Testing continuous spontaneous localization with Fermi liquids

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Collapse models describe phenomenologically the quantum-to-classical transition by adding suitable nonlinear and stochastic terms to the Schrödinger equation, thus (slightly) modifying the dynamics of quantum systems. Experimental bounds on the collapse parameters have been derived from various experiments involving a plethora of different systems, from single atoms to gravitational wave detectors. Here, we give a comprehensive treatment of the continuous spontaneous localization (CSL) model, the most studied among collapse models, for Fermi liquids. We consider both the white and non-white noise case. Application to various astrophysical sources is presented.

I. INTRODUCTION

Collapse models provide a phenomenological description of quantum measurements, by adding stochastic and nonlinear terms to the Schrödinger equation that implement the collapse of the wave function [1]. Such effects are negligible for microscopic systems and become stronger when their mass increases. This is how the quantum-to-classical transition is described and the measurement problem solved, which is the main motivation for which they were formulated in the first place.

The most studied model is the continuous spontaneous localization (CSL) model [2,3]. It applies to identical particles and the collapse, which is implemented by a noise coupled nonlinearly to the mass density of the system, occurs continuously in time. The collapse effects are quantified by two parameters: the collapse rate λ and the correlation length of the noise r_C. Different theoretical proposals for their numerical value were suggested: $\lambda = 10^{-16}\ \mathrm{s}^{-1}$ and $r_C = 10^{-7}$ m by Ghirardi *et al.* [4] and $\lambda = 10^{-8\pm2}\ \mathrm{s}^{-1}$ for $r_C = 10^{-7}$ m and $\lambda = 10^{-6\pm2}\ \mathrm{s}^{-1}$ for $r_C = 10^{-6}$ m by Adler [5]. Experimental data were extensively used to bound the parameters [5–24] and new proposals were presented, suggesting ways to further push these bounds [24–31]. Figure 1 summarizes the state of the art.

In this context, one important question is the origin of the collapse noise. While collapse models do not give an answer, as the collapse is inserted "by hand" into the Schrödinger dynamics (but its mathematical structure is fully constrained by the request of no-superluminal-singling and norm-conservation [1]), several times it has been suggested that it is related to gravity [38–48]. If there is truth in this conjecture, then the gravitational fluctuations responsible for the collapse add to the usual gravitational effects present in matter, in particular in strongly gravitationally bound systems such as those we will consider in this paper.

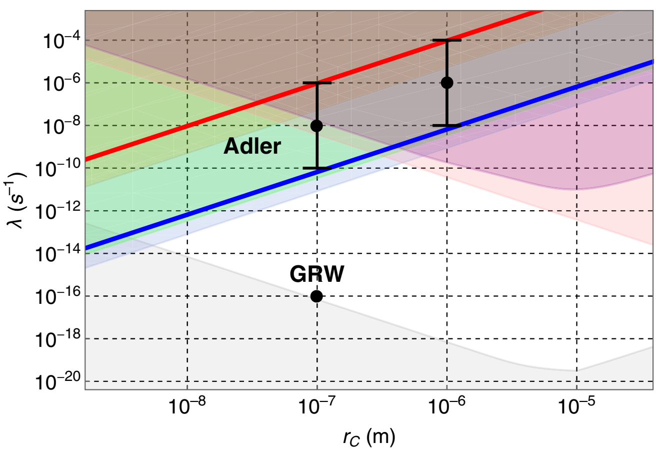

FIG. 1. Bounds on the collapse parameters λ and r_C for the standard (white noise) mass-proportional CSL model. The red and blue lines denote the upper bounds given by Eq. (17) applied to the heat flow from the neutron star `PSR J 1840-1419` and from Neptune. The shaded areas show the already experimentally and theoretically excluded regions: the orange region comes from a cold atom experiment [14,32]; the green region from a phonon analysis in cryogenic experiments [23,33,34]; the blue region from x-ray emission from germanium [7,11,12,21,35]; the purple region from a mechanical cantilever [16,19]; the pink region from LISA Pathfinder [17,24,36,37]; the grey region from theoretical arguments [20,22].

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A consequence of collapse models is a spontaneous heating, induced by the random collapse. This effect has been calculated for many types of systems [13,14,16–19,23,24], but not for Fermi liquids, an issue raised in a recent paper of Tilloy and Stace [49]. Here, we give a comprehensive treatment of CSL induced heating in Fermi liquids, including the experimentally relevant case of non-white noise, and we apply our results to various astrophysical systems, including neutron stars.

