

Rarefaction of beta diversity

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Abstract. Beta diversity has long been used to summarize the amount of variation in species

composition among a set of N sampling units. However, while classical beta diversity provides an

estimate of multiple-site dissimilarity among all sampling units, it is not informative on the changes

30 of multiple-site dissimilarity as a function of sampling effort. For gamma diversity, this pattern is
31 usually represented as a species accumulation curve, which is the graph of the number of observed
32 species when the number of plots varies from 1 to N . Here, we will show that species accumulation
33 curves may also be used to summarize directional and non-directional beta diversity as a function of
34 sampling effort. The behavior of the proposed measures of beta diversity is illustrated with one
35 worked example on plant species in Mediterranean coastal vegetation. We believe this approach to
36 the measurement of beta diversity provides a relevant contribution to summarize multiple-site
37 dissimilarity as the result of a species turnover process, rather than as a static indicator. For
38 directional species accumulation curves, the method, for which we provide a custom R function,
39 further allows summarizing the spatial autocorrelation in species composition among plots along an
40 a-priori defined spatial, temporal or environmental gradient.

41

42 **Keywords:** Community turnover; Directional and non-directional accumulation curves; Effective
43 number of plots; Number of species; Spatial autocorrelation.

44

45 Introduction

46 Whittaker (1960, 1972) introduced the concept of beta diversity (the amount of variation in
47 species composition among a set of sampling units) for linking mean local-scale diversity (or alpha
48 diversity) to the diversity of the broader regional species pool (gamma diversity). Since then beta
49 diversity has become a primary tool for connecting the spatial structure of species assemblages to
50 ecological processes and there is a huge literature on its measurement and summarization (for a
51 review, see e.g. Tuomisto 2010a, 2010b; Anderson et al. 2011). According to Whittaker's proposal,
52 given a community composition matrix of S species in N plots (or sampling units), beta-diversity
53 can be calculated as the ratio of the diversity of the pooled set of plots and mean species diversity
54 within each plot, such that $\beta = \gamma/\alpha$. This metric of beta diversity summarizes multiple-site
55 dissimilarity within the community composition matrix. However, it does not provide any

56 information on the changes of multiple-site dissimilarity when the number of plots varies from 1 to
57 N . For gamma diversity, this variation is traditionally represented as a species accumulation curve, a
58 graph of the number of observed species as a function of sampling effort or the area sampled
59 (Gotelli and Colwell 2001). In this view, the species accumulation curve represents gamma
60 diversity as the outcome of the sampling process rather than as a fixed summary statistic. A
61 distinction is usually made between directional and non-directional accumulation curves: directional
62 curves summarize how species accumulate along an a-priori defined spatial, temporal or
63 environmental gradient. By contrast, non-directional curves measure turnover in community
64 structure without reference to any specific gradient.

65 In this paper, we show that species accumulation curves may also be used for summarizing
66 directional and non-directional beta diversity. While beta diversity measures have been developed
67 for a long time, there is a knowledge gap on how multiple-site dissimilarity changes in relation to
68 sampling effort. The present manuscript fills this gap and shows how species accumulation curves
69 can be used to summarize beta diversity as a function of sampling effort. This can be very useful to
70 researchers attempting to identify the necessary sampling effort to correctly characterize beta-
71 diversity. We also show that comparing the directional beta diversity of a given set of plots with its
72 non-directional counterpart provides a measure of the spatial autocorrelation of species composition
73 among plots induced by the non-random species sorting along the ecological gradient.

74 The paper is organized as follows: first, we present an overview on directional and non-
75 directional species accumulation curves and their associated beta diversity measures. Next, to show
76 the behavior of the proposed measures, a worked example is used with data on plant species cover
77 in 68 plots of Mediterranean coastal vegetation.

