Philosophy of Quantum Mechanics: Dynamical Collapse Theories FREE

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https://doi.org/10.1093/acrefore/9780190871994.013.77
Published online: 23 February 2021

Summary

Quantum Mechanics is one of the most successful theories of nature. It accounts for all known properties of matter and light, and it does so with an unprecedented level of accuracy. On top of this, it generated many new technologies that now are part of daily life. In many ways, it can be said that we live in a quantum world. Yet, quantum theory is subject to an intense debate about its meaning as a theory of nature, which started from the very beginning and has never ended. The essence was captured by Schrödinger with the cat paradox: why do cats behave classically instead of being quantum like the one imagined by Schrödinger? Answering this question digs deep into the foundation of quantum mechanics.

A possible answer is Dynamical Collapse Theories. The fundamental assumption is that the Schrödinger equation, which is supposed to govern all quantum phenomena (at the non-relativistic level) is only approximately correct. It is an approximation of a nonlinear and stochastic dynamics, according to which the wave functions of microscopic objects can be in a superposition of different states because the nonlinear effects are negligible, while those of macroscopic objects are always very well localized in space because the nonlinear effects dominate for increasingly massive systems. Then, microscopic systems behave quantum mechanically, while macroscopic ones such as Schrödinger's cat behave classically simply because the (newly postulated) laws of nature say so.

By changing the dynamics, collapse theories make predictions that are different from quantum-mechanical predictions. Then it becomes interesting to test the various collapse models that have been proposed. Experimental effort is increasing worldwide, so far limiting values of the theory's parameters quantifying the collapse, since no collapse signal was detected, but possibly in the future finding such a signal and opening up a window beyond quantum theory.

Keywords: dynamical collapse theories, wave function collapse, foundations of quantum mechanics, superposition principle, open quantum systems, decoherence, nonlocality, philosophy of quantum mechanics

1. Prelude

The debate about the meaning and validity of quantum mechanics is as old as the theory itself. The reason is that the ontology of the theory is not clear at all. Newtonian mechanics is about (ideally, point-like) particles moving in space subject to forces. Electromagnetism is about charged particles, the electric and the magnetic field, evolving in space and interacting with each other. Then what is quantum theory about? About particles? Not really, or at least not in the usual sense, because of the superposition principle. Is it about waves? Not really, or at least not in the usual sense, because upon measurements, waves disappear. Is it about the wave function? One has to be careful, because the wave function does not live in real space as the electric and magnetic fields do; it lives in configuration space. Then again, what is quantum theory about?

The official answer, which goes under the name of the Copenhagen interpretation, is that the theory is about outcomes of measurements. It does not provide an objective description of the microscopic world; it only tells what happens if one performs a measurement. So far so good; many phenomenological models are like that. Then the question is: does it not provide an objective description of the microscopic world because it is incapable of so doing, being insufficiently rich in detail, or because it is not possible, even in principle, to formulate one? The first option amounts to saying that quantum theory is incomplete, which is what Einstein always claimed, but that means that a more fundamental theory exists. This is what the defenders of quantum theory do not like: they say, quantum theory is the best theory one can have; there is nothing else beyond it. This might be true, but it opens a serious problem: if one cannot have an objective picture of the microscopic world of atoms and molecules, how is one supposed to have an objective picture of the macroscopic world of tables and chairs, which are made of atoms and molecules? Are they not objective?

This conundrum is the essence of the measurement problem in quantum theory, and a lot of work has been devoted to finding a solution to it. This produced Bohmian Mechanics, where quantum systems are collections of particles moving nonlocally in three-dimensional space, guided by the wave function. It produced the Many Worlds interpretation, according to which the wave function does describe reality, but in not in the ordinary three-dimensional sense. It produced also Dynamical Collapse Theories, the subject of the present review.

Collapse theories accept that the Schrödinger equation is not exactly correct. It has to be supplemented, at least phenomenologically, with nonlinear stochastic terms, which tend to localize the wave function in space. The larger the system, the stronger the effect. The picture that emerges is that of a microscopic world where “particles” tend to dissolve in space like ice-cream under the sun; this is the effect of the Schrödinger dynamics. But when particles interact with each other, the collapse terms make them stiffer and stiffer, to the point that when a macroscopic number of them glues together to form a table or a chair, they become rigid. According to collapse theories, there is an objective reality at all scales: their distinctive feature is to show in a mathematically precise way how the deterministic world of classical rigid bodies emerges from the microscopic world of random and wavy systems.

2. The Measurement Problem in Quantum Mechanics

Einstein famously wrote (Einstein, 1971): “Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the Old One. I am at any rate convinced that *He* does not play dice.” More recently, the Nobel laureate Leggett said (Leggett, 2010): “I am inclined to put my money on the idea that if you push quantum mechanics hard enough it will break down and something else will take over—something we can’t envisage at the moment.” The Nobel laureate Weinberg similarity wrote (Weinberg, 2017): “I’m not as sure as I once was about the future of quantum mechanics.” Why such eminent skepticism about quantum theory?

Weinberg himself explains why (Weinberg, 2012, p. 062116):

> The Copenhagen interpretation assumes a mysterious division between the microscopic world governed by quantum mechanics and a macroscopic world of apparatus and observers that obeys classical physics. During measurement the state vector of the microscopic system collapses in a probabilistic way to one of a number of classical states, in a way that is unexplained, and cannot be described by the time-dependent Schrödinger equation.

Bell put it more ironically (Bell, 1993b, p. 117):

> It would seem that the theory is exclusively concerned about ‘results of measurements’, and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of ‘measurer’? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a Ph.D.?

Feynman ridicules the situation even more (Feynman, Morinigo, & Wagner, 1995, p. 14):

> Does this mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the thought that without observer there is no reality? Which observer? Any observer? Is a fly an observer? Is a star an observer? Was there no reality before 10^9 B.C. before life began? Or are you the observer? Then there is no reality to the world after you are dead? I know a number of otherwise respectable physicists who have bought life insurance. By what philosophy will the universe without man be understood?

