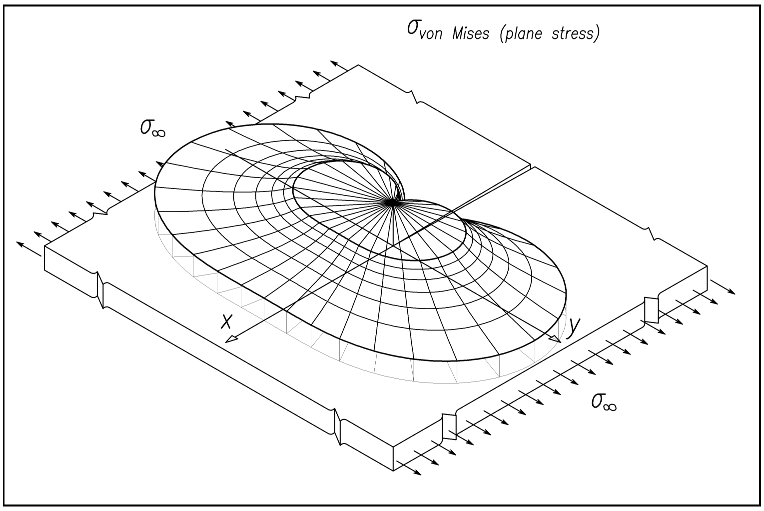

Fig. 4 – von Mises corrected stresses (plane stress state).

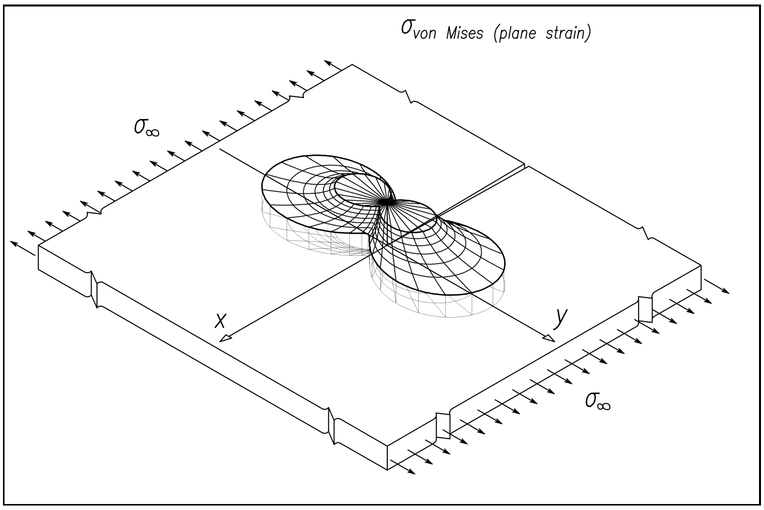

Fig. 5 – von Mises corrected stresses (plane strain state).

In a two-dimensional body the stress gradient through the thickness plays an important role in establishing the state of either plane stress or plane strain. In fact, if the in-plane stresses σ_x and σ_y have a high gradient in the z direction, a self-constraint mechanism arises in relation to the strains ε_z, and consequently a stress σ_z is present. In a thick plate, such a loading condition implies a plane strain state ($\varepsilon_z = 0$), whereas if the plate is thin the self-constraint mechanism is weaker and then a state of plane stress ($\sigma_z = 0$) can be assumed. Moreover, the ductility of the material directly affects this phenomenon: in front of the crack tip a plastic zone forms, and its size influences the state of stress. In particular, low-yield-strength materials promote the development of large plastic zones, and consequently of large displacements of the surrounding elastic material: in this case a state of plane stress prevails.

A crack grows when the stresses in the vicinity of the tip are high enough, that is to say, when the stress intensity factor range is above a certain *threshold* value ΔK_{th}. The crack can become unstable, i.e., its growth can become very rapid without any increase of the load, if the stress field in front of the tip reaches critical values or, in other words, if the stress intensity factor reaches a *critical* value K_c called *fracture toughness*.

K_c depends on the material, temperature, strain rate, environment, and thickness. In particular, the thickness effect is related to the gradual transition from full plane stress to full plane strain, passing from thin to thick plates. In fact, as thickness increases the K_c values go toward an asymptotic minimum called *plane strain fracture toughness* and denoted K_{Ic}. It is worth noting that K_{Ic} can be considered an inherent material property, since it is independent from the thickness: it is experimentally determined through tests performed according to Mode I loading (hence the subscript I).