78

79 **Methods**

80 ***Non-directional species accumulation curves and beta diversity***

81 Sample-based accumulation curves are constructed for a set of N equal-sized plots by plotting
82 the number of detected species S when the number of sampled plots increases from 1 to N (Scheiner

83 2003; Scheiner et al. 2011). For non-directional curves, it is assumed that the position of the plots in
 84 physical or environmental space is irrelevant. Therefore, accumulation curves are generally
 85 constructed via permutation procedures by randomly resampling the N plots and then plotting the
 86 average number of species found in 1, 2, ..., N plots (Gotelli and Colwell 2001; Colwell et al. 2004).
 87 For standard resampling without spatial constraints, a mathematical expression for sample-based
 88 accumulation curves was first derived by Shinozaki (1963) and then independently re-discovered by
 89 several authors in the 1970s and then again in the 2000 (for a review, see Chiarucci et al. 2008). For
 90 species incidence (presence and absence) data, the average number of species detected by sampling
 91 all possible combinations of M equal-sized plots randomly and without replacement from a larger
 92 set of N plots, is given by:

$$S_M = \sum_{i=1}^S \left[1 - \frac{\binom{N-N_i}{M}}{\binom{N}{M}} \right] = S - \sum_{i=1}^S \left[\frac{\binom{N-N_i}{M}}{\binom{N}{M}} \right] \quad (1)$$

96 where S_M is an estimator of the number of species in the M plots, S the total number of species in
 97 the entire set of N plots ($N \geq M$), and N_i the number of plots that contain species i . The binomial

98 ratio $\rho_{iM} = \binom{N-N_i}{M} / \binom{N}{M}$ is the probability that species i is not present in a randomly selected
 99 sample of M plots. Therefore, $\rho_{iM} = 0$ for $M > N - N_i$.

100 Species accumulation curves have been used for comparing species richness in assemblages of
 101 various sizes after rarefaction to a common level sampling effort or sample completeness (Gotelli
 102 and Colwell 2001; Colwell et al. 2004; Chao and Jost 2012), and for extrapolating species richness
 103 to a larger area sampled (Colwell et al. 2012; Chao et al. 2014, 2015). Eq. 1 is also used for
 104 constructing individual-based accumulation curves relating the estimated number of species to the

105 number of sampled individuals. In this case, M and N represent the sampled number of individuals
106 and the overall population size, respectively (Hurlbert 1971; Simberloff 1972).

107 The estimated number of species S_M represents the gamma diversity in the M plots: $\gamma_M = S_M$.

108 By contrast, alpha diversity does not change with M being always equal to S_1 . This is easily shown

109 considering that in Eq. 1 each plot appears an equal number of times for the calculation of S_M ,

110 irrespective of the value of M , such that $\alpha_M = S_1$ (see Ricotta et al. 2012, Appendix S3). From S_M

111 and S_1 we can calculate an index of beta diversity for all values of M as (Chao et al. 2014):

112

$$113 \quad \beta_M = \frac{S_M}{S_1} = \frac{\gamma_M}{\alpha_M} \quad (2)$$

114

115 According to Jost (2007), β_M can be interpreted as an ‘effective number’ of communities or

116 plots, the theoretical number of fully-distinct plots (no shared species) with mean species richness

117 equal to α_M needed to produce the given value of beta diversity (see also Wilson and Shmida

118 1984).

119 Like the original species accumulation curve, the curve of beta diversity built from the β_M

120 values is a concave and increasing function of M (proof in Appendix 1). Therefore, the graph of β_M

121 vs. M shows how the effective number of plots accumulates with sampling effort. Note however

122 that β_M summarizes multiple-site dissimilarity among plots in the range $[1, M]$. This range is

123 intuitively related to the very meaning of β_M as the effective number of maximally distinct plots.

124 For a community composition matrix of S species x N plots, if all plots are compositionally

125 identical, then $\beta_M = 1$ irrespective of the value of M . In that case, since $\alpha_M = \gamma_M$, one single

126 ‘effective’ plot is enough to produce the observed value of β_M . By contrast, if the plots do not have

127 any species in common, $\beta_M = M$, meaning that M maximally distinct plots with mean species
 128 richness α_M are needed to obtain the observed value of β_M . Therefore, since the range of β_M
 129 depends on M , the effective numbers of plots cannot be used for comparing beta diversity for
 130 different values of M . To remove this dependence, beta must first be rescaled onto the unit interval
 131 (Chao et al. 2012; Ricotta and Pavoine 2015). The simplest way to rescale a given quantity X
 132 between zero and one is to use the linear transformation $(X - X_{\min}) / (X_{\max} - X_{\min})$. This
 133 transformation gives a normalized measure of non-directional beta diversity (Harrison et al. 1992;
 134 Jost 2007):

