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ASSICURAZIONE E FINANZA: MATEMATICA E GESTIONE

Development of a pricing methodology for CDS on Small and Medium Enterprises

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ABSTRACT

The present thesis is aimed to investigate the possibility to expand the hedging purpose of Credit Default Swaps (CDS), to hedge credit risk on trade credits of Small and Medium (industrial) Enterprises (SME).

Recent years of severe economic crisis have led to some deep changings in the practices of credit management within industrial entities. Year after year, such entities have stressed their attention on credit risk management.

At the end of the chain of credit risk management we find Trade credit insurance, which is a fast growing sector, with raising interest by the industrial entities.

Conceptually, Trade credit insurance's purpose is not quite different than the purpose of a Credit Default Swap (i.e. hedging the buyer from the insolvency risk of an underlying entity), so it would be its most direct competitor.

The main issue in extending the pricing of CDS to SME is represented by the fact that SME, in general terms, are neither listed nor rated and do not issue debt instruments. This entails the fact that, for such entities, risk neutral Probabilities of Default, needed by standard pricing models, cannot be retrieved from market traded instruments.

To overcome such issue, we constructed an "equivalent risk neutral PD". The approach was similar to the Radon-Nikodym theorem: we defined a corrective factor to be applied to the "real world PDs", in order to obtain an equivalent risk neutral Probability of Default. Obviously, what obtained is a proxy.

Real world PDs for SME were calculated starting by the databases put into our disposal by modeFinance Srl, which is a registered credit rating agency, specialized in the creditworthiness evaluation of Small and Medium Enterprises. On the other hand, CDS risk neutral default probabilities were bootstrapped from real CDS trades, by different maturities and rating classes. The data source was in this case Bloomberg.

Being such hypothetical CDS a new instrument, there is no real benchmark to state whether the obtained spreads resulted into a fair cost, or not.

So we first applied the developed pricing model to a set of 1,000 Italian SME. The results obtained were quite positive, since the average SME-CDS spread obtained by rating class and maturity, other than being monotonically increasing with rating class worsening and maturity increasing, was also in average 42% higher than the (average) spreads observed for real traded CDS, as common sense would suggest it had to be.

As a final step we compared the obtained cost of these new instrument with the cost of 4 standard Trade credit insurance policies (which were kindly disclosed, in anonymous form, by the insurance broker Willis Italia S.p.A.).

Even if CDS and Trade Credit Insurance may have the same conceptual goal (to protect from default risk of an underlying entity), as instruments they are radically different and such cost comparison of the two is not quite straightforward.

In order to make the comparison more plausible, we made two hypotheses on how to calculate the equivalent CDS cost of such trade credit insurance policies.

For all the analyzed policies, the real policy cost was lying between the two CDS cost proxies.

In general terms, at the moment, it is hard to tell whether the CDS prices obtained with the model developed, could be defined "fair". Certainly, the results of the benchmarking tell us that they are at least plausible.

ABSTRACT (Italiano)

La presente tesi si propone di investigare la possibilità di estendere l'utilizzo dei Credit Default Swaps (CDS) alla copertura del rischio di credito sui crediti commerciali delle Piccole e Medie Imprese (PMI).

I recenti anni di crisi economica hanno portato a profondi mutamenti nelle pratiche di credit risk management implementate anche dalle piccole realtà industriali. Alla fine della catena del processo di credit risk management si trova l'assicurazione crediti. Questa, peraltro, sta riscuotendo sempre maggior interesse nel mondo industriale.

Concettualmente, il fine dell'assicurazione crediti non è distante da quello di un Credit Default Swap (ossia proteggere l'acquirente dal rischio di default di una società di riferimento), quindi ne risulterebbe il più diretto competitor.

Il principale problema nell'estendere lo sviluppo di CDS alle PMI (quali società di riferimento) è rappresentato dal fatto che queste ultime, in generale, non sono né quotate né emettono strumenti di debito. Ciò comporta l'impossibilità di ricavare, per tali società, delle probabilità di default neutrali al rischio, che stanno alla base dei modelli di pricing standard dei CDS.

Per superare tale problematica, è stata costruita una "probabilità risk-neutral equivalente". L'approccio utilizzato è simile a quanto espresso dalla derivata di Radon-Nikodym: è stato definito un fattore correttivo da applicare alle PD "real-world" al fine di ottenere un'equivalente "risk-neutral". Ovviamente si tratta di un'approssimazione.

Le PD reali per le PMI sono state calcolate a partire dai dati messi a disposizione da modeFinance Srl, che è un'agenzia di rating registrata e specializzata nella valutazione del merito creditizio delle PMI.

Per quanto riguarda le PD neutrali al rischio, queste sono state ottenute dagli spread dei CDS quotati (clusterizzati per classe di rating e scadenze). In questo caso la fonte dati utilizzata è stata Bloomberg.

Essendo questi ipotetici CDS (su PMI) uno strumento del tutto nuovo, non esiste un benchmark che consenta di stabilire se gli spread ottenuti siano equi, o meno.

Di conseguenza il modello sviluppato è stato inizialmente applicato ad un insieme di 1.000 PMI italiane. I risultati ottenuti sembrano positivi. Gli spread medi ottenuti per le diverse classi di rating e scadenze, oltre che essere risultati monotoni crescenti al peggiorare della classe di rating e all'aumentare delle scadenze, sono anche risultati, in media, il 42% più alti degli spread medi osservati per i CDS "reali". Ciò è concorde con quanto ci si potesse aspettare.

Come step finale, è stato calcolato il costo equivalente in CDS di 4 polizze di assicurazione crediti (gentilmente messe a diposizione, in forma anonima, dal broker assicurativo Willis Italia S.p.A.).

Sebbene gli scopi concettuali dei CDS e dell'assicurazione crediti non siano dissimili (ossia proteggere dal rischio di credito di una società di riferimento), come strumenti essi sono in realtà radicalmente diversi, ed il loro confronto non è per nulla scontato.

Al fine di rendere tale confronto il più plausibile possibile, sono state formulate due ipotesi su come calcolare il costo equivalente in CDS di tali polizze assicurative.

Per tutte le polizze analizzate, il loro costo reale si posizionava sempre in mezzo ai costi ottenuti dalle due ipotesi suddette.

In generale, al momento, è arduo dire se gli spread (e risultante costo) ottenuti col modello sviluppato, possano essere definiti "equi". Di certo i risultati ottenuti ci suggeriscono che siano per lo meno plausibili.

INTRODUCTION

GENERAL OVERVIEW

The present thesis is aimed to investigate the possibility of developing a new financial tool or, to be more precise, to expand the hedging purpose of an existing tool, which are Credit Default Swaps.

Recent years of severe economic crisis have led to some deep changings in the practices of credit management within industrial entities.
Especially Small and Medium Enterprises (SME) have faced, simultaneously, a credit crunch on the financial side and, on the commercial side, increasing difficulties in collection of credits, with consequent negative reflection on cash cycle and, more widely, entailing a domino effect throughout their pertaining sectors.
Year after year firms have stressed their attention on credit risk management. Even Small-Medium enterprises have updated their practices to be more effective. The intervention has been 360° degrees wide, starting by monitoring credit performance of both clients' and suppliers' portfolios, to end with trade credit insurance.
It is exactly on this last aspect that the present work is focused on.

Trade credit insurance is a fast growing sector, with raising interest by the industrial entities.
Conceptually, credit insurance's purpose is not quite different than the purpose of a Credit Default Swap.
While a company that buys a credit insurance policy wants to insure its commercial credits against the potential insolvency of its clients, the buyer of a CDS wants to "insure" its (supposedly hold) bond against the issuer's insolvency. So, in the end, both the instruments are aimed to hedge their buyer from the insolvency risk of a "reference entity".
Even if so described the instruments seem to be already quite similar, there are some key differences that make not straightforward the extension of CDS hedging, to commercial credits for Small and Medium Enterprises.
In first place, after their issuance, CDS are traded on (Over The Counter) markets. Which entail the consequence that it is the market itself to price the contracts and, most of all, the probability of default (PD) of the reference entity (which is a key variable of the pricing methodology) is not the "real world" one, but the risk neutral PD, implied by the market price.
In general, SME entities, do not have any securities traded on any market, thing that makes it impossible to retrieve a risk neutral PD for such entities.
In second place, the maturity of a CDS is usually multi-year, while commercial credit insurance has, in essence, a validity of one year.

As previously stated, the purpose of the present work it is to investigate whether it would be possible an extension of the CDS, to hedging SME's trade credits.
In order to overcome the issues above stated, as several other issues, some assumptions and hypothesis have to be made.

The final step of the thesis will be to compare the price of these new instrument (obtained according to the assumptions and hypothesis made) with the cost of standard credit insurance and see if they could be comparable and competitive, or not.
We will see though, that even if CDS and Trade Credit Insurance may have the same conceptual goal: to protect from default risk of an underlying entity; as instruments, they are radically different and such cost comparison of the two is not quite straightforward.

What are CDS

Credit Default Swaps (CDS) are the most widely used credit derivatives and allow to negotiate-transfer credit risk. They are indeed contracts that offer protection against the insolvency (credit event) of a specific company/sovereign entity (defined as "reference entity").

The CDS buyer is (or is supposed to be, if the contract is not used for trading/speculation) a bond holder of the reference entity. The Face value of the bond is the 'notional principal' of the CDS. The CDS gives the buyer the right to sell at par the Bond of the reference entity in case a 'credit event' is triggered.
Credit events' definition is agreed between buyer and seller, at the time the trade is entered into and are part of the contract. The majority of single-name CDSs include the following credit events as triggers[1]: reference entity bankruptcy, failure to pay, obligation acceleration, repudiation and moratorium.
In the case of a credit event, the CDS seller will liquidate the contract through 'physical delivery' of the bonds, or in cash.
In the first case, the buyer of protection actually delivers a bond to the seller of protection for par. In the second case, a credit event auction is made, according to ISDA[2] global protocol, and it is set a price for cash settle: the *Cheapest To Delivery* price (CTD); the protection buyer receives the difference between the CTD bond value at the time of settlement and the bond's nominal value in cash.

In return for protection, the buyer has the duty to make to the seller periodical payments (quarterly/semiannual/annual), until maturity (usually five years) of the CDS or until a 'credit event' is triggered. The ratio between the annual amount that the buyer pays to the seller over the notional principal of the CDS is defined as the 'Spread' of the CDS.

After been issued, the market value of the CDS depends, among others, on the creditworthiness of its reference entity, being this commercial or sovereign.

Credit Derivatives are classified into two sub sets: "single name" or "multi name".
Credit Default Swap are the most widely used single name credit derivatives, meaning that they hedge against insolvency of a single reference entity.
While the most widely used multi-name contracts are the "Collateralized Debt Obligation" (CDO). In this case there is not a single credit "insured", but a whole portfolio.

A key element of CDS is the definition of "default" (i.e. the credit event). This depends also on countries regulations, usually, the definition of default includes operations of debts restructuration for European reference entities, does not include restructuration for North-American reference entities.

Highlights on the CDS history and market development

CDS invention is widely credited to Blythe Masters in 1994, while she was responsible for credit derivative products at J.P. Morgan.
In their first use, they were intended as a mean to transfer credit exposure for banks' commercial loans and so to free up regulatory capital.

By late 1990s, CDS were starting to be sold with the purpose of hedging corporate bonds and municipal bonds.

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[1] Source: http://www2.isda.org/asset-classes/credit-derivatives/2014-isda-credit-derivatives-definitions/
[2] International Swaps Derivative Association: http://www2.isda.org/

In the early 2000's CDS became increasingly popular: between 2002 and 2007 the gross notional amount outstanding grew from less than 2 trillion USD, to nearly 50 trillion USD[3]. This is due to the desire of financial institutions to manage credit risk, but also and largely, to the fact that CDS were now used with speculative purpose instead of hedging purpose. By the end of 2007, the CDS market had a notional value of $45 trillion, but the corporate bond, municipal bond, and structured investment vehicles market totaled less than $25 trillion[4].

2008 crisis has revealed some shortcomings in CDS markets, first of all an insufficient information regarding the existent open positions.

A clear example of this is Lehman Brothers case: after the collapse of the bank it indeed turned out that the net exposure relative to Lehman was only a small fraction of the gross amount. As a consequence, the potential losses linked to Lehman as a reference entity were largely overstated. On the other hand, Lehman was also a counterparty issuing CDS, and the replacement costs of Lehman as a counterparty turned out to be higher than the credit losses induced by CDS written on Lehman.

Insufficient transparency was probably one of the main reasons for the market over-reaction to Lehman's default.

Immediately after the financial crisis, regulators and sector players started working for defining greater transparency and measures to contain contagion effects in event of important defaults. Such measures include central clearings (a clearing house stands between two parties in a trade, ensuring a deal is completed in the event of a default) and netting processes (for each counterparty it is calculated the net exposure toward a reference entity). These transformations were aimed to translate from an OTC market, to a more-like regulated exchange market structure.

It has to be said that, meanwhile the crisis' fright was fading, such good proposals were fading as well. As reported by the Financial Times by July 2013, "*for now CDS remain the purview of banks. The first credit futures traded on exchange were launched just a fortnight ago – via IntercontinentalExchange – while clearing of OTC trades remains the exception. Around $2.5tn of transactions had been cleared by December 2012, compared to the notional outstanding for the CDS market of $25tn*".

Major players in the CDS market

After 2008 crisis the -already high- concentration of CDS dealers had further increased as the consequence of major participants running out of the business (e.g. Bear Stearns, Lehman Brothers and Merrill Lynch). According to a Fitch survey (2009) the 5 largest CDS dealers were responsible for 88% of the total notional amount bought and sold.

Such five dealers were JPMorgan, the Goldman Sachs Group, Morgan Stanley, Deutsche Bank and the Barclays Group[5].

As per the kind of companies dealing with CDS, there are both banks and insurance companies. The first act on both side of the market as sellers of protection but also as buyers. Banks buy CDS mainly for managing the riskiness of their own loan portfolios.

[3] Deutsche Bank Research, "*Credit default swaps, Heading towards a more stable system*". December 2009.
[4] Robins Kaplan LLP "Credit Default Swaps: From Protection To Speculation", September 2008.
[5] European Central Bank, CREDIT DEFAULT SWAPS AND COUNTERPARTY RISK, August 2009.

Insurance companies (such as Ambac, MBIA, or AIG), provide credit protection but have limited activities as buyers[6].

Literature review

At the beginning of the thesis development (November 2014), a thorough literature review has been conducted aimed to understand whether a similar work had been done before.

Several publications, textbooks and papers on CDS topics were analyzed (see Bibliography) but nothing similar to the goal of this thesis had been found.

The only topic that had some adherence with the present work, was the transformation of real-world probabilities of default into risk-neutral probabilities of default.

Such topic was analyzed in a publication of CONSOB (the Italian securities and exchange commission): "Probabilità reali e probabilità neutrali al rischio nella stima del valore futuro degli strumenti derivati", August 2013.

They were suggesting, as a mean to change the probability measure, to employ the Radon-Nikodym derivative.

For detailed information we remand to Appendix D. Here we limit ourselves stating that they were suggesting to define a variable Z which is the ratio between the risk-neutral probability measure and the real-world one. They demonstrated that the expected value of a random variable Y, under the risk neutral probability measure, is the same as the expected value of the transformed variable ZY under the real world probability measure.

In the end, this is quite similar to what has been done in the present work.

[6] Deutshce Bank Research, "*Credit default swaps, Heading towards a more stable system*".

Chapter 1

CREDIT DEFAULT SWAPS

"A Credit Default Swap (CDS) is a form of insurance against the default of a debt issuing entity. This can be a corporation, a municipality, or sovereign state. The protection lasts for a specified period (e.g. five years), and if the reference entity defaults in this period, the protection buyer receives a payment from the protection seller. In return, the buyer of protection makes regular (e.g. quarterly) premium payments to the protection seller"[7].

The most widely recognized reference institution about credit derivatives is the ISDA (International Swaps and Derivatives Association). ISDA pursues the goal of standardize as much as possible the over-the-counter (OTC) derivatives.

In the literature about CDSs, several pricing models have been presented. Anyway, in the world of practitioners, the market standard is represented by the *hazard rate method* of pricing credit default swaps. ISDA itself has proposed a (hazard rate) pricing model for CDS, but has never released (at least until this pages are being written) an exhaustive documentation on the model proposed.

The pricing model adopted for developing the present thesis-work, and here presented, is a market standard adopted by banks and already "codified" by Mathworks® into its Matlab® programming language.

The theoretical descriptions and formulas hereafter presented are retrieved by a paper by Dominic O'Kane and Stuart Turnbull (Valuation of Credit Default Swaps, *Lehman Brothers | Quantitative Credit Research*, April 2003).

How do CDSs work

Credit Default Swaps (CDS) are the most widely used credit derivatives and allow to negotiate-transfer credit risks. They are intended to insure bond holders against the default of the bond issuer.

Credit Derivatives are classified into two main sub sets: "single name" or "multi name".

The most diffused single name Credit Derivatives are Credit Default Swaps. In CDS, the payoff depends on the "credit behavior" of the single reference entity on which the CDS is issued.

The most widely used multi-name contract is the "Collateralized Debt Obligation" (CDO). In this case there is not a single reference entity, but a whole portfolio.

There are two subjects on the sides of the contract: the protection "buyer" and the protection "seller". If the reference entity does not meet its commitments on the outstanding debt, the protection seller has to cash the buyer a certain amount.

In case of multiname Credit Derivatives, payments to buyers are made according to a very complex regulation.

[7] R. White, The Pricing and Risk Management of Credit Default Swaps, with a Focus on the ISDA Model. OpenGamma Quantitative Research, September 2013.

The "**Spread**" of a CDS is defined as the ratio between the annual amount that the buyer pays as premium to the seller, over the notional value of the CDS.

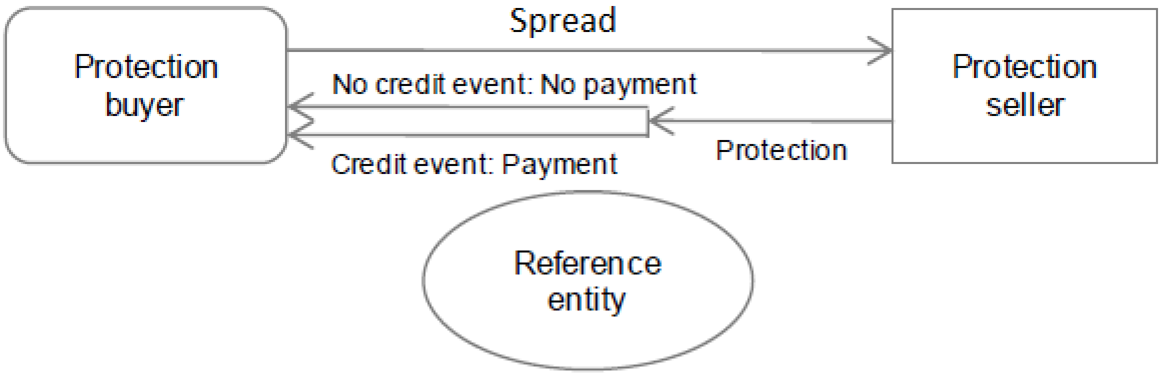


Fig. 1: CDS working schema

Usually payments are made quarterly and the classical maturity of a CDS is 5 years. Off course, there are many different maturities and payments' periods.

CDSs are settled when and if a "*credit event*[8]" triggers the compensation payment (otherwise they simply expire). The settlement can be "physical" or "cash".

In case of physical settlement**,** the protection buyer has to deliver the underlying bond in return for its par value.

In case of cash settlement, the protection buyer receives the difference between the (cheapest to delivery) bond's value at the time of settlement and the bond's nominal value in cash.

A key element of CDSs is the definition of *credit event*. Usually, this includes operations of debts restructuration for European reference entities, while does not include restructuration for North-American reference entities.

The standardized definitions of credit events, and all related legal aspects go beyond the scope of this thesis and will not be treated. Anyhow, some definitions of *credit event* are presented in the "Glossary" chapter, while more detailed information can be found at: http://www.isda.org/publications/isdacredit-deri-def-sup-comm.aspx#isdacrd14.

In this thesis we will focus on single name CDS. The following theory and formulas are retrieved by a publication by Dominic O'Kane and Stuart Turnbull (Valuation of Credit Default Swaps, *Lehman Brothers | Quantitative Credit Research*, April 2003).

CDS pricing

In a CDS contracts, there are two parties: the *protection seller* and the *protection buyer*. The seller has duty to cover the loss (face value of some asset) incurred by the protection buyer in the case a *credit event* is triggered.

[8] Refer to "Glossary" chapter for definitions.

In order to pay for such protection, the buyer makes to the seller a stream of regular payments[9]. Such stream of payments is defined as the *premium leg* (Fig. 2). Payments continue until a credit event, or until maturity, whichever occurs first.

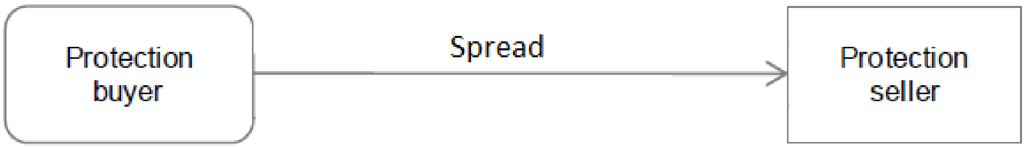


Fig. 2: premium leg

If a *credit event* occurs before maturity, the protection seller has to cash the buyer either the difference between par value and the price of the cheapest to deliver asset of the reference entity (cash settlement), or the face value of such asset, in return for its physical delivery by the protection buyer (physical delivery settlement).

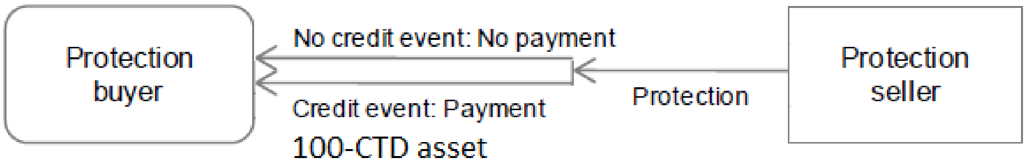


Fig. 3: protection leg

Remark: In case of credit event, the protection buyer receives the protection payment as above described, but is required (for standard single name CDS contracts) to pay the protection seller the premium accrued between the last premium payment and the date of the credit event (accrued premium). For sovereign-linked default swaps there may be no payment of premium accrued.

Example

(*retrieved by Valuation of Credit Default Swaps Marking default, Lehman Brothers | Quantitative Credit Research, April 2003*)

Suppose a protection buyer purchases 5-year protection on a company at a default swap spread of 300bp. The face value of the protection is $10 million. The protection buyer therefore makes quarterly payments approximately equal to $10 million × 0.03 × 0.25 = $75,000. Assume that after a short period the reference entity suffers a credit event and that the CTD asset of the reference entity has a recovery price of $45 per $100 of face value. The payments are as follows:

• The protection seller compensates the protection buyer for the loss on the face value of the asset hold by the protection buyer. This is equal to $10 million × (100% – 45%) = $5.5 million.

• The protection buyer pays the accrued premium from the previous premium payment date to time of the credit event. For example, if the credit event occurs after a month then the protection buyer pays approximately $10 million × 0.03 × 1/12 = $18,750 of premium accrued.

[9] Standard CDS foresee quarterly payments at IMM dates.

Modeling credit event using a reduced-form approach

Unlike bonds, CDS to be valued need the use of a term structure of default swap spreads, a recovery rate assumption and a model for valuing the survival probabilities of the reference entity.

The field of credit modeling is essentially divided into two sub sets: structural models, whose forefather is the Merton model (1974) and all its following variations, and the reduced form models.

For structural models the definition of credit event is tied to the fundamentals of the company analyzed, with the consequence that such models frequently require the valuation of the balance sheets of the company. Also, due to the low-frequency of balance sheet publishing, these models generally lack the flexibility to fit well a term structure of spreads, therefore they cannot be easily extended to the pricing of credit derivative instruments.

In the reduced form models, the fundamentals of the company are not considered for modelling the process of credit events. The probability of the credit event is modeled directly. Such pricing models allow to extract the probability of default directly from market prices. Reduced form models have the flexibility to fit the prices of a variety of credit instruments, also of more exotic kinds. For such reason they are the most widely diffused models for credit derivative pricing.

The majority of reduced-form models are based on the work of Jarrow and Turnbull (1995), who characterize a credit event as the first event of a Poisson counting process, which occurs at some time *t* with a probability defined as:

$$Pr[\tau < t + dt | \tau \geq t] = \lambda(t)dt \tag{1}$$

According to such process, the probability of defaulting within the time interval [t,t+dt), conditional on surviving to time t, is proportional to some time dependent function λ(t), known as the hazard rate, and the length of the time interval dt.

In other words, we can think of modelling (one period) default as a simple binomial tree, with default probability λ(t) and survival probability 1- λ(t).

Within this model it is made the simplifying assumption that the hazard rate process is deterministic, which entails that the hazard rate is independent of interest rates and recovery rates.

Note that for almost all market participants, these assumptions are acceptable, as their pricing impact is well within the typical bid-offer spread for credit default swaps[10].

[10] O'Kane-Turnbull Valuation of Credit Default Swaps Marking default, Lehman Brothers | Quantitative Credit Research, April 2003.

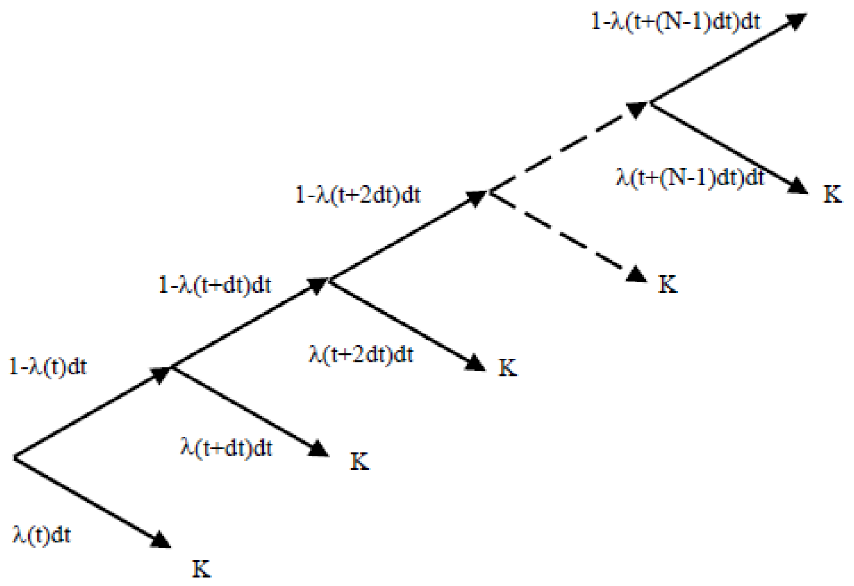


Fig. 4 source: Valuation of Credit Default Swaps Marking default, Lehman Brothers | Quantitative Credit Research

Figure 4 above, shows the extension of the model to multiple time periods, where K indicates the payoff in event of default.

In the extension of the model to continuous time, it can be shown[11] that the survival probability Q, to time T, conditional to surviving until time t_v (considering the limit with dt->0) has the form:

$$Q(t_v,T) = exp\left(-\int_{t_v}^{T} \lambda(s)\,ds\right) \tag{2}$$

Such process for the survival probability will be used in the following paragraphs to value both the premium and the protection leg of a CDS, hence for calculating the break even spread of the default swap. It will also be shown as the process allows to retrieve the risk neutral (arbitrage-free) survival probabilities from market spreads.

Valuing the premium leg

The *premium leg* consists of the series of payments made by the protection buyer until maturity, or until the *credit event* (whichever occurs first). The premium leg also takes into account the (eventual) partial premium accrued between the last premium paid and the (eventual) credit event (we remind that in standard corporate single name CDS, the protection buyer is required to pay for the premium accrued between the last premium payment date and the credit event date).

Assume that there are n=1,...,N contractual payment dates $t_1,\dots,t_N$ where t_N is the maturity date of the default swap. Denoting the contractual default swap spread by $S(t_0,t_N)$ and ignoring premium accrued, we can write the present value of the premium leg of an existing contract as:

[11] See Appendix 'A'. Private communication with S. Ziraldo, PhD

$$Premium\ Leg\ PV(t_{V,}t_N) = S(t_{0,}t_N) \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B\ ) Z(t_V, t_n) Q(t_V, t_n) \tag{3}$$

Where:

- $\Delta(t_{n-1},t_n,B)$ is the day count fraction between premium dates t_{n-1} and t_n in the appropriate basis convention denoted by B;
- $Q(t_V,t_n)$ is the risk-neutral survival probability of the reference entity from valuation time t_V to premium payment time t_n. This factors into the pricing the risk that a reference entity will not survive to a premium payment time;
- $Z(t_V,t_n)$ is the discount factor from valuation date to premium payment date n. Before 2008 crisis, Libor was the standard as a discount curve, after 2008 a variety of discount curves has been developed.

Equation (3) ignores the effect of the partial premium accrued in case of a credit event.

To "price" the accrued premium in event of default, it has to be considered, for every coupon payment interval, the probability of defaulting in each of these periods.

To do so, we have to:

1. Consider each premium accrual period starting at t_{n-1} with the payment date at t_n;
2. Determine the probability of surviving from the valuation date t_V to each time s in the premium period and then defaulting in the next small time interval ds. The probability of this is given by $Q(t_V,s)\lambda(s)ds$;
3. Calculate the accrued payment since the previous premium date to each time;
4. Discount this payment back to the valuation date, using the appropriate discount factor;
5. Integrate over all times in the premium period. Strictly speaking, this is a discrete daily integration since premium payments are only calculated on a daily basis. However, for mathematical simplicity we tend to approximate this as a continuous integral. The difference is essentially negligible;
6. Sum over all premium periods from n=1 to the final premium n=N.

The resulting expression is the following:

$$S(t_{0,}t_N) \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \Delta(t_{n-1}, s, B) Z(t_V, s) Q(t_V, s) \lambda(s) ds \tag{4}$$

where:

- $\Delta(t_{n-1},s,B)$ is the day count fraction between premium dates t_{n-1} and s in the appropriate basis convention denoted by B;
- $Q(t_V,s)$ is the risk neutral survival probability of the reference entity from valuation time t_V to time s;
- λ (s) is the hazard rate of the probability process (i.e. default intensity);
- $Z(t_V,s)$ is the discount factor from valuation date to premium payment date s;
- $S(t_0,t_N)$ is the contractual spread.

