

Nat And His Friends: A Learning Support In Mathematics Education

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Abstract

We describe a path which came into being after it was noted that Italian primary school children (aged 6-11 years) have difficulty in understanding the necessity for using different numerical sets depending on the problem they are dealing with. We decided to use a mostly visual support which would be simple, engaging and able to evolve over time. Thus, the poster “Nat and his friends” was born, where the characters appear, for the first time, in the moment in which a problem requires it in the five years of Italian primary school. Besides “Nat” (the set of natural numbers), “Ra” (the set of rational numbers), “Rel” (the set of relative numbers), other characters were necessary, that is, “Al”, “Geo”, “Misu”, “Alea”. “Geo” and “Alea” represent two areas, respectively that of geometry and that of mathematics of uncertainty; “Al” is the “depository” of procedures, while “Misu” steps in when there are problems of measurement. Lastly, there is “Logic the Traveller”. This character, given that she is a “traveller”, is not on the poster; she is mostly used when dealing with problems of classification.

1. Introduction

At Italian primary school level (5 years: from first to fifth year, with children aged 6-11 years), mathematics aims to provide tools to investigate and explain many of the phenomena of the world which surround us, favouring a rational approach to the problems that reality sets (see MIUR, 2018; see also Canetta, Manara & Marchi, 1986, and Angeli, D’Amore, Di Nunzio & Fascinelli, 2011). This happens for the natural number, in both cardinal and ordinal aspects: The first aspect stems from the spontaneous and “natural” operation of counting and translates, first via concepts and then symbols, the relationship of equipotency between finite sets of concrete objects; the second aspect may be reached through elementary operations of comparison between sets of different cardinality or using “natural” intuition in linear ordering or alignment. The first moment of construction of the natural number is therefore that of carrying out a comparison between finite sets which leads to the construction of an abstract concept.

The aim of primary school is to widen the learners’ fields of interest and, as a consequence, there are more and more things to learn about the world outside: Thus, from handling concrete sets which have a finite number of elements, requiring the use of natural numbers, we arrive at the presentation of problems with solutions in other numerical sets. Therefore, the foundations are created for the necessity for new cognitive and expressive “worlds”, which will be the subject for study after primary school.

Obviously, difficulty in comprehension and, correspondingly, didactic difficulty increase. It should be said that the difficulties encountered by the learners, in their experiences with numbers, are the same as those of human beings through the centuries, in the historical evolution of the concept of the number, as the various numerical systems were introduced in response to differing questions regarding means to control certain situations. If the subject of natural numbers seems to be managed (quite) well, difficulties arise with handling and understanding the significance of rational numbers (and their representations) and continue to increase when, at secondary school level, real numbers, not to mention complex numbers, are dealt with.

2. The Research Project

The project illustrated here was born in the context of our research group (“Nucleo di Ricerca in Didattica della matematica”, Department of Mathematics and Earth Science, University of Trieste, Italy), comprising a mixed group of teachers from nursery, primary, middle and high school level (who teach children and young people between 3 and 19 years old), led by university teachers involved in many activities to promote and improve mathematics teaching at the various school levels. In the course of the periodical group meetings, difficulties, gaps, and misconceptions are highlighted and together we try to find strategies and approaches of “prevention” and “solution”. Our activity is also “outward-looking” whereby we organize events to promote mathematics for children and young people and initial training or refresher courses for teachers; among these activities we should

remember “La matematica dei ragazzi”, which has been held every two years since 1996 this year being the twelfth time (see Leder, Scheriani & Zuccheri, 2002, and Zuccheri & Zudini, 2014).

The idea for the present research came from the observation over a period of years by one of the authors of this paper (a primary school teacher since 1996 in Trieste) that primary school children experience great difficulty in recognizing that an entity belongs to a certain numerical set (in particular, distinguishing between natural and rational numbers) or other mathematical fields (for example, geometry, measurement, ...).

In the specific context of our project, we believe that the research experience presented here has importance both for preventing the formation of misconceptions and for limiting the difficulties encountered by the children. According to the project, the experience would be expected to start right in the first year of primary school and to run throughout the whole period of primary school, ending in the fifth year.

With regard to the present state of the research, begun in the first year of the Primary School “G. Foschiatti” of Trieste (15 pupils) in 2015, we are now at the end of the third year (June 2018).