II. CSL MODEL: PERTURBATIVE CALCULATION

Following [23], we consider the transition amplitude $c_{fi}(t)$ caused by a perturbation, from an initial state $|i\rangle$ of a quantum system to a final state $|f\rangle$, with associated energies $E_i = \hbar\omega_i$ and $E_f = \hbar\omega_f$, respectively. For the sake of simplicity we restrict the problem to the case of one fermion of mass m_A. The result for the N particle case, either fermions or bosons, is given in the Appendix. We have

$$c_{fi}(t) = -\frac{i}{\hbar}\int_0^t \mathrm{d}s\,\langle f|e^{\frac{i}{\hbar}\hat{H}_0 s}\hat{V}(s)e^{-\frac{i}{\hbar}\hat{H}_0 s}|i\rangle, \tag{1}$$

where $\hat{H}_0$ is the free Hamiltonian and the perturbation, for the CSL process applied to a particle of mass m_A, is [23]

$$\hat{V}(t) = \int \mathrm{d}\mathbf{z}\, w_t(\mathbf{z})\hat{\mathcal{V}}(\mathbf{z}),$$
$$\hat{\mathcal{V}}(\mathbf{z}) = -\frac{\hbar}{m_0} m_A g(\mathbf{z} - \hat{\mathbf{x}}_A), \tag{2}$$

where m_0 is the nucleon mass, $w_t(\mathbf{z})$ is a noise with zero mean [$\mathbb{E}[w_t(\mathbf{z})] = 0$], and the correlator is

$$\mathbb{E}[w_t(\mathbf{z})w_s(\mathbf{x})] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}\omega\,\gamma(\omega)e^{-i\omega(t-s)}\delta(\mathbf{x}-\mathbf{z}), \tag{3}$$

where $\gamma(\omega) = \gamma(-\omega)$ is the frequency-dependent collapse strength. We denoted with $\hat{\mathbf{x}}_A$ the position operator of the particle, and

$$g(\mathbf{x}) = \frac{e^{-\mathbf{x}^2/2r_C^2}}{(\sqrt{2\pi}r_C)^3} = \frac{1}{(2\pi)^3}\int \mathrm{d}\mathbf{q}\, e^{-\mathbf{q}^2 r_C^2/2 - i\mathbf{q}\cdot\mathbf{x}}. \tag{4}$$

We assume that the particle is free and confined in a box of side L; the initial and final states read

$$\langle \mathbf{x}|i\rangle = \frac{e^{i\mathbf{k}_i\cdot\mathbf{x}}}{L^{3/2}} \quad \text{and} \quad \langle \mathbf{x}|f\rangle = \frac{e^{i\mathbf{k}_f\cdot\mathbf{x}}}{L^{3/2}}. \tag{5}$$

We then have

$$c_{fi}(t) = \frac{i m_A}{m_0 L^3}\int \mathrm{d}\mathbf{q}\, e^{-\mathbf{q}^2 r_C^2/2}\int_0^t \mathrm{d}s\, e^{i\omega_{fi}s} \times \int \mathrm{d}\mathbf{z}\, w_s(\mathbf{z})e^{-i\mathbf{q}\cdot\mathbf{z}}\delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q}), \tag{6}$$

where $\omega_{fi} = \omega_f - \omega_i$ and $\mathbf{k}_i$, $\mathbf{k}_f$ are the initial and final momenta of the particle, respectively. The transition probability, under stochastic average, is then given by

$$\mathbb{E}[|c_{fi}|^2] = \frac{m_A^2}{m_0^2 L^3}\int \mathrm{d}\mathbf{q}\, e^{-\mathbf{q}^2 r_C^2} \times \int \mathrm{d}\omega\,\gamma(\omega)\delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q})t\delta^{(t)}(\omega_{fi} - \omega), \tag{7}$$

where we used the relations

$$[\delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q})]^2 \sim (L/(2\pi))^3\delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q}),$$
$$\int_0^t \mathrm{d}s\, e^{i(\omega_{fi}-\omega)s} = 2\pi e^{i(\omega_{fi}-\omega)t/2}\delta^{(t)}(\omega_{fi} - \omega),$$
$$[\delta^{(t)}(\omega_{fi} - \omega)]^2 \sim (t/(2\pi))\delta^{(t)}(\omega_{fi} - \omega). \tag{8}$$

