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A Monte Carlo code for the fragmentation of polarized quarks

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Abstract. We describe a Monte Carlo code for the fragmentation of polarized quarks into pseudoscalar mesons. The quark jet is generated by iteration of the splitting $q \rightarrow h + q'$ where q and q' indicate quarks and h a hadron. The splitting function describing the energy sharing between q' and h is calculated on the basis of the Symmetric Lund Model where the quark spin is introduced through spin matrices as foreseen in the 3P_0 mechanism. A complex mass parameter is introduced for the parametrisation of the Collins effect. The results for the Collins analysing power and the comparison with the Collins asymmetries measured by the COMPASS collaboration are presented. For the first time preliminary results on the simulated azimuthal asymmetry due to the Boer-Mulders function are also given.

1. Introduction

Sarting from the recent work of Ref. [1, 2] we have developed a model for the fragmentation of polarized quarks into pseudoscalar mesons and implemented it in a stand alone Monte Carlo program. The polarized splitting function is calculated on the basis of the relativistic string decay as described in the Symmetric Lund Model (SLM) [3], with confinement as a built in feature and the Left-Right (LR) symmetry, namely symmetry under the exchange of the quarks placed at the string endpoints. The spin information is propagated along the fragmentation chain according to a full quantum version of the semi-classical 3P_0 mechanism [4].

Our model for the hadronisation of a polarized quark jet and the Monte Carlo program are described in Section 2 and in Section 3 respectively. In Section 4 we show some results on the Collins effect for transversely polarized u quark jets and compare them with the Collins asymmetries measured by the COMPASS Collaboration in SIDIS off a transversely polarized proton target. In Section 5 we describe how the primordial transverse momentum of the fragmenting quark has been introduced and its effect on the Collins asymmetry and in Section 6 we present for the first time results of the correlation between transverse spin and transverse momentum described by the Boer-Mulders function. Finally in Section 7 we draw our conclusions.



2. polarized splitting with the 3P_0 mechanism

We simulate with a stand alone Monte Carlo program the jet produced by an initial polarized quark q_1

$$q_1 \rightarrow h_1 + q_2, q_2 \rightarrow h_2 + q_3, \dots, q_i \rightarrow h_i + q_{i+1} \dots \quad (1)$$

repeating recursively the splitting

$$q \rightarrow h(q\bar{q}') + q' \quad (2)$$

where q, q' indicate quarks and h represents the $q\bar{q}'$ meson state. The index i labels the rank of h . k, k' and p are the 4-momenta of q, q' and h respectively. They are related by the momentum conservation $k = p + k'$. The splitting in eq. (2) is described by the splitting function $F_{q',h,q}(\mathbf{k}_T, \mathbf{k}'_T, Z)$ which gives the probability of producing a hadron h with transverse momentum $\mathbf{p}_T = \mathbf{k}_T - \mathbf{k}'_T$ and longitudinal momentum fraction $Z = p^+ / k^+$ (the lightcone coordinates are defined as $p^\pm = p^0 \pm p^3$). \mathbf{k}_T and \mathbf{k}'_T are the transverse momenta of q and q' respectively. The reference axis is the jet axis which we take along the momentum of the fragmenting quark q_1 . The quark spin is encoded in the 2×2 spin density matrix $\rho(q) = (\mathbf{1} + \sigma \cdot \mathbf{S}_q) / 2$ where \mathbf{S}_q is the polarization vector of q .

The input of the model is the transition operator $T_{q',h,q}(k, k')$ which gives the transition amplitude in momentum \otimes spin space for the generic splitting:

$$T_{q',h,q}(k, k') = C_{q',h,q} \times e^{-\frac{b_T}{2} \mathbf{k}'_T{}^2} \times (1 - Z)^{a/2} e^{-\frac{b_L}{2} \frac{\varepsilon_h^2}{Z}} \times [\mu + \sigma_z \sigma \cdot \mathbf{k}'_T] \times \sigma_z \times \hat{u}^{-1/2}(\mathbf{k}_T). \quad (3)$$