On these bases, with reference to the characteristics of the materials and to the geometry of the structural elements used in shipbuilding, in most practical cases states of plane stress can be properly assumed. Nevertheless, in engineering applications a conservative approach is usually pursued making use of the lower level of the material toughness K_{Ic}.

The knowledge of the stress intensity factor K in static cases makes it possible to evaluate the residual strength of a cracked body. In addition, with reference to fatigue loads the same factor K can be considered as the main factor governing the crack propagation.

As a matter of fact, cyclic loads imply a variation of stresses ($\Delta\sigma_o = \sigma_{o,max} - \sigma_{o,min}$) and consequently the stress field in front of the crack tip is subjected to a change that can be represented through the *stress intensity factor range* $\Delta K = K_{max} - K_{min}$. It is worth pointing out that in the crack growth mechanism an important role is played by the interaction between the plastic zone in the vicinity of the tip and the surrounding elastic material. Such an interaction is influenced by the extreme values of the stresses during each cycle, and hence by the K_{max} and K_{min} values. Usually, instead of considering the extreme stress intensity factors, reference is made to their difference ΔK and to the relevant *stress ratio* $R = K_{min}/K_{max} = \sigma_{o,min}/\sigma_{o,max}$.

In the case of load cycles that include compressive phases, the latter can be neglected because there is no significant contribution to the crack growth process and K loses its meaning; in other words this situation can be treated as the case of pulsating tension condition $R = 0$. Therefore, if $R < 0$ an *effective* stress intensity factor range ΔK_{eff} can be considered:

$$\Delta K_{eff} = \Delta K \, \frac{1}{1 - R} \tag{3}$$

Focusing now on fatigue crack growth, three phases can be identified: *initiation*, *propagation*, and *final fracture*.

Experiments give evidence that for a given stress ratio R there is a *threshold stress intensity factor range* ΔK_{th} below which no notable crack growth occurs (ΔK_{th} decreases as R increases). If the threshold level is overcome there will be an appreciable growth of the crack length at every load cycle: such a crack growth rate is denoted by $\mathrm{d}a/\mathrm{d}N$.

For a constant amplitude cyclic loading the value of ΔK increases gradually cycle after cycle along with the crack length a.

Eventually, when the stress intensity factor range reaches a critical value ΔK_c an abrupt fracture takes place. More specifically, at the last cycle K_{max} is equal to the fracture toughness K_c. Actually for a given stress ratio R the stress intensity factor range ΔK can be expressed by $\Delta K = K_{max} - K_{min} = K_{max}\,(1 - R)$, and if $K_{max} = K_c$ the critical range ΔK_c is obtained:

$$\Delta K_c = K_c\,(1 - R) \tag{4}$$

Quite a number of semiempirical laws depicting the crack growth rate $\mathrm{d}a/\mathrm{d}N$ as a function of the stress intensity factor range ΔK can be found in the literature. With reference to the region of crack propagation, experimental data show that essentially a linear relationship between $\log(\mathrm{d}a/\mathrm{d}N)$ and $\log(\Delta K)$ exists. On this bases, Paris and Erdogan were the first to propose the power law described by the following formula:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\,(\Delta K)^n \tag{5}$$

where on a log-log plot of $\mathrm{d}a/\mathrm{d}N$ versus ΔK, n is the slope of the curve and C is the intercept found by extending the straight line to $\Delta K = 1$. Such parameters C and n depend on material characteristics, environment (corrosion and temperature), and loading variables (frequency and stress ratio).

For most practical applications concerning structural steels, the slope n is the same for all stress ratios R, while the intercept C is function of the specific stress ratio R.

To represent the effective slope of the crack growth rate curve in the final region (where there is an acceleration of the crack growth as K_{max} approaches the fracture toughness K_c) some adjustment to Paris-Erdogan's law have been proposed. The most commonly used relation is that proposed by Forman et al.:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{C\,(\Delta K)^n}{(1-R)\,K_c - \Delta K} \tag{6}$$

For the prediction of the residual life of a structural particular where a crack has been detected it is necessary to determine the critical crack length a_c for which $K_{max} = K_c$, that is, in accordance with expression (1):

$$\beta_c\,\sigma_{o,max}\,\sqrt{\pi\,a_c} = K_c \tag{7}$$

where $\beta_c = \beta(a_c)$.