What, then, is the trouble with quantum mechanics? The theory is based on the following set of rules:

1. The state of a quantum system is described by a wave function.
2. The wave function evolves according to the Schrödinger equation. Its fundamental property is linearity: the linear combination of two solutions is a new solution. This is the quantum superposition principle.

3. Physical quantities are represented by specific self-adjoint operators. Their (real) eigenvalues represent the possible outcomes of measurements of those quantities.
4. In a measurement, outcomes are random and distributed according to the Born rule $|\langle a_n|\psi\rangle|^2$, where $|a_n\rangle$ is the eigenstate corresponding to the eigenvalues that has been observed when the system is in the state$|\psi\rangle$. After the measurement, the wave function collapses into the eigenstate corresponding to the eigenvalues that have been obtained. This is the von Neumann projection postulate.

It is clear that there is something wrong with this set of rules. A quantum system evolves according to the Schrödinger equation, possibly being superimposed, when left alone, while it randomly collapses when measured. This is perfectly understandable from the phenomenological point of view: the two situations are different—in the first case the system is isolated, in the second case it interacts with the measuring device. The problem is that quantum theory is not supposed to be a phenomenological theory, but a fundamental description of nature. Therefore, if those mentioned are the rules of a fundamental theory, the distinction between the two situations is supposed to be a property of nature. But then it is not clear why there should be such a distinction: measuring devices are made of atoms, which are quantum. As such, it should be possible to describe them quantum mechanically. But if this is done, then the Schrödinger equation will tell that there is no collapse: because of linearity, microscopic quantum superpositions will propagate to the macroscopic world, as first highlighted by Schrödinger himself with the cat paradox (Schrödinger, 1935). There will be no definite outcomes, contrary to ordinary experience. This is the measurement problem of quantum mechanics, which is as old as the theory itself.

Over the last century, several resolutions to the problem have been proposed. The first and best developed one is Bohmian Mechanics (Dürr & Teufel, 2009); perhaps the most popular one is the Many Worlds Interpretation (Wallace, 2012). The list includes the Consistent Histories Approach (Griffiths, 2003) and the Modal Interpretation (Dieks & Vermaas, 1998).

In this article another proposal is outlined: Collapse Models. They were formulated in the 1980s and further developed in the 1990s with the aim of combining the Schrödinger equation and the random collapse of the wave function into a single dynamical framework. In such a framework, microscopic systems evolve quantum mechanically, with very tiny, almost unobservable deviations in the form of spontaneous collapses. For larger systems made of an increasing number of particles, these deviations become stronger and stronger, to the point that they dominate the dynamical evolution of macroscopic objects, whose wave function always remains well localized in space.

Collapse theories provide a unified description of both quantum and classical phenomena, avoiding the emergence of macroscopic superpositions—the root of the quantum measurement problem—and explaining in mathematically and physically clear terms the transition from one domain to the other.

3. The Ghirardi-Rimini-Weber (GRW) Model

The GRW model (Ghirardi, Rimini, & Weber, 1986) is the first consistent model of wave-function collapse, formulated in 1986 by Ghirardi, Rimini, and Weber. The model is based on the following two assumptions:

1. As in quantum theory, the state of a quantum system is described by a wave function.
2. Each constituent of a physical system is subject to spontaneous collapses, in the sense that the system's wave function undergoes the sudden change:

$$\psi(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N, t) \rightarrow \frac{L^i_{\mathbf{x}}\psi(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N, t)}{\|L^i_{\mathbf{x}}\psi(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N), t\|}, \qquad L^i_{\mathbf{x}} = \frac{1}{(\pi r_C)^{3/4}} e^{-(\mathbf{x}_i - \mathbf{x})^2/2r_C^2} \tag{1}$$

at the time when the collapse occurs. In these formulas, $\mathbf{x}$ is a point in space around which the collapse occurs, and r_C is a free parameter of the model, which defines the resolution of the collapse. The collapses are random in time, with frequency λ; they are also random in space, the probability distribution being $p(\mathbf{x}) = \| L^i_{\mathbf{x}}\psi(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N, t)\|^2$, which is normalized to one. The collapses for different constituents are independent. In between collapses, the wave function evolves according to the Schrödinger equation.

Figure 1 illustrates the effect of a collapse on a single particle's wave function (in one dimension). As can be seen, before the collapse the wave function is spread out in space; after the collapse, it is localized with a resolution given by r_C.