It leads to a LR-symmetric model. The factor $C_{q',h,q}$ describes the splitting (2) in flavour space and is determined by the isospin wave function of h . The first exponential is a cutoff in the transverse momentum \mathbf{k}'_T of the quark q' and the factor containing Z comes from the symmetric string fragmentation ($\varepsilon_h^2 = m_h^2 + \mathbf{p}_T^2$ is the hadrons transverse energy squared). The spin term contains the complex mass parameter μ , responsible for the Collins effect and comes from the 3P_0 mechanism. σ_z is the pseudoscalar coupling with h . In the last term $\hat{u}(\mathbf{k}_T)$ is a positive definite single quark density matrix in $\mathbf{k}_T \otimes$ spin space given by

$$\hat{u}(\mathbf{k}_T) = \sum_h |C_{q',h,q}|^2 \int d^2 \mathbf{k}'_T e^{-b_T \mathbf{k}'_T{}^2} \sigma_z [\mu^* + \sigma \cdot \mathbf{k}'_T \sigma_z] [\mu + \sigma_z \sigma \cdot \mathbf{k}'_T] \sigma_z \frac{dZ}{Z} (1 - Z)^a e^{-\frac{b_L \varepsilon_h^2}{Z}}. \quad (4)$$

b_T, b_L, a and μ are the free parameters of the model obtained by fitting the experimental data.

The splitting function $F_{q',h,q}(\mathbf{k}_T, \mathbf{k}'_T, Z)$ is then calculated from eq. (3), up to a normalisation factor, as

$$F_{q',h,q}(\mathbf{k}_T, \mathbf{k}'_T, Z) \propto \text{tr} \left(T_{q',h,q}(\mathbf{k}_T, \mathbf{k}'_T, Z) \rho(q) T_{q',h,q}^\dagger(\mathbf{k}_T, \mathbf{k}'_T, Z) \right) \quad (5)$$

where $\rho(q)$ is the spin density matrix of q .

Finally, the spin density matrix of the quark q' is given by

$$\rho(q') = \frac{T_{q',h,q}(\mathbf{k}_T, \mathbf{k}'_T, Z) \rho(q) T_{q',h,q}^\dagger(\mathbf{k}_T, \mathbf{k}'_T, Z)}{\text{tr} \left(T_{q',h,q}(\mathbf{k}_T, \mathbf{k}'_T, Z) \rho(q) T_{q',h,q}^\dagger(\mathbf{k}_T, \mathbf{k}'_T, Z) \right)}. \quad (6)$$

Equation (5) and eq. (6) are used to simulate the quark fragmentation.

3. Simulation program

The simulation program starts choosing the flavour (u, d, s), the energy and the spin density matrix $\rho(q_1)$ of the initial quark q_1 . The momentum of q_1 defines the jet axis and we assume it to be aligned with the \hat{z} axis in an orthonormal $(\hat{x}, \hat{y}, \hat{z})$ reference system. The jet is generated iteratively as a sequence of splittings. At each step we choose at random the flavour of a new $q'\bar{q}'$ pair (s quarks are suppressed by the factor $s/u = s/d = 0.33$), identify the hadron $h(q\bar{q}')$ and calculate its momentum generating first Z and then \mathbf{p}_T using eq. (5). The momentum of h is completely built imposing the mass-shell condition. Afterwards the spin density matrix of q' is calculated using eq. (6). This procedure is applied until the exit condition is reached, namely that the remaining energy allows to generate one (not simulated) baryon. η^0 meson production is suppressed according to $n(\eta^0)/n(\pi^0) \simeq 0.57$ [6]. A more detailed description of the simulation code implementing an approximated version of the 3P_0 model is given in Ref. [5].

4. Results on the Collins effect

The free parameters of the model have been tuned comparing the simulation results with experimental data on unpolarized hadron multiplicities as function of p_T^2 [7], on unpolarized fragmentation functions [8] and on Collins asymmetries in e^+e^- [9]. The values of the parameters used in these simulations are $a = 0.9$, $b_L = 0.5 \text{ GeV}^{-2}$, $b_T = 5.17 \text{ GeV}^{-2}$ and $\mu = (0.42 + i0.76) \text{ GeV}$.