We now apply Eq. (7) to the system under study, i.e., a particle in a Fermi gas. The heating power $P_{\mathrm{CSL}}(t) = \mathrm{d}E_{\mathrm{TOT}}(t)/\mathrm{d}t$ reads

$$P_{\mathrm{CSL}}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\sum_i\sum_f \mathcal{N}(\mathbf{k}_i)(1 - \mathcal{N}(\mathbf{k}_f))\hbar\omega_{fi}\mathbb{E}|c_{fi}(t)|^2, \tag{9}$$

where $N(\mathbf{k}_i)$ is the probability of the initial state having momentum $\mathbf{k}_i$, and $(1 - \mathcal{N}(\mathbf{k}_f))$ is the probability for the final state with momentum $\mathbf{k}_f$ not to be occupied; otherwise the particle could not jump there because of the Pauli exclusion principle. Since $\mathcal{N}(\mathbf{k}_i)\mathcal{N}(\mathbf{k}_f)$ and $\mathbb{E}[|c_{fi}|^2]$ are even, whereas ω_{fi} is odd, under the interchange $i \leftrightarrow f$, the term containing $\mathcal{N}(\mathbf{k}_i)\mathcal{N}(\mathbf{k}_f)$ makes a vanishing contribution to Eq. (9). The above expression then simplifies to

$$P_{\mathrm{CSL}}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\sum_i\sum_f \mathcal{N}(\mathbf{k}_i)\hbar\omega_{fi}\mathbb{E}|c_{fi}(t)|^2. \tag{10}$$

Using the standard box-normalization prescription, according to which in the limit $L \to +\infty$,

$$\frac{1}{L^3}\sum_{\mathbf{p}} g(\mathbf{p}) \to \frac{1}{(2\pi)^3}\int \mathrm{d}\mathbf{p}\, g(\mathbf{p}), \tag{11}$$

one obtains

$$P_{\mathrm{CSL}}(t) = \frac{L^3}{(2\pi)^3}\frac{\mathrm{d}}{\mathrm{d}t}\sum_i \mathcal{N}(\mathbf{k}_i)\int \mathrm{d}\mathbf{k}_f \hbar\omega_{fi}\mathbb{E}|c_{fi}(t)|^2, \tag{12}$$

which in the long time limit reads

$$P_{\rm CSL}(t) = \frac{m_A^2}{m_0^2(2\pi)^3}\sum_i \mathcal{N}(\mathbf{k}_i)\int d\mathbf{q}\hbar\bar{\omega}_i(\mathbf{q})e^{-\mathbf{q}^2 r_C^2}\gamma(\bar{\omega}_i(\mathbf{q})), \tag{13}$$

where

$$\bar{\omega}_i(\mathbf{q}) = \frac{\hbar}{2m_A}(\mathbf{q}^2 + 2\mathbf{k}_i\cdot\mathbf{q}). \tag{14}$$

In the white noise case, where $\gamma(\omega) = \gamma$, the integration over $\mathbf{q}$ can be easily performed, giving

$$P_{\rm CSL}(t) = \frac{3}{4}\frac{\hbar^2\lambda m_A}{m_0^2 r_C^2}, \tag{15}$$

where we used $\gamma = \lambda(\sqrt{4\pi}r_C)^3$ and $\sum_i\mathcal{N}(\mathbf{k}_i) = 1$. For the N atom case, the calculation in the Appendix shows that m_A in Eq. (15) is replaced by the total mass $M = Nm_A$. This is the same result obtained from the study of phononic modes in matter [23,33,50].

III. NEUTRON STARS

Neutron stars are small (radius $\sim$10 km) and dense (mass $M\sim 1.4$–4.2×10^{30} kg and density $\mu\sim 10^{17}$ kg/m^3), resulting from the collapsed cores of stars with mass above the Chandrasekhar limit [51]. After the first stage next to their formation, where they cool through emission of baryonic matter, the main cooling process is dominated by thermal emission of radiation [52,53], which is described by the Stefan-Boltzmann law:

$$P_{\rm rad} = S\sigma T^4, \tag{16}$$

where S is the surface of the neutron star, $\sigma = 5.6\times 10^{-8}$ W m^{-2} K^{-4} is the Stefan's constant, and T is the effective black-body temperature of the star. As a reference value for the temperature we can consider $T = 0.28^{+0.19}_{-0.12}\times 10^6$ K, which refers to the neutron star PSR J 1840-1419 [54]. The radius is $R = 10$ km and the mass $M = 2\times 10^{30}$ kg, equal to the solar mass, giving a density $\mu = 4.8\times 10^{17}$ kg/m^3. Variations of R and M, for typical dimensions of a neutron star, do not imply significant changes in the bounds on the CSL parameters.