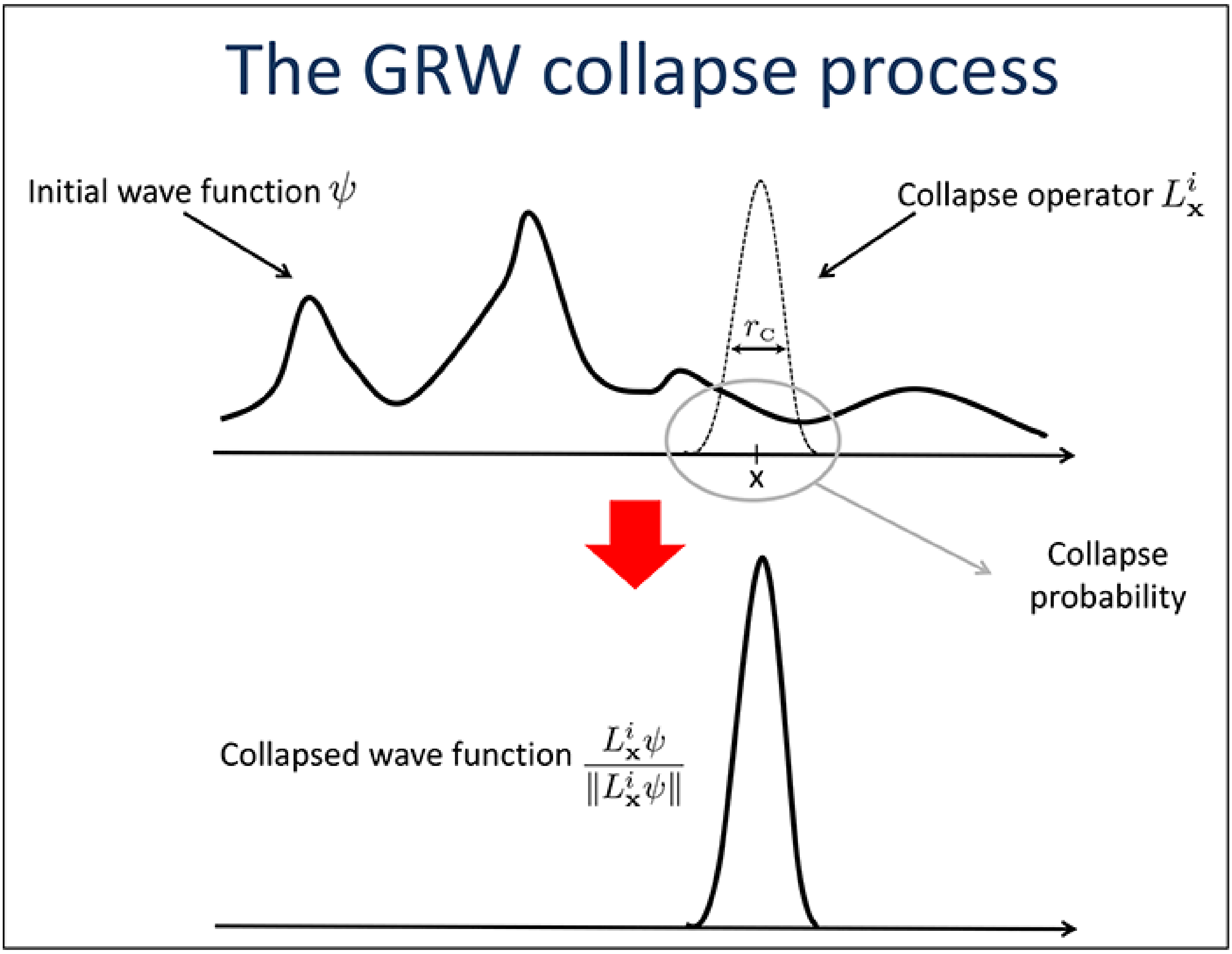

Figure 1. An initially delocalized wave function becomes well localized in space after a collapse. The collapse is mathematically accounted for by multiplying the wave function with a Gaussian function of width , and by re-normalizing the output function. The collapse is more likely to occur where the wave function is larger, similarly to the Born rule.

- At this stage, only non-relativistic systems are considered, and “constituent” refers to the electrons, protons, and neutrons in each atom.
- Regarding the meaning of the wave function in collapse models, which will be discussed later, it is sufficient to consider it (more precisely, its square modulus), as giving the mass density of each constituent.

As can be seen, in collapse models, as in Bohmian Mechanics or any classical theory, only the states (the ontology) and the dynamics are specified, and all the rest follows. In particular, measurements and self-adjoint operators representing observables do not play any fundamental role in GRW. They emerge from the dynamics when the behavior of those specific physical situations are considered, usually refered to as measurement processes. The full process can be analyzed within the theory, which is not a trivial task given the complexity of the situation, since a macroscopic number of degrees of freedom is involved. The result is that, for all practical purposes, the description can be simplified by ignoring the dynamics of the measuring device, by considering the system’s wave function alone, and by assuming that it is subject to the von Neumann collapse process distributed with the Born rule, with physical quantities described in simpler terms by suitable operators. This corresponds to rules 3 and 4 previously listed.

The important conceptual point is that, while in standard quantum mechanics rules 3 and 4 have the status of axioms, and the reason why they exist and when they apply is not really clear, within the GRW model (and the other collapse models as well) they are no longer axioms; they emerge as practical rules for describing effectively measurement situations, which in principle can be described entirely by the newly proposed collapse dynamics. Measurements do not play any special role at the fundamental level; they simply correspond to specific physical situations, which can be used to infer properties about the state of physical systems.

The GRW model is defined in terms of two parameters: The collapse rate λ and the collapse resolution r_{C}. As already anticipated,r_{C} tells how well the wave function is localized after the collapse;λ instead tells how frequent the collapses are, for each particle. At this stage, these parameters have to be considered as two new universal constants of nature. GRW (Ghirardi et al., 1986) proposed the following values:$r_{\mathrm{C}} = 10^{-7}\ \mathrm{m}$, and $\lambda = 10^{-16}\ \mathrm{S}^{-1}$. This choice for r_{C} means that microscopic states, which typically stretch out for distances much smaller than $10^{-7}\ \mathrm{m}$, are basically left unaltered by the collapse, while macroscopic superpositions of the Schrödinger-cat type are suppressed. The choice for λ instead implies that each single particle experiences a collapse once every 0.3 billion years on average: almost never. This means that, at the microscopic level, the new collapse terms can be safely ignored, and the usual quantum properties are recovered, modulo very tiny deviations.

A key feature of the GRW model is the *amplification mechanism*. Instead of a single system, consider a N-particle system like a macroscopic rigid body, whose relevant degree of freedom —for the present discussion—is the position of its center of mass. It can be easily shown that each time a constituent of the rigid body suffers a localization, so does the center of mass. Then a macroscopic superposition of the form $|\text{here}\rangle \mid + |\text{there}\rangle$ will collapse in $1/N\lambda$ seconds, which, for the numerical choice of λ done by GRW and considering that in a macroscopic rigid body there are about $N = 10^{24}$ constituents, amounts to 10^{-8} S. To put it in Bell's words (Bell, 1993c), in GRW "The cat is not both dead and alive for more than a split second." Actually, this is not entirely true. According to GRW there is not even the time to create a macroscopic superposition: the collapses will keep the wave function localized during the entire process.

Adler (Adler, 2007) suggested a value for the collapse rate λ is around 9 orders of magnitude stronger than that earlier suggested by GRW. While the argument behind GRW's choice for λ was to place the quantum-to-classical transition between the mesoscopic and the macroscopic world, meaning that micro- and mesoscopic systems behave quantum mechanically while macroscopic objects behave classically, Adler's choice is dictated by the requirement that any measurement process, also those involving a few atoms or molecules like latent-image formation in photography, come with an effective collapse of the wave function. This choice was reinforced by Adler (Adler, Bassi, & Ferialdi, 2020), who showed that in a "minimal" measurement system recorded on flash memory, reduction in the measurement time cannot be explained by the GRW noise coupling but can be explained by the one suggested by the latent-image formation analysis. Adler's proposal, that reduction should be reached in nanoscale systems, has the great merit to have put experimental testing of collapse models within reach, and has motivated a large body of recent experimental work.