O'Kane and Turnbull (2003) have demonstrated that it is possible to approximate this equation with:

$$\frac{S(t_{0,}t_N)}{2}\sum_{n=1}^{N}\Delta(t_{n-1},t_n,B\,)\,Z\,(t_V,t_n)\big(Q(t_V,t_{n-1})-Q(t_V,t_n)\big) \tag{5}$$

Where it has been made the assumption that the eventual accrued premium is half the normal premium. This because, if a default occurs between two coupon dates, the average accrued premium will be half the full premium due at the end of the premium period.

By merging (3) and (5) we obtain the value of the premium leg:

$$S(t_0,t_N)\times RPV01 \tag{6}$$

Where RPV01 is defined as the "Risky Present Value of 01 bp of spread":

$$RPV01=\sum_{n=1}^{N}\Delta(t_{n-1},t_n,B)Z(t_V,t_n)\left[Q(t_V,t_n)+\frac{1}{2}\big(Q(t_V,t_{n-1})-Q(t_V,t_n)\big)\right] \tag{7}$$

REMARK: the contribution of the accrued premium to the spread is very small, but not negligible: for large contractual spreads it can be wider than the bid-offer spread, hence cannot be ignored.

Valuing the protection leg

The *protection leg* consists of the payment of Notional*(1-R) that the protection seller has to make following a credit event, where R is the expected recovery rate of the underlying bond (even if in reality it will be the Cheapest To Delivery obligation under protection at the time of the credit event). Note that in the following formulas the Notional is omitted, assumed to be unitary.

It is assumed that the payment of the protection leg is made immediately after the credit event.

Within the *hazard rate* approach, it has to be precisely taken into account the timing of the credit event, which can have a significant impact on the present value of the protection leg. We can solve the timing problem by conditioning on each small time interval [s,s+ds] between time t_V and time t_N at which the credit event can occur.

The steps are the following:

1. Calculate the probability of surviving to some future time s which equals $Q(t_V,s)$;
2. Compute the probability of a credit event in the next small time increment ds which is given by $\lambda(s)$*ds;
3. At this point an amount (100% – R) is paid, and we discount this back to evaluation day at the risk-free rate $Z(t_V,s)$;
4. We then consider the probability of this happening at all times from $s = t_V$ to the maturity date t_N. Strictly speaking the timing of a credit event should not be resolved to less than a day. However, assuming that a credit event can occur intra-day has almost no effect on the valuation while simplifying the expression.

The above steps result in the following formula:

$$(1-R)\int_{t_v}^{t_N} Z\,(t_V,s)\,Q\,(t_V,s)\lambda(s)ds \tag{8}$$

where:

- $Q(t_V,s)$ is the survival probability of the reference entity from valuation time t_V to time s;
- $Z(t_V,s)$ is the discount factor from valuation date to time s;
- R is the expected recovery rate of the CTD asset at the time of the credit event;
- $\lambda(s)$ is the default intensity at time s.

'O kane and Turnbull (2003) have demonstrated that it is possible to assume that the credit event can only occur on a finite number M of discrete points per year, without any loss of material accuracy. Such assumption turns (8) into a –easier to solve– discrete formula:

$$(1-R)\sum_{m=1}^{M\,x\,t_N} Z(t_V,t_m)\left(Q(t_V,t_{m-1})-Q(t_V,t_m)\right) \tag{9}$$

Where, for a maturity t_N, it has been assumed that a credit event can occur M times in a year. The lower M the fewer the calculations, but the lower the accuracy.

O'Kane and Turnbull (2003) have shown that, assuming M=12, (corresponding to monthly intervals), the level of accuracy is well inside the typical bid-offer spread.

The breakeven CDS spread

The breakeven CDS spread is such as:

PV of Premium Leg = PV of Protection Leg

Which in the end is given by:

$$S(t_V,t_N)=\frac{(1-R)\sum_{m=1}^{M\,x\,t_N} Z(t_V,t_m)[Q(t_V,t_{m-1})-Q(t_V,t_m)]}{RPV01} \tag{10}$$

In the above formula, the recovery rate R can be assumed, the discount factors $Z(t_v,t_m)$ can be retrieved from the market instruments/Curves. There are, though, up to $M*t_N$ survival probabilities $Q(t_v,t_m)$ that clearly cannot be retrieved by the above equation (even if the spread were to be known from the market).

Obviously, some simplifying assumptions about the term structure of survival probabilities have to be made.

Building a hazard rate term structure

The modeling assumption for the hazard rate term structure, most widely accepted in the market, is a piecewise flat function of maturity time.

A piecewise linear assumption makes little difference and only if we do not have quoted spreads for many maturities and the curve is steeply sloped.

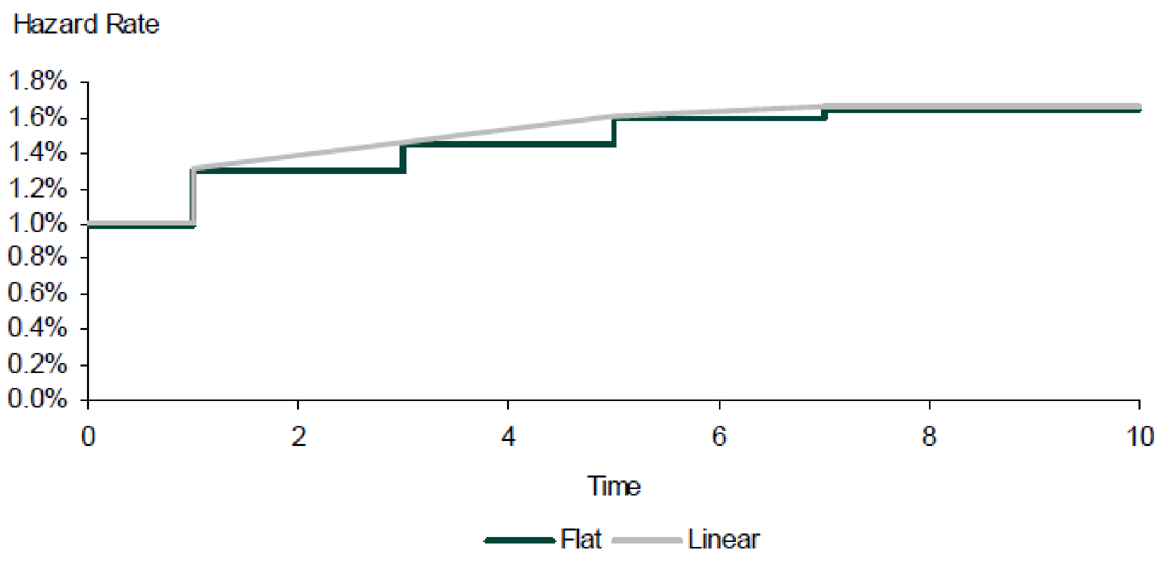


Fig. 5 source: Valuation of Credit Default Swaps Marking default, Lehman Brothers | Quantitative Credit Research

Having 1Y, 3Y, 5Y, 7Y and 10Y default swap spread values, would turn into have a hazard rate term structure with five sections $\lambda_{0,1}$, $\lambda_{1,3}$, $\lambda_{3,5}$, $\lambda_{5,7}$ and $\lambda_{7,10}$, as shown in Fig. 5

Bootstrap of the hazard rate term structure

The process of bootstrapping starts by retrieving, from the market, the spread of the shortest maturity contract available and using it to calculate the first survival probability, i.e. the 1Y default swap spread has to be used to calculate the value of $\lambda_{0,1}$.

Assuming a quarterly premium payment frequency, using a value of M=12, and assuming that premium accrued is not paid, this is achieved by solving:

$$\frac{S(t_V, t_V + 1Y)}{1 - R} \sum_{n=3,6,9,12} \Delta(t_{n-3}, t_n, B) Z(t_v, t_n) e^{-\lambda_{01}\tau_n} = \sum_{m=1}^{12} Z(t_v, t_n)\left(e^{-\lambda_{01}\tau_{m-1}} - e^{-\lambda_{01}\tau_m}\right) \qquad (11)$$

The equation (11) has to be solved numerically (with 1-dimensional root searching algorithm).

Once retrieved the hazard rate $\lambda_{0,1}$ (hence the survival probability, cfr. eq. (2)), this will be used to retrieve the hazard rate $\lambda_{1,3}$ from the market spread of the three years CDS.

The process is then repeated until the maturity of the evaluated contract is reached.

Defining τ=T-t_V, the survival probabilities Q(t_v,T) are given by:

$$Q(t_v, T) = \begin{cases} \exp(-\lambda_{0,1}\tau) & \text{if } 0<\tau<1 \\ \exp(-\lambda_{0,1} - \lambda_{1,3}(\tau - 1) & \text{if } 1<\tau<3 \\ \exp -\lambda_{0,1} - 2\lambda_{1,3} - \lambda_{3,5}(\tau - 3)) & \text{if } 3<\tau<5 \\ \exp(-\lambda_{0,1} - 2\lambda_{1,3} - 2\lambda_{3,5} - \lambda_{5,7}(\tau - 5)) & \text{if } 5<\tau<7 \\ \exp(-\lambda_{0,1} - 2\lambda_{1,3} - 2\lambda_{3,5} - 2\lambda_{5,7} - \lambda_{7,10}(\tau - 7)) & \text{if } \tau>7 \end{cases}$$

For maturities higher than 10 years, hazard rate is assumed to be constant.

The hazard rates so retrieved are risk-neutral. This means that they are the hazard rates required by this model, in order to fit the market spreads (resulting in arbitrage-free pricings).

Risk-neutral hazard rates tend to be higher than those retrieved by historical data, because they include other non default-related pemia, as liquidity-risk premia, risk premia, etc…

Chapter 2

MODEFINANCE RATINGS AND PDS

As explained in the introduction of the present work, a first goal of the thesis is to study whether it is possible to extend the use of CDS as a tool to insure trade credits of Small and Medium Enterprises (SME). A second -conditional to the first- goal is to see if resulting price of such CDS is comparable against the classical trade credit insurance.

To do so, ratings of SME and their probabilities of default are needed and this are (usually) not provided by the “*big three*” rating agencies.

The ratings and (part of) the probabilities of default used in the present thesis, are those assessed by modeFinance Srl.

The information that follows are retrieved from modeFinance internal documentation.

About modeFinance

modeFinance is a registered Credit Rating Agency specialized in companies’ creditworthiness evaluation and financial consulting.

The company was established in late 2009 as a spin-off of the University of Trieste.

The innovative *MORE rating* model (MORE is the acronym of Multi Objective Rating Evaluation), developed by modeFinance, permitted the company to locate its headquarter in AREA SCIENCE PARK which is a multi-sector -worldwide known- Science and Technology Park (located in Friuli Venezia Giulia - Northeastern Italy).

Thanks to the MORE methodology for credit risk analysis and its massive database (gathering more than 20 million companies), modeFinance provides different products and services to satisfy every business need belonging to a modern credit and risk management.

By July 2015 modeFinance was registered by ESMA (European Securities and Markets Authority) as Credit Rating Agency.

modeFinance MORE rating

A credit rating is an opinion of the general creditworthiness of an obligor (issuer rating), or the creditworthiness of an obligor in respect of a specific debt security, or other financial obligation (issue rating), based on relevant risk factors.

A different probability of default is associated with each credit rating category (traditionally indicated by a coded rating scale).

modeFinance adopts a ten classes rating scale, explained in the following figure.

MORE Rating	Rating Macro class	Assessment
AAA	Investment Grade	The company's capacity to meet its financial commitments is extremely strong, The company shows an excellent economic and financial flow and fund equilibrium,
AA		The company has very strong creditworthiness, It also has a good capital structure and economic and financial equilibrium, Difference from 'AAA' is slight,
A		The company has a high solvency, The company is however more susceptible to the adverse effects of changes in circumstances and economic conditions than companies in higher rated categories,
BBB		Capital structure and economic equilibrium are considered adequate, The company's capacity to meet its financial commitments could be affected by serious unfavourable events,
BB	Non Investment Grade	A company rated 'BB' is more vulnerable than companies rated 'BBB', Furthermore the company faces major ongoing uncertainties or exposure to adverse business, financial, or economic conditions,
B		The company presents vulnerable signals with regard to its fundamentals, Adverse business, financial, or economic conditions will be likely to impair the company's capacity or willingness to meet its financial commitments,
CCC		A company rated 'CCC' has a dangerous disequilibrium on the capital structure and on its economic and financial fundamentals, Adverse market events and an inadequate management could affect with high probability the company's solvency,
CC	Distressed	The company shows signals of high vulnerability, In the event of adverse market and economic conditions, the company's strong disequilibrium could increase,
C		The company shows considerable pathological situations, The company's capacity to meet its financial commitments is very low,
D		The company has not any longer the capacity to meet its financial commitments

Figure 6 : modeFinance rating scale

The Multi Objective Rating Evaluation (MORE) model is essentially used to assess the level of distress of industrial companies by using data included in their financial statements.

The model adopts newly developed numerical methodologies, drawing together financial theory, data mining and engineering design methodologies. The heart of MORE is a multi-dimensional and multi-objective algorithm that produces a classification of each company, by taking into account any attributes (such as sector and country) characterizing a firm.

The MORE rating vision is to look at the fundamental economics of the company. The main idea is to evaluate the rating observing every aspect of the economic and financial behavior of the company: the better is the equilibrium between the different aspects, the better will be the final rating.

This is done studying, evaluating and aggregating the most important sections of the financial and economic behavior of a company as: profitability, liquidity, solvency, interest coverage and efficiency.

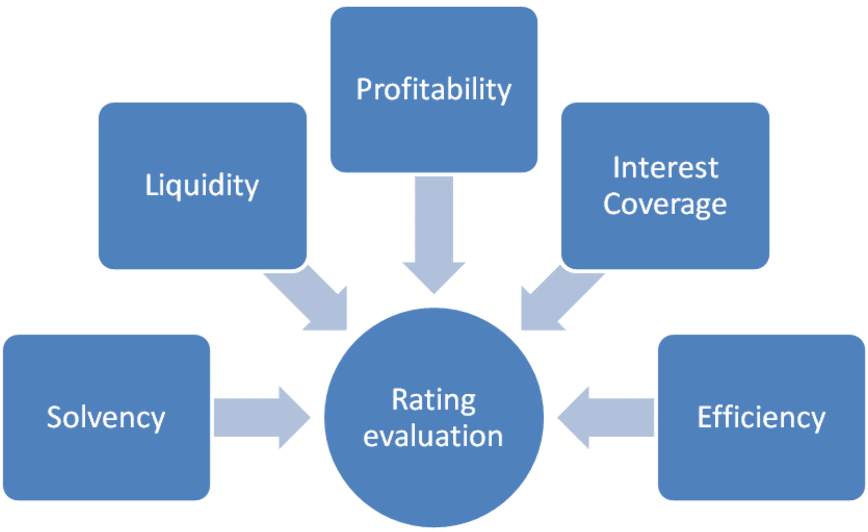


Figure 7 : MORE rating components

The MORE model is developed to take into account the differences between countries and economic sectors, for this reason the model is adapted according to different countries (worldwide) and 8 different economic sectors (Agriculture, Industry, Energy, Construction, Commerce, Service, Finance, Holding).

modeFinance default probability

The MORE model also allows to assess the probability of default, defined as the degree of certainty that the company will go into default, for each company by country and sector.

Considering the lack of information of defaulted firms for many countries, it has not been possible to assess a Probability of Default (as an output of the model) based on default frequency data and related frequency theories.

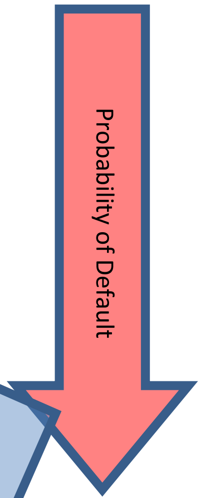
For these reasons it has been used the *transition matrix theory,* using the starting rating for each company in the database to evaluate how the rating changes over the years.

In the use of the rating transition matrix as a default model, default probabilities indicated in the last column of Tab. 1, are assigned to the companies belonging to the corresponding credit rating class of the first column.

Also to note that the definition of D (distressed) companies, includes companies rated CC, C, or D.

It is important to remind that probability of default has been evaluated for each country; this is because the probability of default could slightly change from country to country, depending on the countries' economic development level and legislation.

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	37,88%	43,94%	13,64%	1,52%	1,52%	0,00%	1,52%	0,00%
AA	1,21%	62,51%	27,83%	5,79%	1,24%	1,21%	0,22%	0,12%
A	0,01%	4,66%	62,46%	23,36%	5,09%	3,13%	0,97%	0,33%
BBB	0,00%	0,41%	14,96%	51,74%	23,81%	5,92%	2,32%	0,84%
BB	0,00%	0,05%	1,79%	13,79%	57,99%	18,78%	5,17%	2,43%
B	0,00%	0,03%	0,73%	3,34%	20,03%	55,85%	13,60%	6,42%
CCC	0,00%	0,05%	0,75%	2,67%	11,69%	32,44%	35,14%	17,26%
D	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%



Tab. 1 : transition matrix

MORE rating validation

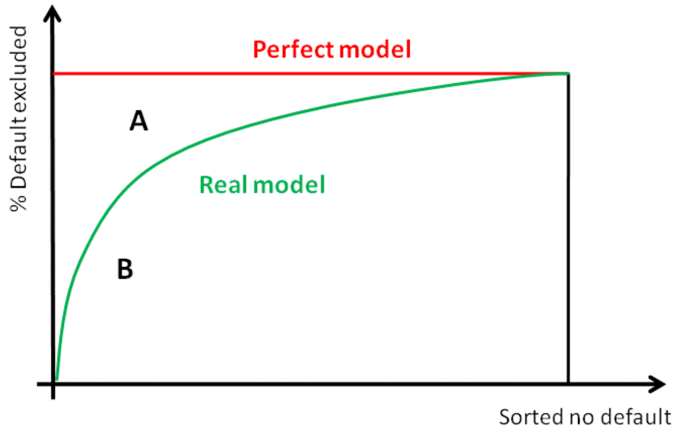
A rating model is effective if:

1. The model discriminates profitable companies from bankrupted companies (*Discriminating power of the model*);
2. For companies in bankruptcy, the assigned rating becomes worse and worse, approaching the default date (*Bankruptcy dynamics*);
3. The rating classes (from AAA to D) are consistent with the probability of default (*Evaluation of the probability of default*).

For the first two steps, and partially for the third, information of the bankrupted companies is needed.

Considering the availability of defaulted companies' data, a first validation of points 1 and 2 has been made on *Belgium, France, Italy*.

The most popular technique used to evaluate the discriminating power of rating model is the so called *Area Under Curve (AUC[12])* of a ROC curve.



AUC	Accuracy
[1.00; 0.90]	Excellent
[0.90; 0.80]	Good
[0.80; 0.70]	Adequate
[0.70; 0.60]	Poor
[0.60; 0.50]	Fail

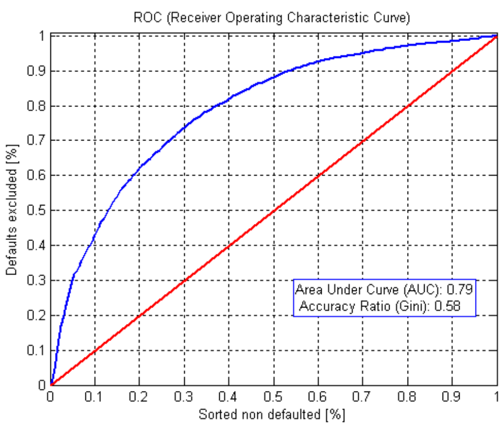
Fig. 8 ROC curve

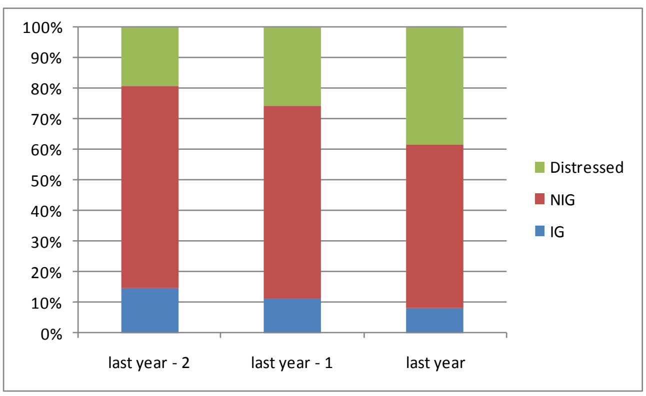
The ratio between the two areas B and A+B gives the AUC value: the closer the ratio is to 1, the more accurate the model is.

As per the bankruptcy dynamic, the rating distribution of only the bankrupted companies has been analyzed during the years N, N-1, N-2; where N is the last available year of financials.

The results for each country at study are displayed below (left side: discriminating power, right side: rating's dynamic).

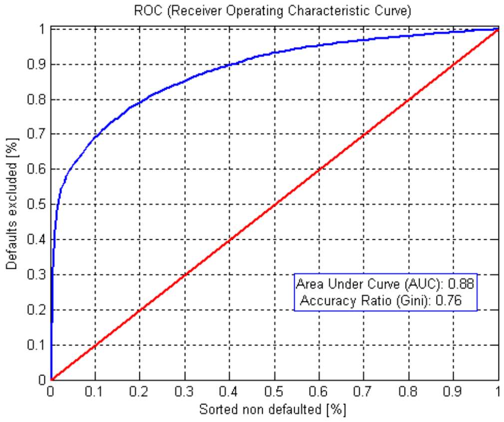
France:

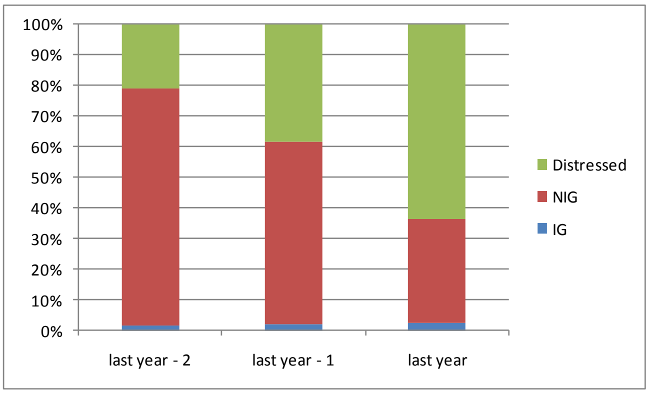




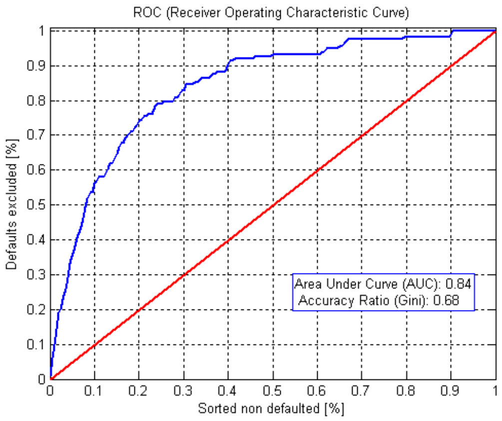
[12] For further information, see Appendix B.

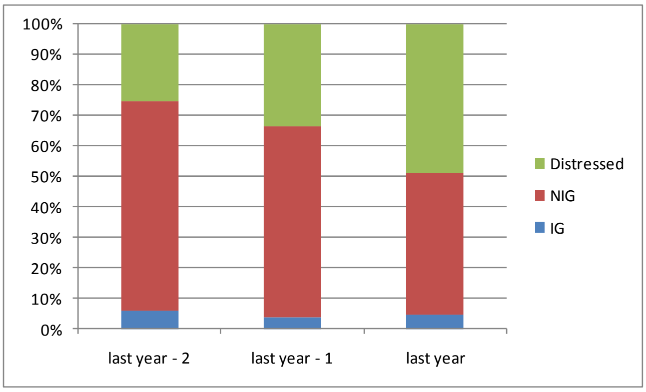
Italy:





Belgium:





All AUC results demonstrate that the MORE rating discriminates with good accuracy the profitable companies from bankrupted companies. From the distributions of ratings, is clear that the frequency of ratings falling into the distressed class increases when nearing the default date.

As per the <u>probability of default</u>, validation is still done applying the Transition Matrix Theory, since in the two previous steps it has been demonstrated that bankrupted companies are concentrated in the D class.

France

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	33,33%	40,00%	10,00%	6,67%	3,33%	6,67%	0,00%	0,00%
AA	0,54%	58,29%	31,06%	7,60%	1,34%	0,54%	0,29%	0,33%
A	0,01%	3,65%	61,87%	25,08%	5,05%	3,02%	0,92%	0,40%
BBB	0,00%	0,42%	16,53%	54,42%	18,61%	6,62%	2,45%	0,94%
BB	0,00%	0,09%	3,15%	20,60%	52,75%	14,07%	5,98%	3,36%
B	0,00%	0,04%	2,64%	9,51%	29,23%	37,79%	12,42%	8,37%
CCC	0,00%	0,05%	1,82%	5,43%	17,40%	20,97%	33,35%	20,98%
D	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Belgium

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	50,00%	50,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
AA	0,54%	58,76%	28,03%	8,89%	1,62%	1,62%	0,54%	0,08%
A	0,00%	9,13%	60,16%	22,82%	4,45%	2,84%	0,45%	0,17%
BBB	0,00%	1,42%	17,81%	56,02%	18,59%	4,29%	1,54%	0,33%
BB	0,00%	0,24%	2,73%	19,27%	59,53%	13,33%	3,53%	1,37%
B	0,00%	0,34%	2,18%	6,70%	28,96%	45,39%	11,98%	4,44%
CCC	0,00%	0,29%	1,45%	4,35%	15,87%	25,43%	33,33%	19,28%
D	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Italy

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	37,88%	43,94%	13,64%	1,52%	1,52%	0,00%	1,52%	0,00%
AA	1,21%	62,51%	27,83%	5,79%	1,24%	1,21%	0,22%	0,12%
A	0,01%	4,66%	62,46%	23,36%	5,09%	3,13%	0,97%	0,33%
BBB	0,00%	0,41%	14,96%	51,74%	23,81%	5,92%	2,32%	0,84%
BB	0,00%	0,05%	1,79%	13,79%	57,99%	18,78%	5,17%	2,43%
B	0,00%	0,03%	0,73%	3,34%	20,03%	55,85%	13,60%	6,42%
CCC	0,00%	0,05%	0,75%	2,67%	11,69%	32,44%	35,14%	17,26%
D	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%

Tab. 2 : probability of default validation

According to financial theory, it has been obtained a growing probability of default (i.e. values of the last column) when the rating class deteriorates. This demonstrates that the definition of rating classes is consistent with the probability of default.

Chapter 3

RETRIEVING RISK NEUTRAL DEFAULT PROBABILITIES FROM CDS MARKET SPREADS

As seen in Chapter 1, in order to obtain the risk neutral default probabilities implied by CDS spreads, we have to boot-strap them from some real market quotes.

As a consequence, a data source (for CDS market quotes) is needed.

Bloomberg's CDS market spreads

Several business information providers do have historical market spreads data available (e.g. GFI Group, Thomson-Reuters, Markit®, Bloomberg, etc), the largest of whom, as per CDS data, is Markit®.

For the present work Bloomberg's database[13] has been employed, as source for the market spread of the Credit Default Swaps.

CDS selection

CDS data have been selected according to the following criteria:

Countries of the reference entities

The Countries of origin of the reference entities are the following:

United States, United Kingdom, France, Germany, Italy, Portugal, Spain, Switzerland, Albania, Australia, Belgium, Bulgaria, Cyprus, Croatia, Denmark, Estonia, Finland, Latvia, Liechtenstein, Lithuania, Luxembourg, Malta, Norway, Nederland, Poland, Czech Republic, Romania, Russia, Serbia, Slovakia, Slovenia, Turkey, Ukraine, Hungary.

Sectors of the reference entities

Primary goods, Communications, Consumer goods, Energy, Industrials, Materials, Health, Public Services, Technology.

Ratings

All.

Debt class

Senior.

Currencies

EUR, USD.

[13] The reason is that the University of Trieste already has an academic subscription to Bloomberg's database.

Tenors

1 Year, 2 Years, 3 Years, 4 Years, 5 Years, 7 Years.

Spread data

The market spreads retrieved is the closing mid spread for each trading day between 31/12/2013 and 31/12/2014, of each ticker selected.

Data analysis

The above selection has produced 764 CDS tickers.

Country distribution

The country-distribution of such tickers is summarized in the following table (countries with at least 10 tickers):

Country	# of ticker	% of ticker	% Cumulative
US	432	57%	57%
GB	98	13%	70%
FR	50	7%	76%
DE	44	6%	82%
NL	26	3%	85%
SE	17	2%	88%
LU	16	2%	90%
CH	14	2%	91%
IT	11	1%	93%
ES	10	1%	94%

Tab. 3: CDS by Country

As it can be seen, only 10 countries gather the 94% of all (available on Bloomberg) CDS tickers. United States alone count for 57% of all tickers.

Liquidity threshold

In order to have meaningful market spreads, it has been imposed a liquidity constraint: only ticker with at least 25 trading days (per annum, i.e. 2014) have been selected.

The resulting numerosity per tenor is the following:

Tenor	# of ticker
CDS 1_Y	291/764;
CDS 2_Y	226/764;
CDS 3_Y	315/764;
CDS 4_Y	237/764;
CDS 5_Y	475/764;
CDS 7_Y	252/764;

Tab. 4: liquid tickers

Bootstrap of the probability of default from CDS market spread

We remind, from chapter 1, that the process of bootstrapping consists of retrieving, from the market spread ($S_{tv,tn}$), the implied survival probabilities $Q(t_v,t_m)$ that solves the equation:

$$S(t_V,t_N) = \frac{(1-R)\sum_{m=1}^{M\,x\,t_N} Z(t_V,t_m)[Q(t_V,t_{m-1}) - Q(t_V,t_m)]}{RPV01} \qquad (12)$$

Where:

- t_v,is the evaluation time;
- t_N is the maturity of the CDS (expressed in years);
- $Q(t_V,t_m)$ is the survival probability of the reference entity from evaluation time t_V to time t_m;
- $Z(t_V, t_m)$ is the discount factor from evaluation time t_V to time s;
- R is the recovery rate of the underlying bond;
- RPV01 is defined as the Risky Present Value of 01 bp of spread[14].

The survival probability so obtained is a risk-neutral survival probability, subject to the hypothesis of a piecewise-flat hazard rate[15] term structure.

To solve the above equation a specific Matlab® function has been employed. More details are provided in the following paragraphs.

Merging Bloomberg's CDS reference entities with modeFinance's ratings and database creation

In order to pursue the goal of the thesis, before bootstrapping the default probabilities, it was needed to clusterize the CDS reference entities according to modeFinance's MORE rating classes.