IV. RESULTS AND DISCUSSION

Assuming that the neutron star's thermal radiation emission is balanced by the heating effect due to the CSL noise, we impose $P_{\rm rad} = P_{\rm CSL}$. This gives an estimate of collapse rate:

TABLE I. Numerical values of the ratio $P_{\rm rad}/M$ for the planets in the Solar System (Sun and Moon included) [61] and the corresponding value of λ/r_C^2 according to Eq. (17). For completeness, we report also the values for the neutron star PSR J 1840-1419 analyzed above.

	$P_{\rm rad}/M$ (W/kg)	λ/r_C^2 (s^{-1} m^{-2})
Mercury	4.74×10^{-7}	1.57×10^{8}
Venus	1.40×10^{-8}	4.62×10^{6}
Earth	2.00×10^{-8}	6.60×10^{6}
Moon	1.55×10^{-7}	5.12×10^{7}
Mars	2.45×10^{-8}	8.10×10^{6}
Jupiter	2.76×10^{-10}	9.14×10^{4}
Saturn	1.94×10^{-10}	6.40×10^{4}
Uranus	6.03×10^{-11}	2.00×10^{4}
Neptune	1.99×10^{-11}	6.57×10^{3}
Pluto	1.50×10^{-10}	4.98×10^{4}
Sun	1.90×10^{-4}	6.29×10^{10}
Neutron star	2.85×10^{-7}	9.43×10^{7}

$$\lambda = \frac{16R^2 m_0^2\pi r_C^2 T^4\sigma}{3M\hbar^2}, \tag{17}$$

where we assumed that the neutron star can be approximated by a sphere of radius R. The corresponding upper bound is shown in red in Fig. 1.

It is interesting to apply Eq. (17) to other objects in the Universe. Table I shows the values of the ratio $P_{\rm rad}/M$ and the corresponding value of λ/r_C^2 for the planets in the Solar System, the Moon, the Sun, and as a comparison that of the neutron star PSR J 1840-1419 analyzed above. Numbers show that Neptune gives the best ratio λ/r_C^2, which is more than 4 orders smaller than the neutron star's one. The corresponding upper bound is identified by a continuous blue line in Fig. 1. These bounds are weaker than the already existing bounds and are further weakened if one assumes a high-frequency cutoff in the noise spectrum following the methods of [23,55–58] or dissipative modification of the CSL model as shown in [14,20,59,60].

ACKNOWLEDGMENTS

S. L. A. acknowledges the hospitality of the Aspen Center for Physics, which is supported by the National Science Foundation Grant No. PHY-1607611. A. B. acknowledges financial support from the COST Action QTSpace (CA15220), INFN, and the University of Trieste. A. B., M. C., and A. V. acknowledge financial support from the H2020 FET Project TEQ (Grant No. 766900).

APPENDIX: FIELD-THEORETICAL CALCULATION

We perform the same analysis presented in the main text, within the framework of quantum field theory. Let us consider the CSL Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{V}_{\text{CSL}}, \tag{A1}$$

where

$$\hat{H}_0 = \sum_i \sum_\tau \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \hat{b}^\dagger_{\mathbf{p}\tau i}(t) \hat{b}_{\mathbf{p}\tau i}(t) \tag{A2}$$

is the free Hamiltonian; the first sum is over the i type of particle, the second sum over the spin τ (ith type of particle), and the third over momentum. Here $\hat{b}^\dagger_{\mathbf{p}\tau i}$ and $\hat{b}_{\mathbf{p}\tau i}$ are creation and annihilation operators, respectively; since the final result is independent from the particle nature, they can be fermionic or bosonic. In fact, the derivation presented below depends only on the commutation relations $[\hat{b}^\dagger_{\mathbf{p}\tau i}\hat{b}_{\mathbf{p}'\tau j}, \hat{b}^\dagger_{\mathbf{k}\tau' l}] = \delta^{(3)}(\mathbf{p}' - \mathbf{k})\delta_{\tau\tau'}\delta_{jl}\hat{b}^\dagger_{\mathbf{p}\tau i}$ and $[\hat{b}^\dagger_{\mathbf{p}\tau i}\hat{b}_{\mathbf{p}'\tau j}, \hat{b}_{\mathbf{k}\tau' l}] = -\delta^{(3)}(\mathbf{p}' - \mathbf{k})\delta_{\tau\tau'}\delta_{jl}\hat{b}_{\mathbf{p}\tau i}$, which are identical for fermions and bosons. The CSL stochastic potential is [7]

$$\hat{V}_{\text{CSL}} = -\hbar \sum_j \sum_{\tau'} \frac{m_j}{m_0} \int d\mathbf{x}\, \hat{\Psi}^\dagger_{\tau' j}(\mathbf{x}, t) \hat{\Psi}_{\tau' j}(\mathbf{x}, t) \xi(\mathbf{x}, t). \tag{A3}$$