By altering the Schrödinger equation, the GRW model makes predictions that differ from standard quantum-mechanical predictions, at least in principle. Since experimental results pass through repeated measurements, the mathematical tool suitable to describe them is the density matrix ρ_t. According to the GRW dynamics (rule 2), the density matrix follows the Lindblad dynamics (Ghirardi et al., 1986):

$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H, \rho_t] - \frac{\lambda}{2}\sum_{i=1}^{N}\int d^3x \, [L_{\mathrm{x}}^i, [L_{\mathrm{x}}^i, \rho_t]], \tag{2}$$

Where H is the standard quantum Hamiltonian of the system. It is important to stress that here the density matrix represents a proper mixture of states that evolve (and collapse) following different realizations of the noise, and not the improper mixture that is obtained by tracing over the degrees of freedom of a hypothetical quantum environment.

There are two interesting predictions of Eq. (2) that are worthwhile highlighting. The first is clear: the off-diagonal elements of the density matrix are exponentially suppressed, meaning that spatial quantum coherence in superpositions is progressively lost. This is the obvious consequence of the fact that the GRW terms collapse the wave function in space. The second prediction is a diffusion process: the mean position diffuses in time more than what is predicted by the Schrödinger equation alone. This is a consequence of the stochastic character of the collapses: they occur randomly in space and each time they occur, they not only collapse the wave function, but also randomly shift it slightly in space.

These two properties lead to two different strategies for testing the GRW model, and collapse models in general: interferometric experiments, aimed at testing the persistence or disappearance of quantum coherence in space, and non-interferometric experiments, which aim at detecting the Brownian motion fluctuations induced by the collapse noise.

4. Continuous Spontaneous Localizations

The GRW model can be modified so that the discrete jumps become a continuous diffusion process: this is the *Continuous Spontaneous Localization* (CSL) model (Ghirardi, Pearle, & Rimini, 1990b; Pearle, 1989), the output of the combined efforts of the Ghirardi group (Ghirardi et al., 1986), and the previous works of Pearle (Pearle, 1976, 1979). This new formulation has two advantages, one aesthetic and one computational. The aesthetic one is that a continuous dynamic might be preferable over a partly continuous (Schrödinger evolution) and partly discrete (random jumps) one, given that all theories so far are expressed in terms of continuous dynamical laws. The practical one is that working with a continuous dynamic is simpler than working with a discrete one.

What follows is a simplified version of a continuous collapse model, first proposed by Diosi (Diosi, 1989, 1990), under the name of *Quantum Mechanics with Universal Position Localization* (QMUPL). The one-particle dynamics is given by the following stochastic differential equation of the Itô type (Amold, 1974):

$$d\psi(x,t) = \left[-\frac{i}{\hbar}H dt + \sqrt{\lambda}(x - \langle x\rangle_t)dW_t - \frac{\lambda}{2}(x - \langle x\rangle_t)^2 dt\right]\psi(x,t), \tag{3}$$

expressed for simplicity in one spatial dimension. H is the standard quantum Hamiltonian, $\langle x\rangle_t = \langle\psi(t)|x|\psi(t)\rangle$ the mean position, and W_t a standard Wiener process (essentially, a noise without drift and with no memory; Arnold, 1974). The constant λ (which here is also dimensionally different from the GRW collapse rate) sets the strength of the collapse process. The generalization to three dimension or to a many-particle system is rather straightforward: terms similar to the second and third on the right-hand side are added, one for each direction in space and/or for each particle.

The properties of the new terms, which are worthwhile highlighting, are:

- Nonlinearity: this is necessary in order to suppress quantum superpositions;
- Stochasticity: the collapse occurs randomly (in accordance with the Born rule); and
- Function of the space coordinates: the collapse occurs in such a way as to localize wave functions in space.

It has been shown (Bassi, Dürr, & Kolb, 2010) that for a free particle ($H = p^2/2m$, where p is the momentum operator and m the mass of the particle), every initial state asymptotically converges to a Gaussian state with a precise spread both in position as well as in momentum, whose mean keeps diffusing in space. It is interesting to note that the two spreads reach almost the minimum allowed by Heisenberg's uncertainty principle (Bassi, 2005). In other words, the collapse in position causes indirectly also a collapse in momentum, compatible with the uncertainty principle. The reason is easy to understand. Take an initially well-localized state with respect to the available or desired accuracy, in the superposition of two different momenta, for example the superposition of two Gaussian states, centered initially around the origin, but with different momenta p_1 and p_2:

$$\psi(x,0) \sim e^{-\alpha x^2}[e^{ip_1 x/\hbar} + e^{ip_2 x/\hbar}]. \tag{4}$$

After some time, the two states will start separating, because of the different momenta: the momentum superposition creates a spatial superposition, which is then suppressed by the collapse process. The final state becomes well-localized both in position as well as in momentum.

The opposite, however, is not true: a collapse in momentum does not cause a collapse in position, not even indirectly. Take a plane wave, which is a momentum eigenstate. This state will remain stable under momentum-collapse dynamics, thus remaining fully delocalized over space. The reason for this difference is that the Hamiltonian term breaks the symmetry between position and momentum: there is always a kinetic term $p^2/2m$ (the one that makes the two Gaussians depart from each other, in the previous example), while the potential term changes depending on the specific situation. This is why in all relevant collapse models, the collapse occurs in position, not in momentum, or energy (although models of this type were formulated in the past [Adler, Brody, Brun, & Hughston, 2001]).