To do so, the CDS reference entities tickers/names/VAT numbers have been cross-matched with the data available on modeFinance's database. 575/764 reference entities have been matched (with recent ratings available).

Restricting the matched reference entities to those respecting the liquidity constraint[16], the number of matched reference entities was the following:

Tenor	# of tickers matched
CDS 1_Y	241/291;
CDS 2_Y	179/226;
CDS 3_Y	261/315;
CDS 4_Y	189/237;
CDS 5_Y	386/475;
CDS 7_Y	219/252;

Tab. 5: CDS matched between Bloomberg and modeFinance databases

Where, for example, CDS 1_Y indicates all CDS with tenor (i.e. maturity) of 1 year.

[14] See chapter 1. $RPV01 = \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_v, t_n) \left[Q(t_V, t_n) + \frac{1}{2}\left(Q(t_v, t_{n-1}) - Q(t_V, t_n)\right)\right]$

[15] See chapter 1. The survival probability Q, to time T, conditional to surviving until time t_v (considering the limit with dt->0) has the form: $Q(t_v, T) = exp\left(-\int_{t_v}^{T} \lambda(s)\, ds\right)$, with λ being the hazard rate.

[16] See Tab.2. Only CDS with at least 25 days of trading have been selected.

After matching the reference entities, to each of them it has been assigned 2013 modeFinance rating.

Database creation

In order to bootstrap the default probabilities implied by the market spreads a database of the matched reference entities has been created.

The structure of the database was the following (spreads in bps):

CDS identifier	Reference entity name	Reference entity's rating*	Daily market spread[17]	Yearly average spread	Monthly average spread
AA CDS USD SR 1Y	Alcoa Inc	CCC	20.559; 20.560; ...	17.995	20.479; 19.503; ...
ABT CDS USD SR 1Y	Abbott Laboratories	A	4.750; 4.207; ...	6.115	4.304; 4.305; ...
ACFP CDS EUR SR 1Y	Accor SA	BB	13.309; 13.159; ...	16.279	15.059; 18.808; ...
APD CDS USD SR 1Y	Air Products & Chemicals Inc	BB	6.577; 5.670; ...	6.729	7.269; 7.402; ...
AIRFP CDS EUR SR 1Y	Airbus Group NV	B	7.456; 6.693; ...	9.345	9.048; 8.374; ...
AKZANANV CDS EUR SR 1Y	Akzo Nobel NV	BBB	6.706; 6.704; ...	12.198	11.370; 12.816; ...
ALCATEL CDS EUR SR 1Y	Alcatel-Lucent	CCC	59.438; 59.453; ...	77.404	69.959; 85.708; ...
…	…	…	…	…	..

*Ratings are not the real ones, those displayed are just for reference.

This operation has been repeated for all tenors from 1 to 5 years, where the 5 years database contains only those CDS with all available tenors, i.e. 1Y, 2Y, …, 5Y; 4 years database contains only those CDS with all available tenors, i.e. 1Y, 2Y,…,4Y; etc.

The creation of such databases was needed in order to bootstrap a full term structure of the default probabilities. According to Eq. (1), in order to bootstrap, for example, the 5 years default probabilities term structure, we need the market spread of the CDS with all available tenors from 1 year to 5 years.

The number of CDS spread data so obtained is summarized in the following table:

Tenor	# of tickers
CDS_1Y	241
CDS_1Y&2Y	166
CDS_1Y&2Y&3Y	165
CDS_1Y&2Y&3Y&4Y	165
CDS_1Y&2Y&3Y&4Y&5Y	165

Tab. 6: CDS reference entities available for PD bootstrapping

[17] See "CDS Selection - Spread data". Closing daily mid (market) spreads from 31/12/2013 to 31/12/2014

Settings and assumptions

The bootstrap of the PDs from the market spreads has been done through a Matlab® function named "*cdsbootstrap.m*". The underlying theory applied by such function is that exposed in chapter 1. Parameters and settings of the function are the following.

Settings and conventions applied for the bootstrap process

The bootstrap process has been done according to the conventions indicated by ISDA[18].

Day Count Convention	"Actual/360"
Business Days Convention	"Following"
Coupon Payment frequency	"Quarterly"
Recovery Rate	0.4
Zero Curve	"USD SWAP" (30/360, Sem) for USD denominated CDS
	"EUR SWAP" (30/360, Sem) for EUR denominated CDS

Tab. 7: settings and conventions

Assumptions

One of the next steps of the thesis will be to compare the "real world" modeFinance's PDs, with the risk neutral PDs retrieved by the market spreads. One of the key differences between these two measures is that modeFinance's PDs (as the ratings) are assessed once a year, when the financial statements of the reference entities are made public; while the market implied PDs can be retrieved from the market spreads for each trading date.

Two hypothesis have been made to overcome such issue and both the hypothesis will be brought forward, in order to see which one will produce the better results, once the PDs will be bootstrapped:

1. Since modeFinance ratings and PDs have a yearly time horizon, for each reference entity, the yearly average market spread is used as the reference spread for bootstrapping the PD.
2. Since for listed companies (which is the case for about 80% of the CDS reference entities) the annual report is published in the first months of the year and that is supposed to have a significant reflection on CDS's spread, we analyze which is the month with the most significant, average, spread variation. We might then use, as reference spread, the spread's monthly average of that specific month (for each reference entity).

As per point 2., for each reference entity, we normalized the spread time series, using the market spread of 31/12/2013 (or the first available spread of the time series), we clusterized the normalized spreads by month, and retrieved the monthly average spread variation. We then averaged across the reference entities for each tenor.

The resulting average spread variation by month is the following:

[18] For further details on *Conventions* see Glossary-CDS Conventions

Tenor	January	February	March	April	May	June
1Y	11.39%	22.44%	13.82%	14.74%	12.41%	2.34%
2Y	4.84%	13.76%	7.12%	6.96%	4.53%	-2.18%
3Y	6.05%	14.02%	8.76%	8.28%	4.79%	-3.45%
4Y	3.25%	8.65%	4.04%	3.97%	2.40%	-4.13%
5Y	2.50%	6.52%	3.31%	2.70%	-0.13%	-7.77%
7Y	2.34%	5.84%	3.04%	1.52%	-1.28%	-8.08%

Tab. 8: Avg. monthly spread variation

From Tab. 8 it can be seen that the month with the highest average spread variation is February (fact that endorse the hypothesis of annual report reflection on Spread, since February is indeed the month when the most of annual reports are published).

We also notice that such "reflection" is weakening its effects with the tenor increasing and this fact also goes along with common sense, since the result of "present" financials will weigh less on a longer time horizon.

Another visualization of what above is provided by the graph in the following figure, where 5 years tenor average yearly spreads and average February spreads have been clusterized in spread bins of 25 basis points, and the distribution has been retrieved (in this case the average spread is referred to the single reference entity):

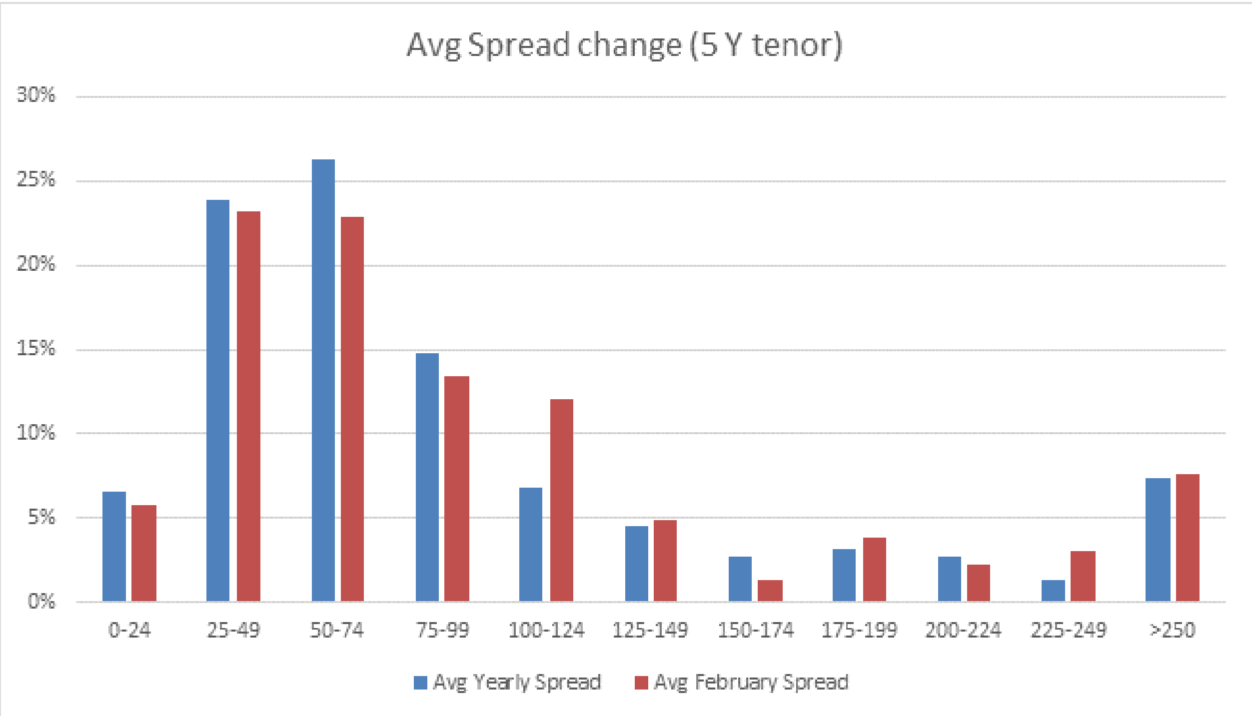


Fig. 9: average yearly and average February spreads distributions

The graph highlights how the distribution tends to move toward slightly higher spreads in the case of February average.

Bootstrap of the PDs

As said, the CDS spreads' data have been clusterized by modeFinance rating classes. Due to a not-wide dataset of CDS reference entities (see Tab.6) the most extreme rating classes are not enough populated to retrieve meaningful PDs. The rating classes for which there was sufficient numerosity of data to bootstrap the PDs are: A, BBB, BB, B, CCC (se chapter 2, Fig.6 for more details on modeFinance rating scale).

We bootstrapped the implicit PDs from market spread on both yearly average spread basis, and February-only average spread basis.

This bootstrap is limited to tenors up to 5 years.

PD bootstrap from yearly average market spread

For each reference entity, of each rating class, it has been bootstrapped the default probability from the reference entity's yearly-average spread.

The retrieved PDs have then been averaged on the reference entities pertaining to each rating class.

The results of such process are displayed in the table below:

Rating class	Num	PD 1 Y (avg)	PD 2 Y (avg)	PD 3 Y (avg)	PD 4 Y (avg)	PD 5 Y (avg)
AA	1	0.12%	0.36%	0.94%	1.72%	2.80%
A	15	0.16%	0.46%	1.05%	1.99%	3.40%
BBB	64	0.26%	0.85%	2.01%	3.86%	6.57%
BB	63	0.31%	1.01%	2.28%	4.37%	7.31%
B	11	0.69%	2.05%	4.33%	7.85%	12.55%
CCC	10	13.38%	17.91%	23.04%	28.98%	34.65%
CC	1	22.20%	43.61%	66.55%	75.61%	83.79%

Tab. 9: PDs bootstrapped from yearly averaged CDS spread

Such PDs are represented, along with their errors below described, in following Fig.10.

In order to investigate the reliability of such results, an error analysis has been made.

Being the PDs above calculated as the average of all CDS across rating classes and tenors, the error has been defined as the standard deviation of the average of the PDs retrieved[19]:

$$\varepsilon = \sigma_{PD}^{class} \Big/ \sqrt{n_{r.e.}^{class}} \qquad (13)$$

Where:

σ^{class}_{PD} is the standard deviation of the PDs by rating class and tenor.

$n^{class}_{r.e.}$ indicates the number of reference entities included in each rating class.

The errors on each of the PDs of Tab. 9 is displayed in the following table:

Rating class	ε 1Y	ε 2Y	ε 3Y	ε 4Y	ε 5Y
AA	--	--	--	--	--
A	0.0001	0.0005	0.0012	0.0025	0.0044
BBB	0.0003	0.0009	0.0021	0.0041	0.0068
BB	0.0003	0.0008	0.0019	0.0037	0.0059
B	0.0016	0.0042	0.0075	0.0123	0.0170
CCC	0.0801	0.0856	0.0909	0.0957	0.0972
CC	--	--	--	--	--

Tab. 10: errors of PDs bootstrapped from yearly averaged CDS spread

[19] This definition of the error is essentially the standard deviation of the distribution of the sample average. See G. Cicchitelli, Probabilità e Statistica, Maggioli Editore (march 2002).

Obviously, due to the inconsistent numerosity, in future elaborations, PDs retrieved for AA and CC class, will be disregarded.

The graphical visualization of the above results is displayed in the following figure:

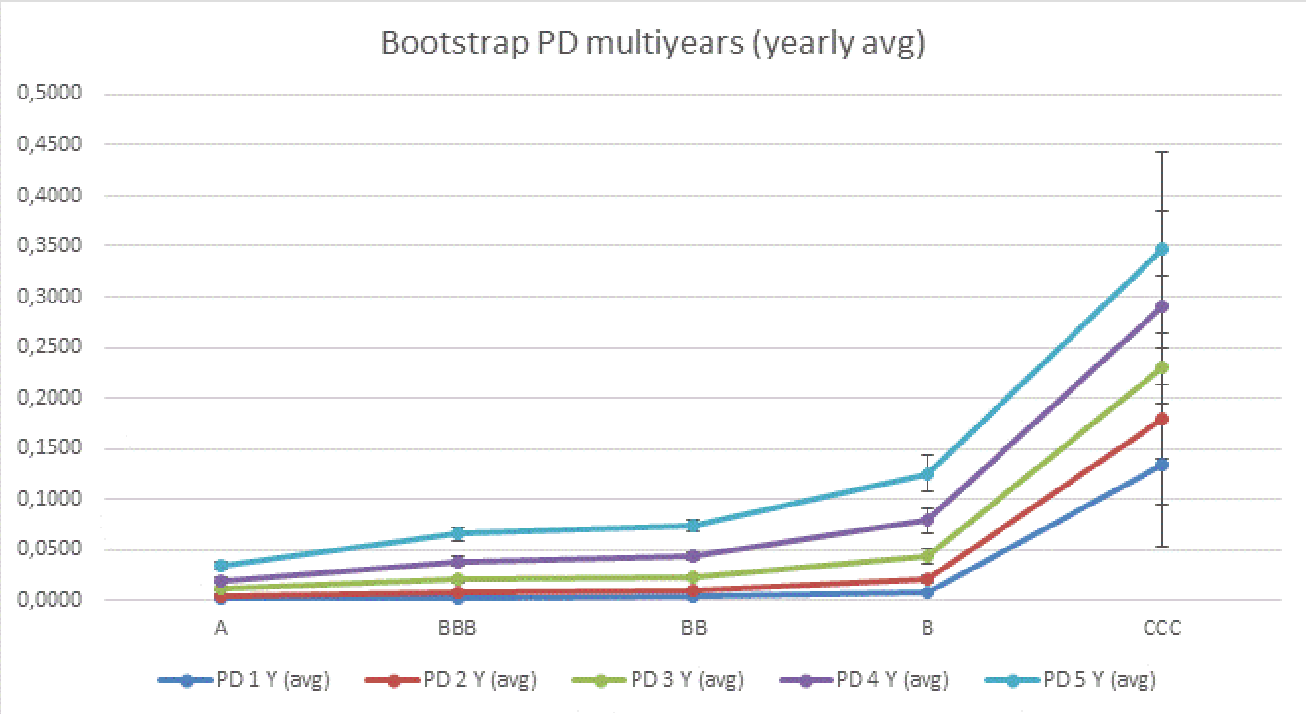


Fig. 10: PDs bootstrapped on yearly averaged market spreads

As it can be observed for all given classes, except CCC, the PDs' differences are significant (with respect to the error).

Analyzing such results, we notice that they reflect "common sense" expectations:

- Probabilities of default increase with rating class decreasing in a monotonic way;
- Probabilities of default increase with tenor increasing in a monotonic way.

We also observe that there is a higher variation of PD on the worst rating class: CCC. This last aspect may have several explanations, but the result is aligned with what found by John Hull, Mirela Predescu, and Alan White[20].

One plausible explanation is that, in modeFinance's CCC class, we are gathering Non-Investment-Grade companies, according to the ratings of those agencies which are adopted as metrics by CDS issuers (i.e. S&P, Moody's and Fitch). Moreover, such explanation is supported by R. White, "The par-spread of a CDS on non-investment grade obligors would typically be very high (sometimes exceeding 10,000bps) to cover the seller of protection from the (high) chance of a quick default"[21].

Another explanation is that the probability of default is not a linear function of the rating (score), but rather a power function. So the PD difference between adjacent low-quality rating classes is much greater than the difference between adjacent high-quality classes.

[20] *Bond Prices*, *Default Probabilities and Risk Premiums*. Journal of Credit Risk, Vol 1, No. 2 (Spring 2005)
[21] The Pricing and Risk Management of Credit Default Swaps, with a Focus on the ISDA Model. OpenGamma Quantitative Research (October, 3 2014).

The following graph underlines the monotonic behavior of the implied PD with respect to the CDS tenor (years on the x-axis).

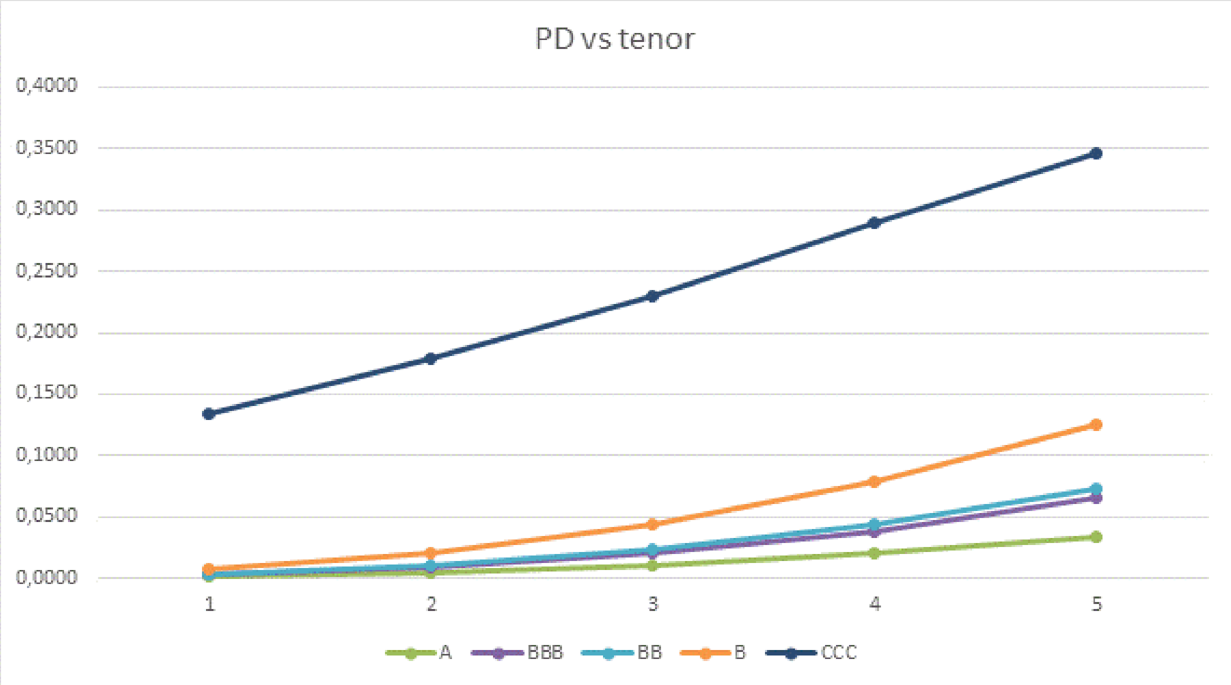


Fig. 11 Risk neutral PDs are monotonic increasing with respect to the tenor

PD bootstrap from February-average market spread

The bootstrap procedure is the same as for the yearly-average spreads. We then go straight to the results obtained.

As for the PDs, results are displayed in the table below:

Rating class	Num	PD 1 Y (avg)	PD 2 Y (avg)	PD 3 Y (avg)	PD 4 Y (avg)	PD 5 Y (avg)
AA	1	0.11%	0.33%	0.91%	1.75%	3.02%
A	14	0.16%	0.49%	1.12%	2.12%	3.64%
BBB	56	0.27%	0.90%	2.09%	3.96%	6.74%
BB	45	0.37%	1.22%	2.76%	5.17%	8.54%
B	9	0.77%	2.44%	5.11%	9.44%	14.84%
CCC	9	9.67%	17.00%	24.30%	30.82%	37.95%
CC	1	21.59%	44.21%	60.12%	69.00%	76.64%

Tab. 11: PDs bootstrapped from February-averaged CDS spread

The errors on each of the PDs of Tab. 11 is displayed in the following table:

Rating class	ε 1Y	ε 2Y	ε 3Y	ε 4Y	ε 5Y
AA	--	--	--	--	--
A	0.0002	0.0006	0.0015	0.0031	0.0055
BBB	0.0003	0.0010	0.0023	0.0043	0.0072
BB	0.0004	0.0012	0.0028	0.0049	0.0080
B	0.0018	0.0051	0.0087	0.0149	0.0207
CCC	0.0456	0.0713	0.0889	0.0933	0.0941
CC	--	--	--	--	--

Tab. 12: errors of PDs bootstrapped from February averaged CDS spread

The graphical visualization of the above results is displayed in the following figure:

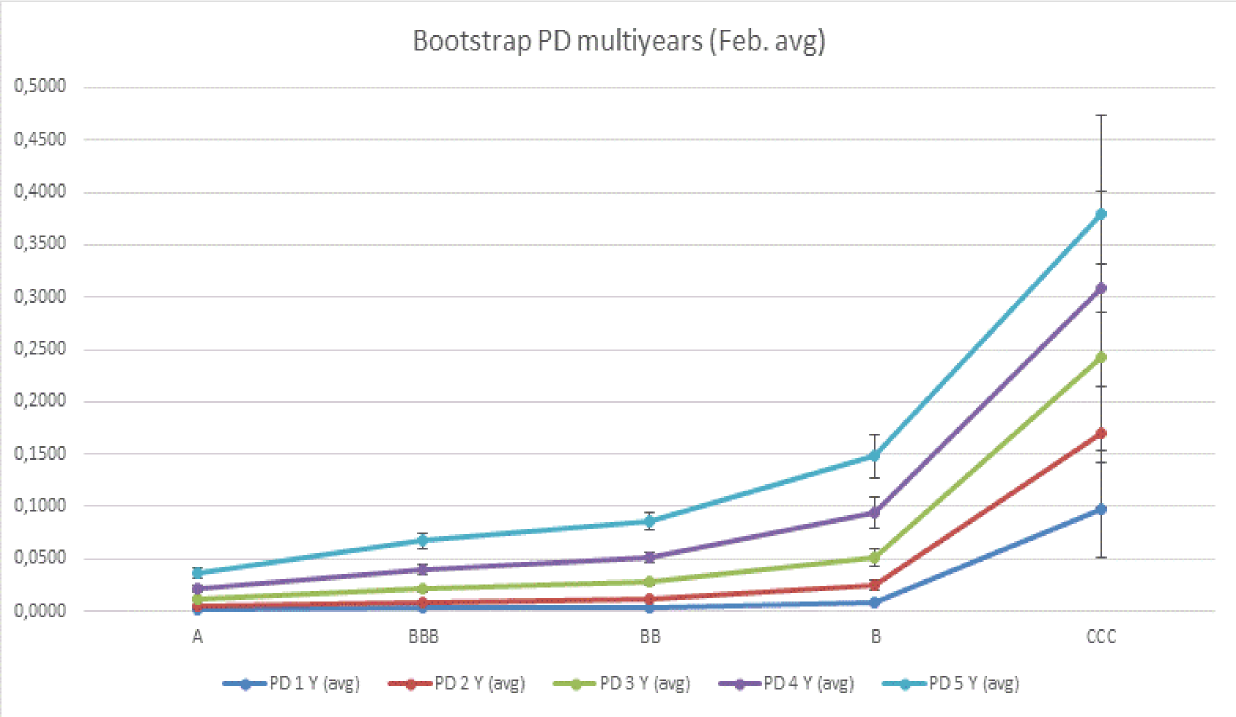


Fig. 12: PDs bootstrapped on February averaged market spreads

As it can be observed for all given classes, except CCC, the PDs' differences are significant (with respect to the error).

Also in this case, the results appear good since they reflect "common sense" expectations:

- Probabilities of default increase with rating class decreasing in a monotonic way;
- Probabilities of default increase with tenor increasing in a monotonic way.

Also in this case there is a higher variation of PD on the worst rating class: CCC.

Results comparison and hypothesis choice

In order to establish which dataset was producing the better results, we compared both the PDs retrieved and the relative errors.

As first step, we compared the implicit PDs retrieved by calculating the ratio between the average PDs retrieved from yearly-average spreads and the average PDs retrieved from February-average spreads:

$$\frac{PD_{avg}^{Year}}{PD_{avg}^{Feb}} - 1 \tag{14}$$

Results are displayed in the table in the following page:

class	PD 1 Y (avg)	PD 2 Y (avg)	PD 3 Y (avg)	PD 4 Y (avg)	PD 5 Y (avg)
A	-0.0250	-0.0545	-0.0548	-0.0611	-0.0648
BBB	-0.0331	-0.0555	-0.0401	-0.0257	-0.0264
BB	-0.1477	-0.1716	-0.1731	-0.1542	-0.1439
B	-0.1068	-0.1609	-0.1541	-0.1684	-0.1539
CCC	0.3835	0.0541	-0.0521	-0.0596	-0.0869

Tab. 13: ratio between yearly avg. based PDs, and February avg. based PDs.

We notice that:

1. Only 1 and 2 years CCC class PDs retrieved by yearly averaged spreads tend to be slightly higher than those retrieved by February averaged spreads;
2. All other PDs are lower in the case of yearly average spreads.

As per the errors comparison, the same ratio (between the PDs' errors retrieved from yearly-average spreads and the February-average spreads) has been calculated:

class	ε 1Y	ε 2Y	ε 3Y	ε 4Y	ε 5Y
A	-0.1512	-0.2049	-0.1681	-0.1737	-0.1872
BBB	-0.2103	-0.1513	-0.1072	-0.0532	-0.0508
BB	-0.3770	-0.3111	-0.3016	-0.2616	-0.2597
B	-0.0832	-0.1638	-0.1373	-0.1761	-0.1763
CCC	0.7571	0.2004	0.0219	0.0257	0.0319

Tab. 14 ratio between errors on yearly avg. based PDs, and February avg. based PDs.

As it can be seen, the error on PDs retrieved from yearly-average spreads are always lower than those retrieved from February-average spreads, except for CCC class. This behavior depends surely also on the higher numerosity of the yearly-averaged spreads and not necessarily discriminates the veridicity of the results. But, *ceteris paribus*, we cannot neglect the higher numerosity as a reliability measure.

Conclusions

In consideration of:

1. the errors-comparison, favorable to the PDs retrieved from yearly-average spreads;
2. the fact that, for the yearly average spreads we had higher numerosity of data (hence higher statistical reliability).

We decided to keep the PDs retrieved from yearly-average spreads as the reference risk neutral (market implied) probabilities of default.

Chapter 4

CALCULATING MODEFINANCE MULTI-YEARS (CUMULATIVE) PROBABILITIES OF DEFAULT

In this section we will calculate the “real world probabilities” of default for CDS reference entities, for a set of listed companies and for a set of small and medium enterprises.

The needed data will be retrieved from modeFinance’s databases[22].

As seen in chapter 2, modeFinance assessed PDs have a yearly time horizon. In the next chapter we will compare the risk neutral PDs retrieved from CDS spreads, for each tenor and rating class, to the real world ones. In order to do so, we have to calculate these last from “real world” data.

As stated in Chapter 2, essentially because of the lack of bankruptcy data in several Countries, the PDs will be assessed using the transition matrix theory.

Calculating multi-years probabilities of default

The theory hereafter exposed is retrieved by a publication of Moody’s Investor Service: *Measuring Corporate Default Rates*. David T. Hamilton, Richard Cantor (November 2006).

The goal of this chapter is to calculate the Probability of Default of a company, with assigned rating, within a given time period T (e.g. 1 year, 2 years,.., 5 years).

Given a pool of companies having assigned rating ‘z’ at calendar date ‘y’, they form a so-called *cohort*. Our goal is to calculate the PD of such a cohort within time horizon T (1 to 5 years in our case).

This probability of default is the cumulative Default Rate over time horizon T referring to cohort ‘z’ (i.e. companies with assigned rating ‘z’). Hereafter we’ll identify it with $D_y^z(T)$.

Such time horizon has to be divided into evenly spaced time intervals ‘t’ (in our case one year each).

In each interval, some fraction of the cohort that has survived up to that time, may default. Hence, the “one-period” default rate is defined as the probability that a company, which has survived up to the beginning of time interval t, will default by the end of this same time interval, hereafter identified by $d_y^z(t)$. Therefore, it is a conditional default probability. Of course, according to this procedure, what we will obtain, is a discrete estimation of the Default Probability.

Mathematically, the (one-period) default rate d(t) of time interval t, for a cohort formed on date ‘y’ holding rating ‘z’ is defined as the number of defaults x(t) of the cohort that occur in the time interval t, divided by the effective size of the cohort n(t), at the beginning of time t:

$$d_y^z(t) = \frac{x_y^z(t)}{n_y^z(t)} \tag{14}$$

[22] For more information about modeFinance see chapter 2 or visit http://www.modefinance.com/en/about-us

Cumulative default rate for time horizon T is built up from the one-period conditional default rates, and is found by subtracting the product of the fraction of surviving cohort members in each of the t time intervals, from unity:

$$D_y^z(T) = 1 - \prod_{t=1}^{T} [1 - d_y^z(t)] \tag{15}$$

Hence, the T-period cumulative default rate is one minus the product of the T one-period survival rates.

Which, expanding the equation and omitting the indexes for brevity, becomes:

$$D(T) = d(1) + d(2)[1 - d(1)] + d(3)[1 - d(2)][1 - d(1)] + \cdots \tag{16}$$

Equation (16) highlights the fact that a cumulative default rate is a sum of conditional probabilities, where default rates in each period are assumed to be independent.

If several cohort-periods are available, a stronger estimation of the expected default probability can be achieved by averaging the one-period default rates across all available cohort dates y in the historical dataset Y. The average cumulative default probability for a period T is calculated by first averaging the period t (one period) default rates, then calculating the cumulative, using equation (15) or (16). These are weighted averages, where each one-period's default rate is weighted by the relative size of the cohort in each time interval t.