Here we introduced

$$\xi(\mathbf{x}, t) = \int d\mathbf{y} \frac{e^{-(\mathbf{x}-\mathbf{y})^2/2r_C^2}}{(\sqrt{2\pi} r_C)^3} w_t(\mathbf{y}), \tag{A4}$$

whose mean and correlator are

$$\mathbb{E}[\xi(\mathbf{x}, t)] = 0, \quad \text{and}$$
$$\mathbb{E}[\xi(\mathbf{x}, t)\xi(\mathbf{y}, s)] = \tilde{\gamma}(t - s) F(\mathbf{x} - \mathbf{y}), \tag{A5}$$

where $\mathbb{E}$ denotes the stochastic average over the noise,

$$F(\mathbf{x}) = \frac{e^{-\mathbf{x}^2/4r_C^2}}{(\sqrt{4\pi} r_C)^3}, \quad \text{and} \quad \tilde{\gamma}(t) = \frac{1}{2\pi} \int d\omega\, \gamma(\omega) e^{-i\omega t}. \tag{A6}$$

The relation between the operator $\hat{\Psi}_{\tau j}(\mathbf{x}, t)$ and $\hat{b}_{\mathbf{p}\tau i}(t)$ is given by

$$\hat{\Psi}_{\tau j}(\mathbf{x}, t) = \sum_{\mathbf{p}} \psi_{\mathbf{p}\tau j}(\mathbf{x}) \hat{b}_{\mathbf{p}\tau j}(t),$$
$$\hat{b}_{\mathbf{p}\tau j}(t) = \int d\mathbf{x}\, \psi^*_{\mathbf{p}\tau j}(\mathbf{x}) \hat{\Psi}_{\tau j}(\mathbf{x}, t), \tag{A7}$$

with $\psi_{\mathbf{p}\tau j}(\mathbf{x})$ denoting the Fourier coefficients of the transformation, spin τ, and momentum $\mathbf{p}$. Below we specify the exact form of $\psi_{\mathbf{p}\tau j}(\mathbf{x})$. The evolution of $\hat{b}_{\mathbf{p}\tau i}(t)$ is determined by the Heisenberg equation $\frac{d\hat{b}_{\mathbf{p}\tau j}(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{b}_{\mathbf{p}\tau j}(t)]$, which gives

$$\frac{d\hat{b}_{\mathbf{p}\tau j}(t)}{dt} = -\frac{i}{\hbar} E_{\mathbf{p}\tau j} \hat{b}_{\mathbf{p}\tau j}(t) + i\frac{m_j}{m_0} \sum_{\mathbf{k}} \int d\mathbf{x}\, \psi^*_{\mathbf{p}\tau j}(\mathbf{x}) \psi_{\mathbf{k}\tau j}(\mathbf{x}) \xi(\mathbf{x}, t) \hat{b}_{\mathbf{k}\tau j}(t). \tag{A8}$$

The solution is

$$\hat{b}_{\mathbf{p}\tau j}(t) = e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}t} \hat{b}_{\mathbf{p}\tau j}(0) + i\frac{m_j}{m_0} \sum_{\mathbf{k}} \int d\mathbf{x}\, \psi^*_{\mathbf{p}\tau j}(\mathbf{x}) \psi_{\mathbf{k}\tau j}(\mathbf{x}) \times \int_0^t ds\, e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)} \xi(\mathbf{x}, s) \hat{b}_{\mathbf{k}\tau j}(s). \tag{A9}$$

Since $\hat{b}_{\mathbf{k}\tau j}(s)$ appears also in the last term, we need to solve perturbatively. We replace $\hat{b}_{\mathbf{k}\tau j}(s)$ with the corresponding form given again by Eq. (A9) and truncate the expression to order γ:

$$\hat{b}_{\mathbf{p}\tau j}(t) = \hat{A}_{\mathbf{p}\tau j}(t) + \hat{B}_{\mathbf{p}\tau j}(t) + \hat{C}_{\mathbf{p}\tau j}(t) + \mathcal{O}(\gamma^{3/2}), \tag{A10}$$

