MODELLING ELASTICITY OF INJECTION MOULDED SHORT FIBRE REINFORCED POLYMERS: COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL APPROACHES

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ABSTRACT

In this work we analysed a sample of short fibre reinforced polyamide extracted from an injection moulded plate. We derived local values of the elastic constants by two different numerical methods, one based on simulation and one based on the reconstruction of the sample's microstructure by micro - CT. Results were compared in terms of moduli of elasticity, assuming an orthotropic material model. Fibre orientation was first simulated by process simulation and results were checked against experimental data obtained by the optical section method. Then, fibre orientation data were used for micro-mechanical modelling of the elastic behaviour by means of mean field homogenisation tools. The experimentally based approach was based on micro computed tomography reconstructions of the inner structure of samples extracted from the injection moulded plate. Using numerical models based on the Cell Method, the elastic behaviour of the reconstructed volume was simulated and results were compared with analytical models based on process simulations and homogenization.

1 INTRODUCTION

The main factor contributing to analytical modelling of elasticity of injection moulded short fibre reinforced polymers is fibre orientation. Fibre orientation distributions can be measured experimentally by different methods and can also be simulated, usually in the framework of process simulation.

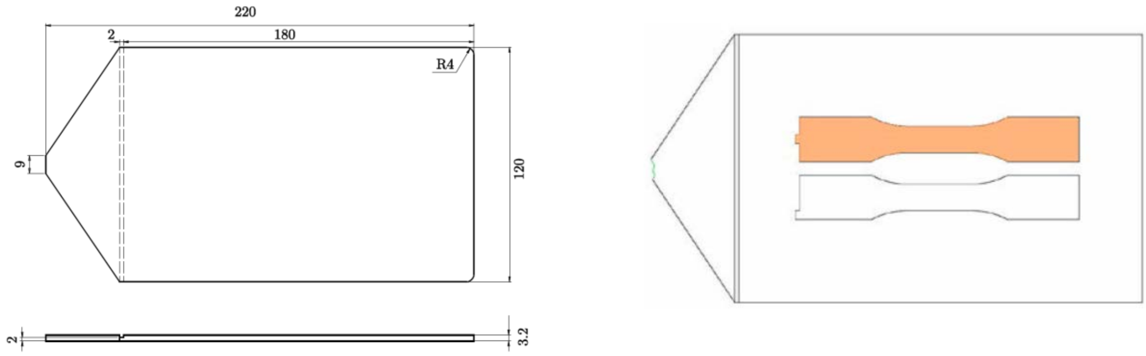
In this work we analyse a sample extracted from an injection moulded plate previously used for the analysis of the effect of fibre orientation upon fatigue behaviour [1], and derive local values of the elastic constants by both numerically and experimentally based approaches. Results are compared in terms of longitudinal moduli of elasticity, assuming an orthotropic material model.

Fibre orientation is first simulated by process simulation and results are checked against experimental data obtained by the optical section method [2, 3]. Then, fibre orientation data are used for micro-mechanical modelling of the elastic behaviour by means of mean field homogenisation tools.

The experimentally based approach stems from micro computed tomography reconstructions [4] of the inner structure of samples extracted from the injection moulded plate. Using numerical models based on the Cell Method [5, 6], the elastic behaviour of the reconstructed volume is simulated and results are compared with analytical models based on process simulations.

2 EXPERIMENTAL

The sample analysed in this work is made of short fibre reinforced (30% by weight) polyamide 6. A specimen was first cut out form an injection moulded plate (Figure 1a), oriented longitudinally, as shown in Figure 1b. Then a smaller sample was extracted from the specimen, at the location shown in Figure 2. This sample was used in [4] for micro-CT analysis and another specimen taken from the same batch of plates was used fibre orientation analysis by the optical method.