Equation (15) becomes:

$$\overline{D^z}(T) = 1 - \prod_{t=1}^{T} [1 - \overline{d^z}(t)] \tag{17}$$

Where

$$\overline{d^z}(t) = \frac{\sum_y x_y^z(t)}{\sum_y n_y^z(t)} \tag{18}$$

Note that this procedure for calculating average cumulative default rates maximizes the existing historical information by using *all* the available ratings and one-period default rate data.

Note that, according to the transition matrix theory exposed in chapter 2, a company will be considered in default when its rating falls below CCC class[23] (i.e. CC, C, or D).

[23] For modeFinance's rating scale definition see chapter 2.

Cumulative default probability for CDS reference entities

For the scope of the Thesis, we need to calculate the "real-world" default probabilities, by rating class, of the CDS reference entities.

To do so, we matched the 764 CDS reference entities retrieved by Bloomberg database, with data available on modeFinance database. In order to calculate PDs according to the theory above exposed, we needed each reference entity to be rated for at least 5 years. But to have more reliable PD estimation, we retrieved ratings on a 10 years time horizon (2004-2014).

A first problem to overcome was the scarce numerosity of data: of 764 reference entities only 597 had at least some ratings. The most extreme rating classes (in particular the highest ones) were not enough populated to calculate the PDs: the transition matrix for those classes was empty on some time horizons.

In order to overcome such issue, we made a correlation analysis of the ratings of the reference entities with respect to "related companies"[24].

A separate analysis has shown that within a corporate group structure, ratings are correlated, in some cases, highly correlated.

The analysis was made on a sample of European (EU 15 members) industrial entities, consisting of 29,990 companies.

The correlation analysis results are displayed in the following table:

Related companies	Correlation
Ref. entity – Global ultimate owner	40.3%
Ref. entity – Direct major shareholder	27.5%
Ref. entity – Owned[25] subsidiaries	19.8%

Tab. 15: related companies rating correlation

Where the correlation has been calculated according to:

$$R(i,j) = \frac{C(i,j)}{\sqrt{C(i,i)C(j,j)}} \tag{19}$$

Where C is the covariance matrix.

In order to enlarge the dataset, we retrieved the data of the reference entities' ultimate owners, major shareholders and owned subsidiaries. Numerosity is displayed in the table below:

Dataset	numerosity[26]	rated[27]
Ref. entity + Global ultimate owner + Direct major shareholder	2,188	904
Ref. entity + Global ultimate owner + Direct major shareholder + Owned subsidiaries	49,289	42,552

Tab. 16: enlarged dataset

[24] By related company we mean: Global ultimate owner, shareholders and subsidiaries.
[25] Control equal or above 50%.
[26] Note that the numerosity is only indicative. The number is referred to 2014, but the sample is opened.
[27] As above.

Only the full data set, which includes subsidiaries, produced a numerosity of ratings capable to ensure full transition matrix for all time horizons and all rating classes.

Real world default probabilities for CDS reference entities

The cumulative (multi-years) probabilities of default obtained from the above dataset, by applying equation (17) are displayed in the following table (for sake of readability we restrict the visualization only to those classes for which we retrieved also the risk neutral PDs):

Class	**1 year**	**2 years**	**3 years**	**4 years**	**5 years**
A	0.70%	1.61%	2.58%	3.62%	4.57%
BBB	1.00%	2.54%	4.15%	5.53%	6.86%
BB	2.75%	5.34%	7.71%	9.55%	11.22%
B	6.52%	11.28%	14.97%	18.11%	20.32%
CCC	17.21%	25.46%	30.92%	34.52%	36.95%

Tab. 17: "real world" reference entities cumulative PDs.

A graphical visualization of the above results is shown in the figure below:

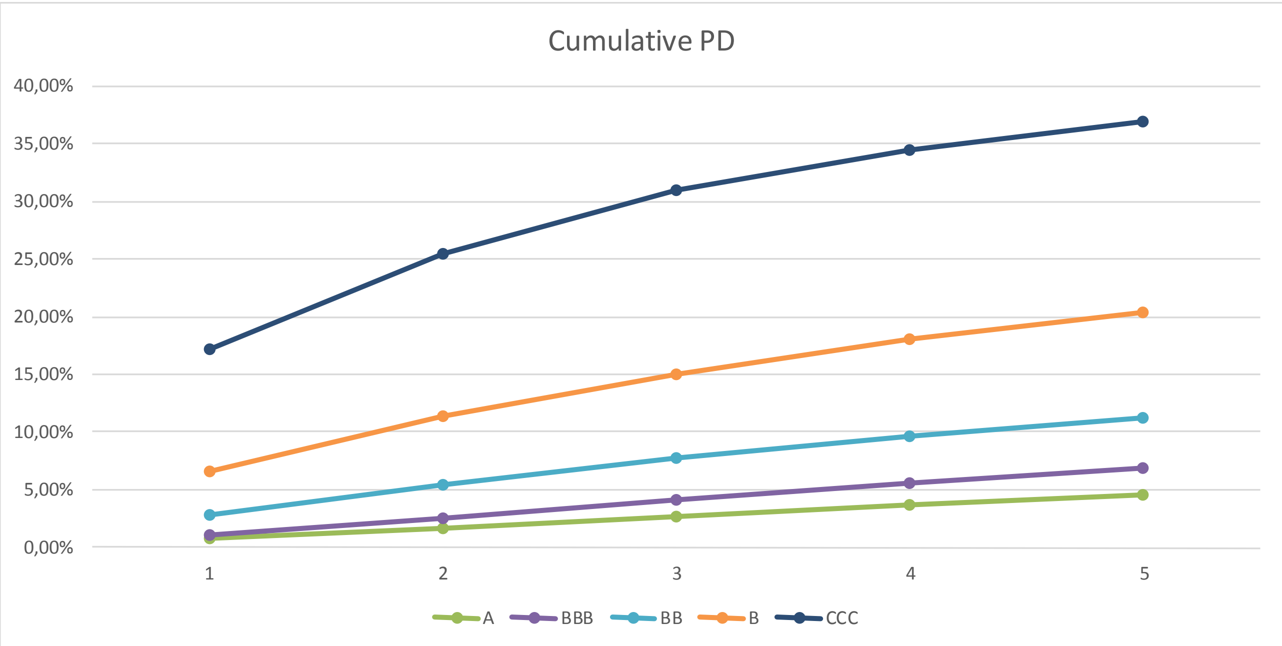


Fig. 13: CDS ref. entities cumulative PDs

Also for the real world PDs, we observe:

1. A monotonic increase with rating class decreasing;
2. A monotonic increase with time increasing;
3. For a given time horizon, a more intense PD increase, with rating class decreasing, for the worst rating classes;
4. For higher rating classes, PD increases in an almost-linear way with respect to time, for lower rating classes the increase has a negative convexity.

Points 1. and 2. reflect common sense expectations.

Point 3. goes along with what found for CDS implied PDs (explained in chapter 3), therefore the same considerations hold true.

While the negative convexity (point 4.) of the lower rating classes can be interpreted as a decreasing default intensity with time passing. In other words: companies with very low ratings, have a high probability to default within short time, but the probability increases less intensely with time passing. Such phenomenon is common for low rating companies, as reported among others by J.C. Hull[28] and R. White[29].

Such interpretation is confirmed by the analysis of the conditional default probabilities (i.e. default intensity on annual basis), whose graphical representation is provided in the figure below:

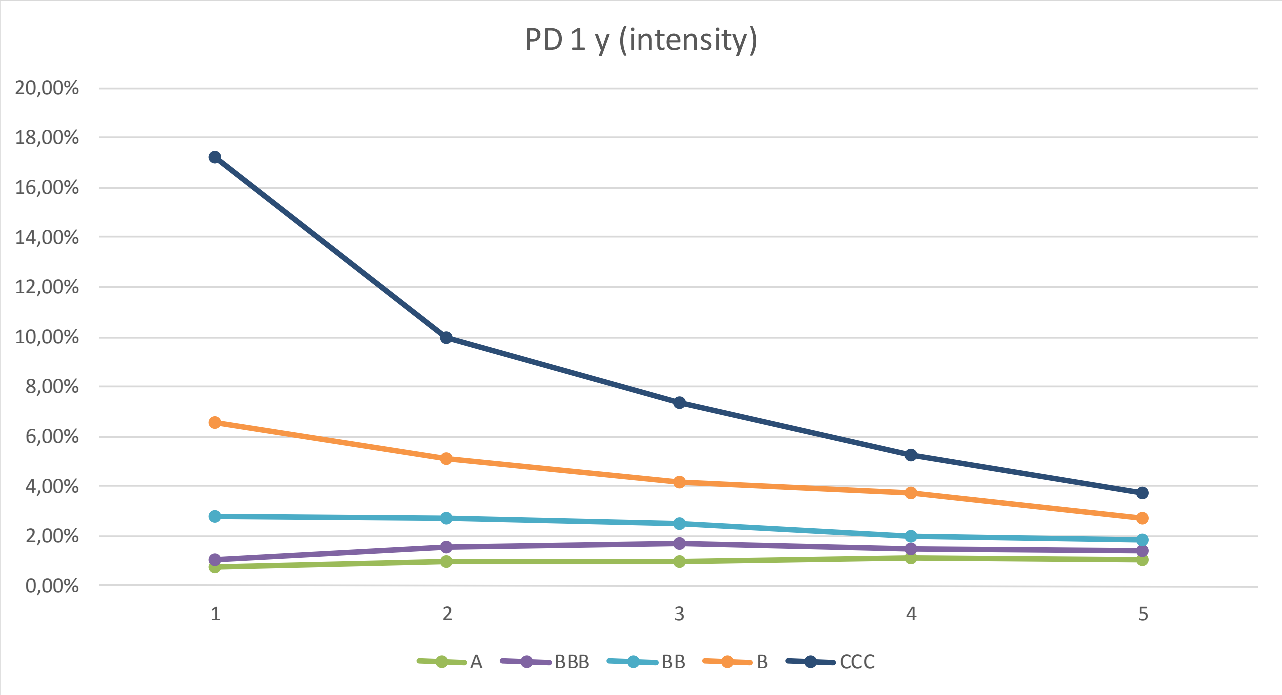


Fig. 14: marginal (mono-annual) default intensity of CDS reference entities

The graph of Fig.14 confirms that a reference entity holding a low rating (e.g. CCC) at time y, has a very high probability of defaulting within the first year, but (conditioned to survive for 1 year) the probability of defaulting during the second year is lower, and so on.

This characteristic becomes more pronounced with the rating class worsening.

Cumulative default probability for publicly listed companies

A next step is to retrieve the cumulative default probabilities specifically for listed companies.

The theoretical basis is the same as for the CDS reference entities.

Dataset created

As per the dataset of listed companies, the following selection criteria have been used:

[28] J.C Hull, *Opzioni, futures e altri derivati*, Pearson, Prentice Hall, 2009.

[29] The Pricing and Risk Management of Credit Default Swaps, with a Focus on the ISDA Model. OpenGamma Quantitative Research (October, 3 2014).

Selection criteria (only listed companies)	
1- Countries	US, GB, FR, DE, NL, SE, LU, IT;
2- Operating revenues	Above 50 million EUR (2013 and 2014 FS);
3- Total assets	Above 43 million EUR (2013 and 2014 FS);
4- MORE rating	Available on at least 5 years.

Tab. 18: listed companies' selection criteria

As per the selected Countries, they are the same of the CDS reference entities' ones. This criterion has been implemented in order to ensure geographical coherence with the CDS reference entities.

Criteria 2 and 3 ensure to select only large companies (no SME[30]).

The last criterion materially allows to calculate the Default probabilities by rating class.

4,906 companies have been selected.

Real world default probabilities obtained from the selected listed companies

The cumulative (multi-years) probabilities of default obtained from the dataset of listed companies, by applying equation (17) are displayed in the following table (also in this case, there are displayed only those classes for which we had data also for CDS):

Class	**1 year**	**2 years**	**3 years**	**4 years**	**5 years**
A	0.24%	0.60%	1.10%	1.54%	2.00%
BBB	0.36%	0.97%	1.75%	2.28%	2.67%
BB	1.11%	2.48%	3.87%	4.73%	5.45%
B	3.36%	6.26%	8.98%	10.88%	12.28%
CCC	11.12%	16.57%	20.33%	22.86%	24.56%

Tab. 19: listed companies' cumulative PDs.

Even if not displayed, we point out that also for such set of companies, the most extreme rating classes (AAA, D) were not enough populated to calculate multi-years PDs.

A graphical visualization of the above results is shown in the figure below:

[30] "*The new SME definition, user guide and model declaration*", European Commission 2003.

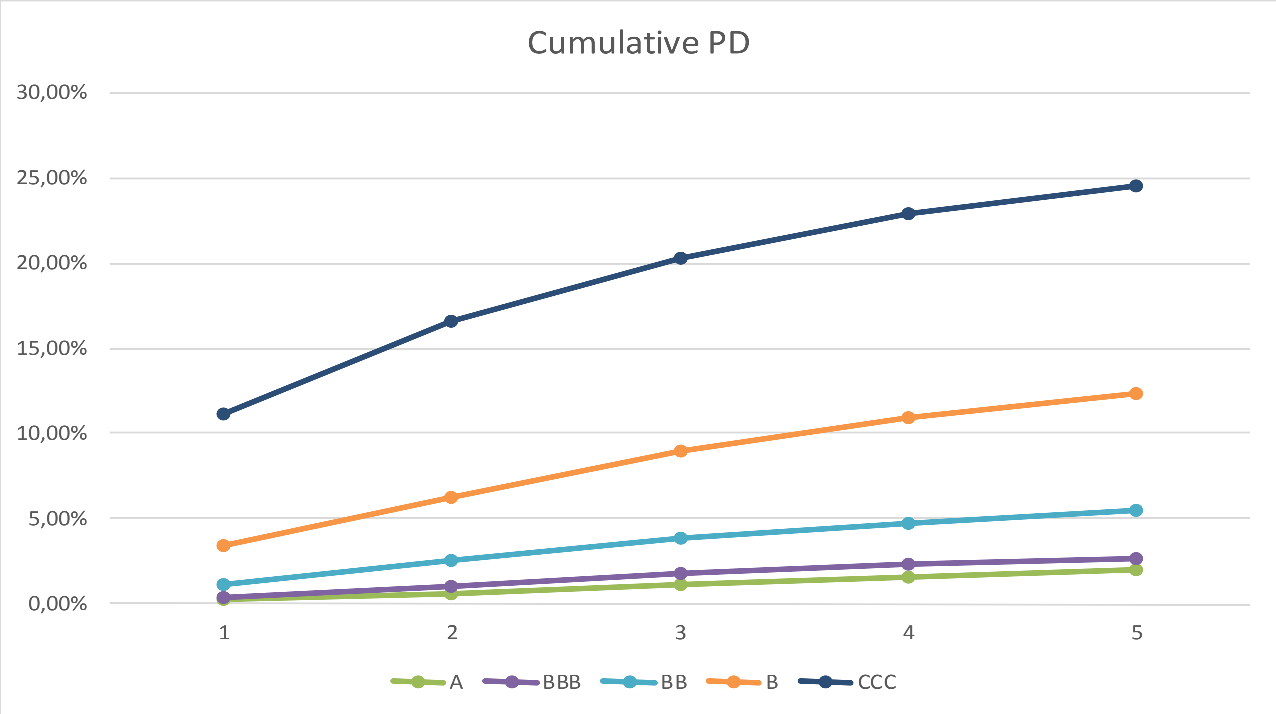


Fig. 15: listed companies' cumulative PDs

The same considerations made for the CDS reference entities' PDs hold true.

The only further observation is that listed companies' PDs tend to be lower than CDS reference entities' (real world) PDs. To justify such behavior, we may hypnotize two reasons:

1. Even if 80% of the reference entities are indeed listed companies, the data set created to have enough numerosity, include a large fraction of un-listed companies (96% of the total sample). For this last, PDs tend to be higher, in average, than for listed companies. An explanation of such phenomenon is provided in the next chapter.
2. Arguably, the existence itself of a CDS over a reference entity suggests that the market perceives a not negligible possibility that the reference entity might go bankrupted. Meaning that, with respect to similar companies, CDS reference entities might by characterized by higher PDs.

Unlisted Small and Medium Enterprises cumulative default probability

The final step as per "real-world" PDs calculation, is to retrieve the cumulative default probabilities specifically for unlisted Small and Medium Enterprises.

The theoretical basis for the cumulative PD calculation is the same as for the CDS reference entities.

Dataset created

In order to create a dataset of SME, the following selection criteria have been used:

Selection criteria (only unlisted companies)	
1- Countries	US, GB, FR, DE, NL, SE, LU, IT;
2- Operating revenues	Between 2 and 50 million EUR (2013 and 2014 FS);
3- Total assets	Between 2 and 43 million EUR (2013 and 2014 FS);
4- MORE rating	Available on at least 5 years.

Tab. 20 unlisted companies selection criteria

As per the selected Countries, they are the same of the CDS reference entities' ones. Once again the criterion has been implemented in order to ensure geographical coherence with the CDS reference entities.

Criteria 2 and 3 ensure to select only (unlisted) SME[31].

The last criterion materially allows to calculate the default probabilities by rating class.

493,727 companies have been selected.

Real world default probabilities obtained from the selected unlisted SME

The cumulative (multi-years) probabilities of default obtained from the dataset of unlisted small and medium enterprises, by applying equation (17) are displayed in the following table:

Class	1 year	2 years	3 years	4 years	5 years
A	0.47%	1.14%	1.94%	2.74%	3.52%
BBB	0.84%	1.99%	3.20%	4.40%	5.56%
BB	2.33%	4.81%	7.18%	9.38%	11.33%
B	5.32%	9.96%	14.00%	17.50%	20.39%
CCC	15.22%	23.83%	30.06%	34.89%	38.44%

Tab. 21: unlisted SMEs cumulative PDs.

A graphical visualization of the above results is shown in the figure below:

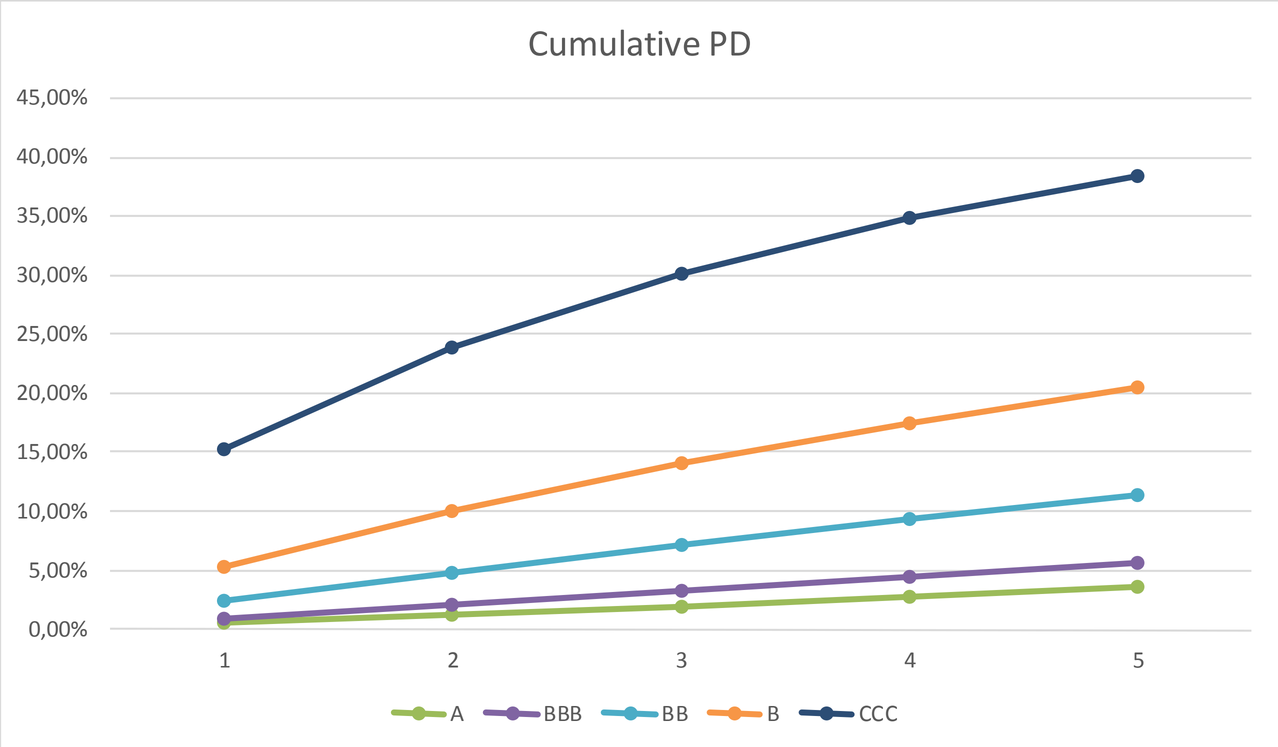


Fig. 16: listed SMEs cumulative PDs

[31] "*The new SME definition, user guide and model declaration*", European Commission 2003.

The same considerations made for the real-world PDs calculated for listed companies hold true.

What we observe as a difference is that the PDs of unlisted SMEs are higher than the PDs of listed companies.

External analysis conducted by modeFinance have shown that listed companies, other than aiming (for the fact itself of being public) to generate higher profits, they also tend to have lower indebtedness levels than unlisted companies. Under a creditworthiness perspective, this surely is a favorable point.

Another key aspect is that, being a listed company, usually of large size, there are other stake-holders that may prevent the company from defaulting.

Consequence of what above, is that there is no surprise to have obtained higher PDs for small and medium enterprises than for publicly listed companies.

As per the negative convexity of lower rating classes, the same considerations made for the CDS reference entities (real world) PDs persist: the default intensity for lower rating classes has a decreasing trend with time:

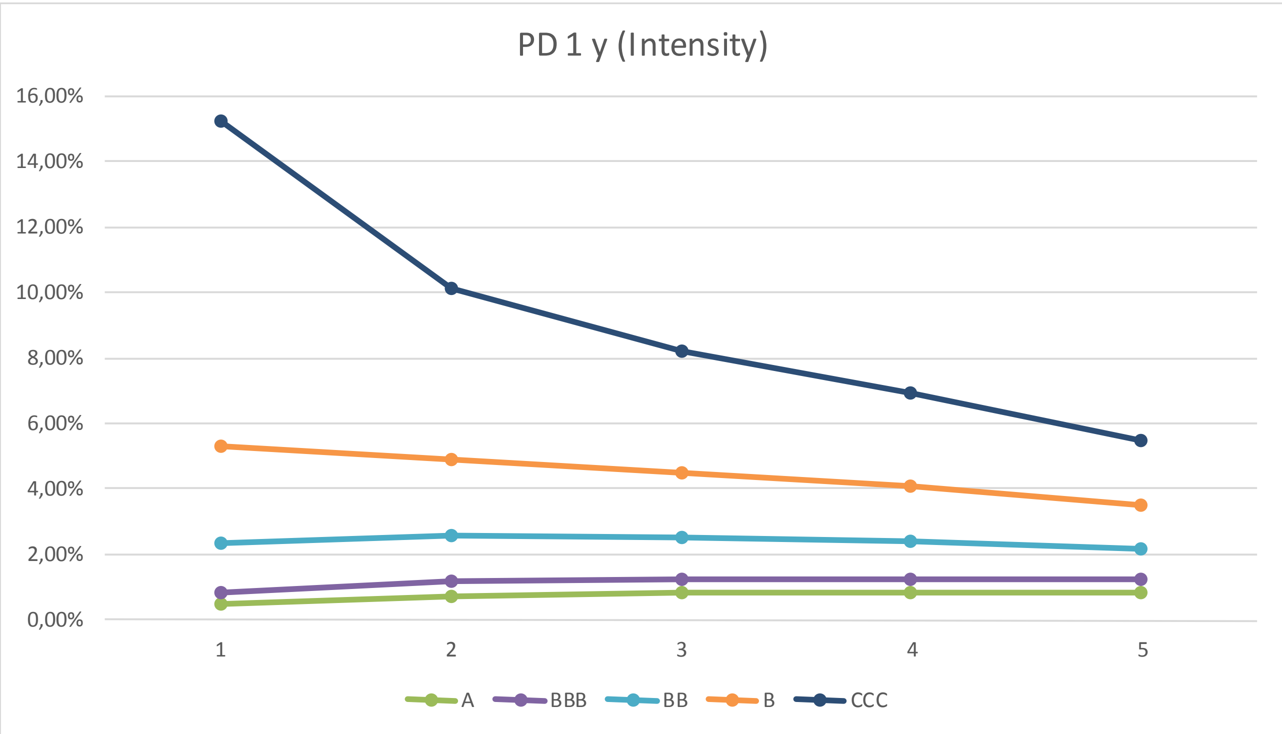


Fig. 17: Default intensity of SME

Chapter 5

EQUIVALENT DEFAULT PROBABILITY DEFINITION AND COMPUTATION OF CDS SPREAD ON SMALL MEDIUM ENTERPRISES

Creation of an "equivalent probability of default"

We briefly remind that the final goal of the thesis is to evaluate the possibility to extend the use of CDS as an instrument to insure SMEs' trade credits and see if the resulting costs (i.e. spreads) are comparable with the conventional trade credit insurance.

To price CDS, theoretically speaking, risk neutral (r-n) probabilities of default are employed. Since there is no public market for any instrument issued by SME (which usually do not issue any financial instrument[32] at all), there is no chance to bootstrap PDs from instruments' market prices for such companies, according to a non-arbitrage principle.

All the elaborations made in the previous chapters are finalized to create an *adjusting factor* that allows to translate a real-world probability of default into an "equivalent" risk neutral one, specifically fitted for the utilized CDS pricing model (see chapter 1 for details on the model employed). Of course, what obtained is a proxy.

The starting real-world probability of default ($mFPD^{xj}$) will be the PD retrieved (by rating class and tenor) for unlisted SME (see chapter 4), selected according to the class of the specific reference entity (intended to be a SME) and adjusted to modeFinance's score.

The *equivalent risk neutral PD* will be obtained according to the following formula:

$$PD_{eq}^{xj} = mFPD^{xj} * Factor^{xj} \tag{20}$$

The *"Factor"* translates, by rating class 'x' and time horizon 'j', the real-world PD of a reference entity into an equivalent risk-neutral PD of an hypothetical CDS, written on the same reference entity.

The approach for defining such Factor, is similar to what suggested by Radon-Nikodym in the definition of the Radon-Nikodym derivative. See appendix D for further information.

Also worth to mention that, even if, as stated, the obtained r-n PD is a proxy; in the world of practitioners, the PDs employed for the first issuance of a CDS are retrieved by the CDS spreads of comparable reference entities (in terms of maturities, ratings, etc). So the concept of approximating a PD by retrieving it from a peer of comparable seems a reasonable approach.

[Notice that for the definition of the "Factor", we originally wanted to compare the risk neutral PDs of the sole reference entities with their real-world ones, and 80% of the considered CDS reference entities are listed companies. Due to scarce numerosity, we had to enlarge the dataset, including other un-listed companies, but selected so that they are still related (in terms of ownership) to the original reference entities].

[32] In Italy as well as in Europe there is a growing "Minibond" market (ExtraMOT electronic market managed by BorsaItaliana®). Minibonds were firstly created in response to the credit crunch followed to the sovereign debt crisis of 2011. The market is rapidly growing but, as per July 2015, this counts "only" 109 minibonds, whose liquidity is scarce. See Appendix C for more information

mFPD

The input Probability of default of Formula (20) is obtained starting from the average cumulative PDs (by rating class, and time horizon) obtained for the Small and Medium Enterprises, as explained in chapter 4 - *Real world default probabilities obtained from the selected unlisted SME* (Tab. 21).

In the same chapter we have retrieved also the average cumulative real world PDs (by rating class, and time horizon) of a set of listed companies: "Cumulative default probability for publicly listed companies" (Tab. 19).

We have noticed already that the average cumulative PDs of SME, are higher than the PDs obtained for listed companies.

Specifically, the ratio between the two measures, defined as:

$$Ratio^{xj} = \frac{mean(mFPD_{nq}^{xj})}{mean(mFPD_{q}^{xj})} \tag{21}$$

Where:

- Index 'x' represents the rating class;
- Index 'j' represents the time horizon of the PD;
- Index 'nq' indicates unlisted SME[33];
- Index 'q' indicates listed "large" companies;

Assumes the following values:

Rating class	1 year	2 years	3 years	4 years	5 years
A	1.95	1.89	1.76	1.79	1.76
BBB	2.34	2.04	1.83	1.93	2.09
BB	2.10	1.94	1.85	1.98	2.08
B	1.58	1.59	1.56	1.61	1.66
CCC	1.37	1.44	1.48	1.53	1.57

Tab. 22: Ratio between PDs of unlisted and listed companies

Such values confirm and quantify the fact that SMEs' PDs are higher than publicly listed companies' PDs. We see that such phenomenon holds true for each rating class and each time horizon, with an average value of 1.79.

Such higher input PDs will translate into an extra CDS spread (respect to the "real" reference entities) that will take into account the higher risk intrinsically entailed by Small and Medium Enterprises, respect large (listed) companies (as 80% of "real" CDS reference entities are).

A further clarification is needed. The PDs mentioned above are indeed average values for rating class and time horizons (hence they are constant), but modeFinance's ratings are assessed on a continuous basis represented by a "Score".

In order to keep a coherence between the equivalent risk-neutral PDs (calculated according to formula 20) and the score assessed by modeFinance, on a continuous basis, for each time horizon (so seizing more precisely the company's creditworthiness) a further step is needed.

[33] "*The new SME definition, user guide and model declaration*", European Commission 2003.

Real-world PD as a function of the score

In modeFinance's MORE[34] methodology, both Ratings and Probabilities of Default are function of the score calculated for the rated entity by the MORE model. The score is a 0-1 continuous variable and ratings are assessed on decimal intervals of the score. For example, a score between 0.5 and 0.6 indicates a rating class BB. Furthermore, we define as "central score" of a rating class the middle score for that class; with reference to previous example, the central score of class BB will be 0.55.