where

$$\begin{aligned}
\hat{A}_{\mathbf{p}\tau j}(t) &= e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}t} \hat{b}_{\mathbf{p}\tau j}(0), \\
\hat{B}_{\mathbf{p}\tau j}(t) &= i\frac{m_j}{m_0} \sum_{\mathbf{k}} \int d\mathbf{x}\, \psi^*_{\mathbf{p}\tau j}(\mathbf{x}) \psi_{\mathbf{k}\tau j}(\mathbf{x}) \\
&\quad \times \int_0^t ds\, e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)} \xi(\mathbf{x}, s) e^{-\frac{i}{\hbar}E_{\mathbf{k}\tau j}s} \hat{b}_{\mathbf{k}\tau j}(0), \\
\hat{C}_{\mathbf{p}\tau j}(t) &= -\left(\frac{m_j}{m_0}\right)^2 \sum_{\mathbf{k}\mathbf{k}'} \int d\mathbf{x}\, \psi^*_{\mathbf{p}\tau j}(\mathbf{x}) \psi_{\mathbf{k}\tau j}(\mathbf{x}) \\
&\quad \times \int_0^t ds\, e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)} \xi(\mathbf{x}, s) \int d\mathbf{y}\, \psi^*_{\mathbf{k}\tau j}(\mathbf{y}) \psi_{\mathbf{k}'\tau j}(\mathbf{y}) \\
&\quad \times \int_0^s ds' e^{-\frac{i}{\hbar}E_{\mathbf{k}\tau j}(s-s')} \xi(\mathbf{y}, s') e^{-\frac{i}{\hbar}E_{\mathbf{k}'\tau j}s'} \hat{b}_{\mathbf{k}'\tau j}(0).
\end{aligned} \tag{A11}$$

Given these expressions, we can compute the evolution of the energy expectation value, which is given by

$$E_{\text{TOT}}(t) = \mathbb{E}[\langle \hat{H} \rangle]. \tag{A12}$$

Due to the stochastic properties in Eq. (A5), we have $\mathbb{E}[\hat{V}_{\text{CSL}}] = 0$; therefore only $\hat{H}_0$ contributes to $E_{\text{TOT}}(t)$. In particular

$$E_{\text{TOT}}(t) = E_{\text{TOT}}(0) + E^{\text{CSL},1}_{\text{TOT}}(t) + E^{\text{CSL},2}_{\text{TOT}}(t) + \mathcal{O}(\gamma^{3/2}), \tag{A13}$$

where

$$E_{\rm TOT}(0)=\sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\langle\hat{A}^\dagger_{\mathbf{p}\tau i}(t)\hat{A}_{\mathbf{p}\tau i}(t)\rangle,$$
$$E^{\rm CSL,1}_{\rm TOT}(t)=\sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\langle\hat{B}^\dagger_{\mathbf{p}\tau i}(t)\hat{B}_{\mathbf{p}\tau i}(t)\rangle,$$
$$E^{\rm CSL,2}_{\rm TOT}(t)=\sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\langle\hat{A}^\dagger_{\mathbf{p}\tau i}(t)\hat{C}_{\mathbf{p}\tau i}(t)+{\rm H.c.}\rangle, \quad \text{(A14)}$$

where H.c. stands for Hermitian conjugate. We notice that there is no contribution from terms like $\hat{A}^\dagger_{\mathbf{p}\tau i}(t)\hat{B}_{\mathbf{p}\tau i}(t)$ or $\hat{B}^\dagger_{\mathbf{p}\tau i}(t)\hat{C}_{\mathbf{p}\tau i}(t)$: the first is zero under stochastic average and the second scales with $\gamma^{3/2}$ and can be then neglected. The above expressions, together with Eq. (A11), give