(a) (b)

Figure 1: shape and dimension of the injection moulded plate (a); position of the specimen (b)

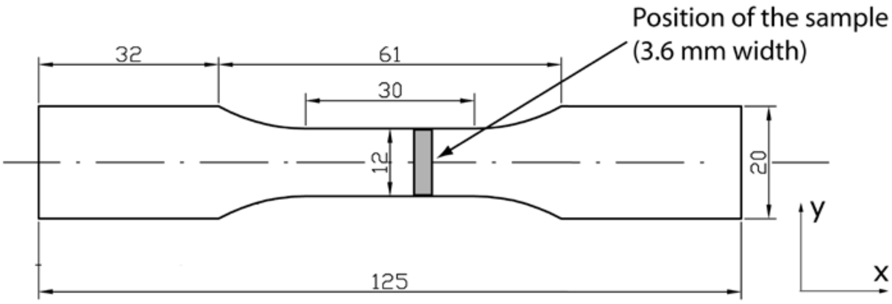


Figure 2: shape and dimensions of the specimen and position of the sample used for fibre orientation analysis and micro-CT analysis

2.1 Use of micro-CT data

The micro-CT scans of the smaller sample were acquired at Elettra, the synchrotron facility in Trieste, and the images were reconstructed with a software developed in-house at Elettra. Details of the micro-CT reconstructions are reported in [4]. Micro-CT data were used to extract several smaller Volumes of Interest (VOIs) through the thickness of the plate, in order to estimate the local values of the elastic moduli by the Cell Method, as described later. A total of 20 VOIs were extracted along the plate’s thickness direction. The examined VOIs are cubic, 40 pixel side and 40 slices, with a resolution of 9 micron. An overlap of adjacent VOIs of 16 pixel was required in order to span the whole 3.2 mm thickness with a spatial resolution similar to that obtained by process simulation.

2.1 Fibre orientation analysis

Fibre orientation was analysed by the optical method at the University of Leeds, using the method described in [3], based on the analysis of the elliptical footprints left by the fibres on a polished section. The sides of entire gauge length of the specimen were analysed and maps of the values of the components of the fibre orientation tensor were obtained. Values of the fibre orientation tensor component were then averaged through the width of the observed surfaces and reported as a function of the position through the thickness of the plate. Measurements confirmed the presence of the well known shell-core-shell microstructure, typical of injection moulded plates, also visible in the volume reconstructed by micro-CT and characterized by fibres oriented mainly perpendicular to the flow in a thin core region, and parallel to the flow in the outer layers.

3 MODELLING

Two models were developed: the first was based on fibre orientation data obtained by process simulation combined with a mean field homogenization analytical model, the second was based on experimental data about the microstructure as obtained by micro-CT reconstructions and processed by the Cell Method.

3.1 Process simulation

Fibre orientation was obtained numerically by process simulation using Autodesk Moldflow 2015. The model geometry is shown in Figure 3, where also the partitions used to identify the specimen and the sample are shown. A 2D mid-plane model was built, with 21 layers through the thickness. Fibre orientation was evaluated using both the Folgar-Tucker model and the Reduced Strain Closure method. Based on the fibre orientation analysis, the RSC method was finally chosen, as values of the first component of the orientation tensor better matched experimental data, as shown in Figure 4

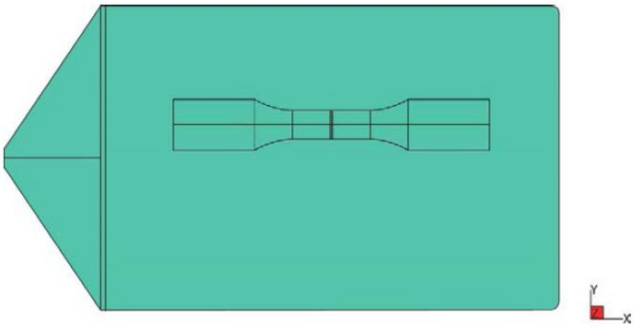


Figure 3: Position of the partitions in the Moldflow model, corresponding to the sample