Another important feature of the collapse is nonlocality. Take the superposition of two states, localized far away from each other. A local theory of collapse would require that the wave function from one side starts "flowing" to the other side, or the two meet somewhere, at some finite speed, possibly less than the speed of light. Technically, the (square of the) wave function should satisfy a continuity equation, which is satisfied by the Schrödinger equation. This is not true for Eq. (3). What happens there is that on a faster timescale, one of the two states starts decreasing and the other one starts increasing *simultaneously* (in order for the squared norm to remain constant and equal to unity), however far apart they are; then on a longer timescale, the two terms converge, and merge into a Gaussian state, which is the asymptotic destiny of any wave function.

This kind of nonlocality, although unpleasant from the relativistic point of view, is necessary in order to account for the violation of Bell's inequalities (Bell, 1993a). Because of quantum nonlocality, outcomes of measurements have to be "instantly" (within experimental error) correlated however far apart they are. Only nonlocal collapses can account for this behavior, which has been verified by countless experiments. But this is precisely the obstacle in combining collapse models and relativity.

As explained before, for a many-particle system, the QMUPL dynamics includes collapse terms for each particle. Assuming that the Hamiltonian H can be written as the sum of a center-of-mass term and a term relative to the internal degrees of freedom, then the overall dynamics also factorizes into a center-of-mass term and an internal dynamics. The center-of-mass dynamics is given by Eq. (3) with λ replaced by $N\lambda$ (assuming for simplicity that the many-body system is made of N constituents, all with the same mass). This is again the manifestation of the amplification mechanism, already discussed in connection with the GRW model: the collapses on the single constituents add coherently and amplify N times the collapse of the center of mass. Taking an isolated system, so that the center-of-mass Hamiltonian is simply that of a free particle, the state toward which any wave function rapidly converges is again a Gaussian state, which now is very well localized in position (spread $< 10^{-13}$ m, for reasonable choices of the constant λ [Bassi, 2005]), so well localized that it can be considered pointlike for all practical purposes. This is how the Newtonian world of rigid billiard balls emerges from a microscopic wave theory. It can also be shown that the diffusion becomes smaller and smaller for larger objects: at the macroscopic level it can hardly be detected, thus explaining macroscopic determinism emerging from a stochastic theory.

The collapse dynamics (3) was introduced in order to provide a consistent unified dynamics for microscopic quantum systems and macroscopic classical ones. It was later understood that if (a) state vector normalization and (b) no faster than light signaling are required, that is a linear (typically, Lindblad) equation for the density matrix (Gisin, 1989; Polchinski, 1991), then the form of the stochastic modified Schrödinger equation is uniquely specified and of the form given in (3). Moreover, this unique form is precisely the one for which the Born rule can be proved (Adler, 2004; Adler & Bassi, 2009; Caiaffa, Smirne, & Bassi, 2017).

5. Quantum Measurements and Collapse Models

It is informative to analyze a quantum measurement process within collapse models a bit more in detail. This was done (Bassi & Salvetti, 2007) for the QMUPL model, for an idealized von Neumann measurement scheme.

Take a pointer moving along a scale, in one direction. It is a rigid object, therefore the only relevant degree of freedom is its center-of-mass, which evolves according to Eq. (3) with $\lambda \to N\lambda$, with N the total number of constituents in the pointer. Its initial state is the asymptotic Gaussian state it would have reached, if left alone for a sufficiently long time. As explained before, this is a very well-localized state in position.

The experiment is designed to measure the spin of a spin-1/2 particle, in such a way that if the spin is $|\mathbf{up}\rangle$ along a chosen direction, then the pointer moves to the left, while if the spin is $|\mathbf{down}\rangle$ along the same direction, the pointer moves to the right. The spin particle is microscopic; therefore, the collapse effects on it can be neglected.

The usual question is: what happens when the initial spin state is a superposition of the two states $|\mathbf{up}\rangle$ and $|\mathbf{down}\rangle$? The answer is disappointingly simple: the wave function of the (center-of-mass of the) pointer will move either to the left or to the right, with a probability given by the Born rule. At the end of the measurement process, the spin state will be collapsed to either the state $|\mathbf{up}\rangle$ or the state $|\mathbf{down}\rangle$, depending on the outcome of the measurement, which is exactly what happens in real experiments.

Some comments are in order. First, it was possible to derive these results by solving the collapse dynamics, without introducing extra axioms, as standard quantum theory needs to do. This is another way of saying that collapse models solve the quantum-measurement problem. The second comment is that never does the pointer experience a superposition during the measurement process, not even for a split second, as a naive application of the GRW formalism might suggest; the pointer's wave function remains well localized during the whole process. The third comment is that the collapses act directly only on the pointer of the measuring apparatus, although its wave function is already localized, not on the micro system, which is the one in a superposition state; yet, the combined effect of the collapse plus the measurement interaction among the two systems, which entangles them, is such that the microscopic superposition of the spin states of the micro-system is rapidly reduced during the measurement.

The previous discussion shows that collapse models describe quantum measurements more or less as standard quantum theory does, with the conceptually crucial difference that now everything is contained in the collapse dynamical law. In this sense, collapse models make precise what quantum theory introduces ad hoc.

6. Collapse Models and Experiments

By modifying the Schrödinger equation, collapse models make predictions that differ from standard quantum-mechanical predictions. It is not easy to detect them, because physical systems have to be properly isolated from environmental noise in order to see possible collapse effects. Still, in recent years an increasing number of experiments has been proposed and/or performed, placing stronger and stronger bounds on the collapse parameters. The most relevant results are listed, which all refer to the CSL model.

As already anticipated, experiments can be divided into two class: interferometric and non interferometric. In the first class of experiments, the aim is to create a spatial superposition of the center of mass of progressively larger objects, for progressively larger delocalization distances and longer times. At the end, the two states are recombined and quantum interference, or the lack of it, is detected. Clearly, during the experiment, surrounding noise sources have to be reduced as much as possible.

In is difficult to maximize all three relevant quantities, mass, distance, and time, in one single experiment. For example, experiments with cold atoms (Kovachy et al., 2015) allow the delocalization distance (tens of centimeters) and times (one second) to be maximized; however, the masses involved are relatively small (below 100 nucleons). Experiments creating superpositions of phononic states (Lee et al., 2011) involve much larger masses (about 10^{16} nucleons), at the price of having much smaller delocalization distances (below Angstroms) and coherence times (picoseconds). Interference experiments with macro-molecules (Eibenberger, Gerlich, Arndt, Mayor, & Tuexen, 2013; Fein et al., 2019) are in between: masses are of the order of 25 thousand nucleons, delocalization distances of the order of hundreds of nanometers, and delocalization times of the order of tenths of seconds.