If a_i is the detected length of the crack, the problem is now that of assessing the number of cycles N_c leading to the final fracture. Starting from the Paris-Erdogan's expression (5), since $K = \beta\,\sigma_o\,\sqrt{\pi\,a}$, the residual number of cycles till the critical crack length a_c is given by:

$$N_c = \int_{a_i}^{a_c} \frac{1}{C\,(\beta\,\Delta\sigma_o\sqrt{\pi\,a})^n}\,\mathrm{d}a \tag{8}$$

while, adopting the Forman expression (6), N_c becomes:

$$N_c = \int_{a_i}^{a_c} \frac{(1-R)\,K_c - \beta\,\Delta\sigma_o\sqrt{\pi\,a}}{C\,(\beta\,\Delta\sigma_o\sqrt{\pi\,a})^n}\,\mathrm{d}a \tag{9}$$

The integration needs to be performed through a numerical procedure since $\Delta\sigma_o$ and β are both functions of the actual crack length a.

It is worth pointing out that in the case of ship structures usually there are not striking high positive overload cycles that may cause delays in fatigue crack growth. It is then reasonable to neglect possible retardation phenomena.

ASSESSMENT OF THE STRESS INTENSITY FACTOR

To assess the crack growth it is essential to start from a proper evaluation of the stress intensity factor K as a function of the actual crack length a.

As a practical matter, it is more convenient to rely on the shape function $\beta(a)$, since the procedure is thus independent of the specific value assumed by the nominal stress σ_o. Clearly, the nominal stress σ_o is evaluated once and for all in the uncracked body on the expected crack growth line.

The evaluation of the stress intensity factor K can be carried out using different approaches based either on the knowledge of the stress–displacement fields near the crack tip, or on the determination of the strain energy fields.

Among the different approaches, a versatile and at the same time accurate procedure is that based on the determination of the distance between facing points of the crack edges, i.e., the so called *Crack Opening Displacement* (COD).

The stress field near the crack tip can be determined on the basis of the stresses given by expressions (2). In particular, the displacements of the crack faces can be derived, and, for the state of plane stress, the distance δ_o between two facing points is given by:

$$\delta_o = u_y(r, -\pi) - u_y(r, \pi) = \frac{K}{E}\sqrt{\frac{8\,r}{\pi}} \tag{10}$$

From this expression the stress intensity factor K can be found once δ_o has been evaluated. A suitable procedure can be based on a FEM analysis and starts from the determination of the displacements δ_{o_i} at a certain number of pairs of nodes facing each other along the crack edges. The value K_i^* obtained making use of expression (10) will be different for each pair of nodes: indeed, expression (10) is rigorously valid only in the region very near to the crack tip. The actual value K shall then be derived through a proper extrapolation at the crack tip (where $r = 0$) of the K_i^* values.

AN EXAMPLE OF CRACK GROWTH CALCULATION

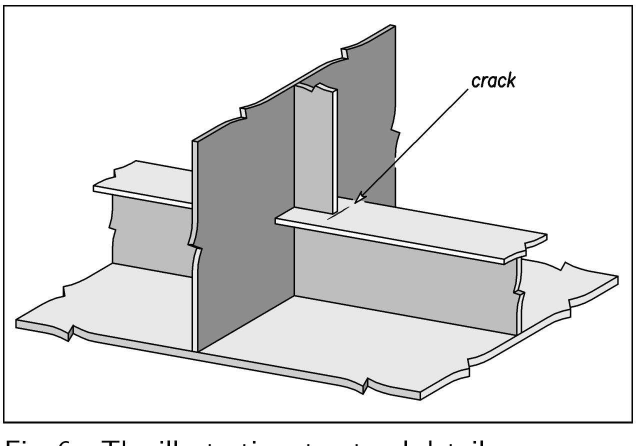

Fig. 6 – The illustrative structural detail.

The procedure outlined above has been applied to various structural configurations in order to check its validity and to properly calibrate the different parameters through a comparison with the results of some known cases.

As an example, the case of a typical intersection between an ordinary stiffener and a primary supporting member like that shown in Fig. 6 is now presented.

In front of the toe of the flat bar stiffener there is a stress concentration area where a crack is likely to initiate. Indeed, in many inspections the presence of transverse cracks on such a spot has been detected. Therefore, a finite element model has been set up with very fine mesh right in the region of the expected crack growth line (Fig. 7).

First, the nominal stresses σ_o are determined on the uncracked body at the expected positions of the crack tip.

The second phase implies the assessment of the shape function $\beta(a)$ through systematic FEM analyses with reference to different crack lengths.