4.1. Single hadron Collins asymmetry

In Fig.1 the solid lines show the Monte Carlo prediction for the Collins analysing power $a_P^{C,u \rightarrow \pi^\pm}$ for charged pions obtained in simulations of fully transversely polarized (along \hat{y} axis) u jets, as function of $z_h = E_h/E_{q_1}$ (left plot) and as function of p_T (right plot). The analysing power is calculated in bins of z_h or p_T as $a_P^{C,u \rightarrow h} = 2\langle \sin \phi_C \rangle$, ϕ_C being the Collins angle defined as $\phi_C = \phi_h - \phi_{\mathbf{S}_{q_1}}$, where ϕ_h is the azimuth of \mathbf{p}_T and $\phi_{\mathbf{S}_{q_1}} = \pi/2$ is the azimuth of the polarisation vector of q_1 . The points represent the Collins asymmetries for charged pions as obtained from COMPASS in SIDIS off transversely polarized protons [10]. The Monte Carlo prediction is rescaled by a factor $\lambda \simeq 0.05$ calculated from the comparison with the Collins asymmetry as function of p_T for π^- measured in COMPASS. In the u -dominance hypothesis, λ gives a rough estimation of the quark transverse polarization inside a transversely polarized proton.

Looking at Fig. 1 we notice that the model reproduces correctly the opposite sign of the Collins asymmetry for oppositely charged pions. It suggests also a mirror symmetry between π^+ and π^- and the analysing power decays linearly with z_h . For large z_h the MC overestimates the π^- data and the linear trend, not seen in the data, is due to the contribution of different effects like depolarisation of the fragmenting quark, the opposite sign of the analysing powers of even and odd rank hadrons and a somewhat stronger analysing power of the second rank hadron with respect to the first rank. The comparison between MC and data as function of p_T , as can be seen from the right plot of Fig. 1, is quite satisfactory. An interesting feature of the simulated analysing power as function of p_T for π^+ is the change of sign at $p_T \sim 0.9 \text{ GeV}$ due to the contribution of π^+ generated immediately after a first π^0 . The data seem not to exclude this possibility.

4.2. Di-hadron asymmetries

In Fig. 2 we show the comparison between the simulated asymmetry $a_P^{u \rightarrow h^+h^-}$ (solid lines) for pairs of charged hadrons in transversely polarized u jets, as in the previous section, and the asymmetries measured by COMPASS [11] (points) in SIDIS off transversely polarized protons. In both simulations and data the asymmetry is estimated as $2\langle \sin \phi_{RT} \rangle$ where ϕ_{RT} is the azimuth

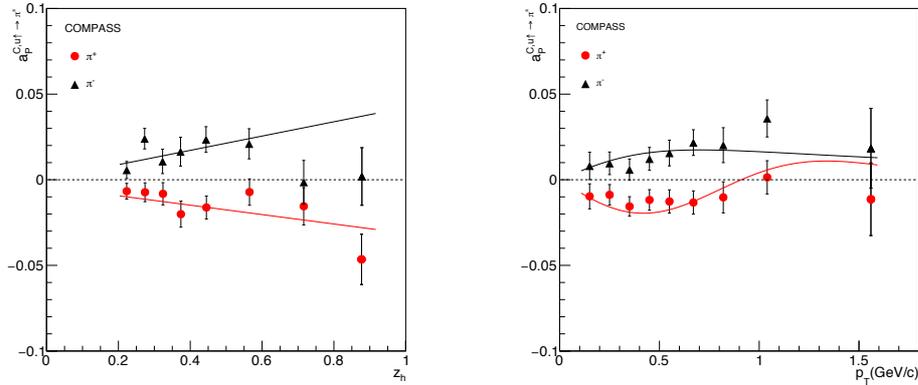


Figure 1. Comparison between the Collins analysing power for charged pions as function of z_h (left plot) and as function of p_T (right plot) predicted by simulations of transversely polarized u quark jets (solid lines) and the Collins asymmetry in SIDIS off transversely polarized protons from COMPASS [10] (points).

of $\mathbf{R}_T = (z_{h_2}\mathbf{p}_{1T} - z_{h_1}\mathbf{p}_{2T})/(z_{h_1} + z_{h_2})$ and 1 (2) label the positive (negative) hadron. The cuts $x_F > 0.1$, $z_h > 0.1$ and $R_T > 0.07 \text{ GeV}$ have been applied.

The left plot shows the comparison as function of $z = z_{h_1} + z_{h_2}$ and the right plot as function of their invariant mass M_{inv} . The MC prediction is in satisfactory agreement with data after rescaling it by the same factor λ as for the single Collins asymmetries. Hence the same 3P_0 mechanism which produces the Collins asymmetries for single hadrons can explain the di-hadron asymmetry.