In spite of being very different experiments, the outcome is roughly the same in all three cases (Belli et al., 2016; Toros, Gasbarri, & Bassi, 2017): The bound on lambda is $\lambda < 10^{-6}\ \mathrm{S}^{-1}$, for a large range of values of r_{C}. Recall that the value proposed by GRW is $\lambda = 10^{-16}\ \mathrm{S}^{-1}$ and that of Adler is about $\lambda = 10^{-7}\ \mathrm{S}^{-1}$, meaning that there is a vast gap to fill before collapse models can be refuted. This conclusion has two implications. The first one is that it is technologically very difficult to create and detect larger/more massive quantum superpositions. Several improvements and breakthroughs will be necessary before arriving at a decisive interferometric experiment, possibly performing the experiment in space (Kaltenbaek et al., 2012). The other implication is profound ignorance about the validity of the most fundamental property of quantum theory, that is, the superposition principle. It is often said that quantum theory is the most successful theory in the history of physics, and in some precise sense it is. But as far as the superposition principle is concerned, it is proven to be true for light, for the elementary constituents of nature, for atoms and for relatively light molecules, and nothing else. Does it still hold true for more complex systems like DNA, a virus, a dust-grain . . . Schrödinger's cat? No one knows, and there is enough room for quantum theory to be the approximation of a deeper-level nonlinear theory (somehow like Newtonian gravity is a limiting case of General Relativity), perhaps in the form envisaged by collapse models.

The second class of experiments is non-interferometric (Bahrami, Paternostro, Bassi, & Ulbricht; 2014; Collett & Pearle, 2003; Diosi, 2015; Nimmrichter, Hornberger, & Hammerer, 2014): no superposition is created, because the goal here is to detect the collapse-induced Brownian-motion fluctuations in particles' motion by very accurate position measurements. As such, these experiments are easier to perform, and place stronger bounds on the collapse parameters. For the rest of this section, the bounds for λ will be relative to the value $r_{\mathrm{C}} = 10^{-7}$ m (the bounds varies for different values of r_{C}); the reader may refer to the cited literature for further information.

The first type of non-interferometric experiment is with cold atoms. It is possible to create in a lab a cloud of cold atoms, whose effective temperature is picokelvin, one of the smallest ever reached with such systems. This means that these atoms are almost still in space, while spontaneous collapses would diffuse them, the diffusion being larger the stronger the collapse rate. The experiment did not register any significant diffusion, placing the bound $\lambda < 10^{-7}\ \mathrm{S}^{-1}$ (Bilardello, Donadi, Vinante, & Bassi, 2016; Laloë, Mullin, & Pearle, 2014). It represents already an improvement of one order of magnitude with respect to interferometric experiments.

Another way to test collapse models is via optomechanical setups. They consist of a mechanical system coupled to light, in such a way that, among the many things that can be done both at the classical as well as quantum level, the motion of the mechanical system can be monitored very accurately. The gravitational wave detector LIGO is one such example, and one can easily have an intuition of how sensitive the system must be in order to detect very faint gravitational waves (the mirror spacing induced by a gravitational wave is less than one-thousandth the charge diameter of a proton). In particular, the mirrors must be very stable, but they are made of atoms, which are subject to spontaneous collapses according to collapse models; therefore, the mirror should tremble slightly, but it does not, thus placing a bound almost one order of magnitude better than the previous one from cold atoms (Carlesso, Bassi, Falferi, & Vinante, 2016; Helou, Slagmolen, McClelland, & Chen, 2017).

Cantilevers form another class of systems. They consist of a small mechanical system, for example a little ball, attached to an oscillating bar. The whole setup behaves like a harmonic oscillator, and its motion can be measured precisely. According to standard physics, the system's motion should themalize to the environmental temperature—down to some millikelvin in some cases. Collapse models instead predict a progressive heating induced by the associated Brownian motion. This has not been detected: the results place the bound $\lambda < 10^{-9}\ \mathrm{S}^{-1}$ (Vinante et al., 2016, 2017), which is more than one order of magnitude stronger than that coming from gravitational wave detectors, challenging Adler's proposal for the collapse rate, but not fully excluding it, because there is an intrinsic uncertainty in the proposed value for the collapse rate, which can vary by about two orders of magnitude. Experiments with cantilevers are the first example of dedicated experiments testing collapse models. Future proposals aim at modifying and optimizing the experiment to arrive at even tighter bounds (Carlesso, Paternostro, Ulbricht, Vinante, & Bassi, 2018a; Carlesso, Vinante, & Bassi, 2018b; Komori et al., 2020; Schrinski, Stickler, & Hornberger, 2017).

As a last example of non-interferometric experiments, there are underground searches for rare events, like dark-matter experiments. Their application to collapse models is simple: the diffusion induced by the collapse on charge particles, being a particular form of accelerated motion, makes them radiate; therefore, photons are constantly emitted by matter subject to spontaneous collapses (Adler & Ramazanoglu, 2007; Adler, Bassi, & Donadi, 2013; Bassi & Donadi, 2014; Donadi & Bassi, 2014; Donadi, Deckert, & Bassi, 2014; Fu, 1997). Clearly, the number of emitted photons is very low, mostly in the deep infrared; still, from time to time an energetic photon might happen to be emitted, which would be detected as a signal in underground experiments, located in a very low-noise environment suitably designed to measure very rare events. Such collapse-induced events would add to the omnipresent background noise, increasing it. The measured background remains low, thus limiting the possible number of collapse events, and giving the bound $\lambda < 10^{-11}\ \mathrm{S}^{-1}$, so far the strongest bound on the CSL model (Curceanu, Hiesmayr, & Piscicchia, 2015; Curceanu et al., 2016; Piscicchia et al., 2017). It is worthwhile mentioning that this bound can be evaded if the noise responsible for the collapse is not a white noise, as implicitly assumed in all models previously discussed, but is colored with a high frequency cut off. Colored-noise collapse models have been developed (Adler & Bassi, 2007a, 2007b; Bassi & Ferialdi, 2009a, 2009b), showing that the collapse features are left basically unaltered, while specific physical predictions, like the spectrum of spontaneously emitted radiation, may change significantly.