135

$$136 \quad \beta_\eta = \frac{\beta_M - 1}{M - 1} \quad (3)$$

137

138 which can be interpreted as the average proportion of non-overlapping species in the M plots. When
 139 $M = 2$, it reduces to the classic Sørensen dissimilarity coefficient. The normalized measure β_η
 140 summarizes beta diversity in a fixed range of values ($0 \leq \beta_\eta \leq 1$) that is independent of M . That is,
 141 the minimum and maximum values that β_η can take are fixed constants that do not depend on the
 142 number of accumulated plots M . This lack of systematic constraints between β_η and M ensures that
 143 β_η can be used for comparing multiple-site dissimilarity at different values of M . For additional
 144 normalized dissimilarity measures of beta diversity, see Chao and Chiu (2016).

145 Normalization removes the (undesired) dependence of the range of β_η on M . However, it does
 146 not eliminate the dependence of beta on sampling effort, which is the main assumption of this
 147 paper. Therefore, for a given set of plots N , the variation in species composition β_η decreases as a
 148 function of the number of plots sampled (proof in Appendix 1). Due to this decreasing pattern,

149 calling the graph of β_n vs. M an ‘accumulation curve’ is not the most appropriate choice.
150 Therefore, like in Wilson and Shmida (1984), we use for these curves the more neutral term of
151 ‘community turnover’ curves.

152 ***Directional species accumulation curves and beta diversity***

153 The examination of turnover in species composition along a gradient requires explicit
154 consideration of the order of plots in physical or environmental space (Vellend 2001). To this end,
155 given N plots ordered along a predefined gradient of interest, starting from one end of the gradient
156 we can construct a directional species accumulation curve by sequentially adding the plots one after
157 the other along that gradient and plotting the cumulative number of species found in 1, 2, ..., N
158 plots.

159 Because each directional accumulation curve is case-specific and depends on the distribution of
160 plots along a user-defined gradient, it cannot be derived theoretically and a mathematical expression
161 for this curve does not exist. Therefore, the values of alpha and gamma diversity along the gradient
162 need to be calculated separately for each curve based on the specific order of plots. Beta diversity
163 for all values of M can then be calculated as above:

164

$$165 \quad \vec{\beta}_M = \frac{\vec{\gamma}_M}{\vec{\alpha}_M} \quad (4)$$

166 and

$$167 \quad \vec{\beta}_\eta = \frac{\vec{\beta}_M - 1}{M - 1} \quad (5)$$

168

169 where the arrows on alpha, beta and gamma denote the directional nature of the turnover process.

170 Unlike a non-directional accumulation curve, in the directional case the value of $\vec{\alpha}_M$ (i.e. the mean
171 number of species in the M plots) is not constant. Rather, like $\vec{\gamma}_M$, it depends on the order of the
172 plots along the gradient. Accordingly, the order in which individual plots are added affects the
173 shape of the resulting accumulation curve, such that directional beta $\vec{\beta}_\eta$ is not necessarily a
174 monotonically decreasing function of M .

175 To complete the picture, ‘constrained’ species accumulation curves which account for the
 176 arrangement of plots in physical or environmental space were introduced by Scheiner (2003, his
 177 Type IIIA curves) and first explored by Chiarucci et al. (2009). Order-free accumulation curves
 178 tend to overestimate the rate of increase in species richness because they ignore the autocorrelation
 179 of species composition among the sampling units. In contrast, constrained accumulation curves
 180 describe the increase in species richness as a function of both the sampling effort and the
 181 autocorrelation in species composition among plots.

182 In building a constrained accumulation curve, adjacent sampling units are combined step by step
 183 using the spatial or environmental proximity among plots as a constraining factor. Given a set of N
 184 plots, for each plot a directional species accumulation curve is constructed by sequentially adding
 185 the first, second, ..., k -th nearest neighbor. This procedure is repeated for all N plots, generating N
 186 directional accumulation curves from which a mean constrained curve is calculated (Chiarucci et al.
 187 2009; Bacaro et al. 2016). The resulting curve is thus an intermediate solution between a non-
 188 directional accumulation curve in which all possible combinations of 1, 2, ..., M plots are used for
 189 building the curve and a pure directional curve in which the N plots are ordered along a single
 190 spatial or environmental gradient.