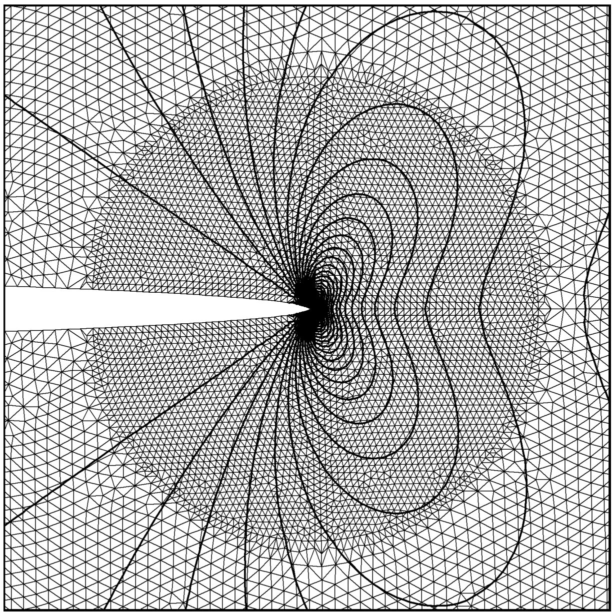

Fig. 7 – FEM model near the crack tip, and isostress contour lines.

For a given length a, a plot like that of Fig. 8 shall be drawn: each point represents the value of the parameter $\beta_i^* = K_i^*/\sigma_o \sqrt{\pi\, a}$, where K_i^* is determined through equation (10) making use of the displacement δ_{o_i} as derived from the FEM analysis for the facing nodes along the crack edges.

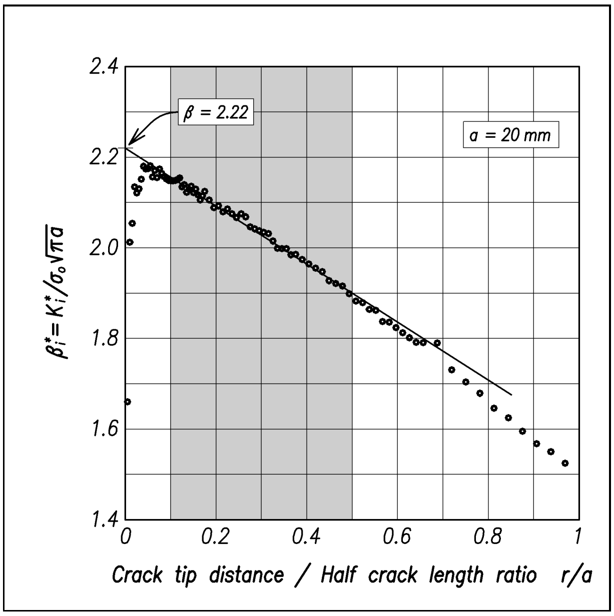

Fig. 8 – Assessment of β for a given crack length.

The actual value of β, corresponding to the given crack length, is attained through a linear extrapolation that shall be performed considering only the region near the crack tip (for example, r/a ranging from 0.1 to 0.5) excluding the points very close to the crack tip, since their values are unreliable because they are strongly influenced by the mesh size. As a matter of fact, in order to get reliable values for such points it would be necessary to resort to even smaller elements than the ones used (that are of about 0.1 mm within a radius of 3.5 mm around the tip), but such a procedure would not give a better final result.

The result of these systematic analyses is a set of β values that are a function of the crack length. Fig. 9 shows the marks ($\triangle$) that represent such β values, along with the relevant 4th order fitting curve. In the same figure are also plotted the values of the stress intensity factor K (marks $\circ$) derived for the particular load case that has been analyzed.