The PD is assessed as a continuous function of the score; specifically, it is a linear interpolation between the central score for the rating classes, and the average PD of the rating classes, as depicted in the figure below:

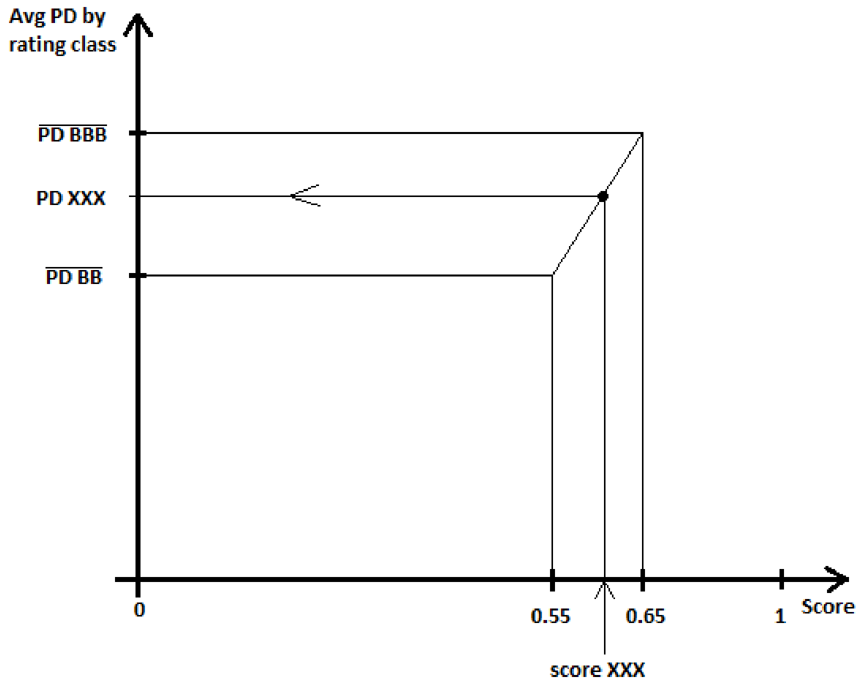


Fig. 18: linear interpolation of the PD as a function of the score

The resulting PD is normally assessed by modeFinance with 1-year time horizon, but we extended such procedure for each of the time horizon (1 to 5 years), using as average-rating class PDs, the real-world probabilities of default, obtained for unlisted SME (see Chapter 5, "Real world default probabilities obtained from the selected unlisted SME" - Tab.21):

The PDs of each time horizon (1 to 5 years) so obtained for any company analyzed (i.e. as a function of the score), will then been used as an input of eq. (20), to retrieve the *Equivalent risk neutral PD*.

[34] See chapter 2, or visit http://www.modefinance.com/en/the-more-philosophy

Factor

The "*Factor*" is meant to convert the real-world probabilities of default, of a reference entity into an equivalent risk-neutral PD of a CDS.

It has the form:

$$Factor^{xj} = \frac{mean(PD^{xj}_{CDS})}{mean(mFPD^{xj}_{CDS\_q})} \tag{22}$$

Where:

- Index 'x' represents the rating class;
- Index 'j' represents the time horizon of the PD;
- Index 'CDS' indicates CDS reference entities;
- Index 'CDS_q' indicates CDS reference entities and related companies;

The numerator of *Factor* contains the average cumulative risk neutral probabilities of default (by rating class, and time horizon) of CDS reference entities obtained according to the process exposed in Chapter 3 - paragraph "PD bootstrap from yearly average market spread", whose values are shown in Tab. 9 of the same chapter.

The denominator contains the average cumulative real world probabilities of default (by rating class, and time horizon) of CDS' reference entities and related companies, according to the process exposed in Chapter 4 - paragraph "Cumulative default probability for CDS reference entities", whose values are shown in Tab. 17 of the same chapter.

The resulting values for the Factor are shown in the table below:

Rating class	1 year	2 years	3 years	4 years	5 years
A	0.222	0.285	0.409	0.550	0.744
BBB	0.258	0.334	0.484	0.698	0.958
BB	0.114	0.190	0.296	0.458	0.651
B	0.105	0.182	0.289	0.434	0.618
CCC	0.777	0.704	0.745	0.839	0.938

Tab. 23: Factor

At the opposite of what mentioned in Chapter 3 ("Conclusions"), the Factor values go against the fact that, among others, Hull, Predescu and White[35], sustain that risk neutral PDs are usually higher than real world ones (even if they refer to PDs backed out from Bonds prices and not from CDS spreads).

This is most probably related to the fact that, even if we employed real world data, the Default Probabilities are calculated on a transition matrix theory basis (see Chapter 2), which provides higher estimates than a frequency approach.

Furthermore, we observe how the ratio between the market implied PD and the real-world one increases for the lower CCC rating class and for longer tenors. Analogous results have been found by Hull (Options, Futures and other derivatives, Pearson Prentice Hall, 2009).

[35]"Bond Prices, Default Probabilities and Risk Premiums", J. Hull, M. Predescu, A. White.

SME spread computation and comparison

The step immediately following the "equivalent-risk-neutral" PDs assessment, is to use these as an input to calculate the spread of a hypothetical CDS over a SME reference entity.

As per the back testing, we actually have no benchmark: as said, unless rare exceptions, SMEs do not issue any financial instrument, nor there is any well-established public market for these.

We can only say that, being SMEs generally riskier than large publicly listed enterprises (we have discussed this fact in the previous chapter), as CDS reference entities usually are, we'd expect higher spreads but, reasonably, well within one order difference with respect to real CDS spreads (at the same rating level and tenor).

As a consequence, we compared the average spreads obtained for a sample of Italian small and medium enterprises, with the average spreads of the CDS downloaded from Bloomberg's database.

Spread computation

The spread calculation has been performed through a Matlab® function named "*cdsspread.m*". The underlying theory applied by such function is that exposed in chapter 1.

Parameters and settings of the function are the same exposed in Chapter 3, of which we report below a summary table:

Day Count Convention	"Actual/360"
Business Days Convention	"Following"
Coupon Payment frequency	"Quarterly"
Recovery Rate	0.4
Zero Curve	"EUR SWAP" (30/360, Sem) for EUR denominated CDS

Tab. 24: CDS computation's settings and conventions

The inputs of Matlab's function are:

1. The Zero Curve (see Tab. 24);
2. Default probabilities data;
3. Settlement date: assumed to be December 31st , 2013;
4. Maturities: assumed to be at June 30th for each tenor.

As per points 3 and 4, these are subjective choices, but the first one is supported by the fact that modeFinance's ratings employed, are assessed on 2013 financials (which mostly have closing date at 31/12). The second choice is tied to the fact that we employed for all previous calculations average yearly spreads, hence a maturity set at mid-year seems fair.

Furthermore, such choices will be kept constant for all of the following elaborations so allowing results' comparisons.

As per the input default probabilities data, they are the "*equivalent risk-neutral*", obtained according to:

$$PD_{eq}^{xj} = mFPD^{j} * Factor^{xj} \tag{23}$$

As described in this same chapter – *Creation of an "equivalent probability of default"* -.

The dataset of Italian SMEs

In order to compare the spread of SMEs to that of regular CDS, we downloaded modeFinance's ratings and scores of a sample of 1,000 Italian SME[36]. The selection included only companies with rating between A and CCC (since only for those rating classes we were able to define the adjusting *Factor*).

For each of them, we calculated the 1 to 5 years time-horizon probabilities of default as function of the score, as described in "*Real-world PD as a function of the score*" of this same chapter and used these as starting PD to calculate the equivalent risk-neutral PDs, by mean of formula (23).

For each company, we used the equivalent PDs as the input PD to calculate the CDS spread.

Spreads obtained and comparison

Results

As a result of the above elaborations, for Italian SMEs we obtained the spreads whose average, by tenor and rating class, is displayed in the table below (spread in basis points):

SME	1Y	2Y	3Y	4Y	5Y
A	8.93	13.90	20.39	28.45	35.85
BBB	20.53	30.29	44.68	62.67	78.46
BB	26.26	40.15	60.21	84.95	106.13
B	51.91	75.68	106.76	144.39	177.13
CCC	645.53	537.51	519.32	526.68	535.44

Tab. 25: average spreads obtained for a sample of 1,000 Italian SME

For clarity we report also the average CDS spread (by tenor and rating class) of the same reference entities (real CDS data) used to retrieve the risk neutral PDs[37].

CDS	1Y	2Y	3Y	4Y	5Y
A	6.23	11.02	18.03	26.35	36.76
BBB	10.33	20.45	34.55	51.75	72.37
BB	12.61	24.40	39.34	58.73	80.64
B	27.71	49.69	75.37	107.23	141.75
CCC	960.68	894.47	866.86	875.38	891.31

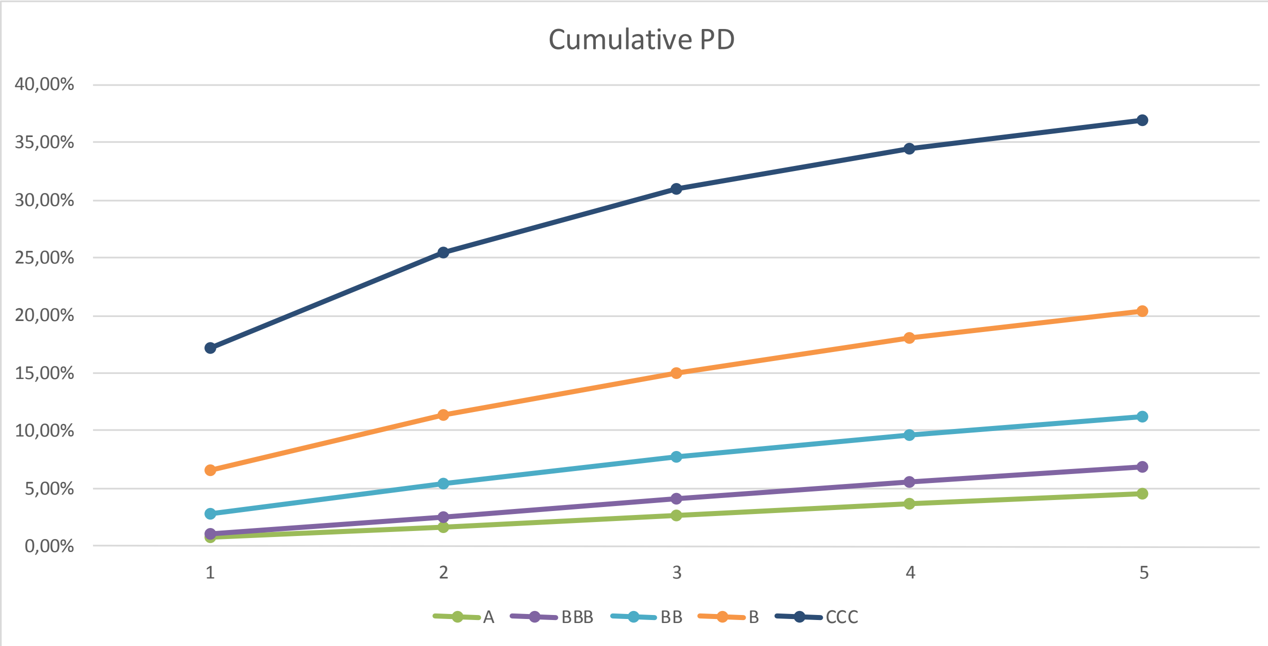
Tab. 26: average spreads obtained from CDS on Bloomberg's database

We see that in both cases spreads are monotonically increasing with rating class worsening and with tenor increasing, except for the worst (considered) rating class: CCC. For this class we observe how the higher spread is observed for the shortest tenor; then it is decreasing up to 3 years tenor, and then increasing again.

This behavior is certainly linked to the decreasing default intensity for the worse rating classes. We remind what observed in chapter 4, as per the cumulative default probabilities: these are always monotonically increasing but, for the worse rating classes, they have a negative convexity, which is due to a decreasing default intensity as shown in the graphs below (retrieved from Chapter 4, Fig. 13 and 14):

[36] "*The new SME definition, user guide and model declaration*", European Commission 2003.

[37] See chapter 3, paragraph "Merging Bloomberg's CDS reference entities with modeFinance's ratings and database creation".



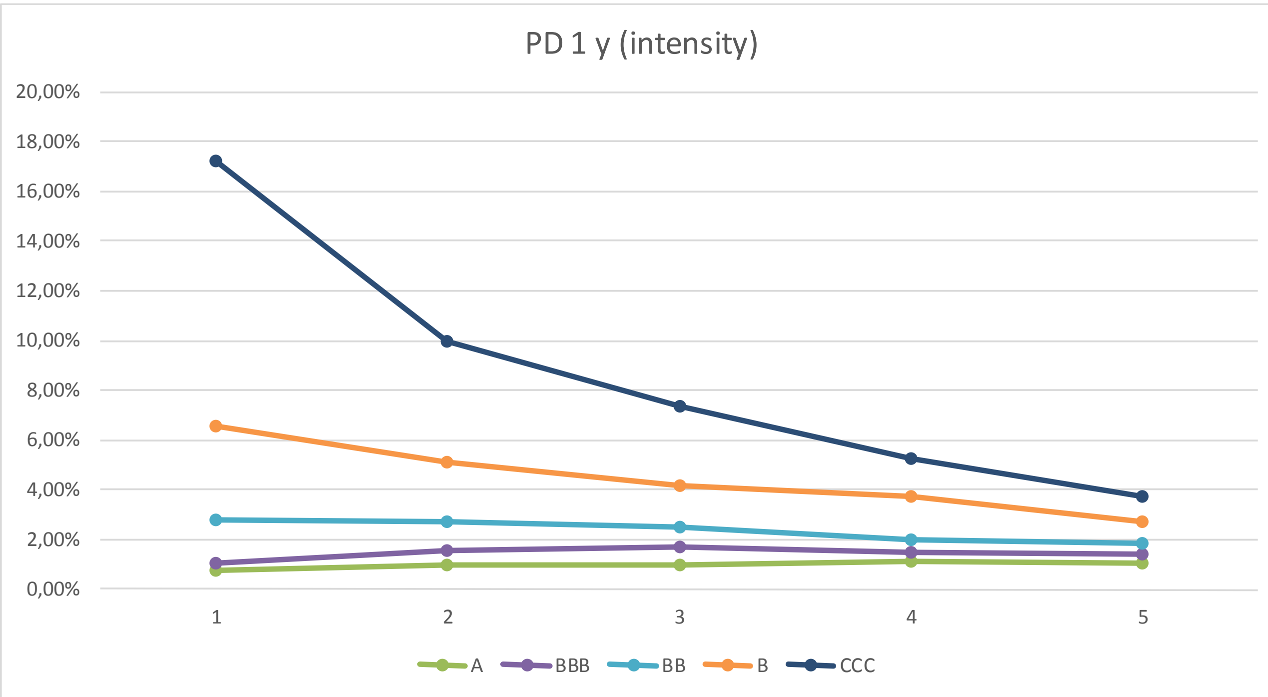


Fig. 19: Cumulative PD and PD intensity for CDS reference entities.

We also remind that the interpretation of such behavior is the following: companies with very low rating, have a high probability to default within short time, but the probability increases less intensely with time passing.

Comparison

Calculating the ratio between the average spread obtained for the Italian SME sample and those of the real CDS, we obtain the results presented in the table in the following page:

	1Y	2Y	3Y	4Y	5Y
A	1.43	1.26	1.13	1.08	0.98
BBB	1.99	1.48	1.29	1.21	1.08
BB	2.08	1.65	1.53	1.45	1.32
B	1.87	1.52	1.42	1.35	1.25
CCC	0.67	0.60	0.59	0.60	0.60

Tab. 27: ratio between avg spread of Italian SME and regular CDS

As expected, the spread obtained for Italian SMEs is in average 42% higher than that obtained for real CDS. Only in the case of CCC rating class, the spread observed for regular CDS is higher than that calculated for SMEs.

One more thing we notice is that the (relative) difference between the spreads is decreasing with tenor increasing. This holds true for all considered rating classes except, once again, for CCC class, where the ratio is quite constant across tenors. Such behavior goes along with the Factor definition, whose value was getting closer to 1 with tenor increasing (see tab. 23)

Sensitivity analysis

As already said, the final aim of the thesis is to compare whether the spread (and resulting cost) of a hypothetical CDS on SME commercial credit, calculated according to the developed model, results comparable to the real price of the trade credit insurance.

For many previous and future elaborations, many parameters have been assumed (will be assumed) according to ISDA[38] standards (see Tab.28).

Day Count Convention	"Actual/360"
Business Days Convention	"Following"
Coupon Payment frequency	"Quarterly"
Settlement date	December 31st 2013
Recovery Rate	0.4
Zero Curve	"EUR SWAP" (30/360, Sem) for EUR denominated CDS

Tab. 28: input parameters

Some other parameters are instead the result of subjective choices.

In previous chapter 3, we've retrieved the risk neutral default probabilities, from the market spreads. In the retrieving process we made an assumption of the maturity of each CDS, for the different tenors.

While in chapter 7 we will have to adjust another parameter, the recovery rate (so far kept at its standard value of 0.4), to align it to the assumptions required by trade credit insurance comparison.

For these reasons, we deemed appropriate to perform a sensitivity analysis with respect to these two parameters: maturity (sensitivity to –less than one year– changes), and recovery rate.

Maturity sensitivity

Even if, standard CDS mature on IMM (International Monetary Market) dates, for the PDs retrieving process, average annual spread of each CDS have been used. Hence our choice to set the maturity at the tenor +0.5 years.

Keeping all other parameters constant (see Tab.29) we analyzed the spread behavior for different maturities, being these last a fraction of the year:

1. Maturity = Tenor +0.0 years (e.g. for 3 years tenor, the maturity date was settlement date +3 years);
2. Maturity = Tenor +0.25 years (e.g. for 3 years tenor, the maturity date was settlement date +tenor +0.25 years);
3. Maturity = Tenor +0.50 years (e.g. for 3 years tenor, the maturity date was settlement date +tenor +0.5 years). This maturity is the standard maturity chosen for the PD retrieving process.

We then computed the Spread for a sample of 1,000 Italian SME (same sample used for the "Spread comparison" paragraph). Afterwards we clusterized the spreads by modeFinance's rating class and calculated the average spread for each rating class. The process was repeated for each tenor from 1 to 5 years.

The results obtained are displayed in the following table (the percent variation is obtained with reference to the "Base maturity" = tenor + 0.5 years, according to the formula: 1-Analyzed maturity/Base maturity), spreads are expressed in basis points:

[38] ISDA - International Swaps and Derivatives Association, Inc. - http://www2.isda.org/

	Absolute spread						*Difference respect the "Base maturity"*				
tenor +0,0y	1Y	2Y	3Y	4Y	5Y		*1Y*	*2Y*	*3Y*	*4Y*	*5Y*
A	6.5	10.2	16.4	23.4	32.6		-27%	-27%	-20%	-18%	-9%
BBB	15.6	23.0	35.2	51.9	71.5		-24%	-24%	-21%	-17%	-9%
BB	18.1	30.3	46.7	70.5	96.8		-31%	-24%	-22%	-17%	-9%
B	37.9	58.9	86.8	121.9	162.8		-27%	-22%	-19%	-16%	-8%
CCC	790.3	566.1	515.8	521.1	531.7		22%	5%	-1%	-1%	-1%
tenor +0,25y	**1Y**	**2Y**	**3Y**	**4Y**	**5Y**		***1Y***	***2Y***	***3Y***	***4Y***	***5Y***
A	7.9	12.2	18.5	26.1	34.3		-11%	-12%	-9%	-8%	-4%
BBB	18.6	27.1	40.3	57.5	75.1		-10%	-11%	-10%	-8%	-4%
BB	23.0	35.8	53.9	78.1	101.6		-13%	-11%	-10%	-8%	-4%
B	46.3	68.3	97.5	133.7	170.2		-11%	-10%	-9%	-7%	-4%
CCC	705.8	551.3	518.1	523.9	533.6		9%	3%	0%	-1%	0%
tenor +0,5y	**1Y**	**2Y**	**3Y**	**4Y**	**5Y**						
A	8.9	13.9	20.4	28.5	35.9						
BBB	20.5	30.3	44.7	62.7	78.5						
BB	26.3	40.1	60.2	85.0	106.1						
B	51.9	75.7	106.8	144.4	177.1						
CCC	645.5	537.5	519.3	526.7	535.4						

Tab. 29: spread sensitivity to maturity changes of less than one year.

As it can be seen, the sensitivity to maturity changes of the extent *quarter-year* to *half-year* is not negligible. As obvious, the spread change is increasing with maturity difference increasing.

In both tenor +0.0 year case and tenor +0.25 year case, it can be noticed a tendency, for the spread relative change, to decrease with tenor increasing (i.e. row-wise). That is most probably due to the fact that the extra time lag added (+0.25 and +0.5 years) has relatively less impact on higher tenors than on smaller (i.e. buying protection for 1.5 years is a 50% longer period that buying it for 1 years, but is only a 10% longer period for a 5 years tenor). From a numerical perspective, we've a much higher PD on a (for example) 1.5 years time horizon than just one year, but PD is not as much higher on a 5.5 year time horizon, respect a 5 years time horizon (in relative terms).

This sensitivity analysis suggests that, when the SMEs' spread will be computed, with the scope to be compared to a real credit insurance price, exactly the same protection time horizon will have to be used, even with maturity differences well shorter than 1 year.

Below we display such spread variation graphically (for readability of the graph, we limit it only to the central BB rating class).

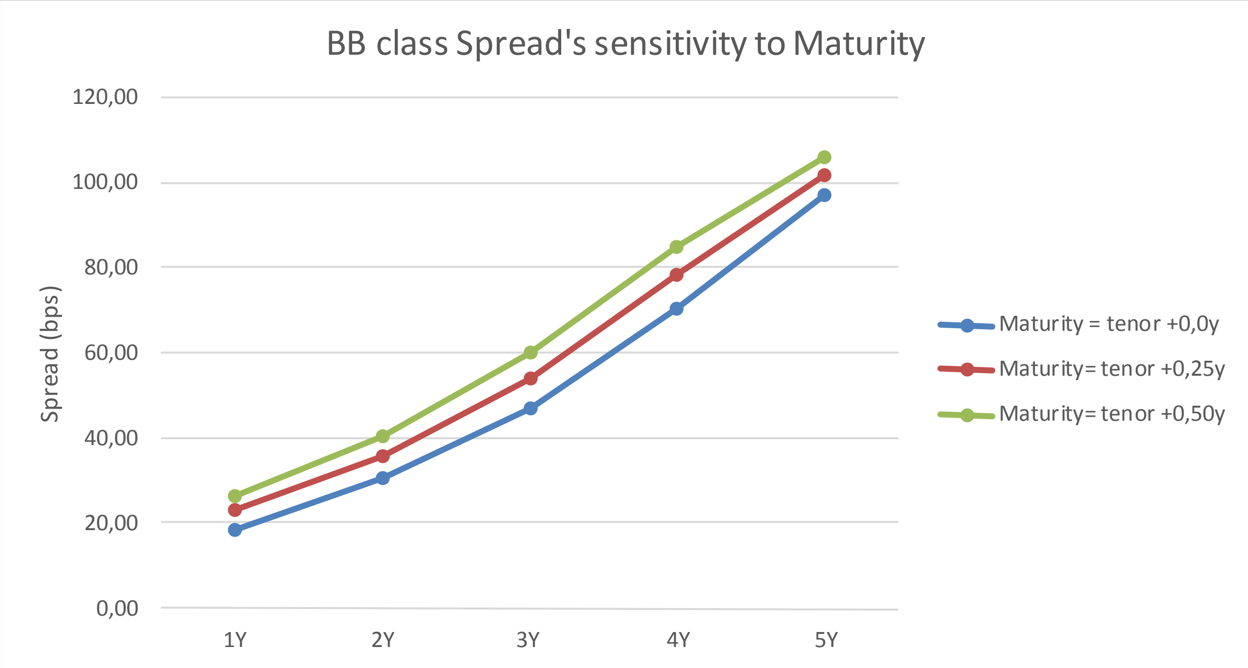


Fig. 20: spread sensitivity to maturity changes of less than one year.

Recovery rate sensitivity

As said another input parameter which is deemed to be subject to possible variations, as per the final step of the thesis (while computing the SME-CDS spreads) is the recovery rate. We'll indeed see that in order to compare CDS spreads to Trade Credit Insurance, some assumptions, which include Recovery rate, will have to be made.

For this reason, we deem appropriate to investigate the sensitivity of the pricing formula to the Recovery rate parameter.

In this case, the result is quite straightforward. According to the spread formula:

$$S(t_V, t_N) = \frac{(1-R)\sum_{m=1}^{M\,x\,t_N} Z(t_V, t_m)[Q(t_V, t_{m-1}) - Q(t_V, t_m)]}{RPV01} \tag{24}$$

Where:

- R is the recovery rate;
- RPV01 is defined as the Risky Present Value of 1 bp of spread and does not depend on the recovery rate.

Accordingly, in the spread formula the Recovery rate only appears in the numerator. As a consequence, the impact of the recovery rate variation on the spread is constant (across different tenors and rating classes), and equal to:

$$\Delta S(t_V, t_N) = \frac{(1-R_1)}{(1-R_2)} \tag{25}$$

Taking as reference "Base Recovery rate" the standard 0.40, we analyzed the spread variation for the following recovery rates:

1. Recovery rate= 0.20;
2. Recovery rate= 0.50;

Even if the relative spread variation is constant, for sake of completeness we report the results in the same way as per the maturity analysis:

	Absolute spread (bps)						***Difference respect the "Base Recovery rate"***				
RR= 0.20	1Y	2Y	3Y	4Y	5Y		*1Y*	*2Y*	*3Y*	*4Y*	*5Y*
A	11.9	18.5	27.2	37.9	47.8		33%	33%	33%	33%	33%
BBB	27.4	40.4	59.6	83.6	104.6		33%	33%	33%	33%	33%
BB	35.0	53.5	80.3	113.3	141.5		33%	33%	33%	33%	33%
B	69.2	100.9	142.4	192.5	236.2		33%	33%	33%	33%	33%
CCC	860.7	716.7	692.4	702.2	713.9		33%	33%	33%	33%	33%
RR= 0.40	**1Y**	**2Y**	**3Y**	**4Y**	**5Y**						
A	8.9	13.9	20.4	28.5	35.9						
BBB	20.5	30.3	44.7	62.7	78.5						
BB	26.3	40.1	60.2	85.0	106.1						
B	51.9	75.7	106.8	144.4	177.1						
CCC	645.5	537.5	519.3	526.7	535.4						
RR= 0.50	**1Y**	**2Y**	**3Y**	**4Y**	**5Y**		***1Y***	***2Y***	***3Y***	***4Y***	***5Y***
A	7.4	11.6	17.0	23.7	29.9		-17%	-17%	-17%	-17%	-17%
BBB	17.1	25.2	37.2	52.2	65.4		-17%	-17%	-17%	-17%	-17%
BB	21.9	33.5	50.2	70.8	88.4		-17%	-17%	-17%	-17%	-17%
B	43.3	63.1	89.0	120.3	147.6		-17%	-17%	-17%	-17%	-17%
CCC	537.9	447.9	432.8	438.9	446.2		-17%	-17%	-17%	-17%	-17%

Tab. 30: Spread sensitivity to recovery rate

As we can see, a Recovery rate increase of 10% (respect the standard 40%) produces a spread diminution of 16.7%, while a recovery rate decrease of 20% produces a spread increase of 33.3%. Such results go well along with common sense: the more it is expected to recover from a defaulted bond, the less will cost protection over such default.

Of course, a constant relative spread variation, translates in higher spread difference, in absolute terms (i.e. bps), for higher spreads. That means higher (absolute) spread difference for longer tenors and for lower rating classes.

What just stated is exemplified in the following figure (taking as reference the rating class "A" and "B").

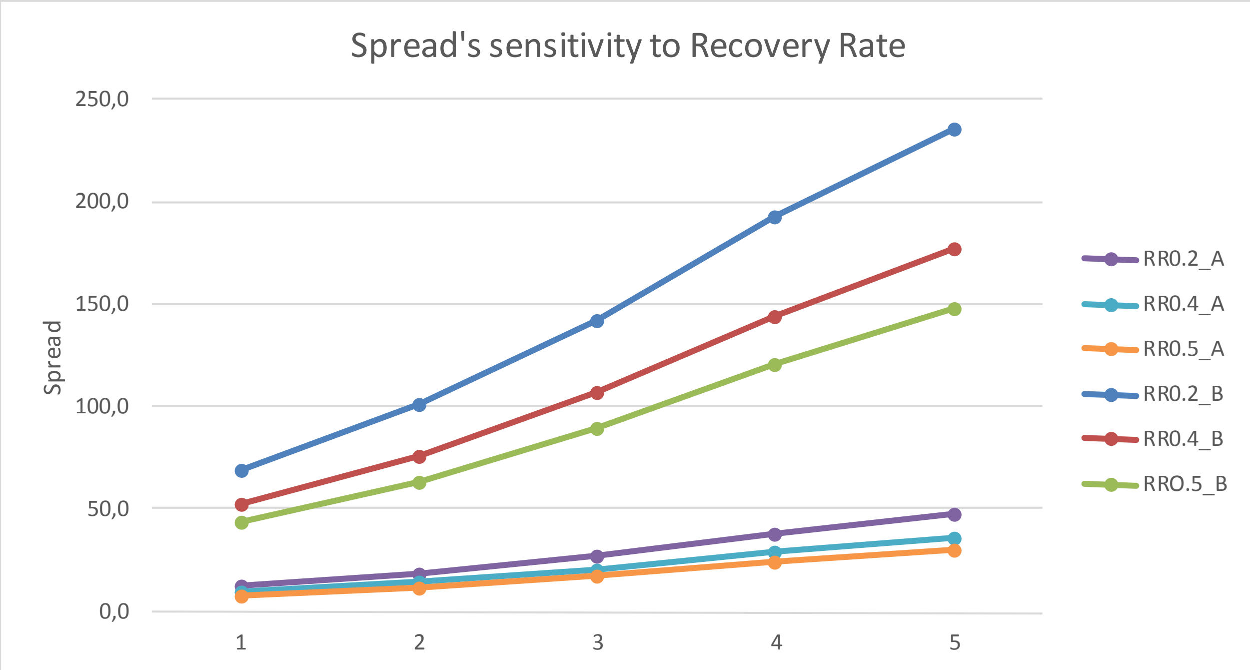


Fig. 21: Spread sensitivity to recovery rate

The conclusion of such sensitivity analysis is that, with the goal of obtaining a fair CDS pricing on SME commercial credits, it will be important to assume a recovery rate equal to what assumed by trade credit insurances for their products.

Chapter 6

TRADE CREDIT INSURANCE

Once more we remind that the final step of the thesis is to compare the price of Credit Default Swaps, as here theoretically developed, to the price of a real trade credit insurance policy. At this point of the thesis, we then introduce the concepts and basics of Trade credit insurance.

What is the Trade Credit Insurance

"Trade credit insurance, insures suppliers against the risk of non-payment of goods or services by their clients (defined as buyers). These may be buyers situated in the same country as the supplier (domestic risk) or buyers situated in another country (export risk). The insurance covers nonpayment as a result of insolvency of a buyer, or non-payment after an agreed number of months after due-date (protracted default). It may also insure the risk of non-payment following an event outside the control of the buyer or the seller (political risk cover), for example the risk that money owed cannot be transferred from the buyer's country to the supplier's country"[39].