191 Denoting by $\vec{\gamma}_{jM}$ and $\vec{\alpha}_{jM}$ the gamma and alpha diversity of the j -th directional community
 192 turnover curve ($j = 1, 2, \dots, N$), the constrained beta diversity $\bar{\beta}_M$ for a sequence of M plots can be
 193 calculated in analogy to Eq. 2 as the ratio of mean gamma to mean alpha:

194

$$195 \quad \bar{\beta}_M = \frac{\bar{\gamma}_M}{\bar{\alpha}_M} = \frac{\sum_{j=1}^N \vec{\gamma}_{jM} / N}{\sum_{j=1}^N \vec{\alpha}_{jM} / N} \quad (6)$$

196 such that

$$197 \quad \bar{\beta}_\eta = \frac{\bar{\beta}_M - 1}{M - 1} \quad (7)$$

198

199 The reasons for calculating beta as $\bar{\beta}_M = \bar{\gamma}_M / \bar{\alpha}_M$ instead of $\bar{\beta}_M = 1/N \sum_{j=1}^N \bar{\gamma}_{jM} / \bar{\alpha}_{jM}$ were
 200 discussed by Chao et al. (2014) in the context of the rarefaction of Hill numbers, and the reader is
 201 addressed to this paper for additional details.

202 The plot of $\bar{\beta}_\eta$ vs. M describes how ‘constrained’ beta diversity varies with increasing sampling
 203 effort. For each value of M , the difference between $\bar{\beta}_\eta$ (or $\bar{\beta}_\eta$) and the non-directional beta reflects
 204 the amount of autocorrelation in community composition among plots. This is because directional
 205 beta summarizes turnover in community structure along an a-priori ecological gradient and is thus
 206 directly influenced by the similarity in community composition among plots. By contrast, non-
 207 directional turnover is calculated with permutation procedures which ignore any autocorrelation in
 208 community structure. Therefore, a measure of autocorrelation for directional beta diversity can be
 209 calculated as the normalized difference between directional and non-directional beta:

210

$$211 \quad \bar{\beta}_\phi = \frac{\beta_\eta - \bar{\beta}_\eta}{\beta_\eta + \bar{\beta}_\eta} \quad (8)$$

212

213 with $-1 \leq \bar{\beta}_\phi \leq 1$. Since the compositional dissimilarity among plots will often increase more slowly
 214 when sampling units are close to each other, usually $\beta_\eta > \bar{\beta}_\eta$ and $\bar{\beta}_\phi > 0$. Compositional
 215 autocorrelation occurs due to two processes: localized dispersal mechanisms and autocorrelation in
 216 environmental conditions. We therefore expect greater habitat homogeneity to lead to lower
 217 compositional heterogeneity among adjacent plots. If adjacent plots are compositionally very
 218 similar to each other, the difference $\beta_\eta - \bar{\beta}_\eta$ will be high, such that the normalized difference $\bar{\beta}_\phi$
 219 will be high too. By contrast, if autocorrelation in community composition among adjacent plots is
 220 low, $\bar{\beta}_\eta$ will approach its random expectation β_η and $\bar{\beta}_\phi$ will be close to zero.

221 **Worked example**

222 We used vegetation plots sampled on recent (Holocene) coastal dunes occupying a narrow strip
223 (< 150 m) of about 16 km length along the seashore on the Tyrrhenian coast of central Italy (Figure
224 1). The vegetation of the dune profile follows a compressed zonation of habitats along a clearly
225 defined sea-inland environmental gradient that drives considerable turnover in species composition
226 and functioning over short distances: from the pioneer communities of the upper beach to the
227 woody communities (Mediterranean macchia and evergreen forests) of the inland fixed dunes
228 (Acosta et al. 2003).

229 Vegetation sampling was undertaken in spring (April-May) of 2006. Randomly generated GPS
230 coordinates were used to define the sampling location of 2 x 2m vegetation plots in the study area.
231 In each plot, the cover of all vascular plant species was recorded using a 10% interval rank scale. A
232 total of 68 plots were sampled and 61 species recorded (Appendix 2). For the 68 plots of coastal
233 dune vegetation used in this study we produced:

234
235 i) A non-directional community turnover curve of β_η vs. M constructed according to Eq. (1-3).