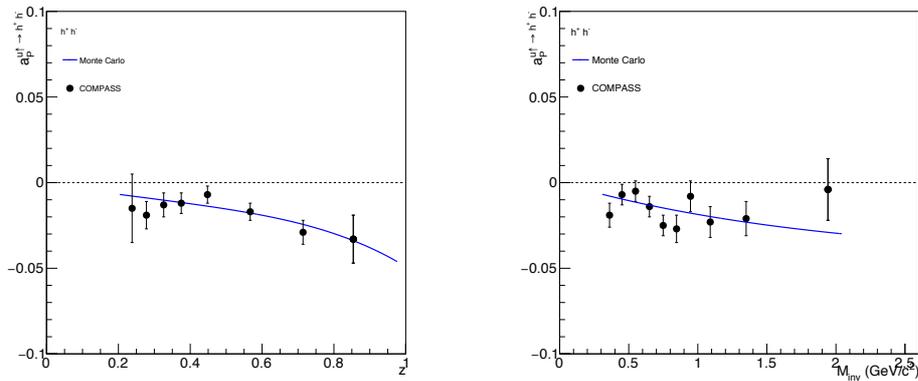


Figure 2. Comparison between the di-hadron analysing power for charged mesons as function of z (left plot) and as function of M_{inv} (right plot) obtained in simulations of transversely polarized u quark jets (solid lines) and the di-hadron asymmetries in SIDIS off transversely polarized protons from COMPASS [11] (points).

5. Effect of the primordial transverse momentum

In simulations we have included the possibility for the initial quark to have some "primordial transverse momentum" $\mathbf{k}_\perp = (k_x, k_y)$, representing for instance the intrinsic transverse momentum of quarks inside the nucleon. It changes the direction of the jet axis and we take

it into account by rotating the hadrons accordingly, in the string center of mass frame. The primordial \mathbf{k}_\perp is generated according to the gaussian distribution $\exp(-\mathbf{k}_\perp^2/\langle k_\perp^2 \rangle)$. Results for $\langle k_\perp^2 \rangle = 0.3 \text{ GeV}^2$ show that the smearing of the Collins asymmetries for single hadrons is a small effect and visible at the highest z_h only. Due to the primordial k_\perp the integrated Collins analysing power is reduced by 14% for π^+ and by 6% for π^- . There is no effect on the di-hadron asymmetries, as expected.

6. The Boer-Mulders asymmetry

In this section we present some new results from simulations of quark jets where the primordial transverse momentum and the transverse polarisation of the fragmenting quark are correlated as described by the Boer-Mulders function.

Considering the SIDIS process, in the γ^* -nucleon center of mass frame the distribution of quarks of flavour q and with transverse momentum \mathbf{k}_\perp inside an unpolarized proton moving along $-\hat{\mathbf{z}}$ axis is described by the Transverse Momentum Dependent (TMD) PDF

$$f_{q\uparrow/P}(x, \mathbf{k}_\perp) = \frac{1}{2} \left(f_{q/P}(x, k_\perp^2) + \frac{h_1^{\perp q}(x, k_\perp^2)}{M_P} (\hat{\mathbf{z}} \times \mathbf{k}_\perp) \cdot \hat{\mathbf{S}}_{Tq} \right). \quad (7)$$

where $f_{q/P}$ and $h_1^{\perp q}$ are the unpolarized and the Boer-Mulders TMD PDFs respectively and M is the nucleon mass. The transverse polarization of the quarks is then

$$\mathbf{S}_{Tq} = \frac{h_1^{\perp q}(x, k_\perp^2)}{M_P f_{q/P}(x, k_\perp^2)} (\hat{\mathbf{z}} \times \mathbf{k}_\perp). \quad (8)$$

After the interaction with the γ^* , the polarization vector of the struck (fragmenting) quark is partly depolarized and reflected with respect to the normal of the scattering plane, which we assume here to be the $(\hat{\mathbf{x}}, \hat{\mathbf{z}})$ plane. It is

$$\mathbf{S}'_{qT} = D_{NN} \frac{k_\perp h_1^{\perp q}(x, k_\perp^2)}{M_P f_{q/P}(x, k_\perp^2)} \left(\frac{k_y}{k_\perp}, \frac{k_x}{k_\perp} \right) \quad (9)$$

where D_{NN} is the depolarisation factor.