3. “Nat And His Friends”

3.1 The Birth Of The Poster

Previous experience of one of the authors of this paper (over a period of more than 15 years) demonstrates that the use, right from the first year of primary school, of a poster which shows a picture of nine children, the “Children of Mother Grammar”, allows all pupils, even the ones who have some difficulty, to recognize the nine parts of speech of Italian grammar.

The drawing (see Figure 1) is based on the fact that every child-character, who represents a part of speech (noun, article, verb, adjective, adverb, preposition, pronoun, exclamation, and conjunction), has his/her own colour and “is doing” something related to his/her grammatical function. For example, “Article” (drawn slightly smaller than “Noun”) holds hands with “Noun”, who, in turn, holds hands with “Adjective”: In Italian grammar, these agree, or “get on with each other”.



Figure 1. The poster “Mother Grammar” (Figure and photo by D. Leder)

If the methodology works for learning Italian grammar, why not then use the same sort of visual aid (poster) for mathematics and try creating a poster with characters relating to it?

3.2 Characters Of The Poster

In the first year of the Primary School “G. Foschiatti of Trieste, the following characters were born in 2015 (listed in order of appearance), each of whom are depicted as a boy/girl, identified with a colour and the characteristics accessible to the pupils at the present state of the research (that is, at the end of the third year, June 2018):

1. “Nat” (coloured blue), representing natural numbers; in line with his prerogative, he does not write anything and counts pointing (cardinal and ordinal aspects of the natural number) (see Figure 2).



Figure 2. “Nat” (Figure and photo by D. Leder)

2. “Al” (coloured white, perhaps because he is acknowledged as “cutting across” various fields), representing the writing of numbers; Nat’s precise and fussy brother, in line with his prerogative, he puts everything in order and records by writing down (he compares, orders, writes, and enables the consultation of the algorithms of operations) (see Figure 3). The children, for the time being, have understood more the operative aspect rather than that of the writing.



Figure 3. “Al” (Figure and photo by D. Leder)

3. “Ra” (coloured red), representing rational numbers; in line with her prerogative, she shares out a bar of chocolate (see Figure 4). “Ra” has appeared a little earlier than usual in the school syllabus but in this way the learners arrive, with greater familiarity, at decimal numbers and fractions.



Figure 4. “Ra” (Figure and photo by D. Leder)

4. “Misu” (coloured green), representing measurement. In line with his prerogative, he compares two equal baskets which contain quantities of fruit to decide if they are equal or, if not, which one is lesser or greater

than the other (see Figure 5); when he has decided, he might need “Nat” and then, if “Nat” cannot help him, he calls “Ra”.



Figure 5. “Misu” (Figure and photo by D. Leder)

5. “Geo” (coloured brown), representing geometry; in line with her prerogative, she traces paths and identifies the shapes of the fruit and leaves and dreams about an ice cream and a witch’s hat (in order to have solid figures, such as cones and cylinders, represented also in other contexts) (see Figure 6). “Nat” and “Ra” are very friendly and close with her, even if she often has to ask for help also from “Al” (for example, for calculating a perimeter).



Figure 6. “Geo” (Figure and photo by D. Leder)

All these characters are drawn in a wood, on the poster of “Nat and his friends” (see Figure 7)



Figure 7. The poster “Nat and his friends” (Figure and photo by D. Leder)

Starting from the first year, whenever there is a need or a necessity, these child-characters are “called”. For example, if I have a cake and I want to divide it up into 6 equal parts, who shall I call? Surely “Nat”! And also “Al” and maybe even “Ra”.

Further characters have been added:

6. In the third year (this year), Mrs. “Logic” has appeared; she is a traveller. She is the twin sister of the “Logic” used in Italian and is multi-coloured, like a rainbow (as she is recognized as “cutting across” various fields and therefore not part of the poster) (see Figure 8). The choice of her name is due to her interventions with “and”, “or”, “not”, while remaining clear that she is to be distinguished from her twin sister who intervenes in Italian.



Figure 8. “Logic the Traveller” (Figure and photo by D. Leder)

7. At the end of the third year, also “Alea” (“die” in Latin) has appeared as an objective necessity for the problematic situation to be solved, representing probability and statistics; in fact, she plays with dice. “Alea” does not have a colour, but her clothes have the same markings as a die (see Figure 9). She is not yet on the poster, but she will appear at the beginning of the fourth year (September 2018).