236
237 ii) A spatially-constrained turnover curve of $\bar{\beta}_\eta$ vs. M (hereafter distance-based turnover curve), in
238 which all plots are ordered according to their distance from the seashore. Carboni et al. (2011)
239 showed that for coastal vegetation, distance from the seashore is related to a directional stress
240 gradient of wind and soil parameters, ranging from the exposed conditions of the upper beach to the
241 more sheltered conditions of the back dune. Therefore, plant communities located closer to the sea
242 are generally subject to higher environmental stress and disturbance.

243
244 iii) A functionally-constrained turnover curve (hereafter trait-based turnover curve), in which all
245 plots are ordered along an ecological gradient according to the functional characters of the most
246 abundant species. For building this functional turnover curve, we defined an a-priori directional

247 gradient in functional trait space. To do so, we used a set of plant functional traits measured for a
248 subset of 42 dominant species. This subset of species was chosen by selecting the most common
249 and abundant species within each habitat along the sea-inland gradient, that collectively account for
250 ~80% of the standing live biomass (see Carboni et al. 2013 for details). This threshold has been
251 shown to ensure a satisfactory description of overall community properties (Pakeman and Queded
252 2007).

253 Three quantitative life-history traits were chosen based on their relevance to the functional
254 ecology of plants in coastal dune environments (Carboni et al. 2016): plant height (cm), leaf dry
255 mass content (LDMC; $\text{mg}\cdot\text{g}^{-1}$) and seed mass (mg). These traits provide a good representation of the
256 species global spectrum of form and function (Diáz et al. 2016) and are also correlated with the
257 directional stress gradient of wind and soil parameters which shape the zonation of the coastal dune
258 vegetation (Carboni et al. 2011, 2013). Because of environmental sorting of species according to
259 habitat preferences along the sea-inland stress gradient, we expect that specific species
260 combinations with particular suites of traits (e.g. short annual species with succulent leaves on the
261 upper beach) should gradually be replaced along the gradient by other dominant species with
262 different traits (e.g. tall perennial species on the main dune ridge). Carboni et al. (2013) showed that
263 there was a significant relationship between sea-inland stress and community functional
264 composition.

265 First, we calculated the Bray-Curtis pairwise dissimilarity among species based on the selected
266 functional traits. Before calculations, traits were square root-transformed when necessary and scaled
267 to unit length. We next calculated the pairwise functional dissimilarity among plots according to
268 Pavoine and Ricotta (2014, Eq. 3) The resulting plot-to-plot functional distance matrix was then
269 used for constructing the constrained turnover curve of $\bar{\beta}_\eta$ vs. M . All curves were constructed using
270 a custom R function available as an electronic appendix to this paper (Appendix 3).

271 **Results**

272 The directional and non-directional species accumulation curves of γ_M and $\bar{\gamma}_M$ vs. M , together
273 with the corresponding community turnover curves of β_η and $\bar{\beta}_\eta$ vs. M . are shown in Figure 2 and
274 3, respectively. Figure 4 shows the amount of autocorrelation of directional beta diversity $\bar{\beta}_\phi$ as a
275 function of sampling effort. As expected, the shape of the curves differs depending on which
276 methods are used to construct the curves, thus providing a useful tool to characterize biodiversity
277 patterns and to explore the relevance of selected functional traits or environmental variables in
278 shaping community structure. Due to the very high autocorrelation of plots in functional space, the
279 trait-based accumulation curve increases less steeply than the corresponding non-directional curve.
280 That is, functionally similar plots are on average compositionally more similar to each other than
281 expected by chance alone. As shown in Figure 3, the lower rate of species accumulation of the trait-
282 based curve compared to the random non-directional curve results in a lower rate of turnover in
283 community structure and thus in lower beta diversity.