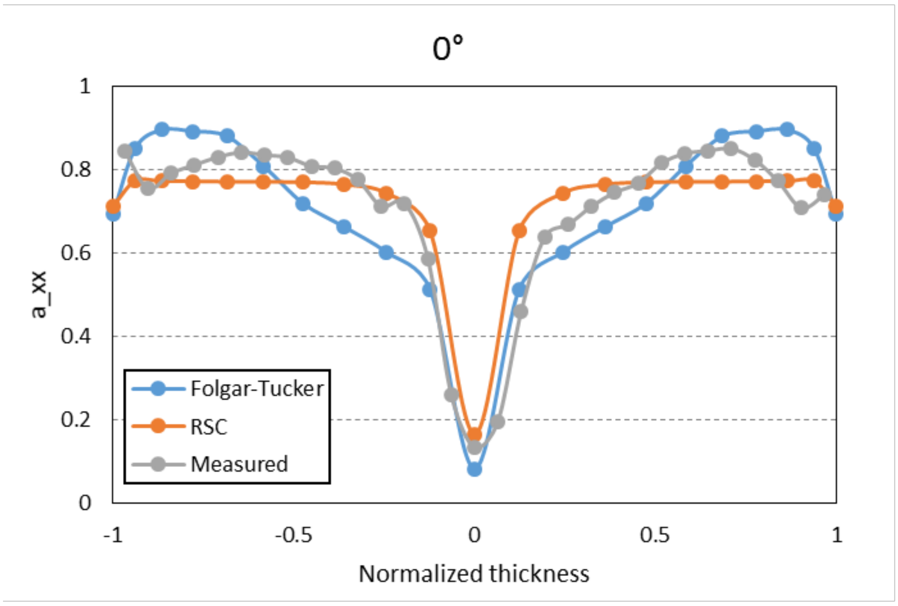


Figure 4: Experimentally and numerically determined values of the first component of the fibre orientation tensor.

3.2 Mean field homogenization

Values of the components of the fibre orientation tensor were extracted from the Modflow model and averaged over the entire gauge area of the specimen. Values were extracted from each of the 21 layers used for the discretization of the plate through its thickness. Based on the values of the fibre orientation tensor, a linear elastic mean field homogenization scheme was applied, based on the Mori-Tanaka model, with a pseudo-grain approximation (for an overview of these methods see [7]), as implemented in Digimat 5.1 by e-Xtream. The parameters used in the homogenization step are reported in Table 1.

Parameter		Value
Fibre elastic modulus E_f	[GPa]	72
Matrix elastic modulus E_m	[GPa]	1.17
Average diameter	[mm]	0.01
Average fibre length	[mm]	0.22

Table 1: Parameters used for the homogenization scheme.

3.3 Cell Method

The apparent elastic moduli values of the reconstructed volumes, E_x, E_y, E_z, along the three coordinate axes, were obtained by a numerical model based on the Cell Method. The interested reader can find detailed descriptions of the CM formulation for several physical fields in [8-12]. Without going into the details of the formulation, it can be stated that the Cell Method defines an influence region (dual cell) for each node and writes the balance equations directly in a discrete form by adding up the surface, the volume and the external loads acting on the dual cell. The resulting linear system can be solved with the usual methods, and the Cell Method results are in general comparable with FEM ones, despite the different approach.

Since the Cell Method is founded on a direct discrete formulation of laws, there is no need to maintain in the field equations a condition of continuity across the different cells and the size of discretization and the dimensions of the heterogeneities can have the same characteristic length [13]. The model adopted here was introduced for SFRP in [5] and [6] where it was used to evaluate the local apparent elastic moduli values in different VOIs reconstructed by micro-CT. In this work, the Cell Method is used to evaluate the apparent elastic moduli of the material, i.e. the elastic moduli along the longitudinal (E_x), in plane transverse (E_y) and out-of-plane transverse (E_z) directions, with respect to the specimen’s longitudinal axis.

A Young’s modulus of 1.17 GPa for matrix and of 72 GPa for glass fibre was assumed as in the mean field homogenization and single mesh of 812905 tetrahedral cells and 141982 nodes was used. The different fibre patterns were reproduced by allocating the elastic properties of each cell according to the matrix/fibre distribution in each VOI. The characteristic length of the mesh in the simulation corresponds to approximately 7 micron.