General aspects of trade credit insurance

For trading companies, which act as suppliers, trade receivables can represent 30-40% of their balance sheet[40]; for such companies it is a wide spread practice to sell goods/services on open account. It is obvious that one or more insolvencies among their clients can lead to catastrophic consequences, the worse of which is the default of the supplier itself. Anyhow, more commonly, the immediate consequence of an insolvency event is a negative reflection on liquidity and/or working capital.

It is straightforward that a Seller company will do its best to reduce such risk. The first step is certainly to set up an efficient credit risk management. But to further enhance the hedging of trade credit risk, the most effective measure (not necessarily the most –economically– convenient) is to insure the trade credits.

In a trade credit insurance scheme, the players involved are the following:

- A "Seller" company: sells goods, or services to its portfolio of customers;
- The "Buyers" companies: the customers, they buy goods or services from the "Seller";
- The "Credit Insurer" (or just insurer), which insures the trade receivables of the "Seller".

If the Seller wants to hedge the risk of insolvency of its customers (Buyers), it will then insure its "commercial relationships" with them. It will define, with an Insurance company, an insurance policy, on which it will pay a premium. In return for the premium, the Insurer will cover (part of) the credit owed by a Buyer in the event this defaults, or in the event of a "protracted default" (i.e. payment delay beyond a predefined period).

The conceptual scheme of a credit insurance process is displayed in the following figure 22.

In the event a buyer defaults, the credit insurer may have the right of "Subrogation", which means that the insurer has the contractual right to claim from the defaulted buyer the credit it has covered.

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[39] The World Bank, "Trade Credit Insurance", Peter M. Jones, Feb. 2010

[40] Source: The World Bank, "Trade Credit Insurance", Petr M. Jones, Feb. 2010.

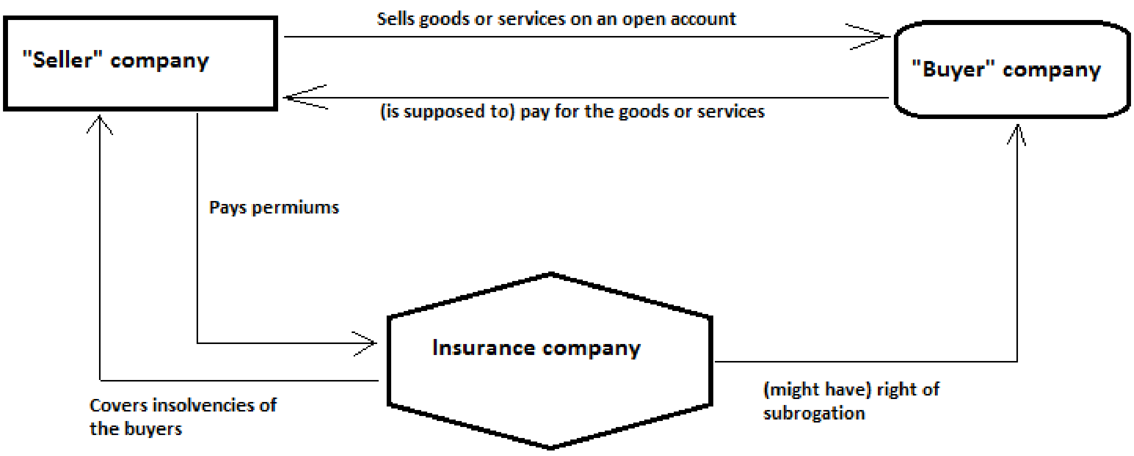

Fig. 22: trade credit insurance scheme, source: "Trade Credit Insurance", Peter M. Jones, The World Bank, Feb. 2010

Outlines of a standard trade credit insurance policy

General contractual terms

A trade credit insurance policy normally covers short-term commercial credit risks. That means, the standard protection period lasts one year (365 days).

The most diffused short-term policies are "**whole-turnover**" kind, meaning that they cover for the entire portfolio of clients of the policy holder (i.e. the "Seller company").

In general terms, within a whole turnover contract, a company cannot choose to insure credits of a specific pool of buyers, which may consider to be riskier. This protects the insurer from "adverse selection", that is, a –too high– concentration of risk on a single contract. Companies can still decide to insure only a specific pool of byers, but premia will higher.

In order to define a contract (and the relative premium) the insurer will need to know the entire portfolio of Buyers of the (to be) insured company. It will then gather information on them, aimed to evaluate their creditworthiness. On the basis of the creditworthiness level assessed, for each Buyer it will define a credit limit. Such credit limit is the insured amount for all transactions with the specific Buyer. Note that the assigned credit limit may vary during the year, if adverse information is found. In this sense, the insurance plays also the role of monitoring the portfolio of buyers of its client, warning him on adverse signs by some of the clients.

The insured company can ask for the increase of a specific credit limit; such increase has to be approved by the insurer. On the other hand, the insurer can unilaterally decide to reduce, or zero the credit limit for a specific buyer.

Usually Insurers offer standard policies, but any contract can be tailored to the customer needs. Many trade credit insurers have developed policies aimed to suit SMEs' needs, which are competitively priced and have lower administrative requirements.

Another kind of credit insurance policy is the "**Excess of Loss**". This kind of contract is best-suited to mid-large companies.

In this case, the insurer defines a maximum coverage amount and a deductible threshold (which generally is the average loss recorded by the Seller in the last years).

It is not defined a credit limit for each of the Buyers, neither monitored their creditworthiness, since deductible and maximum coverage are predefined. As a consequence, this kind of policy does not provide such a service.

Differently from the "Whole turnover" kind, it is not foreseen a coverage in the event of protracted-default, but only in case of legal default.

Premium

The premia of such policies are calculated on the basis of actuarial techniques. Essentially they follow the procedures applied for the broader "*insurances for damage*". The premia are calculated so to cover the expected loss (from the insurance perspective), the return on risk capital and the expenses sustained by the insurance.

The first two components (expected loss, return on risk capital) need an estimation of the distribution of the future losses. In this sense, the employed models do not differ from those applied by banks (as for example CreditRisk+ of First Boston – Credit Suisse). It does not exist a "predominant" model, since the choice depends on the specific products and the information available on the subjects insured. In general terms, are preferred models that deal separately the loss frequency and the severity (according to the principles of Basel II). Specifically, credit risk is classified as "low frequency/high severity".

The third component (expenses sustained by the insurance) are dealt separately, as it is specific of the insurer.

Worth to note that premia do not depend –directly– from the creditworthiness of the buyers of the insured company. Such variable (creditworthiness), is taken into account when defining the credit limit granted for every buyer of the insured company.

There may exist some clauses for premium reduction/increase, such as: no-claims bonus clauses, maximum loss coverage, excess clause, etc…

For (the most diffused) "whole-turnover" contracts, premia are generally expressed as a percentage of the forecasted annual turnover (i.e. their exact dollar value is not known in advance), with a minimum premium due. When the effective yearly turnover is known (or on a quarterly basis), the excess (if any) respect the minimum premium is settled between the insurer and the insured company.

For the "excess of loss" contracts, the premium is predefined and calculated on the basis of the Maximum contractual coverage.

Coverage

The insurer does not cover the entire credit limits granted to the buyers of the policy holder but, usually, a percentage between 80% and 90% of it. This is to make sure that the policy holder is still committed to manage its credit risk carefully.

The insurer will cover its client's loss either if one (or some) of its buyers defaults, or if this does not honor its debt within a predefined period ("Protracted default"), usually between 60 and 180 days. The "Protracted default" is not covered by the "Excess of loss" policies.

The insurer may have, as said, a "Subrogation" right-claim on the buyer un-honored debt.

Common trade credit insurance policies do not cover intra-group sales, or sales made to government, or governmental entities.

Trade credit which are under a dispute, are not covered for, until and unless they are resolved in favor of the Seller company.

Finally, it is not covered the entire sum of credit limits granted, but it is defined a policy's *maximum liability limit*, which generally is considerably (about one order) lower than the sum of all credit limits granted.

Benefits

For a supplier, the trade credit insurance brings several benefits (at a cost, of course). The main of which are:

- a predetermined and sure cash flow;
- no need to take care of the collection of un-honored receivables;
- easier access to bank borrowing, as the insurance policy can be assigned to the bank as a security (and consequently it can borrow at more favorable conditions).

A few figures of trade credit insurance worldwide

The main players in trade credit insurance are Euler Hermes, Atradius, Coface. Their market share (updated at 2013) is presented in the following table.

Insurer	Market share
Euler Hermes	34%
Atradius	25%
Coface	23%
Others	18%

Tab. 31: main Insurer and market share. Source: AON presentation, 2013.

The volumes of the trade credit insurance in the world and their evolution is represented in the graph at the following page:

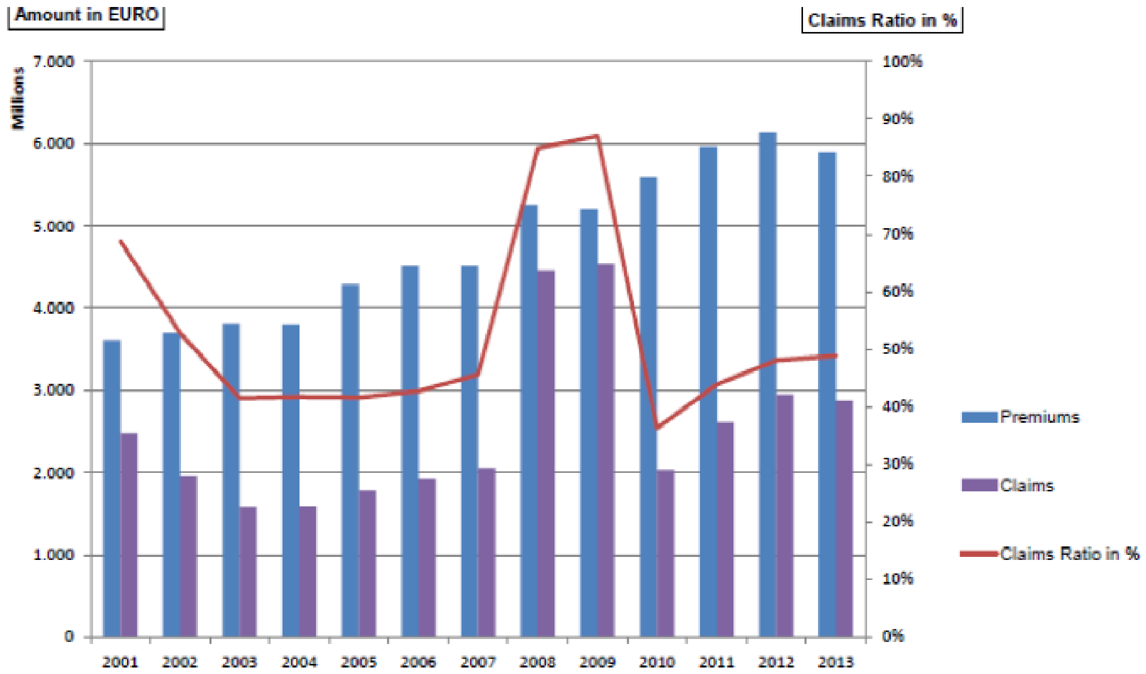

Fig. 23: evolution of the business of Trade credit insurance. Source: AON presentation 2013.

As it can be seen, trade credit insurance business has known a constant growth since 2001. It is worth to note the peaks of claims recorded in 2008 and 2009, the years of most severe impact of the economic crisis.

Trade credit insurance alternatives

The main "competitors" of trade credit insurance are bank letters of credit and factoring.

Letters of credit

A letter of credit is a bank guarantee over a debtor obligation (in our case, the buyer's debt). It is a task of the buyer to ask a bank for a Letter of credit. They represent a "disadvantage" for the buyer, as its borrowing capacity from the bank will decrease. At the opposite, on a borrowing perspective, a credit insurance is a favorable element for the Seller company, as it is perceived from the bank as a sort of guarantee.

In relative terms, a letter of credit is more expensive than trade credit insurance and such product is less and less utilized[41].

Factoring

Factoring is a transaction set in place between a financial intermediary, the Factor, and a business company, according to which the business company sells its receivables to the Factor at discount.

This allows the business company to meet its immediate cash commitments, but factoring assumes an insurance profile only in case of "non-recourse" factoring. In this case it is the Factor to bear the consequences of a default by the debtor company.

[41] Source: private discussion with Willis Italia SpA.

In case of "with-recourse" Factoring, credit risk is not transferred to the Factor. As a consequence, in event of default of the debtor, the creditor will have to refund the Factor.

Obviously, "non-recourse" factoring is more expensive of "with-recourse" factoring and also (in relative terms) of trade credit insurance.

Trade Credit Insurance in Italy

Trade credit insurance in Italy was firstly regulated by "Circolare n° 145", promulgated on January 7th, 1960, and it is classified as an "*insurance for damage*". As per the export risk, this is regulated by DLGS 143/98 (ex L. 227/77-"Legge Ossola").

Such kind of insurance has not known the diffusion as it has had in other European countries. It is common opinion that it is "wrongly classified" (as insurance for damage) and such fact has contributed to restrain its diffusion.

However, in recent years, trade credit insurance has known a substantial growth. The financial and economic crisis, began in 2008-2009, has led to a growth of insolvencies in the industrial environment. Since in Italy trade credits can represent the 30% of a trading company balance sheet, credit managers have become more careful on assessing the credit risk of clients and, as a last resort, they have begun to see credit insurance as an effective mean to hedge the insolvency risk.

Furthermore, after Basel II, companies view the trade credit insurance also as a mean to improve their rating and consequently to ease their access to the bank facilities.

Despite this, according to AON[42], in 2010, Italian companies were still "under-insured", compared to the other European countries.

In 2013, with the crisis (re-)harshening (+7% of insolvencies), companies have become more aware of the credit insurance tool, to the extent that 52% of companies[43] deemed such tool as an effective mean to mitigate the credit risk. The perceived importance of trade credit insurance increases dramatically in the case of foreign credits.

Always with reference to 2013, the sectors most hit by insolvencies were: Commerce (30%), Industry (22%) and Constructions (16%)[44].

Sector's statistics

Source: Ania, ITALIAN INSURANCE IN FIGURES, 2015.

In 2014, non-life insurance premia were up to 32.8 billion (-2.7% respect 2013). The credit and suretyship branch counted for 0.5 billion EUR, with a market share of 1.4% (decreased by 2.3% respect 2013).

[42] AON is an insurance broker: http://www.aon.com/italy/
[43] ANRA, "Aumenta l'interesse delle imprese nell'assicurazione del credito", July 2013.
[44] Euler Hermes, Press release, July 2013.

Non-life classes	direct premiums (€ bn)	market share (%)	change 2014/2013** (%)
motor and marine liability	15.2	46.4	-6.5
property*	5.1	15.5	2.5
accident and sickness	5.0	15.3	0.0
general T.P.L.	2.8	8.6	-0.6
land vehicles	2.4	7.3	-1.1
credit and suretyship	0.5	1.4	-2.3
transport*	0.4	1.4	-6.8
other non-life classes*	1.4	4.2	9.2
TOTAL	**32.8**	**100.0**	**-2.7**
EU branches***:			
motor and marine liability	0.8	18.0	-15.8
other non-life classes (excl. motor and marine liability)	3.7	82.0	4.7
Total	4.5	100.0	0.3

Fig. 24: non-life Italian direct business premiums.

The loss ratio (claims cost/earned premiums) for credit and suretyship has worsened from 71.2% to 77.4%.

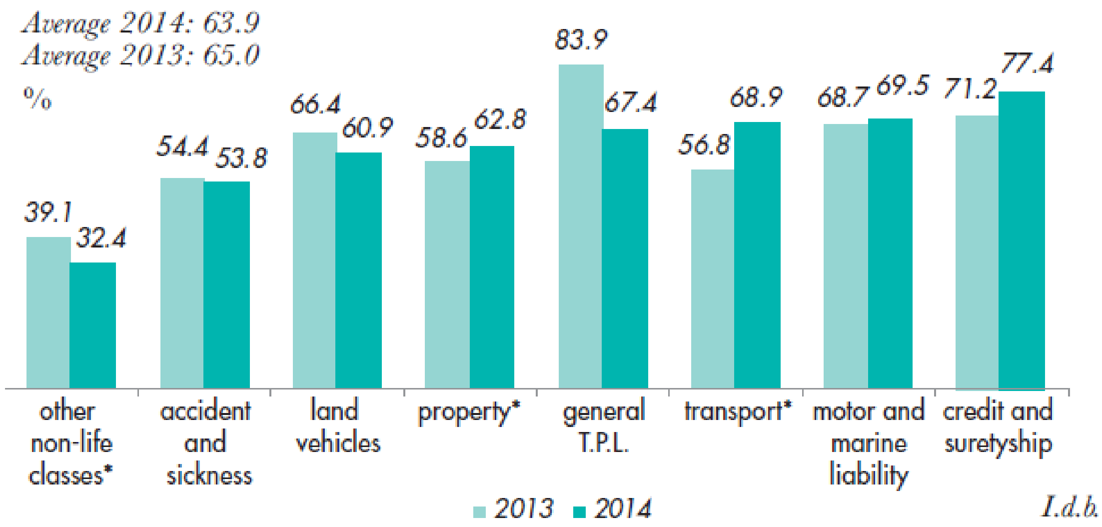

Fig. 25: Loss ratio (claims cost/earned premiums) non-life classes

The overall non-life technical result was positive for 3.6 billion, while property, credit and suretyship results were negative (both for 0.1 billion).

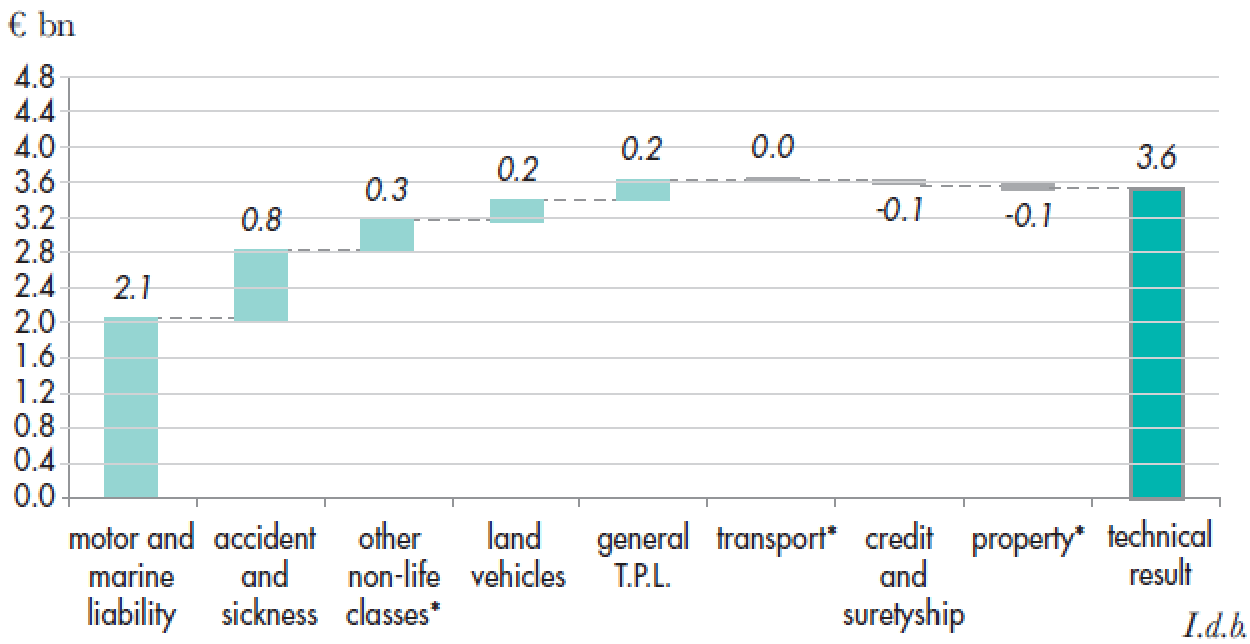

Fig. 26: Non-life insurance technical result breakdown

Considering both the technical and the non-technical results, the non-life industry registered in 2014 a 2.5 billion return, which determined a Return on Equity (ROE) positive and equal to 10.4% (9.8% in 2013).

Chapter 7

COMPARISON BETWEEN DEVELOPED CDS ON SMALL AND MEDIUM ENTERPRISES AND REAL TRADE CREDIT INSURANCE POLICIES

In this last step of the thesis, we are going to compare the hypothetical costs implied by the developed model for CDS over Small and Medium Enterprises with the costs of some real policies of Trade Credit Insurance (from now on "*TCI*").

As seen in the previous chapter, the (far) most common TCI policy is the "Whole-turnover" kind. In such a policy, the entire portfolio of "buyers" of the policyholder is insured, to the extent of each Credit limit granted by the Insurer (which can also be zero), and within a maximum liability limit.

In light of this, to make the two products comparable, we make the hypothesis of holding a portfolio of single name CDS, as described in more detail in the following paragraph.

In order to be able to perform such comparison, we have confidentially obtained from the insurance broker *Willis Italia SpA* (part of Willis Group Corp Ltd.), four TCI policies complete with the following data:

1. The premium rate;
2. The (forecasted) insurable turnover of the insured company;
3. The minimum premium;
4. The maximum liability limit;
5. The credit limit granted for each company of each portfolio;
6. A unique identifier (e.g. Vat number) of each company of each portfolio;
7. The coverage of the policy (i.e. the amount of the granted credit limit eventually reimbursed).

The *premium rate:* has to be applied to the insured turnover in order to obtain the policy cost.

The *insured turnover:* is the turnover of the insured company net of sales paid in advance, sales made to public institutions and intra-group sales and adjusted to the non-zero Credit Limits granted. The insured turnover is not known 'till the beginning of the following Fiscal Year.

The *insurable turnover:* is an estimation of the Insured turnover, based on:

- the previous year sales amount (net of sales paid in advance, sales made to public institutions and intra-group sales);
- the non-zero Credit Limits granted.

It is a proxy for calculating both the policy cost and the Maximum liability limit, before the -real- insured turnover is known.

Maximum liability limit: it is the maximum exposure (loss) covered by the insurance and it is defined as a multiplier (usually between 25 and 35 times) applied to the policy premium.

All of the four TCI policies subscriber are Italian companies (or Italian branches of foreign corporations). The large majority, but not all, of the portfolio of buyers are Italian companies.

<u>Both the policy subscribers and the constituents of their portfolio will not be disclosed in this thesis, due to a Non-Disclosure-Agreement signed between the author of the present work and Willis</u>.

Issues and assumptions

In the present work, it has been developed a model for evaluating the spread of a single name CDS, whose reference entity is intended to be a Small or Medium Enterprise.

We face the fact that the vast majority of TCI policies (as the ones obtained by Willis) are whole-turnover kind. Such a situation could be interpreted as holding a basket Credit Default Swap. Usually[45], baskets of CDS are *add-up* kind. Meaning that the CDS settlement is triggered by the n^{th} defaulter (e.g. in a second-to-default basket CDS, settlement is made only at the time the second default takes place, regardless of the reference entity defaulting) and then the contract expires.

For such kind of derivatives, the pricing methodologies are far more complex than for single name ones, among other differences, for such products it plays an important role the default correlation among constituents of the basket of reference entities.

Moreover, our situation is different: firstly, because the payoff in our case does depend on the defaulting reference entity, secondly, because the contract must not expire, whatever the number of default.

For these reasons, we will assume that we hold a portfolio of single name CDS, each written on a different reference entity and that there is no correlation among defaults.

Another issue we are facing in this comparison, is that a not negligible fraction of the companies constituting the portfolio of a "Seller", is represented by partnerships. Such companies (in the majority of countries, among which Italy) are exempted from filing (or make public disclosure of) their financial statements. This raises a problem of rating (and score) evaluation.

As explained in previous chapters, modeFinance's ratings and scores are here employed. The same company has developed a rating model for partnerships, whose scale is the same as the "standard" modeFinance's rating (see chapter 2). Anyhow, the continuous credit risk measure, which is the score (employed by the developed model to evaluate the CDS spread via the "Equivalent risk-neutral default probability") is not provided for partnerships.

Assumptions for partnerships

For each partnership of each portfolio we will employ, as usual, modeFinance's rating. As per the score, we will assume the central score for the rating class (see chapter 5 – "*Real world PDs as a function of the score*" for further clarifications).

We remind that the model takes as input both the rating class and the score assessed for each company.

We also remind that the corrective "Factor" of the model (see chapter 5 - eq. (20) - for further clarifications), has been developed on sole limited companies and corporations.

As a consequence, the assumptions made to solve the "Partnerships issue" represent an approximation among the inputs of the model, entailing the introduction of an error.

[45] John C. Hull "Options, Futures and Other Derivative", Pearson Prentice Hall 2009.

Assumptions for CDS spread computation

Under the perspective of employing CDS as a hedging instrument against defaults on trade credits, we made the following assumptions:

1. *CDS Notional*: for each reference entity set equal to Credit Limit granted for the same company, net of eventual absolute deductibles[46];
2. *Recovery rate*: set equal to (1 - Insurance Coverage);
3. *Maturity*: 1 year;
4. For Ratings lower than CCC, the rating applied will be CCC and the score 0.301 (the lowest limit for CCC class);
5. For Ratings higher than A, the rating applied will be A and the score 0.799 (the upper limit for A class);

As per the points 4 and 5, we remind that, due to the scarce numerosity of real CDS in these rating classes, it had not been possible to define the corrective "Factor" for these classes. We are aware that the application of points 4 and 5 introduce an error as well, in the Policy's CDS cost calculation; but the two approximations, to some extent, will offset each other.

The total cost of the portfolio protection through CDS will be obtained as the sum of each calculated spread, applied to the corresponding notional (i.e. Credit Limit).

We though face the further issue that each of the given policies foresees a Maximum Liability Limit of 30 times the annual premium. While, assuming a portfolio of single names CDS, no such limit exists. In light of this, the two costs do not result fully comparable.

We will then also compute a weighted average portfolio spread, to be applied to the maximum liability limit, in order to have another proxy of the equivalent CDS-hedging cost. The spread of each reference entity will be weighed according to the contribution of the Credit limit for the specific reference entity, to the sum of all Credit limits.

This is done assuming that the contribution of each reference entity to the reaching of the maximum liability limit of the policy, is proportional to the Credit Limit ensured. For example, a credit limit of 1.5 million EUR in face of a Maximum liability limit of 5 million EUR, will weigh much more than a credit limit of 50 th EUR.

Under such hypothesis, the average spread S to be applied to the Maximum liability limit is obtained as follows:

$$S = \sum_{i=1}^{N} s_i * w_i \tag{26}$$

Where:

- s_i is the spread obtained for the i-th reference entity of the portfolio;
- N is the number of buyers of the insured portfolio;
- w_i is the weight, calculated as

$$w_i = \frac{CL_i}{\sum_{i=1}^{N} CL_i}$$

With CL_i being the Credit Limit of the i-th reference entity.

[46] This has been adjusted to take into consideration that absolute deductibles are not going to be reimbursed.

The total cost of the portfolio protection through CDS, obtained for each of the four TCI portfolios, according to each of the two approaches, will be finally compared to the cost of the corresponding trade credit insurance policy.

Note that the policy cost has been obtained as the premium rate applied to the *Insurable turnover*, not being known (by the end of October 2015) the real *Insured turnover* of the insured companies. This represent a further approximation, but the insurable turnover as per calculated at the end of October is a fair proxy of the Insured turnover.

Trade credit insurance policy 1

As said, due to a Non-Disclosure Agreement with the data provider, both the insured company and details of the constituents of its portfolio of buyers will be omitted.

The contractual terms of such policy and the portfolio characteristics are displayed in the following table:

Trade credit insurance policy 1	
Overall Premium Rate	0.12%
Minimum premium	180,000 €
Maximum Liability Limit	30*premium
Coverage	90%
Deductible (relative)	5,000 €
Insurable Turnover	220,151,000 €
Expected policy cost	**264,181 €**

Portfolio 1	
# of buyers	335
# of non-zero Credit limits	301
Average CL	415,135 €
Average non-zero CL	462,388 €
Median ratio CL/Turnover	0.00150

Tab. 32: Details of policy and portfolio1

Before performing the CDS spread calculations, we performed some portfolio analysis.

Portfolio analysis

The distribution of credit limits, by bins of 50 th EUR width, is displayed in the graph below:

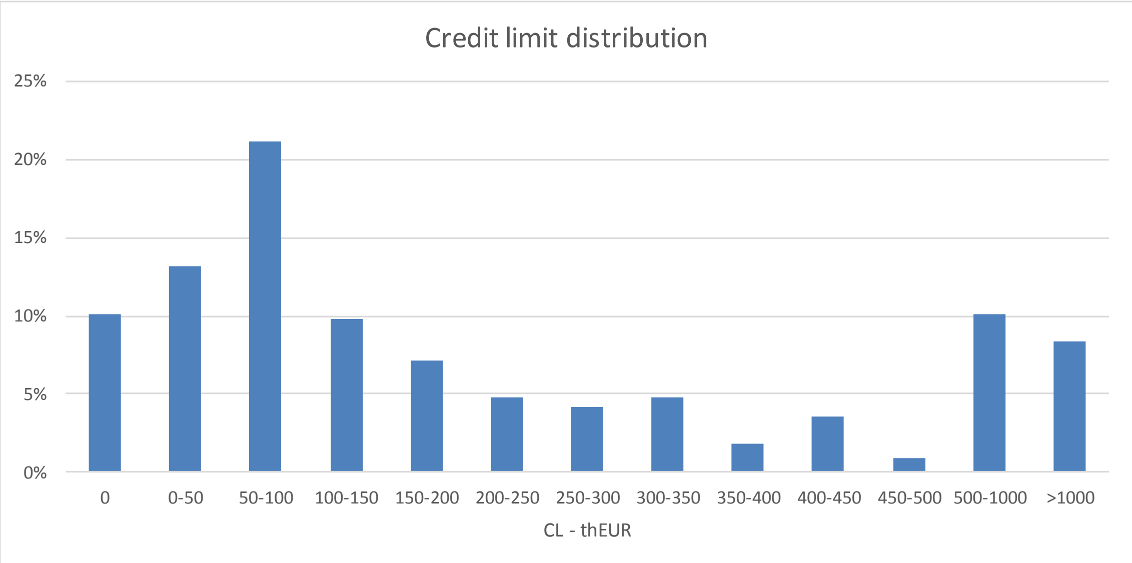

Fig. 27: Credit limit distribution, portfolio 1

The distribution of credit limits granted by such policy presents high frequencies also on the side of higher Credit limits.
Even if the (relative) majority of Credit limits is gathered in the class between 50 and 100 th EUR, there is a 10% of Credit limits very high: between 500 th EUR and 1 million and an 8% of credit limits higher than 1 million EUR.
Also interesting to note, that a 10% of companies is excluded from the insurance protection (10% of zero credit limits).