284 On the other hand, the plot of the distance-based accumulation curve is very similar to that of the
285 non-directional curve. That is, at least for our case study, distance from the seashore is a less
286 effective indicator of the species sorting than trait-based differences. This is probably due to the
287 effects of natural and anthropogenic disturbance, which tend to modify the natural zonation of
288 vegetation along the dune profile giving rise to a complex mosaic of fragmented and intermingled
289 communities. As shown in Figure 4, for the distance-based curve, the autocorrelation of directional
290 beta diversity is much lower than that of the trait-based curve and becomes negligible for $M > 45$,
291 meaning that after this threshold the compositional turnover of the constrained distance-based curve
292 does not substantially differ from random expectation. This example clearly shows how different
293 degrees of autocorrelation in geographical or functional space can affect the estimates of gamma
294 and beta diversity of community turnover curves.

295 **Discussion**

296 In this paper, we showed that community turnover curves provide a tool for summarizing various
297 types of directional and non-directional beta diversity as a function of sampling effort. Azovsky
298 (2011) separated two conceptually different curves: species-area and species-sampling effort
299 relationships. The present manuscript deals mostly with species-sampling effort relationship,
300 although directional analysis of real gradients with ordered plots goes a step further simple species-
301 sampling effort relationships *sensu* Azovsky (2011). The same approach can be extended for
302 constructing incidence- or abundance-based community turnover curves either mathematically or
303 with resampling methods for any taxonomic, functional or phylogenetic diversity metric (Walker et
304 al. 2008; Ricotta et al. 2012; Chao et al. 2014, 2015). The resulting measures of beta diversity can
305 then be compared among habitats sampled with different effort after rarefaction to the sample size
306 of the smaller datasets. For examples, see e.g. Bacaro et al. (2016). In addition, if directional or non-
307 directional beta diversity is calculated for a nested series of plots of different size, the resulting
308 turnover curves can be used to analyze how beta diversity scales with local sample area, thus
309 extending our results to species-area relationships (see Palmer and White 1994; Mokany et al.
310 2013).

311 Finally, turnover curves can also be used for calculating the amount of autocorrelation in
312 community composition for directional beta diversity compared to its non-directional counterpart.
313 Species are almost never randomly distributed, either due to heterogeneity of environmental factors
314 or to non-random dispersal mechanisms. Therefore, autocorrelation is a crucial aspect of directional
315 turnover curves and appropriate methods should be used in diversity studies if there is evidence of
316 significant clustering of species in physical or environmental space (Legendre 1993; Fortin and
317 Dale 2009; Bacaro et al. 2016).

318 Previous methods to assess multivariate spatial community structure include distance-decay plots
319 (Nekola and White 1999; Qian and Ricklefs 2007), multivariate variograms (Wagner 2003) and
320 Mantel correlograms (Borcard and Legendre 2012). Our proposal adds a new tool to the ecologist

321 toolbox as it allows to summarize autocorrelation in species composition along spatial, temporal or
322 environmental gradients from community turnover curves.

323 Directional species accumulation curves have been typically used to summarize the decrease in
324 species accumulation due to spatial autocorrelation along an environmental gradient compared to
325 non-directional curves (e.g. Chiarucci et al. 2009; Bacaro et al. 2016). The same ‘directional’
326 approach can also be used to maximize the rate of species accumulation with a given effort so as to
327 improve the efficiency of species inventories (Palmer et al. 2002; Rocchini et al. 2005). Because the
328 sampling order of plots determines the slope of the accumulation curve, an optimal strategy to
329 maximize species accumulation consists of using external data to choose the order of plots so that
330 the resulting accumulation curve is the steepest possible (Rocchini et al. 2005). That curve is a
331 special kind of directional turnover curve, which accumulates species at a faster rate compared to
332 the corresponding random non-directional curves. As a consequence of this increased accumulation
333 rate, the normalized difference between the non-directional and directional beta diversity measures
334 takes negative values showing a general tendency towards a negative correlation in the
335 compositional similarity of subsequent plots. This illustrates how our proposed metric of
336 autocorrelation based on turnover curves can also be used as a synthetic criterion for defining
337 optimization strategies in vegetation sampling, for example to show at what distance species
338 composition is no longer autocorrelated. Thus, one can use this critical distance between sampling
339 units to avoid spending time and resources on redundant information.

