Model Learning with Personalized Interpretability Estimation (ML-🥧)

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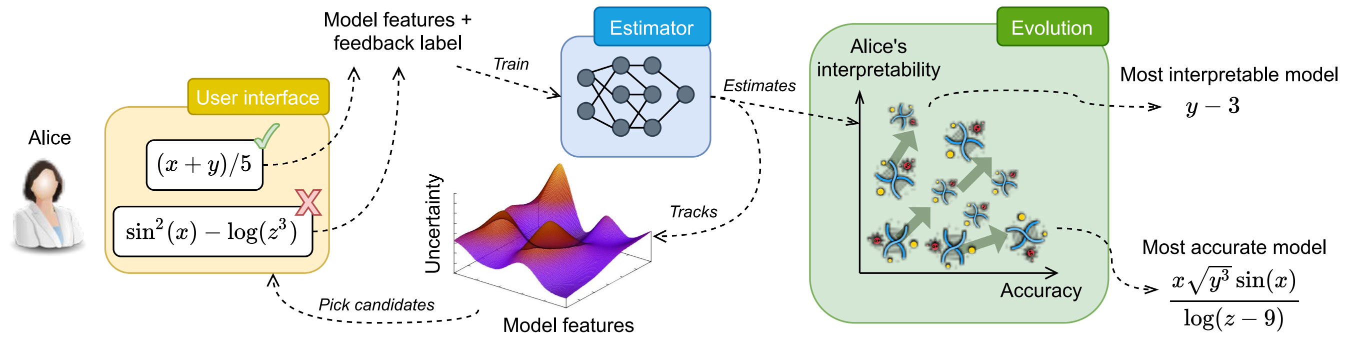

Figure 1: Schematic view of the proposed approach, ML-PIE. In the implementation proposed in this paper, the user provides feedback on models that are being discovered by an evolutionary algorithm. This feedback is used to train an estimator which, in turn, shapes one of the objective functions used by the evolution. Ultimately, this steers the evolution towards discovering models that are interpretable according to the specific user. To minimize the amount of feedback needed, ML-PIE keeps track of which models cause the estimator to be most uncertain, and submits these models for user assessment.

ABSTRACT

High-stakes applications require AI-generated models to be interpretable. Current algorithms for the synthesis of *potentially* interpretable models rely on objectives or regularization terms that represent interpretability only coarsely (e.g., model size) and are not designed for a specific user. Yet, interpretability is intrinsically subjective. In this paper, we propose an approach for the synthesis of models that are tailored to the user by enabling the user to steer the model synthesis process according to her or his preferences. We use a bi-objective evolutionary algorithm to synthesize models with trade-offs between accuracy and a user-specific notion of interpretability. The latter is estimated by a neural network that is trained concurrently to the evolution using the feedback of the user, which is collected using uncertainty-based active learning. To maximize usability, the user is only asked to tell, given two models at the time, which one is less complex. With experiments on two real-world datasets involving 61 participants, we find that our approach is capable of learning estimations of interpretability that can be very different for different users. Moreover, the users tend to prefer models found using the proposed approach over models found using non-personalized interpretability indices.

CCS CONCEPTS

• **Computing methodologies** → **Active learning settings**; **Neural networks**; **Genetic programming**.

KEYWORDS

Explainable artificial intelligence, interpretable machine learning, active learning, neural networks, genetic programming

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1 INTRODUCTION

World-wide policies concerning the fair and responsible use of artificial intelligence (AI) increasingly demand AI-made decisions to be explainable [27]. To answer this call, the field of *eXplainable AI* (XAI) studies methods to provide explanations of the decisions taken by unintelligible, *black box* models, such as deep neural networks [1]. However, explanation methods can be limited and should be used with care. For example, these methods may produce explanations that are only valid for a narrow neighborhood of the input data; mislead about feature relevance when importance scores are scattered across correlated features; and provide seemingly sensible explanations even when the black box is blatantly wrong [37, 45]. For these reasons, XAI also studies how to develop methods that can produce *interpretable AI*, i.e., models which can be inspected by humans as to provide a complete picture of their workings (including, e.g., edge cases). When capable of good performance, interpretable models should be preferred over black box ones [38, 45].