The ratings have been evaluated on the last available financial account (almost always 2014). The distribution of ratings of the portfolio (for non-zero credit limits) is displayed in the graph below:

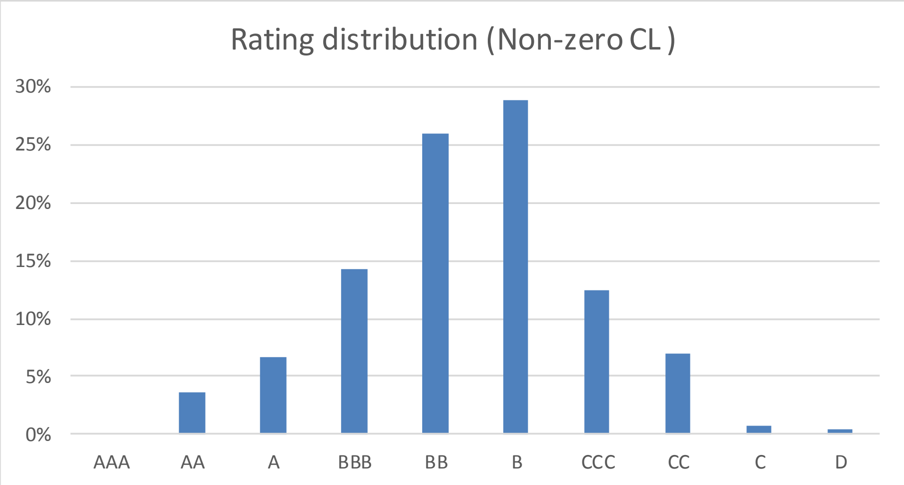

Fig. 28: rating distribution of non-zero Credit limits, portfolio 1

As we can see, the rating distribution of non-zero credit limits is quite symmetrical, implying that there is a 20% of risky companies (rating lower or equal than CCC).

We can interpret such distribution as the fact that, on each side of the policy contract, there is no evidence of risk selection, after some buyers have been excluded (i.e. zero credit limit).

Computation of the equivalent CDS portfolio and relative cost

After applying the assumptions and processes described in previous paragraph, we obtained the following results:

Real Policy Cost	**264,180 €**
CDS Policy Price	**3,869,845 €**
Maximum Liability Limit	7,925,400 €
Sum of credit limits	*139,303,000 €*
Minimum premium	180,000 €
CDS Maximum Liability Price	**220,169 €**

Tab. 33: Results TCI policy 1

Such insurance policy is characterized by a mid-low premium rate but, compared to the other policies, the credit limits granted in this case are very high.

From the portfolio rating distribution, we notice a relatively high presence of ratings equal or lower than CCC. Recalling Chapter 3, we remind that for CCC class there is a very high "jump" in the CDS spread, being it more than 10 times higher than that of a B class. We also remind that such a "jump" is most probably related to the fact that a modeFinance's CCC rated company is probably regarded (in terms of CDS reference entities) as a Non-Investment-Grade company by CDS issuers (see Chapter 3 for further details).

The above considerations (very high credit limits, and high percentage of CCC-or lower ratings), can probably explain much of the relative deviation from the policy price obtained through a CDS portfolio, respect to the *Real Policy Cost*: 14 times.

Another aspect to point out for such policy is the huge difference between the *Maximum Liability Limit* and the *Sum of credit limits* (1 to 18). As said, a portfolio of CDS would not imply any liability limit. If we apply the (weighted) average CDS spread to the *Maximum Liability Limit*, we obtain a *CDS Policy Price* that is "only" 17% lower than the *Real Policy Cost*.

Indeed, the ratio between "*CDS Policy Price*" and "*Sum of Credit Limits*" is 2.78%, which is quite aligned to the ratio between "*Real Policy Cost*" and the "*Maximum Liability Limit*": 3.33%

In the end, we observe that the *Real Policy Cost* lays between the *CDS Policy Price* and the *CDS Maximum Liability Price*.

Trade credit insurance policy 2

The contractual terms of this policy and the portfolio characteristics are displayed in the following table:

Trade credit insurance policy 2	
Overall Premium Rate	0.2588%
Minimum premium	101,250 €
Maximum Liability Limit	30*premium
Coverage	90%
Deductible (absolute)	1,000 €
Insurable Turnover (estimated)	67,000,000 €
Expected policy cost	**173,396 €**

Portfolio 2	
# of buyers	591
# of non-zero Credit limits	379
Average CL	42,444 €
Average non-zero CL	65,963 €
Median ratio CL/Turnover	0.00157

Tab. 34: Details of policy and portfolio 2

Portfolio analysis

The distribution of credit limits, by bins of 50 th EUR width, is displayed in the graph below:

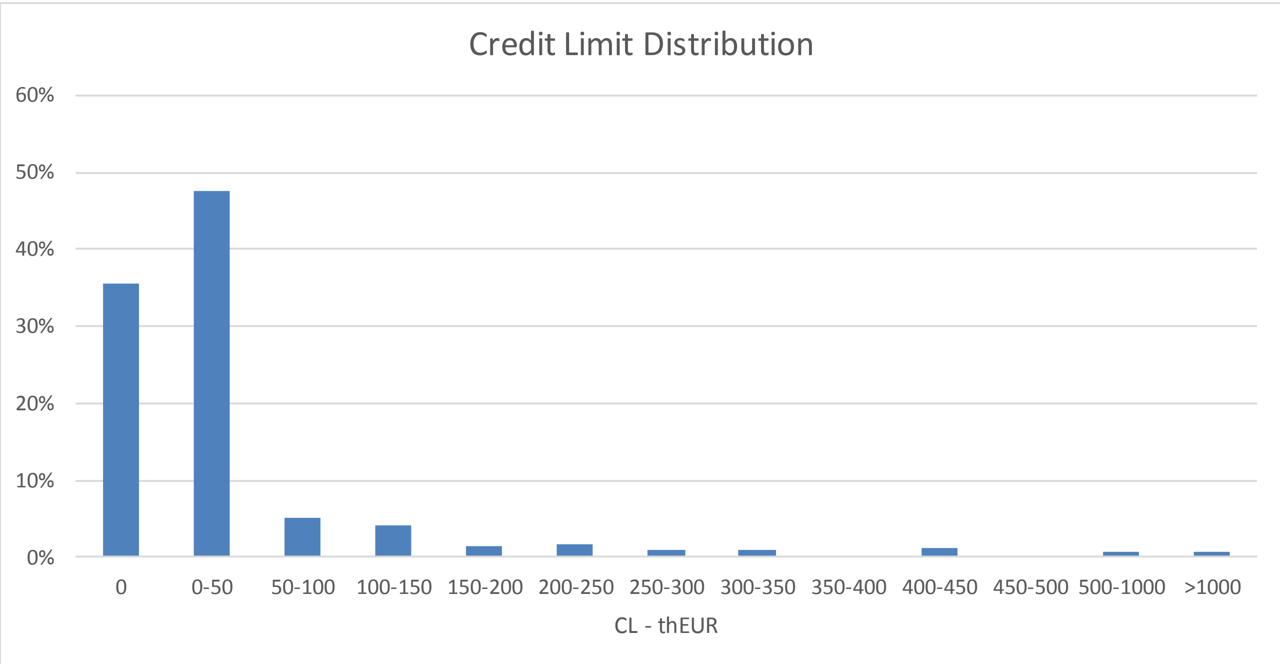

Fig. 29: Credit limit distribution, portfolio 2

As it can be seen, for such portfolio, the most of the credit limits are very low: 48% of them is lower than 50 th EUR and, in 36% of cases, no credit limit has been granted. Only 2% of Limits are higher than 500 th EUR.

The ratings have been evaluated on the last available financial account (almost always 2014). The distribution of ratings of the portfolio (for non-zero credit limits) is displayed in the graph below:

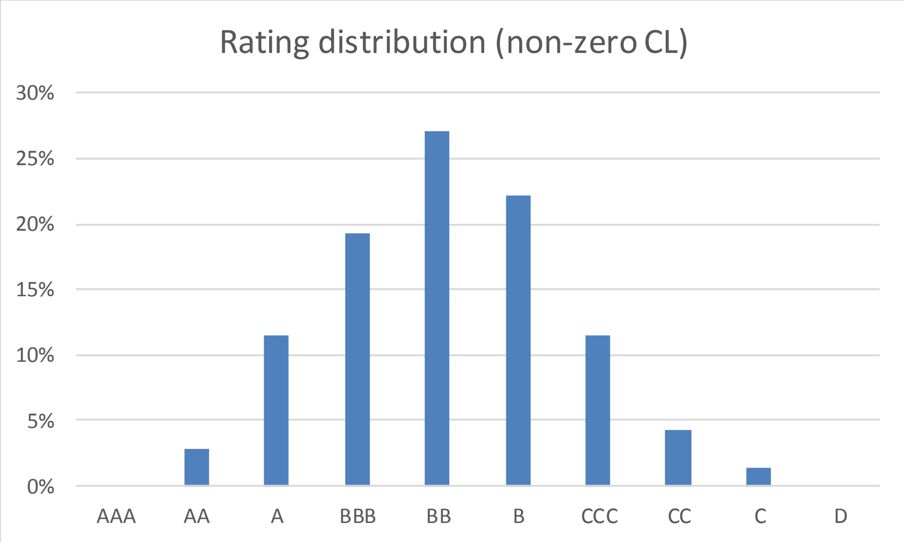

Fig. 30: rating distribution of non-zero Credit limits, portfolio 2

For this portfolio we could say that the rating distribution of non-zero credit limits is almost perfectly symmetrical, meaning that, net of Zero Credit Limits, the portfolio results free of any selection for both sides of the contract.

Computation of the equivalent CDS portfolio and relative cost

After applying the assumptions and processes previously described, we obtained the following results:

Real Policy Cost	**173,396 €**
CDS Policy Price	**196,339 €**
Maximum Liability Limit	5,201,880 €
Sum of credit limits	*25,000,000 €*
Minimum premium	101,250 €
CDS Maximum Liability Price	**41,482 €**

Tab. 35: Results TCI policy 2

This insurance policy is characterized by a high premium rate but the credit limits granted are in this case very low: almost 50% of credit limits are lower than 50 thousand Euro.

The rating distribution of non-zero credit limits underlines a balanced presence of ratings equal, or higher than A and equal, or lower than CCC.

The above factors turn out into a *CDS Policy Price* which is quite aligned to the *Real Policy Cost*: 13% higher.

For such policy instead, the greater difference is between the CDS cost of the *Maximum Liability Limit* and the *Real Policy Cost*: 75% lower, 41.5 th EUR vs 173.4 th EUR.

Finally, we notice that the *Real Policy Cost* lays between the *CDS Policy Price* and the *CDS Maximum Liability Price*.

Trade credit insurance policy 3

The contractual terms of this policy and the portfolio characteristics are displayed in the following table:

Trade credit insurance policy 3	
Overall Premium Rate	0.09902%
Minimum premium	276,303 €
Maximum Liability Limit	30*premium
Coverage	90%
Deductible (absolute)	none
Insurable Turnover	467,194,170 €
Expected policy cost	**426,615 €**

Portfolio 3	
# of buyers	203
# of non-zero Credit limits	165
Average CL	363,502 €
Average non-zero CL	447,218 €
Median ratio CL/Turnover	0.01650

Tab. 36: Details of policy and portfolio 3

Portfolio analysis

The distribution of credit limits, by bins of 50 th EUR width, is displayed in the graph below:

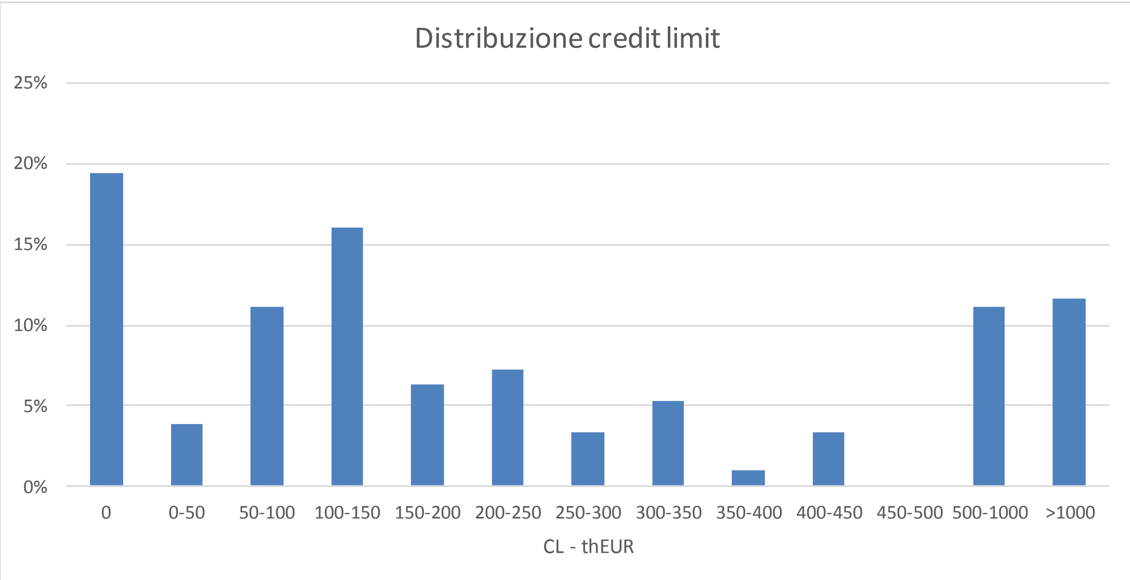

Fig. 31: Credit limit distribution, portfolio 3

The credit limit distribution for such portfolio is quite diversified. Besides the excluded buyers (19% of total receive zero credit limit), the relative majority of credit limits is between 100 and 150 th EUR (16%). We also notice a conspicuous amount of very high credit limits: above 1 million EUR.

The ratings have been evaluated on the last available financial account (almost always 2014). The distribution of ratings of the portfolio (for non-zero credit limits) is displayed in the graph below:

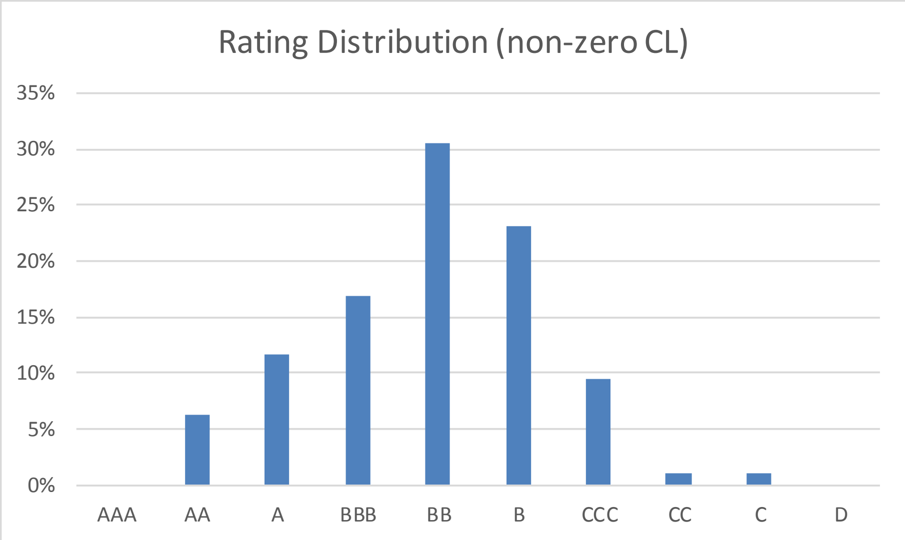

Fig. 32: rating distribution of non-zero Credit limits, portfolio 3

This portfolio is characterized by a discrete amount of rather high rating classes: equal or higher than BBB (35% of total). The prevailing class is the central BB: 31% of total. Only 12% of portfolio constituents can be considered risky: rating equal or lower than CCC.

Computation of the equivalent CDS portfolio and relative cost

After applying the assumptions and processes earlier described, we obtained the following results:

Real Policy Cost	**462,615 €**
CDS Policy Price	**1,108,821 €**
Maximum Liability Limit	13,878,470 €
Sum of credit limits	*73,791,000 €*
Minimum premium	276,303 €
CDS Maximum Liability Price	**208,545 €**

Tab. 37: Results TCI policy 3

This TCI policy is characterized, as the first one, by high credit limits granted: 12% above 1 million EUR and 11% between 500 thousand and 1 million EUR. Unlike the first policy, we have a rather higher percentage of ratings equal or above A class (18%) than ratings equal or lower than CCC, which are 12% of total.

The above characteristics imply a situation similar to that encountered for the first policy: the equivalent CDS cost is considerably higher than the *Real Policy Cost*: 2.4 times higher, 1.11 million EUR vs 463 th EUR.

Alike the first policy, the ratio between "*CDS Policy Price*" and "*Sum of Credit Limits*" is 1.50%, which is somehow aligned to the ratio between "*Real Policy Cost*" and the "*Maximum Liability Limit*": 3.33%

If we compare the equivalent CDS cost of the Maximum Liability limit, this results 55% lower than the *Real Policy Cost*: 209 th EUR vs 462 th EUR.

Also for this portfolio, the *Real Policy Cost* lays almost perfectly in the middle of the two CDS proxies.

Trade credit insurance policy 4

The contractual terms of this policy and the portfolio characteristics are displayed in the following table:

Trade credit insurance policy 4	
Overall Premium Rate	0.29%
Minimum premium	25,000 €
Maximum Liability Limit	30*premium
Coverage	85%
Deductible (absolute)	1,000 €
Insurable Turnover	22,842,143 €
Expected policy cost	66,242 €

Portfolio 3	
# of buyers	879
# of non-zero Credit limits	535
Average CL	16,831 €
Average non-zero CL	27,654 €
Median ratio CL/Turnover	0.01185

Tab. 38: Details of policy and portfolio 4

Portfolio analysis

The distribution of credit limits, by bins of 50 th EUR width, is displayed in the graph below:

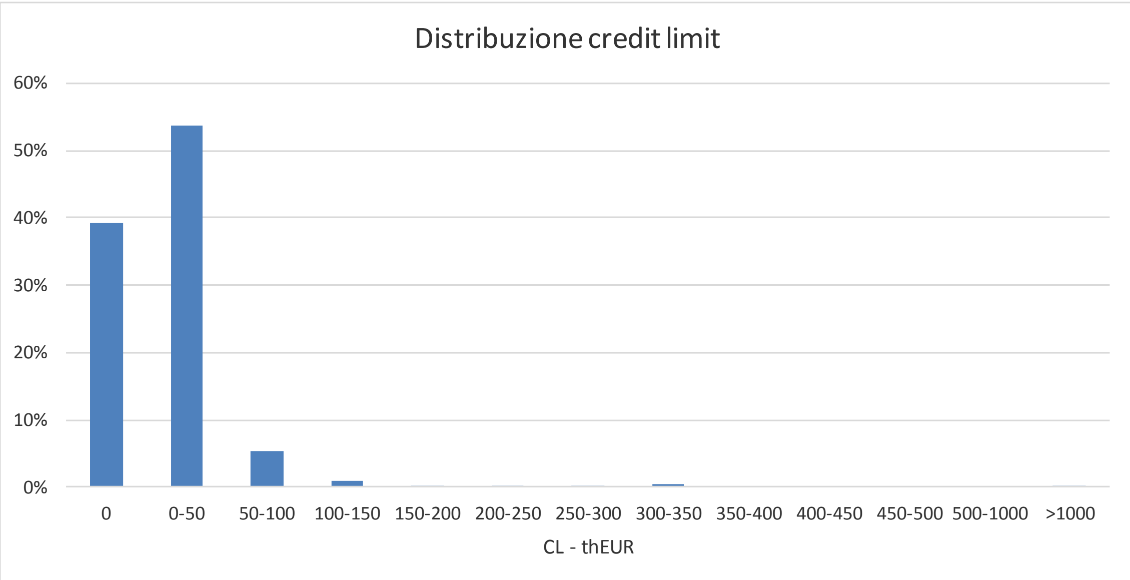

Fig. 33: Credit limit distribution, portfolio 4

For this specific portfolio, we see that almost all of the non-zero credit limits are very low: between 0 and 50 th EUR.
We also notice that 39% of the required credit limits have not been granted.

The ratings have been evaluated on the last available financial account (almost always 2014). The distribution of ratings of the portfolio (for non-zero credit limits) is displayed in the graph below:

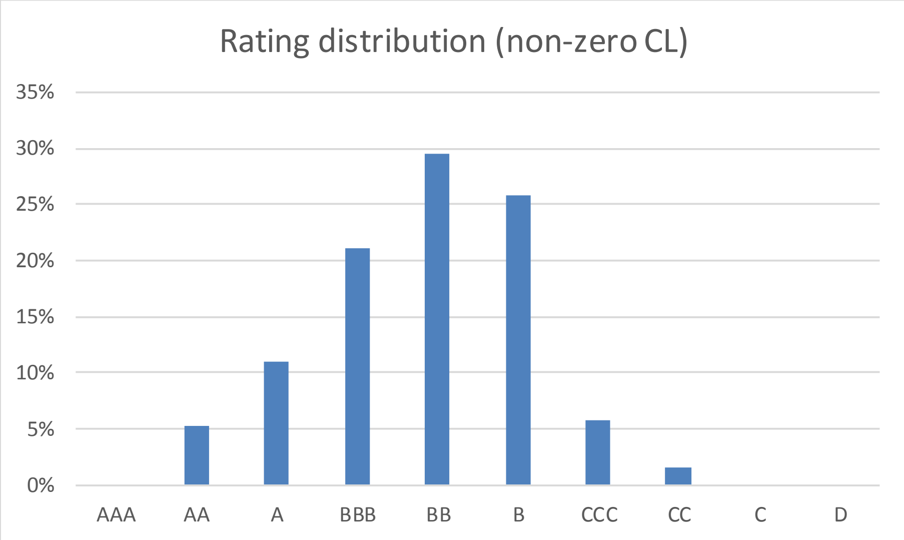

Fig. 34: rating distribution of non-zero Credit limits, portfolio 4

We see that, eliminated the zero credit limits, the resulting rating distribution is not quite symmetrical: we could argue that there are a little healthier buyers insured, rather than risky ones. Indeed 11% of buyers hold a rating equal or higher than A versus an 8% of buyers which hold a rating equal or lower than CCC. In general, the distribution appears skewed toward the higher classes.

Computation of the equivalent CDS portfolio and relative cost

After applying the assumptions and processes earlier described, we obtained the following results:

Real Policy Cost	**66,242 €**
CDS Policy Price	**101,104 €**
Maximum Liability Limit	1,987,266 €
Sum of credit limits	*14,795,000 €*
Minimum premium	25,000 €
CDS Maximum Liability Price	**13,580 €**

Tab. 39: Results TCI policy 3

Alike the TCI policy number 2, this insurance policy is characterized by very low credit limits: almost all of them is below 50 thousand EUR. Furthermore, there is a low amount of ratings equal or lower than CCC.

Therefore, as for the second policy, we observe how the Equivalent *CDS Policy Price* is higher than the *Real Policy Cost*, but in the order of 53%.

Also in this policy we notice the very large difference between the *Maximum Liability Limit* and the *Sum of credit limits* granted: 1 to 7.

The CDS price of the Maximum Liability Limit results indeed very low: 13.58 thousand EUR, which is the one fifth of the *Real Policy Cost*.

Also for this policy, the real cost lays between the two CDS proxies.

Summary of results

In the following table, we display a summary of the results, containing the variables, which play the central role in both the CDS cost determination and the real policy cost.

	Premium rate	**Avg non-zero CL**	**% of low ratings***	**Sum of Credit limits**	**Max Liability limit**	**CDS Max Liability Price**	**Real Policy Cost**	**CDS Policy Price**
Policy 1	0.120%	462,388 €	20%	139,303,000 €	7,925,400 €	220,169 €	264,180 €	3,869,845 €
Policy 2	0.259%	65,963 €	17%	25,000,000 €	5,201,880 €	41,482 €	173,396 €	196,339 €
Policy 3	0.099%	447,218 €	12%	73,791,000 €	13,878,470 €	208,545 €	462,615 €	1,108,821 €
Policy 4	0.290%	27,654 €	7%	14,795,000 €	1,987,266 €	13,580 €	66,242 €	101,104 €

*Tab. 40: summary of results -*Ratings equal or lower than CCC*

From this summary table is easier to notice that, to average low credit limits it corresponds a rather higher premium rate. Also the opposite sentence holds true: high average credit limits entail low premium rates (but also lower Maximum liability limits, in relative terms) and this case is less favorable to the CDS model developed. We also notice that the Maximum liability amount (which depends directly on the premium rate) is about one order lower than the sum of Credit limits, in the case of Policy 1, it is 1/18th of the Insurable turnover.

It is straightforward clear that it is not always fair a direct comparison between the CDS equivalent policy price and the real policy cost, which, under the circumstance of high misalignments between insurable turnover (or more precisely the sum of credit limits) and maximum liability limit, should rather be compared to the CDS maximum liability price. In these cases (policies 1 and 3), we have also noticed how the ratio between *CDS Policy Price* and *Sum of Credit Limits* is somehow aligned to the ratio between *Real Policy Cost* and the *Maximum Liability Limit.*

Furthermore, for the reasons already stated, we have the confirmation that low ratings deeply affect the CDS policy price: the major misalignments are indeed encountered in the case of Policy 1, which counts a 20% of low ratings.

Single risk protection

Even though the vast majority of the TCI policies are whole-turnover kind, some Trade credit insurances offer the possibility to ensure a single foreign[47] transaction under some conditions.

The premium rate applied to these kind of insurance is established by law (in Italy: DLGS 143/98 - "Legge Ossola"). Premium is applied to the insured turnover.

The determinant of the premium rate are essentially the Country risk and the payment delay granted to the customer. There are three levels of country risk. According to the country risk, also the Coverage rate is different (see table below).

[47] Meaning that ONLY export sales are insured

Country risk level:	1, 2 or 3
Coverage:	90%, 80% or 70%
Maximum payment delay grantable:	12 months
Minimum premium:	50% of expected premium
Maximum Liability Limit:	50*premium
Maximum insurable transactions per policy:	10 clients

Tab. 41: single risk protection conditions

Such product is radically different from a CDS, except for the facts that allows credit protection against a single (or a small pool of) reference entity.

At the opposite, there are some major differences:

1. The only risk metric is the Country risk level;
2. The premium rate only depends on the Country risk level and on the payment delay granted;
3. The Coverage decreases with Country risk increasing (under the perspective of CDS as here developed, this would mean that the recovery rate is increasing with country risk increasing, which represents a paradox);
4. There still is a Maximum liability limit.

Aware of all these major differences, always with the goal of benchmarking our results, we tried a comparison between such kind of insurance and the CDS developed.

For each of our (policies) portfolios, the large majority of buyers belongs to countries of Country risk group one. We then compared the premium rate of Country risk group one, with the average spread retrieved for each portfolio:

<table>
<tr><th>Country risk level</th><th>Recovery rate</th><th colspan="2">Premium rate</th><th colspan="2">Avg. Spread (in bps)</th></tr>
<tr><td rowspan="4">Group 1</td><td rowspan="4">10%</td><td rowspan="2">7 to 12 months</td><td rowspan="2">1.55%</td><td>Policy 1</td><td>278 (2.78%)</td></tr>
<tr><td>Policy 2</td><td>118 (1.18%)</td></tr>
<tr><td rowspan="2">1 to 6 months</td><td rowspan="2">1.10%</td><td>Policy 3</td><td>145 (1.45%)</td></tr>
<tr><td>Policy 4</td><td>64 (0.64%)</td></tr>
</table>

Tab. 42: single risk protection premia and comparison

It is not known how these premium rates were defined by such law (“legge Ossola”). Despite this, all differences considered, as it can be seen our average portfolio spreads are close to the fix rate imposed by law, in the case of single risk protection.

Always considering the existent differences between the two hedging products, these last results might suggest that the CDS pricing here developed could be a plausible price for the kind of protection bought.

Chapter 8

CONCLUSIONS

General conclusions

Goal of the thesis

The whole thesis was aimed to evaluate the possibility of developing a pricing methodology for Credit Default Swaps written on Small and Medium Enterprises (SME), intended as a mean to hedge credit risk on commercial transactions with the reference entity.

The main issues in doing such development are represented by the fact that SME, in general terms, are neither listed nor rated. This entails the fact that risk neutral Probabilities of Default (needed by standard pricing models), for such entities, cannot be retrieved from market traded instruments.

To overcome such issue, we constructed an “equivalent risk neutral PD”. The approach was similar to the Radon-Nikodym derivative: we defined a corrective factor to be applied to the “real world PD”, in order to obtain a proxy of the risk neutral Probability of Default (defined “equivalent risk-neutral probability”). This last was in first place bootstrapped from real CDS trades, by different maturities and rating classes.

The real world PDs were specifically calculated on a large set of Small and Medium Enterprises. We have demonstrated that, on an average basis, such PDs are higher than those of larger listed companies (as 80% of CDS reference entities are). Such difference enables to factor in the resulting spread, the higher risk entailed by a SME.

The equivalent risk neutral PD measure has been used to calculate a CDS spread, according to the wide-accepted CDS pricing methodology explained in Chapter 1.

Probably, the Spreads obtained, even though considerably higher in average terms, than those of the real CDS, are not high enough to take into account other risk factors entailed by SME (other than the higher PD), such as, for example, the eventual probable illiquidity of such instruments.

Results

We applied the developed pricing model to a set of 1,000 Italian Small and Medium Eneterprises.

The results obtained were quite positive, since the average SME-CDS spread obtained by rating class and maturity, other than being monotonically increasing with rating class worsening (except for the lowest rating class) and maturity increasing, was also in average 42% higher than the (average) spreads observed for real traded CDS. Only the lowest CCC rating class presents average spreads, which are lower than the real CDS spreads and decreasing with tenor increasing (‘till 3 years tenor), this last behavior was observed also for real CDS. A justification of such phenomenon is provided in Chapter 4.