The most interesting bounds on the CSL model were considered. Most of them were derived in recent years, showing that the interest in testing these models has increased. There are well grounded reasons for that: any test of collapse models is ultimately a test of the quantum superposition principle, and it is of paramount importance to assess whether the building block of the most fundamental physical theory is correct or not.

7. The Meaning of the Wave Function and the Ontology of Collapse Models

There are three possible interpretations of the wave function within collapse models, which can be called ontological, nomological, and phenomenological. These correspond to different prescriptions for the primitive ontology (Dürr, Goldstein, & Zanghi, 1992) of collapse models (Allori, Goldstein, Tumulka, & Zanghi, 2008).

Ontological interpretation. As already argued, the fundamental motivation to consider collapse models in the first place, is to avoid the dynamical formation of macroscopic superpositions, which contradict our empirical evidence. Having achieved this, an attempt at attributing an ontological meaning to the wave function can be made (Albert, 1992, 1996; Lewis, 2005; Nicrosini & Rimini, 2003).

According to this interpretation, particles as such, strictly speaking, do not exist, though this term was used regularly in the previous sections. The constituents of matter are extended in space, possibly over large distances. For example, when passing through a double slit, they do split in two as water would do, passing through both slits at the same time. Better than water, they behave like a jelly, which can be more or less fluid depending on the context (the temperature, for a jelly). In fact, when left alone, the particle-jelly spreads out in space, while when interacting with a larger object such as a measuring device, it shrinks in size: its extension rapidly reduces in space to almost a point—depending on the type of interaction with the device. Measurements, according to collapse models, are always invasive and unavoidably alter the state of the quantum system.

In such a situation, classical physical quantities such as position and momentum have to be reconsidered. Quantum systems in general will not occupy a specific point in space, like true particles do. Yet they can occupy a definite location to the extent to which the wave function is contained within that location, similar to how a laser pulse is focused in space, although its electromagnetic field technically has tails stretching out at the speed of light. Momentum, energy, angular momentum, and so forth all have to be reformulated appropriately.

The outcome of quantum measurements does not necessarily reveal properties previously possessed by the system: depending on the specific situation, they are the result of a compromise between the state of the system prior to the measurement and the interaction with the device during the measurement. The measuring apparatus then participates in the definition of the outcome of the measurement, also in principle, contrary to what happens in the classical case, where the device in principle is neutral. This does not mean that the quantum system does not possess definite properties before the measurement, as dictated by the Copenhagen interpretation. The system always possesses definite properties, suitable for a wave theory, yet they cannot be observed without being modified.

So far, the ontological interpretation seems quite natural, once the fundamentally wavy nature of the constituents of matter is accepted. However, there is a serious difficulty, which characterizes any attempt at attributing an ontological meaning to the quantum wave function: it lives in configuration space, not in the Newtonian three-dimensional space. Hence these "quantum" waves are not like water waves, or electromagnetic waves. There is a way to cope with this, at a price: this interpretation requires that the true space is not the three-dimensional space of daily experience, but the configuration space, where the wave function lives. Collapse models then imply that the wave function will always contain peaks—corresponding to what are usually referred to as macroscopic objects—whose motion and interactions can be equally well represented (with negligible error) by points moving in a three-dimensional space, thus giving the illusion that this is the real physical space.

This is to some extent the inverse process to what occurs in classical statistical mechanics. There, particles are assumed to live in three-dimensional space, but when their number is too large, it is more convenient to represent them as a point in phase space. Here, configuration space is the real space of events, but they can equally well be seen as occurring in three-dimensional space, where things are more familiar.

The price to pay is the necessity of establishing a clear mathematical bridge between the physics in configuration space and the resulting physics in three-dimensional space, in order to prove that collapse models are capable of describing the classical world of our experience. Although a tentative framework can be easily sketched, the precise definition of it seems rather difficult.

Nomological interpretation. In 1995 Ghirardi and collaborators (Ghirardi, Grassi, & Benatti, 1995) suggested considering the mass density of the system as the appropriate ontology for collapse models. The mass density is defined from a multiparticle wave function via the relation:

$$\rho(\mathbf{x},t) \equiv \sum_{i=1}^{N} m_i \int d^3x_1 d^3x_2 \ldots d^3x_N \left|\psi(\mathbf{x}_1, \mathbf{x}_2 \ldots \mathbf{x}_N, t)\right|^2 \delta^{(3)}(\mathbf{x}-\mathbf{x}_i). \tag{5}$$

The reason for introducing the mass density is that it lives in three-dimensional space, and as such it does not require any further justification of why Newtonian space, rather than the configuration space, is the space of experience contrary to the previously discussed ontological interpretation of the wave function. The worldview that emerges is again that of a wavy microscopic world where particles are not particles but really matter waves: they diffuse, diffract around obstacles, and break in parts against them. But when many interact with each other, they become stiffer and stiffer, to the point that a macroscopic number of them will behave as a rigid and well-localized object, moving in space according to Newton’s laws (apart from very tiny deviations).

In the discrete GRW model, the mass density can be replaced by the “flashes,” that is, the space-time points where the collapses occur. This was first suggested by Bell (Bell, 1993c, 1989; Kent, 1989) and later reconsidered in (Tumulka, 2006a, 2006b, 2009). The worldview then changes quite substantially. At the microscopic level, isolated particles almost never exist, since statistically a collapse is so rare; in a double-slit experiment, nothing passes through the slits; the hydrogen atom is a mathematical artifact. But when the macroscopic world is considered, the flashes are so frequent in each small region of space (because matter is dense) that they give the illusion of a continuous mass distributed around the shape of the object, moving in space according to classical laws.

The two ontologies are then opposite to each other at the microscopic level, but converge when larger and larger objects are considered, as must be the case or they would not be appropriate ontologies for our world.