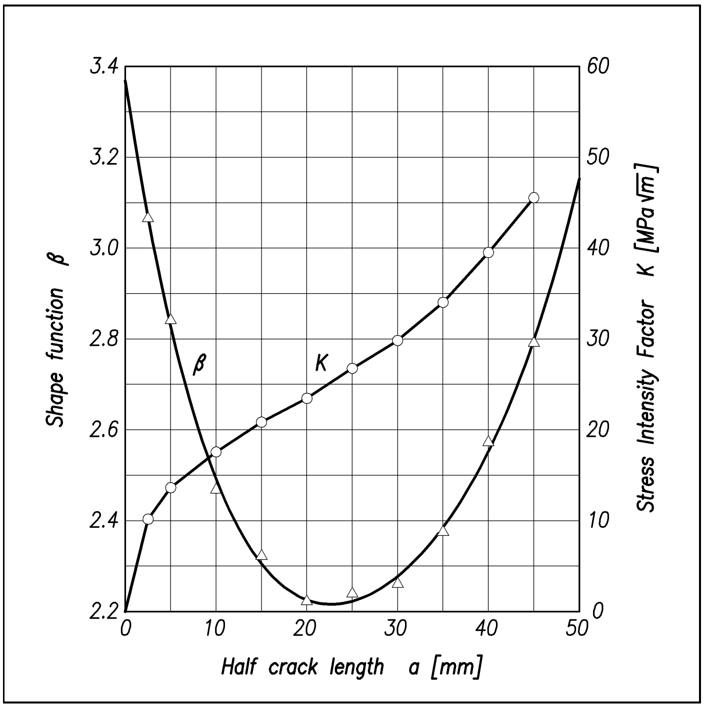

Fig. 9 – The β function for the considered detail, and the K values corresponding to a particular load case.

The analysis outlined above is preparatory to assessing the crack growth as a function of the applied loads (in terms of stress levels, and of number of cycles).

In the case here considered, the stress ratio of the applied load is $R = -1$, and the crack growth is estimated making use of Paris-Erdogan's law considering only the tensile phase (i.e., $\Delta K_{eff} = \Delta K/2$). Reference has been made to the equations suggested by the International Institute of Welding (IIW), and by J.M. Barsom. Specifically, for ferritic steels the equations proposed (units in MPa, m) are:

$$\text{IIW:} \qquad \frac{da}{dN} = 9.5 \cdot 10^{-12}\,(\Delta K)^{3.0} \tag{11}$$

$$\text{Barsom:} \qquad \frac{da}{dN} = 6.9 \cdot 10^{-12}\,(\Delta K)^{3.0} \tag{12}$$

From these expressions the two crack growth curves for the case in question are then calculated. In Fig. 10 the two curves obtained with reference to a detected initial crack length $a_i = 0.5$ mm are plotted.

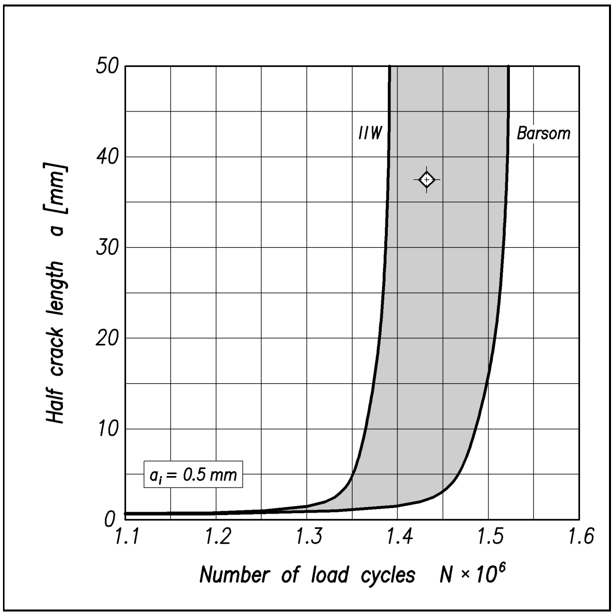

Fig. 10 – Crack growth curves.

These curves make it possible to determine the number of load cycles beyond which the crack growth rate leads to the final collapse of the structure. This means that, when a crack of a certain length has been detected, the procedure outlined here makes it possible to estimate its future behaviour, and further surveys and overhauling can be properly planned.

It is worth pointing out that the two curves, even though different, have in common the slow growth phase, and the critical phase initiates from points that are not so far apart (even if the unstable behaviour is predicted at different number of cycles).

From a practical point of view, it is more important to know when the crack growth rate becomes dangerous than to know exactly when the final collapse will take place, so in effect both curves give quite similar information.

Experimental data are also available for the case presented here: the mark inside the shaded area of Fig. 10 represents the failure point of the tested specimen.

CONCLUSIONS

The present work has been developed bearing in mind that ship structures are subjected to periodic surveys. During the surveys special attention should be given to the hot spot areas of all crucial structural members.

When a fatigue crack is detected, it is important to know what its future behaviour will be. The procedure outlined above – based on concepts of Fracture Mechanics – makes it possible to plan, on a sound basis, possible maintenance or replacement measures.

The whole procedure has been methodically described in all its steps, and practical suggestions have been given. Even more reliable assessments will certainly be possible when more experimental results concerning specimens of materials and thicknesses typical of ships structures become available.

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