The average CDS spreads, obtained for Small and Medium Enterprises, by rating class and tenor, and their comparison with respect real CDS average spreads, are displayed in the graph and table in the following page:

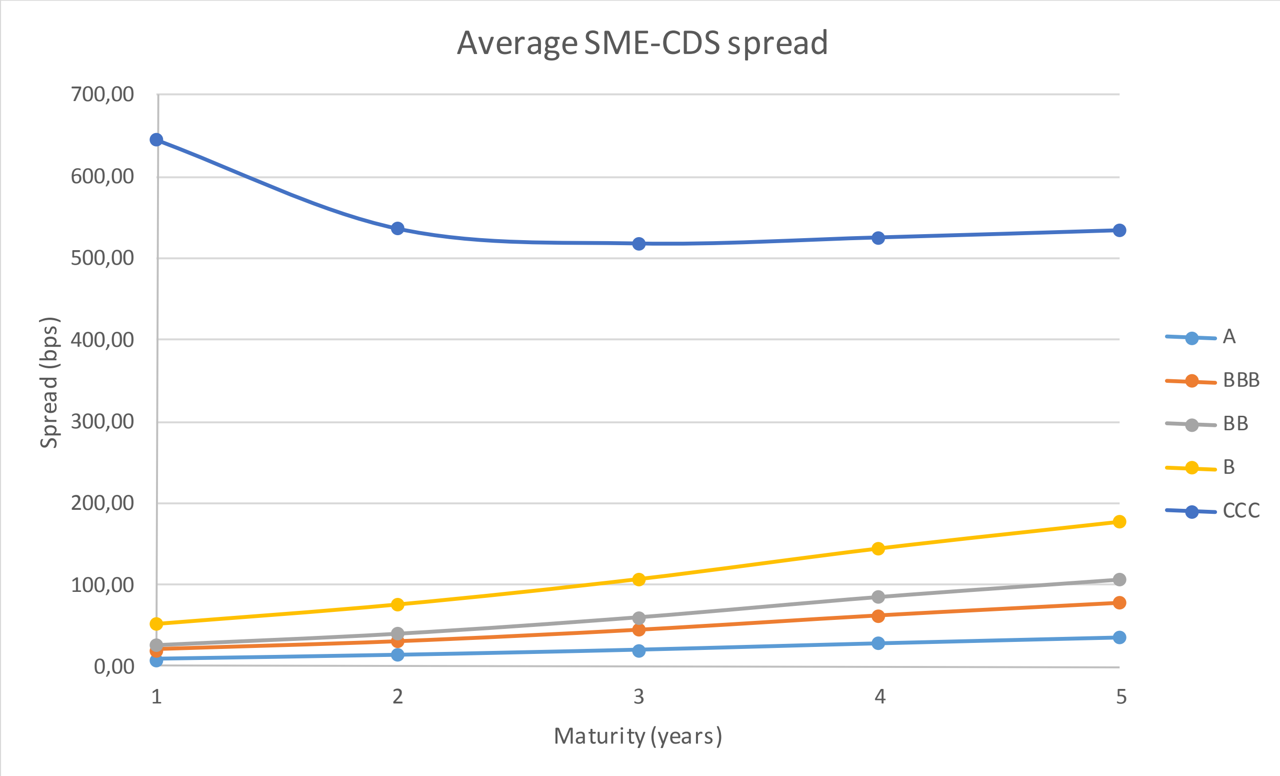

Fig. 35: SME average CDS spread

	1Y	2Y	3Y	4Y	5Y
A	1.43	1.26	1.13	1.08	0.98
BBB	1.99	1.48	1.29	1.21	1.08
BB	2.08	1.65	1.53	1.45	1.32
B	1.87	1.52	1.42	1.35	1.25
CCC	0.67	0.60	0.60	0.60	0.60

Tab. 43: ratio between average spread of Italian SME and regular CDS

The large spread variation between the average spread of B rating class and CCC class, is most probably ascribable to the inclusion of Non-Investment Grade CDS reference entities into the cluster from which we bootstrapped the PDs for such modeFinance's rating class (see Chapter 3 for further details). Another reason is that the PD is not a linear function of the rating, but rather a power one. Furthermore, the same phenomenon was observed by John Hull, Mirela Predescu, and Alan White[48].

As per the decreasing behavior of the CDS spread curve of the CCC class, this is ascribable to the very high default intensity of companies holding such rating, over a short term time horizon (e.g. one year). Further information can be found in Chapter 4.

As previously mentioned, even though we obtained higher spreads for CDS over SME, these only factor the higher PD of such kind of companies. But, were these instrument to become real, their spreads should also consider other risk factors as, for example, an extra premium for probable scarce liquidity of this instrument (written on those particular reference entities).

[48] *Bond Prices, Default Probabilities and Risk Premiums*. Journal of Credit Risk, Vol 1, No. 2 (Spring 2005)

Comparison with Trade Credit Insurance Cost

As seen in chapter 6, Trade credit insurance, is an insurance form that represents a niche within the insurance world, even if has known a significant growth since the beginning of the crisis in 2008.

Even though the final goal of a Trade credit insurance is not conceptually far from the goal of a CDS: i.e. hedging the holder of the policy/CDS from the insolvency risk of an underlying entity, the way they are structured is extremely different.

Despite such differences, since no CDS on a SME has ever been developed (at least until the time in which the literature review was made) and since no traded instruments issued from SME exist (except for “minibonds” -see appendix C- which, despite rapidly growing, still represent a niche and illiquid market), trade credit insurance policies represent the only “instrument” that allows a minimal chance of benchmarking the CDS prices obtained.

As seen in Chapter 7, in general terms, the costs of the two instruments appear quite distant, but we cannot neglect the differences of the two instruments and the assumptions and approximations made:

1. The far most common Trade credit insurance policy is the “whole-turnover” one, where the entire portfolio of the policy holder is insured, without possibility of selecting only one, or a pool of clients. This restriction would not apply for CDS, as long as the CDS buyer (insured company) would be willing to pay the defined –potentially very high– CDS spread. Actually, in reality, it is unthinkable that a SME industrial company would buy a portfolio of hundreds of CDS (so to hedge the entire portfolio), as assumed for the comparison of the two instruments.
2. As per the CDS side, we introduced the scoring approximation for partnerships and the approximations for very high/very low ratings (as clearly described in chapter 7).
3. The insurer can zero the Credit Limits assessed for the trade credit policy, at any time, unilaterally. The result is that those entities would be (suddenly) excluded from protection. Such event does not take place for CDS.
4. In every whole-turnover Trade credit insurance policy it is foreseen a maximum liability limit for the insurance coverage, and that is far lower (even more than ten times) than the sum of insured credit limits. This characteristic does not persist with CDS.

In effect, the first point above is the exactly hypothetical added value of such CDS instruments, i.e. the possibility of buy protection on just one -or a few- specific buyers (reference entities), without being “obliged” to subscribe an insurance policy on the entire portfolio of buyers. This is even more true, taking into account that some of the buyers could be excluded from the insurance coverage, based on a unilateral decision of the insurance company.

Another advantage of such hypothetical credit derivatives is that, if present market conditions would be deemed favorable, a company could buy longer term protection over its “long term, or core” buyers (i.e.clients).

Among the above factors, it seems that the prices obtained are highly affected by the “maximum liability limit effect”. When we have compared the CDS cost obtained from the average spread applied to the maximum liability limit (see chapter 7), we always obtained a lower cost than the real one. Furthermore, when we compared the ratio between “*CDS Policy Price*” and “*Sum of Credit Limits*” to the ratio between “*Real Policy Cost*” and the “*Maximum Liability Limit*” we observed how they were quite aligned.

Moreover, we have to point out that trade credit insurance, despite a growth in recent years (due to the economic crisis) is still a niche in the insurance world and is a field “under development”. Apparently, according to the literature research made, it still lacks a “well-defined” scientific approach. Even when directly asked by the writer, Trade Credit Insurance representatives did not provide any documentation, or satisfactory explanations, of the pricing methodologies adopted.

Paradoxically, even if Trade Credit Insurance is an instrument to hedge the credit risk of a pool of counterparties (being these the clients of the insurance holder), their creditworthiness is only an indirect variable of the policy pricing (being a determinant of the credit limits assigned, but not, directly, of the premium rate).

As a last comparison, we benchmarked the average spread obtained for each insurance portfolio, to the -law defined- premium rate of the single risk protection policies (applicable only for foreign transaction). Aware of all the limits of such comparison (see chapter 7), the results obtained were quite coherent.

Anyhow, for all the considered policies, the real policy cost was lying between the two CDS cost proxies. In general terms at the moment, it is hard to tell whether the CDS prices obtained with the model developed, could be defined “fair”. Certainly, the results of the benchmarking tell us that they are at least plausible.

In the end, we remind that all the legal and fiscal aspect of both CDS and Trade credit insurance, which at this stage are deemed beyond the scope of this thesis, have been neglected.

Possible improvements and further developments

A possible improvement could arise from the re-definition of the corrective “Factor” based on more specific sub-clusters, like by Country or by sector. Unfortunately, such improvement faces the scarce numerosity of CDS liquid trades, which hardly enables to obtain statistically meaningful results.

Alternatively to what above, instead of defining a (set of) corrective factors (i.e. by rating class and tenor) it might lead to more precise results to attempt a fitting between the Factor (always defined as the ratio between the risk neutral PD and the real world one) and the reference entities’ score (intended as that assessed by modeFinance, or any other rating agency). This would turn into a continuous –instead of discrete- corrective factor. Still, though, we face the issue of scarce data numerosity for the most extreme scores (corresponding to high/low ratings).

Another margin of improvement comes from a more thorough application of the discount curves in the bootstrap process: since in the process of PD bootstrapping we used average yearly spreads (for each CDS), we employed average discount factors. It might be more precise (even if computationally much more expensive) to bootstrap the PD on each trading day and then calculate the needed statistics on such PDs (as for example the yearly average PD, by rating class and tenor).

Such improvement attempts would though need a reliable benchmark to understand if they bring to factual improvements and, as known, at the moment, for such instruments we do not have any reliable benchmark.

The only foreseeable benchmarking source is, right now, the mini-bond market (see appendix C). At the moment, in Europe, such market is still a niche and the instruments traded are highly illiquid. If in the incoming future there will be a growth of such market, a risk neutral probability for Small and Medium Enterprises could be more easily retrieved from such market trades.

Final considerations

The mini-bond market said above, would be the natural cradle for the development of these “new” CDS. As said, the development of the mini-bond market would enable to retrieve more easily a risk neutral PD for the SME and eventually extend the use of such CDS to reference entities beyond the mini-bond market.

A further consideration is that we’ve always thought of such new instrument, as a mean for an industrial entity, to hedge against the default risk of a business partner (imagined to be one of its clients). Actually, another potentially interested player for such product, might well be a bank. It’s well known, especially in Italy, how banks have dramatically reduced their exposition toward (and the willingness to fund) Small and Medium Enterprises. Even more troublesome is the amount of Nonperforming Loans hold. On this regard, the possibility to buy CDS over SME, would give banks the chance to free-up regulatory capital, by transferring credit risk and maybe, in the mid-term, they might reconsider the SME financing. Even if that consideration would probably not apply to the NPL because of the probable-very-high cost of CDS protection for such reference entities.

Finally, 2008 financial crisis, where CDS have played a central role, has (or should have) taught that the use these instruments needs to be strictly regulated by authorities, in order to prevent their speculative -instead of hedging- use.

Being the “SME universe” enormously wider than the Corporates’ one, also the potential damages such instruments could cause are enormously greater. For this reason, it is definitely desirable that, were such CDS over SME to become real, a complete and precise regulation over their use would be applied.

Appendix

A. SURVIVAL PROBABILITY Q(T_V,T)

Demonstration of formula (2), (by S. Ziraldo, PhD: theory and numerical simulation of condense matter):

$$Q(t_v,T) = exp\left(-\int_{t_v}^{T} \lambda(s)\, ds\right)$$

Given the default probability in time interval dt, conditional to the company surviving at t:

$$Pr[\tau < t + dt | \tau \geq t] = \lambda(t)dt$$

Defining the survival probability Q(t_0,T) as the probability to survive between t_0 and T, we can divide the time interval [t_0,T] into infinitesimal sub intervals: t_0 , t_1 , t_2 , ...t_n=T, where t_i-t_{i-1}=dt.

Then, in every time sub interval, the conditional survival probability is given by (1-λ(t_i)*dt).

The survival probability Q(t_0,T) is then given by:

$$Q(t_0,T) = \prod_{i=0}^{n-1}(1 - \lambda(t_i)\mathrm{dt})$$

$$\ln\left(Q(t_0,T)\right) = \sum_{i=0}^{n-1} ln(1 - \lambda(t_i)\mathrm{dt})$$

Reminding that $\lim_{x\to 0}\left(ln(1-x)\right) = -x$, taking the limit for n->∞, we have

$$\ln\left(Q(t_0,T)\right) = -\int_{t0}^{tn} \lambda_\tau d\tau$$

hence:

$$Q(t_0,T) = \exp(-\int_{t0}^{tn} \lambda_\tau d\tau)$$

B. CAP AND ROC CURVES

"The quality of a rating system is determined by its discriminatory power between non-defaulting obligors and defaulters, ex ante, for a specific time horizon (usually a year). The CAP (Cumulative Accuracy Profile) measure and ROC (Relative or receiver Operating Characteristic) provide statistical measures to assess the discriminatory power of various rating models based on historical data"[49].

CAP curves are constructed by sorting all (credit) scores from the worst to the best on the x axis and then, for each x (score) values, plotting on the y axis the percentage *d(x)* of defaulters included within that credit score.

It is natural to understand that a "perfect" rating model will assign the lowest scores to the defaulters. Such model is described in the figure 36 below, by the curve which is increasing linearly and then staying at one.

For a random model (with no discriminative power), a given fraction x of debtors with the lowest rating scores will contain the same amount (x percent) of all defaulters.

A real rating system will fit between these two extreme models.

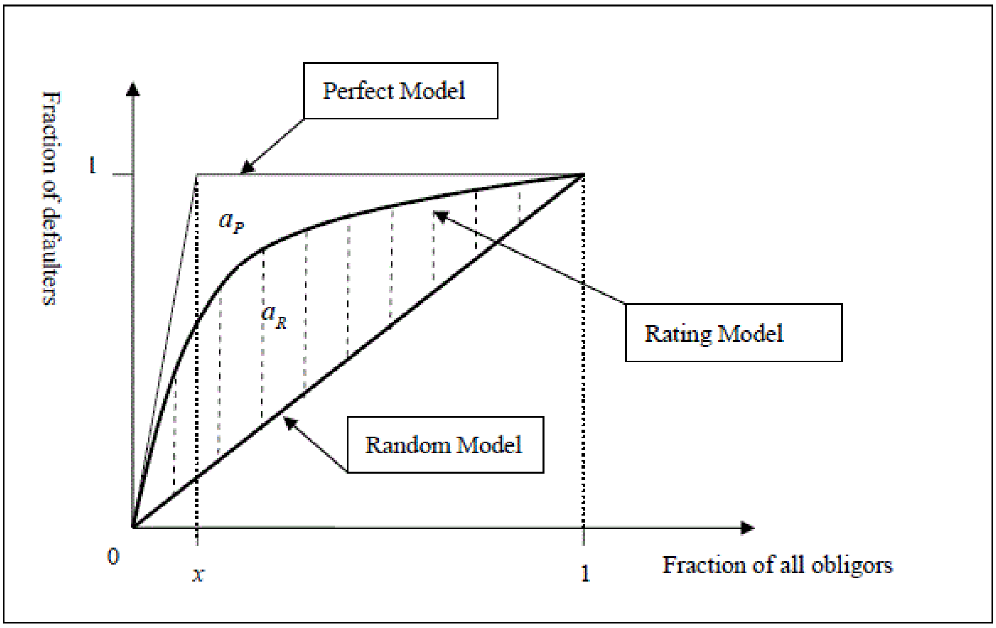

Fig. 36 : source: Applications to Credit Rating Model Validation, University of technology Sydney.

ROC curves are constructed by computing for all possible cut-off values C of the (credit) scores, the quantities Hit Rate HR(C) versus False Alarm Rate FAR(C).

Where HR(C) is defined as the hit rate:

$$HR(C) = \frac{DEF\ (C)}{DEF\ total}$$

DEF(C) is the number of defaulters predicted correctly at cut-off C;
DEF total is the total number of defaulter in the sample.

[49] Steve Satchell, Wei Xia, Analytic Models of the ROC Curve: Applications to Credit Rating Model Validation, QUANTITATIVE FINANCE RESEARCH CENTRE, University of technology Sydney, Aug. 2006.

And FAR(C) is false alarm rate defined as:

$$FAR(C) = \frac{F(C)}{ND\ total}$$

F(C) is the number of non-defaulters that were classified incorrectly as defaulters at cut-off C;
ND total is the total number of non-defaulters in the sample.

The resulting plot of HR(C) vs FAR (C) represents the ROC curve.

A convenient measure for summarizing the performance of the graph of the ROC is the Area Under the Curve (AUC), which is calculated as the proportion of the area below the ROC relative to the total area of the unit square. A value of 0.5 indicates the random model, and a value of 1.0 indicates perfect model.

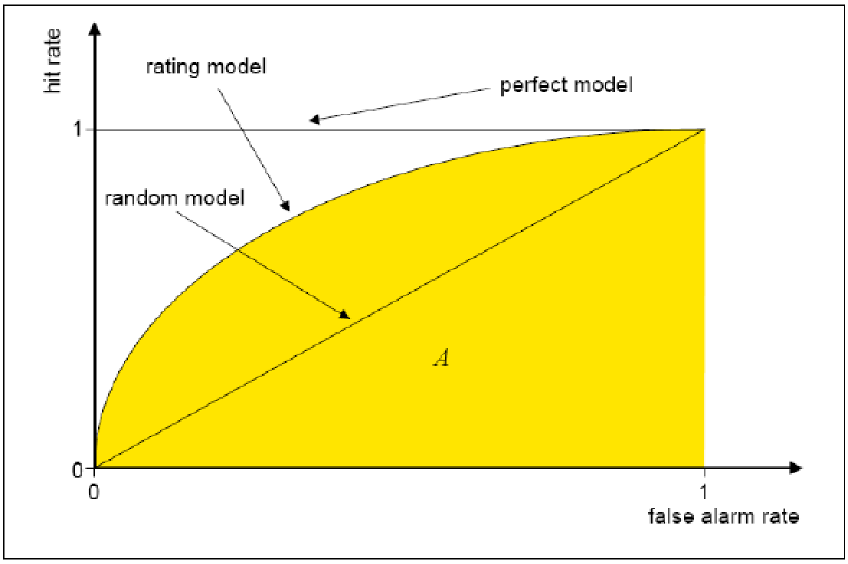

Fig. 37: source: Tasche (2003).

AUC	Accuracy
[1.00;0.90]	Excellent
[0.90;0.80]	Good
[0.80;0.70]	Adequate
[0.70;0.60]	Poor
[0.60;0.50]	Fail

Tab. 44 : source modeFinance srl

C. MINIBONDS

Source: Roberto Calugi, Valentina Morelli e Gianmarco Paglietti I mini-bond Istruzioni per l'uso, Consorzio camerale per il Credito e la Finanza, 2014.

Over the years, Italian small and medium enterprises have relied on bank debts as main funding source, remaining closed to different external sources of funds. This practice has led to a generalized under capitalization of Italian small enterprises.

2008 financial crisis has brought to a credit crunch and the following economic crisis has even worsened such phenomenon. The result has been that small and medium enterprises have seen almost vanish their chances of obtain funds from banks. On the other hand, the private equity sector in Italy has never actually reached significant levels.

In 2012 special laws were promulgated (D.L. 'Sviluppo', June 22, 2012) aimed to legally allow unlisted SME to issue bonds to the capital market. Such bonds were named "Minibond". In 2013 and 2014 further laws have been promulgated (D.L. 'Destinazione Italia', December 23, 2013 and D.L. 'Competitività', June 24, 2014) in order to ease the emission of such debt instruments and enhance their diffusion among institutional investors. The prefix "Mini" is intended to give a hint on the fact that the size of the bond can be very small, according to the needs of a small enterprise.

In order to issue a Minibond, a company is subject to a due-diligence process, aimed to analyze the value and strength of the company.

Even if not specifically required by law, it is highly recommended that a company that wants to issue a Minibond provides a rating issued by a registered Credit Rating Agency[50].

Issued Minibonds are traded in *Borsa Italiana* (the Italian stock exchange), within the ExtraMOT market (a market specifically dedicated to debt instruments for unlisted companies, established in February 2013). This is a secondary market, where only institutional investors can operate. Privates and retail investors are excluded. It is not a regulated market (MOT), but a multilateral trading facility.

For the issuing company, the overall cost of a Minibond is generally higher than a bank loan of same size. Besides the pure bond costs (e.g. interests), the company has to face collateral costs such as advisors, consultants, rating agency, ect. But these costs also have a return in terms of image and reliability of the company toward the market. Moreover, the bond's coupon itself tends to be higher than for large listed corporates, because of the scarce liquidity of this type of instruments.

Market size

Source: Politecnico di Milano – Dipartimento di Ingegneria Gestionale, Osservatorio Mini-Bond I° Report italiano sui Mini-Bond, February 2015.

As per May 2015, 109 Minibonds have been issued, for the total amount of 996 million Eur. At 31/12/2014 they were 96, but only 90 were traded on the stock markets, 86 of which are traded on Italian ExtraMOT. The 48% of the issuers can be classified as SME companies and "only" the 14% of the issuer has benefited

[50] The European authority in charge of certifying and supervising the rating agencies is the ESMA, which provides as well the list of Credit Rating Agencies: http://www.esma.europa.eu/page/List-registered-and-certified-CRAs

of the modified legisaltion as above described (meaning that only 14% as been allowed to issue Minibonds specifically thanks to the new legislation).

The average maturity is 6.2 years, the median maturity is 5 years. The average coupon is 6.14%, the median is 6%.

For large issuers the principal's payoff is usually at maturity, while SME issuers tend to amortize the debt over the entire life of the bond.

The market development since its inception is shown in the figure below:

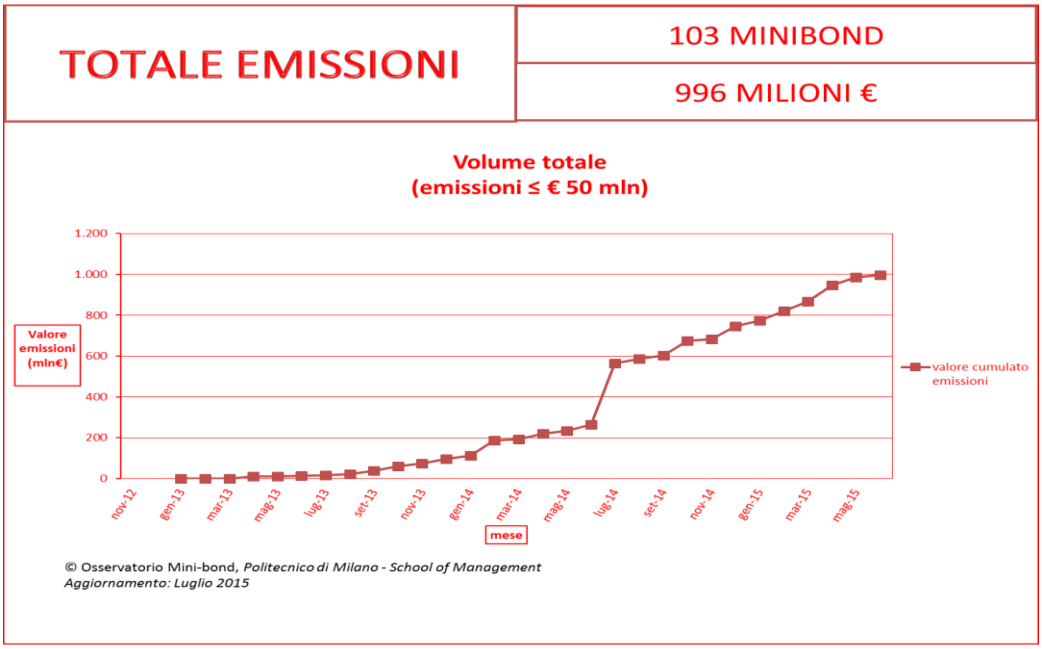

Fig. 38 : minibonds' market size (source: Osservatorio Mini Bond)

Even though growing, the market is still at an embryonic stage: during the whole 2014 there have been 1.125 trades, for a gross amount of € 127.660.958 (corresponding to only 2,76% of the notional amount of the debt securities). As a consequence, the downside of such instruments is their scarce liquidity[51].

A further boost to the market development will be given by private debt funds, by February 2015, already 29 entities were starting some initiatives to enter this market. Only at the end of this year (2015) it will be possible to draw a balance of the market evolution.

Minibond in Europe

In other European Countries such as Germany, France and United Kingdom, the Minibond market were born earlier than in Italy. In all those countries they were born in 2010-2011. In Spain the market was established in 2013.

In all of the said countries there is at least a specific market place opened to retail investors. Anyhow, in all these countries such markets count on a few tents of traded bonds and they're all characterized by scarce liquidity. Only the German "Boerse Stuttgart", has a dedicated market (Bondm) with market makers that ensure liquidity of the securities traded.

[51] Politecnico di Milano – Dipartimento di Ingegneria Gestionale, Osservatorio Mini-Bond I° Report italiano sui Mini-Bond, February 2015

D. FROM REAL WORLD TO RISK NEUTRAL PROBABILITY: THE RADON-NIKODYM DERIVATIVE

Source: L. Giordano, G. Siciliano, "Probabilità reali e probabilità neutrali al rischio nella stima del valore futuro degli strumenti derivati", Quaderni di finanza, CONSOB, Aug. 2013.

The measure change, from real world probability P, to risk neutral probability Q, is fundamental for the stochastical simulation of future payoffs of a derivative instrument. Such transformation enables to discount cash flows with a known risk free rate, rather than a -subjective- expected rate.

Given P and Q as probability measures of a finite event space Ω, we assume $P(\omega)>0$ and $Q(\omega)>0$ for every $\omega \in \Omega$.

If we define the stochastic variable:

$$Z(\omega) = \frac{Q(\omega)}{P(\omega)}$$

Then the following properties hold true:

(i) $P(Z>0) = 1$;
(ii) $E^P[Z]=1$;
(iii) For every random variable Y, $E^Q[Y]=E^P[ZY]$; i.e., the Y and the transformed variable ZY have the same expected value, if the two different probability measures are employed.

The first two are obvious. Regard the third property we have:

$$E^Q[\mathrm{Y}] = \sum_{\omega \in \Omega.} Y(\omega)Q(\omega) = \sum_{\omega \in \Omega.} Y(\omega)\frac{Q(\omega)}{P(\omega)}P(\omega) = \sum_{\omega \in \Omega} Y(\omega)Z(\omega)P(\omega) = E^P[\mathrm{ZY}]$$

The passage to the continuous time is straightforward and the transform variable Z becomes:

$$Z(\omega) = \frac{dQ}{dP}$$

Which is defined as the Radon-Nikodym derivative.

Remark: the above definition (and application) of the Radon-Nikodym derivative was developed for derivative products on market traded instruments (under the hypothesis, among other, that the underlying follows a Brownian motion), so to allow to discount at a risk-free rate. A literature research made has failed to confirm its extendibility to CDS derivatives, which are radically different respect, for example, options.

Glossary

CDS Conventions[52]

Conventions applied to ISDA standardized CDS contracts.

Business Days

For non-JPY currencies all computations are based on a business day calendar of weekdays only, i.e., weekends (Saturday and Sunday) are the only non-business days.

Business Day Convention

Business day convention is for adjusting dates when a specified date is not a business day. For single-name CDS the business day convention is "*following*", i.e., the adjusted date is the first following day that is a business day.

Day Count Conventions

Day count convention to define an accrual factor between two dates is "*ACT/360"*, which is also called Actual/360 or A/360. The accrual factor under this day count convention is

$$\frac{Days\ (d_1, d_2)}{360} \tag{27}$$

IMM dates

The maturity dates of CDS contacts are standardized to the International Monetary Market (IMM) dates: March 20th, June 20th, September 20th and December 20th.

Contract Specifications[53]

A single-name CDS contract is specified by trade date, maturity date and coupon.

Trade Date

Trade date is the current business day. Hereafter we denote the trade date by T. Thus T + n represents n days after the trade date.

Maturity Date

Maturity Date is also called end date or protection end date. Scheduled maturities are rolled to the next IMM date and unadjusted by the business day convention. For example, a 5-year trade dealt on June 13th 2013 will terminate on June 20th 2018, whereas a 5-year trade after June 20th 2013 will terminate September 20th 2018.

[52] Yukinori Iwashita, Conventions for Single-Name Credit Default Swaps, OpenGamma Quantitative Research 2013
[53] See above.

Assumptions[54]

Protection Leg

Protection leg is the contingent payment which the protection seller makes to the protection buyer if a credit event occurs.

- *Protection effective date* or *step-in date* is when protection starts and set to be T +1.
- *Protection maturity* date is the same as maturity date. Thus the number of days of protection is
  (Protection maturity date) - (Protection effective date) + 1
- *Protection payoff* can be expressed as
  (Notional) x(100% -Recovery rate)
  There are two ways to make the payment of the protection leg, physical settlement and cash settlement (refer to Chapter 1 for details).

Premium Leg

Premium leg is a series of payments which the protection buyer makes to the protection seller. These payments terminate at the maturity of contract, or following a credit event.

- Payment frequency: coupon is paid on a quarterly basis.
- Regardless of when the CDS trade is executed the first coupon payment date is earliest IMM date after T + 1 adjusted by the business day convention.
- Accrued payment is made in the event of a default.
- Accrual begin date, also called start date, is the latest adjusted IMM date prior to T+1, or if T+1 itself is an adjusted IMM date then it is T+1.
- Accrual dates are IMM dates adjusted by the business day convention. Note that the last accrual date, i.e., maturity date, remains unadjusted.
- Accrual periods, or payment intervals are the interval between the previous accrual date inclusive, to the next accrual date exclusive. For the last accrual period the accrual end date, i.e., maturity date, is included. Payment amount at each accrual date is:

  (Notional) x (Year fraction of accrual period) x (Coupon)

Credit Curve and Discount Curve

- Both survival probability and discount factor are assumed to be 1 at T.
- A yield curve is constructed from money market rates and swap rates.

[54] Yukinori Iwashita, Conventions for Single-Name Credit Default Swaps, OpenGamma Quantitative Research 2013

Credit events[55]

Bankruptcy

Relevant only for corporate entities.

Obligation acceleration

Obligation becomes due and payable before its normal expiration date.

Obligation default

Refers to a technical default, such as violation of a bond covenant.

Failure to pay

Failure of the reference entity to make any due payments.

Repudiation/Moratorium

Provides for compensation after specified actions of a government (e.g. delay in payment).

Restructuring

Reduction and renegotiation of delinquent debts in order to improve or restore liquidity. In 2009, US contracts eliminated restructuring as a potential trigger event.

[55] Deutshce Bank Research, "*Credit default swaps, Heading towards a more stable system*".

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