$$\begin{aligned}
E_{\rm TOT}(0) &= \sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\langle\hat{b}^\dagger_{\mathbf{p}\tau i}(0)\hat{b}_{\mathbf{p}\tau i}(0)\rangle,\\
E^{\rm CSL,1}_{\rm TOT}(t) &= \sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\left(\frac{m_i}{m_0}\right)^2\sum_{\mathbf{k}\mathbf{k}'}\int{\rm d}\mathbf{x}\int{\rm d}\mathbf{y}\,\psi_{\mathbf{p}\tau i}(\mathbf{x})\psi^*_{\mathbf{k}\tau i}(\mathbf{x})\psi^*_{\mathbf{p}\tau i}(\mathbf{y})\psi_{\mathbf{k}'\tau i}(\mathbf{y})F(\mathbf{x}-\mathbf{y})\\
&\quad\times\int_0^t{\rm d}s\int_0^t{\rm d}s'\tilde{\gamma}(s-s')e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau i}(s-s')}e^{\frac{i}{\hbar}E_{\mathbf{k}\tau i}s}e^{-\frac{i}{\hbar}E_{\mathbf{k}'\tau i}s'}\langle\hat{b}^\dagger_{\mathbf{k}\tau i}(0)b_{\mathbf{k}'\tau i}(0)\rangle,\\
E^{\rm CSL,2}_{\rm TOT}(t) &= -\sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\left(\frac{m_i}{m_0}\right)^2\sum_{\mathbf{k}\mathbf{k}'}\int{\rm d}\mathbf{x}\int{\rm d}\mathbf{y}\,F(\mathbf{x}-\mathbf{y})\psi_{\mathbf{k}\tau i}(\mathbf{x})\psi^*_{\mathbf{k}\tau i}(\mathbf{y})\int_0^t{\rm d}s\int_0^s{\rm d}s'\tilde{\gamma}(s-s')\\
&\quad\times[\psi^*_{\mathbf{p}\tau i}(\mathbf{x})\psi_{\mathbf{k}'\tau i}(\mathbf{y})e^{\frac{i}{\hbar}E_{\mathbf{p}\tau i}s}e^{-\frac{i}{\hbar}E_{\mathbf{k}\tau i}(s-s')}e^{-\frac{i}{\hbar}E_{\mathbf{k}'\tau i}s'}\langle\hat{b}^\dagger_{\mathbf{p}\tau i}(0)\hat{b}_{\mathbf{k}'\tau i}(0)\rangle\\
&\quad+\psi_{\mathbf{p}\tau i}(\mathbf{y})\psi^*_{\mathbf{k}'\tau i}(\mathbf{x})e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau i}s}e^{\frac{i}{\hbar}E_{\mathbf{k}\tau i}(s-s')}e^{\frac{i}{\hbar}E_{\mathbf{k}'\tau i}s'}\langle\hat{b}^\dagger_{\mathbf{k}'\tau i}(0)\hat{b}_{\mathbf{p}\tau i}(0)\rangle].
\end{aligned} \quad \text{(A15)}$$

The above terms contain $\langle\hat{b}^\dagger_{\mathbf{p}\tau i}(0)\hat{b}_{\mathbf{k}\tau i}(0)\rangle$. To compute it we consider a state of N particles with density matrix diagonal in momentum and weight given by the occupation number $\mathcal{N}(\mathbf{p})$. Then we have

$$\langle\hat{b}^\dagger_{\mathbf{p}\tau i}(0)\hat{b}_{\mathbf{k}\tau i}(0)\rangle=\delta_{\mathbf{p}\mathbf{k}}\mathcal{N}(\mathbf{p}). \quad \text{(A16)}$$

Although $\mathcal{N}(\mathbf{p})$ is different in the fermionic and the bosonic case, as it should be clear from the calculations, the final result is independent from the type of statistics. Applying this result we obtain

$$\begin{aligned}
E_{\rm TOT}(0) &= \sum_i\sum_\tau\sum_{\mathbf{p}}E_{\mathbf{p}\tau i}\mathcal{N}(\mathbf{p}),\\
E^{\rm CSL,1}_{\rm TOT}(t) &= t\sum_i\sum_\tau\sum_{\mathbf{p}\mathbf{k}}E_{\mathbf{p}\tau i}\left(\frac{m_i}{m_0}\right)^2\mathcal{N}(\mathbf{k})\int{\rm d}\mathbf{x}\int{\rm d}\mathbf{y}\,\psi_{\mathbf{p}\tau i}(\mathbf{x})\psi^*_{\mathbf{k}\tau i}(\mathbf{x})\psi^*_{\mathbf{p}\tau i}(\mathbf{y})\psi_{\mathbf{k}\tau i}(\mathbf{y})F(\mathbf{x}-\mathbf{y})\\
&\quad\times\int{\rm d}\omega\,\gamma(\omega)\delta^{(t)}\left(\frac{E_{\mathbf{p}\tau i}-E_{\mathbf{k}\tau i}}{\hbar}-\omega\right),\\
E^{\rm CSL,2}_{\rm TOT}(t) &= -t\sum_i\sum_\tau\sum_{\mathbf{p},\mathbf{k}}E_{\mathbf{p}\tau i}\mathcal{N}(\mathbf{p})\left(\frac{m_i}{m_0}\right)^2\int{\rm d}\mathbf{x}\int{\rm d}\mathbf{y}\,\psi_{\mathbf{k}\tau i}(\mathbf{x})\psi^*_{\mathbf{p}\tau i}(\mathbf{x})\psi_{\mathbf{p}\tau i}(\mathbf{y})\psi^*_{\mathbf{k}\tau i}(\mathbf{y})F(\mathbf{x}-\mathbf{y})\\
&\quad\times\int{\rm d}\omega\,\gamma(\omega)\delta^{(t)}\left(\frac{E_{\mathbf{p}\tau i}-E_{\mathbf{k}\tau i}}{\hbar}-\omega\right),
\end{aligned} \quad \text{(A17)}$$

where we exploited the relations in Eqs. (8) and (A6).