The question of the role of the wave function remains open. The most obvious answer is that the wave function has a nomological role: it defines the law for the time evolution of the mass density (or of the flashes). A similar role is taken by the wave function in Bohmian Mechanics.

Phenomenological interpretation. The previous two interpretations implicitly assume that collapse models are alterative theories, which in principle are correct descriptions of nature until they are proven wrong. Another approach, which emerged in recent years, is to consider them as phenomenological models of an underlying deeper-level theory, out of which quantum theory in the form described by collapse models emerges as an effective theory, in the same way as thermodynamics emerges from classical statistical mechanics. A framework of this kind has been elaborated by Adler (Adler, 2004), although no detailed working model has been developed yet.

Within such an approach, the wave function will acquire a role similar to temperature in thermodynamics: temperature is real because a body at 500°C burns. But it is not fundamental: what is fundamental are the velocities (and the positions) of all particles in the body. The wave function can have a similar status. The hope is that its living in the configuration space can be reconciled with the three-dimensional world of daily life, because it is only a phenomenological description of a different type of reality. Not much more can be said, until this program is consistently worked out.

8. Open Questions

Collapse models were proposed as a way to combine the linear, deterministic character of the Schrödinger dynamics and the nonlinear, stochastic character of the collapse process into a single dynamical framework, capable of explaining both the wavy indeterministic character of microscopic quantum phenomena as well as the particle-like deterministic behavior of macroscopic objects. Collapse models successfully achieve this goal, at least at the nonrelativistic level.

The first open question is how to extend these models to make them compatible with special relativity. Relativistic collapse models have been presented (Bedingham, 2011; Dowker & Henson, 2004; Ghirardi, Grassi, & Pearle, 1990a; Nicrosini & Rimini, 2003; Pearle, 1990, 1999a; Tumulka, 2006a), but they have a limited validity and/or their status as truly relativistic theories is still disputed. The reason for the difficulty is somehow simple, and has already been mentioned: the collapse is nonlocal—though it does not allow for superluminal signaling—and this is fundamentally incompatible with relativity. Can this apparent contradiction be resolved?

The answer largely depends on how the dynamical collapse program is considered. On the one side, collapse models can be considered as a candidate theory of nature, like, for example, electromagnetism. Then, if special relativity is also a theory of nature, these two theories need to be merged into a single framework.

The fact that formulating relativistic extensions of collapse models is so difficult might suggest that this is actually not the right direction to follow. There are two arguments supporting this. The first is that the collapse equations have very much the flavor of phenomenological equations, emerging from an underlying theory yet to be discovered, as suggested, for example, by Adler (Adler, 2004). In general, stochastic theories have always called for a underlying explanation for the stochasticity, and there is no reason to think that it should be any different with collapse models. Then, since often phenomenological models lose some of the symmetries of the underlying theories (for example the phenomenology of a classical particle in a gas does not exhibit Galilean invariance, while Newtonian physics does), there is no a priori reason to demand that collapse models be Lorentz invariant.

The second argument is nonlocality. If quantum nonlocality is taken seriously, then relativity, which is based on nonlocality, is wrong. This is nothing to be worried about: many quantum-gravity theories assume that relativity (special and general) is an emergent phenomenon. Therefore, it is plausible that quantum theory with the spontaneous collapses attached to it, as well as relativity, emerge from a completely different framework, yet to be discovered.

It is important to stress that this tension between collapse models and relativity comes on top of the well-known problems in formulating rigorously a relativistic quantum-field theory, independently from the issue with the collapse of the wave function.

A second open question is the violation of energy conservation. Mathematically, it is straightforward to show that the mean energy is not conserved by the collapse dynamics; according to both the GRW and CSL model, it steadily increases, eventually diverging, even though very slowly (Ghirardi et al., 1986, 1990b). Physically, the reason for this is very simple: the collapse noise kicks particles here and there, thus changing their energy. Several people consider this a serious drawback of collapse theories. This should not be the case: as a matter of fact, modifications of the GRW and CSL model have been formulated (Smirne & Bassi, 2015; Smirne, Vacchini, & Bassi, 2014), where the collapse properties are left unchanged, while the energy asymptotically approaches a finite value. Actually, a system while collapsing can even lose energy, if its initial energy was higher than the asymptotic one.

In these new models, the energy is still not fully conserved, but the path to follow to recover full energy conservation is clear: the combined quantum system + noise dynamics, where the noise acts on the quantum system, and the quantum system acts back on the noise (this second part is missing in all collapse models formulated so far) must be considered. Then it is expected that the total energy is conserved, with the two systems exchanging it back and forth.

A third open question is the origin of the collapse noise. Two options seem possible. The first one is to consider it as an intrinsic property of nature: at the very fundamental level, nature is not deterministic but stochastic, and the mathematical way to describe a stochastic world is via equations containing noise, which is what collapse models do. After all, one lesson that may be taken from quantum theory is that determinism should not be given for granted. Yet, as mentioned before, stochasticity instinctively calls for an explanation: Where does the noise come from?

This is the second option: the noise is a physical one, originating somewhere. The fact that it couples to the mass density (the stress-energy tensor, in a relativistic world) and that the larger the system, the stronger it is, suggests that it might be linked to gravity. This idea is strongly supported by Penrose (Penrose, 1996, 2014), and more recently reconsidered by Adler (Adler, 2016; Gasbarri et al., 2017); it was expressed in terms of a dynamical equation by Diosi (Diósi, 1987, 1989), an equation that is structurally similar to the CSL equation previously described.

Acknowledgments

The author acknowledges financial support from the EU H2020 Research and Innovation Program under Grant Agreement No. 766900 (TEQ), INFN, and the COST Action CA15220 QTSpace.

Further Reading

At present, there are no textbooks devoted to dynamical collapse theories. The best sources are the three extensive review articles (Bassi & Ghirardi, 2003; Bassi, Lochan, Satin, Singh, & Ulbricht, 2013; Pearle, 1999b). Another introduction to the subject is the article "Collapse Theories" in the *Stanford Encyclopedia of Physics*.

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