In our standalone MC, in which only the fragmentation process is described, after having introduced the intrinsic transverse momentum of the initial quark, we consider it to be fully polarized with

$$\mathbf{S}_{Tq_1} = \left(\frac{k_y}{k_\perp}, \frac{k_x}{k_\perp} \right). \quad (10)$$

In the simulation we first draw the \mathbf{k}_\perp of q_1 as in Section 5 and then calculate the spin density matrix $\rho(q_1) = (\mathbf{1} + \sigma \cdot \mathbf{S}_{Tq_1})/2$ which is taken as input in the generation of the jet in the reference frame where the jet axis lies along the $\hat{\mathbf{z}}$ axis. Afterwards, to take into account the primordial \mathbf{k}_\perp , the produced hadrons are rotated accordingly in the string center of mass frame.

The azimuthal distribution of charged hadrons (with $z_h > 0.2$ and $p_T > 0.1 \text{ GeV}$) produced in simulations of u jets has a strong modulation, as can be seen in the left plot of Fig. 3. The solid line represents the results of the fit using the function $N(1 + a_{\cos \phi_h}^{BM,u \rightarrow h} \cos \phi_h + a_{\cos 2\phi_h}^{BM,u \rightarrow h} \cos 2\phi_h)$ where N is a normalisation factor. While the amplitude of the $\cos \phi_h$ modulation is compatible with zero, we obtain $a_{\cos 2\phi_h}^{BM,u \rightarrow h^+} = 0.120 \pm 0.003$ for h^+ , a factor of 2 smaller than the original (not rescaled by λ) Collins case. In the h^- case it is $a_{\cos 2\phi_h}^{BM,u \rightarrow h^-} = -0.084 \pm 0.004$, a factor of 3 smaller than the original Collins case, of opposite sign with respect to h^+ and somewhat smaller in magnitude. In the right plot of Fig. 3 we show the dependence of $a_{\cos 2\phi_h}^{BM,u \rightarrow h^+}$ on p_T which turns out to be similar to the Collins analysing power in shape but with smaller magnitude.

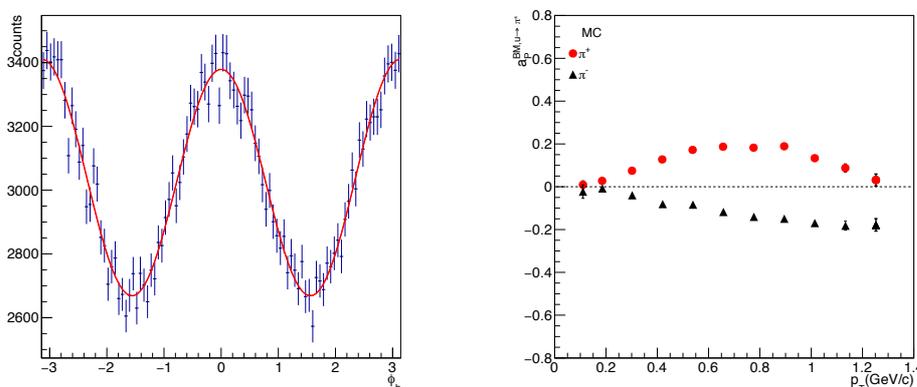


Figure 3. Azimuthal spectrum (left plot) and amplitude of the $\cos 2\phi_h$ modulation as function of p_T (right plot) for positive hadrons obtained in jets of u quarks fully polarized according to the correlation described by the Boer-Mulders function.

7. Conclusions

We have implemented in a stand alone Monte Carlo program a model for the fragmentation of polarized quarks using the Symmetric Lund Model and the 3P_0 mechanism.

The kinematical dependence of the Collins analysing power has been studied and the comparison with the experimental Collins asymmetry measured in SIDIS processes is very promising. We find a satisfactory comparison between simulation and experimental data also for the di-hadron asymmetries. The results show that the same mechanism can be responsible for the Collins effect and the di-hadron asymmetry.

The primordial transverse momentum of the fragmenting quarks is also introduced showing that the effect on the single hadron Collins analysing power is weak. Finally we have presented for the first time results on the $\cos 2\phi_h$ azimuthal asymmetry obtained when assuming a correlation between the transverse spin and the transverse momentum of the fragmenting quark as described by the Boer-Mulders function, taken of maximum amplitude.

The work will continue, in order to study the experimental $\cos 2\phi_h$ asymmetries measured by the different experiments.

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