So far the result is general. We now apply it to the case of interest, i.e., N particles in a cube box of length L. We apply the periodic boundary conditions and the box-normalization prescription

$$\psi_{\mathbf{p}\tau i}(\mathbf{x})\to\phi_{\mathbf{q}\tau i}(\mathbf{x})=\frac{e^{i\mathbf{q}_{\tau i}\cdot\mathbf{x}}}{L^{3/2}},\quad \text{with}\quad \mathbf{q}_{\tau i}=\frac{2\pi}{L}\mathbf{n}_{\tau i}, \quad \text{(A18)}$$

where $\mathbf{n}_{\tau i}\in\mathbb{Z}^3$. The wave functions $\phi_{\mathbf{q}\tau i}(\mathbf{x})$ are orthonormal:

$$\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{L}{2}}^{\frac{L}{2}} \mathrm{d}\mathbf{x}\, \phi_{\mathbf{q}}(\mathbf{x})\phi^*_{\mathbf{q}'}(\mathbf{x}) = \delta_{\mathbf{n}\mathbf{n}'}. \tag{A19}$$

In the $L \to +\infty$ limit (so that space integrals extend over the whole space and can be performed exactly), we have

$$E^{\mathrm{CSL},1}_{\mathrm{TOT}}(t) = t\sum_i\sum_\tau\sum_{\mathbf{pk}} E_{\mathbf{p}\tau i}\left(\frac{m_i}{m_0}\right)^2 \mathcal{N}(\mathbf{k})\frac{e^{-(\mathbf{p}-\mathbf{k})^2 r_C^2}}{L^3}\int \mathrm{d}\omega\,\gamma(\omega)\delta^{(t)}\left(\frac{E_{\mathbf{p}\tau i}-E_{\mathbf{k}\tau i}}{\hbar}-\omega\right),$$
$$E^{\mathrm{CSL},2}_{\mathrm{TOT}}(t) = -t\sum_i\sum_\tau\sum_{\mathbf{p},\mathbf{k}} E_{\mathbf{p}\tau i}\mathcal{N}(\mathbf{p})\left(\frac{m_i}{m_0}\right)^2 \frac{e^{-(\mathbf{p}-\mathbf{k})^2 r_C^2}}{L^3}\int \mathrm{d}\omega\,\gamma(\omega)\delta^{(t)}\left(\frac{E_{\mathbf{p}\tau i}-E_{\mathbf{k}\tau i}}{\hbar}-\omega\right). \tag{A20}$$

The CSL heating power $P_{\mathrm{CSL}} = \frac{\mathrm{d}}{\mathrm{d}t}E_{\mathrm{TOT}}(t)$ in the long time limit is then given by

$$P_{\mathrm{CSL}} = \sum_i\sum_\tau\sum_{\mathbf{p}}\left(\frac{m_i}{m_0}\right)^2\mathcal{N}(\mathbf{p})\frac{1}{L^3}\sum_{\mathbf{k}} e^{-(\mathbf{p}-\mathbf{k})^2 r_C^2}(E_{\mathbf{k}\tau i}-E_{\mathbf{p}\tau i})\gamma\left(\frac{E_{\mathbf{p}\tau i}-E_{\mathbf{k}\tau i}}{\hbar}\right). \tag{A21}$$

In the white noise case, where $\gamma(\omega) = \gamma = \lambda(2\sqrt{\pi}r_C)^3$, by taking $E_{\mathbf{k}\tau i} = \hbar^2\mathbf{k}^2/(2m_i)$ we find

$$\frac{\gamma}{L^3}\sum_{\mathbf{k}} e^{-(\mathbf{p}-\mathbf{k})^2 r_C^2}(E_{\mathbf{k}\tau i}-E_{\mathbf{p}\tau i}) \underset{[L\to+\infty]}{\longrightarrow} \frac{3\hbar^2\lambda}{4m_i r_C^2}. \tag{A22}$$

By merging with Eq. (A21) we have

$$P_{\mathrm{CSL}} = \frac{3\hbar^2\lambda}{4m_0^2 r_C^2}\sum_i m_i\sum_\tau\sum_{\mathbf{p}}\mathcal{N}(\mathbf{p}) = \frac{3\hbar^2\lambda M}{4m_0^2 r_C^2}, \tag{A23}$$

since $\sum_\tau\sum_{\mathbf{p}}\mathcal{N}(\mathbf{p})$ gives the number of particles of type i.

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