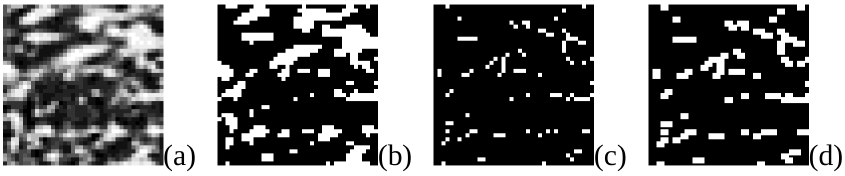


Figure 5: Steps of image processing of micro-CT images

As an example, a reconstructed slice from one of the VOIs is shown in Fig.5a and consists of 256 grey tone levels (8bit). In order to separate the two phases for the numerical analysis, the image is segmented according to the procedure introduced in [4].The result is shown in Fig5b, where fibre is white and matrix is black. It is easily recognizable that the reinforcing short fibres appear to be connected to each other in lumps. Consequently, the computed apparent elastic moduli will result artificially increased, although transverse isotropy is conserved.

A skeleton filter was therefore applied to separate the fibres from each other. As shown in Fig.5c, the fibres are now well isolated, but their diameter has been strongly reduced by the filter. Therefore, the computed apparent elastic moduli become excessively low. In order to compensate for this effect, the images were scaled by a factor of 2 and then segmented again, as shown in Fig.5d. This correction was able to compensate for the skeletonize effect while maintaining the fibres well separated. The computed apparent elastic moduli after this correction in the same VOI resulted 20% lower than those obtained previously without this correction.

4 RESULTS AND DISCUSSION

The results of the mean field homogenization are presented in Figure 6, where values of the apparent elastic modulus in the longitudinal (E_x) and in-plane transverse (E_y) direction are reported. The out-of-plane (E_z) modulus obtained by mean field homogenization based on a 2D mid-plane Moldflow simulation practically does not depend on fibre orientation and was found equal to 1.95 GPa throughout the whole thickness of the sample.

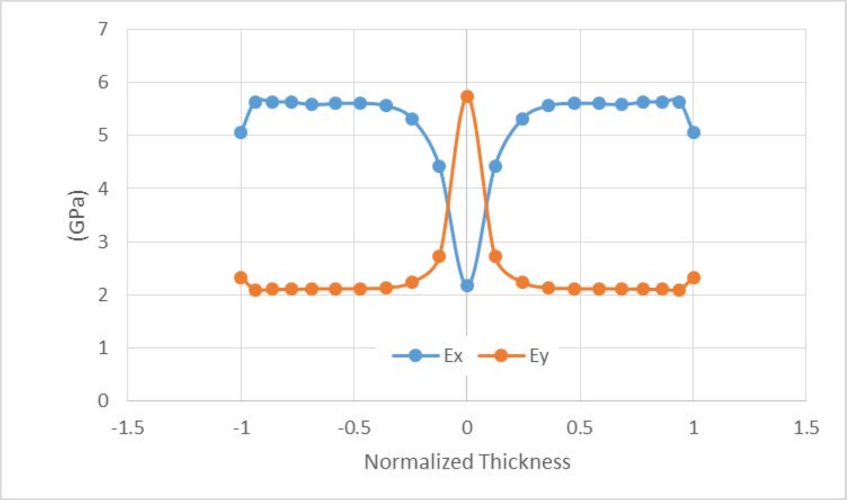


Figure 6 – Values of the apparent moduli obtained by mean field homogenization

The values of the apparent moduli obtained by the Cell Method are reported in Figure 7. Results are compared in Figure 8.