Figure 9. “Alea” (Figure and photo by D. Leder)

8. Instead, “Rel” has shyly appeared in the third year, representing relative numbers; he wears red and black clothes and has in his hand a notebook and a pen to record losses and gains (see Figure 10). He has been informally introduced because the children became curious about “numbers with minus in front”, mentioned by adults during some very cold days, therefore linked to measuring temperature in degrees Celsius. This provided an opportunity to explain that the numbers on the thermometer and the numbers represented by “Rel” are two different things. “Rel” has shyly appeared because it was not yet the right moment to introduce him mathematically and it has not yet been explained why he is dressed as he is, nor why he is holding that notebook...that will be for future discussion... for the moment we have only ruled out that he is a Milan football fan (whose players and fans are identified by the colours red and black). We can disclose that the colours red and black relate to the method of accounting (red is used for outgoings and black for incomings).



Figure 10. “Rel” (Figure and photo by D. Leder)

It should be made clear that the names of the characters make way for the “real” names which the learners will encounter (formally) later on, at middle and high school (for example, “Nat” leads to “natural numbers”, “Ra” to “rational numbers”).

4. First Assessment Of The Poster As A Teaching Support

It is still early to make an objective assessment inasmuch as we are at the end of the third year, only a little over halfway through the experimental period. The children have had time to get to know the characters better and better as they themselves consolidated their knowledge in the field of mathematics.

The fact that they could discuss together in class has allowed for possible doubts to come out and then new stimuli to introduce or develop the subject (see Pontecorvo, Ajello & Zucchermaglio, 1999). To give an example, we have discussed who we should “call” when meeting an apparent fraction: “Ra” or “Nat”? Or again, for zero...is “Nat” alright or “Ra”, or both? Or, furthermore, can “Al” stand alone?

Certainly, the children have been very taken by these characters, so much so that at the end of the third year (June 2018), of their own volition, they performed a short show for the parents, playing the parts of the characters and showing how they “function”, i.e. setting problematic situations which the parents had to solve “calling” the correct (appropriate) characters. It was a useful experience where everybody, even the pupils who have difficulty, took active part.

4.1 Recognition Of The Character Required By The Situation

At the end of the third year, first specific tests were held to ascertain if the children were able to link correctly the character/s with the context provided, showing awareness of the problematic situation that they were dealing with. There were two types of test: In the first type (“PROVA 1”) the children were presented with numbers, operations, and measurements which they had to circle, using the colours of the characters and thus showing that they recognized by colour the character/s which needed to be “called” (graphic test); in the second type of test (“PROVA 2”), which dealt with problematic situations (in the form of a text), the pupils were required to choose the characters (more than one) to “call” putting an “X” on the characters which were written beside.

If, during the curricular lessons, there was always someone in the group who verbally found the right correspondence, in the written tests, in the situations where the pupils just have to call a single character (it does not matter whether it is “Nat” or “Ra”, given that “Al” cannot stand alone, and “Misu” and “Geo” rarely appear alone), 98% of the children identify “Nat”, while 87% identify “Ra”.

Clearly, a problematic situation which requires the consideration of more than one aspect at the same time (i.e. calling more than one character) can be more difficult for 8-year-old children (as is seen from the results obtained regarding, for example, “Nat” and “Ra”: only 84% of the children recognize “Nat” and 53% “Ra”). The ability to take into account simultaneously various aspects is, after all, a skill which should be fostered in relation to the pupils’ cognitive development; we can work towards this to help acquire a greater awareness of the difficulties of the problematic situations and to develop a positive attitude towards it (without fear).

4.2 Future Developments In Research

We have seen how the characters and their personalities have evolved since the beginning of the research, acquiring over time features and prerogatives as the children, bit by bit, encountered increasingly complex problematic situations.

This will presumably also happen as the research continues (starting up again in September 2018, with the beginning of the fourth year, and continuing until the end of the fifth year), where we might reasonably suppose that the characters will be better defined and discussed in their peculiarities in relation to new learning situations. It will be useful to evaluate step by step, through suitably-prepared tests, the competencies which the pupils develop in recognizing that an entity belongs to a certain numerical set or to other mathematical fields. After modifying their text accordingly, the same tests may be given, with the cooperation of teachers who are part of our research group, also to learners in schools of higher level, in order to check the correspondence with our results.

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