340 The idea of calculating an index of beta diversity from species accumulation curves has already
341 been proposed in the past. Ricotta et al. (2002) suggested calculating beta diversity as the slope of
342 the linearized species accumulation curve in semilogarithmic space according to Gleason (1922);
343 Crist and Veech (2006) proposed an additive model of diversity decomposition for which beta
344 diversity is obtained as the difference in species richness between the last and first points of the
345 species accumulation curve: $\beta = \gamma - \alpha = S_N - S_1$. The main difference between these previous
346 works and ours is that we do not treat beta diversity as a fixed quantity: the graph of beta vs. M

347 shows how community turnover varies when M increases from 1 to N . Therefore, using
348 accumulation curves, multiple-site dissimilarity or beta diversity is represented as the result of a
349 species turnover process, rather than as a static indicator. For $M = N$, β_M reduces to the traditional
350 Whittaker's (static) beta diversity for a set of N sampling units $\beta = \gamma_N / \alpha_N$.

351 We also showed that using the classical multiplicative formula $\beta_M = \gamma_M / \alpha_M$, a measure of beta
352 is obtained which is easily interpretable in terms of 'effective number of plots'. However, due to the
353 constraint $1 \leq \beta_M \leq M$ we cannot directly use this measure for comparing beta diversity for
354 different numbers of plots. To remove the dependence on M , we can use the normalized measure
355 $\beta_\eta = (\beta_M - 1) / (M - 1)$. This scaled version of beta diversity can be used for comparing community
356 turnover for different values of M .

357 In principle, because of the fundamental constraint $M\alpha_M \geq \gamma_M \geq \alpha_M$ (Chao et al. 2012), a
358 similar scaling can also be used for defining a normalized measure of gamma diversity
359 $\gamma_\eta = (\gamma_M - \alpha_M) / (M\alpha_M - \alpha_M)$. However, since $\gamma_M = \alpha_M \times \beta_M$, a little algebra shows that
360 $\gamma_\eta = \beta_\eta = (\beta_M - 1) / (M - 1)$, meaning that there is no fundamental difference between normalized
361 beta diversity and normalized gamma diversity. For a given number of plots, the rescaled version of
362 gamma diversity is *per se* a measure of community turnover, such that normalized beta diversity
363 can be also expressed in terms of the relationship between gamma diversity and its extreme values:
364 if $\gamma_M = \alpha_M$ then $\beta_\eta = 0$; At the other extreme, if $\gamma_M = M\alpha_M$ then $\beta_\eta = 1$. Therefore, the scaled
365 versions of beta diversity and gamma diversity lead to the same normalized measure of species
366 turnover.

367 **To conclude**, by its very nature, community ecology usually deals with high-dimensional
368 multivariate data and calls for summarizing methods and indices. However, by condensing the
369 structure of a multidimensional community composition matrix into a single indicator, information
370 is inevitably lost, such that a variety of indices is needed to answer a variety of ecological questions

371 (Pavoine 2016). Therefore, accurate knowledge of the basic properties of diversity measures
372 becomes a necessary condition for comparing different habitats in ecologically meaningful ways.
373 Given this, relying on metrics standardized for sampling effort can provide better means for
374 identifying the most relevant biotic and abiotic drivers of community composition in different
375 habitats and taking appropriate management decisions.

376

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380

381 **Supporting Information**

382 **Appendix 1.** Proofs.

383 **Appendix 2.** Composition matrix and functional trait values of the dominant species for the coastal
384 dune communities of central Italy.

385 **Appendix 3.** R function for the calculation of directional and non-directional accumulation curves,
386 beta diversity and autocorrelation as a function of sampling effort M .

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496 **Figure captions**

497

498 **Figure 1.** Location of the study area.

499

500 **Figure 2.** Directional and non-directional species accumulation curves $\bar{\gamma}_M$ and γ_M vs. M . for the
501 coastal dune communities of central Italy.

502

503 **Figure 3.** Directional and non-directional community turnover curves $\bar{\beta}_\eta$ and β_η vs. M for the
504 coastal dune communities of central Italy.

505

506 **Figure 4.** Plot of the amount of autocorrelation of directional beta diversity $\bar{\beta}_\phi$ as a function of
507 sampling effort M for the coastal dune communities of central Italy.