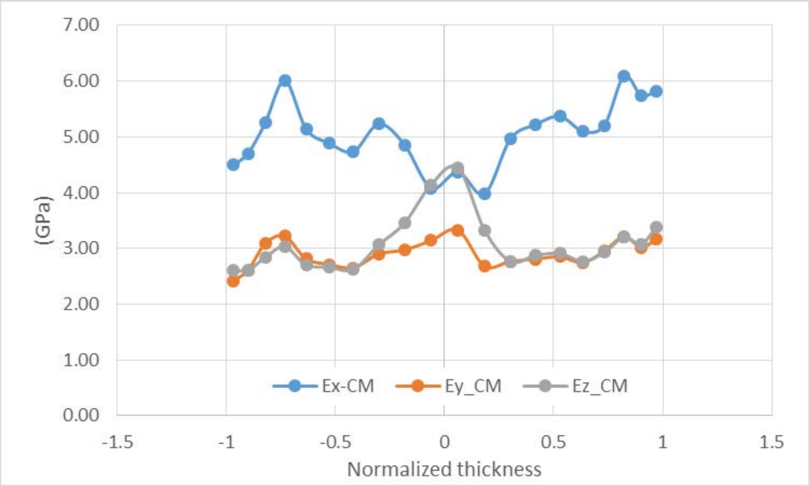
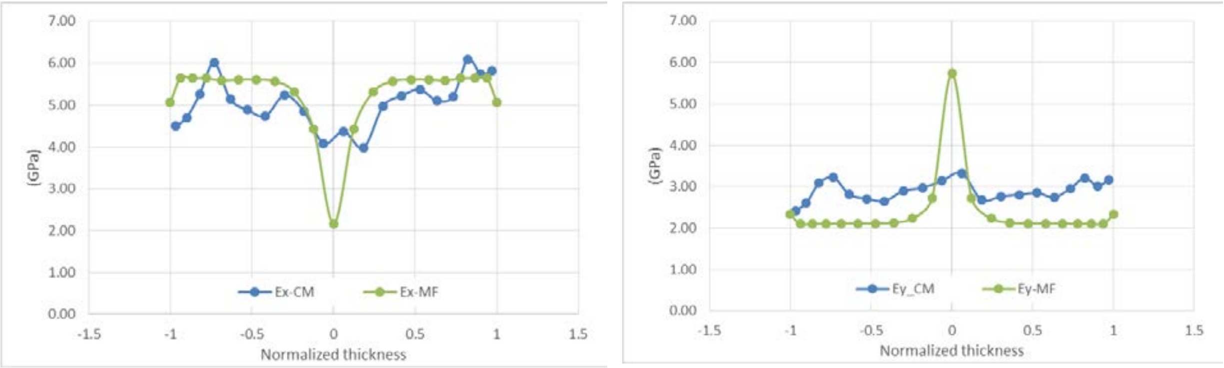


Figure 7 – Values of the apparent moduli obtained by the Cell Method simulations



(a) (b)

Figure 8 – Comparison between values obtained by mean field homogenization (MF) and by the Cell Method (CM)

A good agreement is found in the shell layer in terms of average and also local values of E_x, although the mean field method seems to provide a smoother variation of the elastic moduli, while the Cell Method appears to reflect the local variations inside the portion of the sample used for the analysis. The sharper gradients in the moduli values predicted in the core layer by the mean field homogenization method appear less pronounced in the Cell Method results.

In explaining these findings, first of all it must be considered that the gradients of E_x and E_y in the core layer reach their peak values over a length which is smaller than the characteristic length used for the Cell Method simulations. The Cell Method results necessarily reflect an average over the size of the VOIs used. Additionally, the mean field method results are computed over layers that are shifted with respect to the mid-plane of the VOIs used in the micro-CT based simulations, so that local effects can be appreciated with one method and less evident in the other for purely geometrical reasons.

Moreover, it must be pointed out that the mean field homogenization method results are obtained on the basis of simulated and averaged orientation tensors, which reflect a somehow ideal behaviour of fibres during the injection molding process, as highlighted, for example, by the perfect symmetry of the elastic moduli values around the sample axis. On the other hand, the Cell Method takes into account the actual and local fibre spatial arrangement, which is determined by the specific process parameters encountered during the real sample manufacturing and which obviously differs from the nominal situation considered in the simulations.

Finally, it must be considered that the dimensionality of the problem is different in the two approaches: the mean field homogenization is based on an essentially 2D model, whereas the results obtained starting form micro-CT reconstructions are intrinsically three dimensional.

A further refinement of the comparison could be obtained by 3D modelling of fibre orientation, by increasing the spatial resolution of results of the Cell Method using smaller VOIs and by extending the analysis by the Cell Method on a larger portion of the sample, to better capture the scatter of the actual local moduli.

9 CONCLUSIONS

Two methods of analysis of the local stiffness of an injection-moulded sample of glass reinforced polyamide were presented. The first method was based on simulated fibre orientation data and applied a mean filed homogenization scheme. The second method derived local stiffness values on the basis of samples reconstructed by micro-CT by the Cell Method. Based on the results presented, the following conclusions can be drawn:

- The order of magnitude of elastic moduli obtained by the two different methods are comparable.
- Both methods capture the presence of a core layer with lower values of E_x and higher values of E_y obtained by mean filed homogenization

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