Evolutionary computation provides effective methods to seek interpretable AI models, and genetic programming (GP) [29] is a prime candidate in this sense. GP evolves a population of programs, such as AI models (or, more specifically, machine learning models), made of arbitrary primitive instructions. If these instructions are chosen to be procedures that perform high-level computations and have a clear meaning, GP has the *potential* to discover models that are, themselves, capable of complex functional behavior while also being interpretable. Nevertheless, GP potential to discover interpretable models is likely to remain unexpressed if left to chance. In other words, it is desirable to encapsulate the concept of interpretability into a search objective, so that the search process of GP can be steered towards most promising models.

The design of an objective that represents interpretability is an open problem. To begin with, there exists no clear-cut definition of what interpretability is [3, 34]. Whether a person finds a model interpretable depends on that person's background and, further adding to the complexity, the sensitiveness of the application at hand plays a role in deciding what *degree* of interpretability may be sought [18, 22]. Paradoxically, to seek clear, interpretable models, we need an objective that cannot be clearly defined.

In this paper, we tackle the problem of capturing the subjective notion of interpretability using GP and *active learning*. Instead of attempting to decide beforehand what the user deems interpretable, we train a personalized estimator of interpretability (in the form of a neural network) from feedback that is given by the user during the model synthesis process (in our case, GP). The user provides feedback by telling, given two models at the time, which one is more interpretable; this feedback refines the estimator. Concurrently, the estimator steers the model synthesis process (by implementing an objective of GP). To train the estimator sufficiently well from limited feedback, we keep track of what models cause the estimator to be most uncertain and submit those models for human assessment. Fig. 1 shows the overall approach, which we call *Model Learning with Personalized Interpretability Estimation* (ML-PIE).

We describe how we realize ML-PIE and show that it is capable of tackling the problem of discovering models with personalized interpretability, potentially paving the way for a new generation of personalized XAI methods.

2 RELATED WORK

Linear models, decision tables, decision trees, and other models that consist of high-level rules, are considered to have good chances of being interpretable [20, 24]. For linear models, promoting interpretability essentially corresponds to reducing the number of features [43, 53, 54, 63]. For decision trees and decision rules, besides reducing the number of features, approaches exist to restrict model size, prune unnecessary parts [5, 26], aggregate local models in a hierarchy [48], or promote a trade-off between accuracy and complexity by means of loss functions [30, 52] or prior distributions [33, 60, 61]. Regarding GP (and close relatives like grammatical evolution), perhaps the most simple and popular strategy to favor interpretability is to restrain the number of model components [16, 31, 57], sometimes in elaborate ways or particular settings [6, 32, 40, 49, 56]. Another strategy consists of penalizing models according to a weighted sum of the components that they include, after having pre-determined a weighing scheme [23, 36]. Alternatively, information from model approximators such as polynomials can be used to determine the level of interpretability [59].

Recently, an estimator of human-interpretability for symbolic models expressed as formulae was machine-learned from human feedback [58]. A survey was used to make users simulate the calculations of (random) formulae (an implementation of the XAI concept of *simulatability* [34]), and to identify the behavior of the formula when part of it would vary in some interval (an implementation of the XAI concept of *decomposability* [34]). Data gathered from 334 users was then used to fit a linear estimator comprised of four features extracted from the formulae, as, e.g., the subsequent composition of non-arithmetic operations. The estimator of interpretability was finally incorporated into a bi-objective GP, to evaluate the interpretability of evolving models. This estimator has also been used in another recent work [9].

In this paper we build upon [58] and extend it in three ways: (1) we *concurrently* learn a model of intepretability that is specific to the user; (2) we use a more complex, non-linear estimator instead of a linear one; and (3) we exploit uncertainty estimation to require a small amount of feedback to train the estimator.

Several works have explored including humans in the training loop of AI system synthesis for different applications, e.g., [8, 35, 46]. Very recently, in [40] it is hypothesized that querying humans during an (evolutionary) model synthesis process may help discovering interpretable models: this is what ML-PIE realizes. Furthermore, since ML-PIE actively seeks feedback about the models that make the estimator most uncertain, our work is a form of *active learning*. Broadly speaking, active learning entails techniques to automatically identify what few data items should be labeled, when labeling is expensive (e.g., when a user is involved) [47, 62]. Here we use a relatively simple strategy: we submit for user assessment those models that cause the estimator to be most uncertain. There are a few works that used active learning together with GP, similarly to us, e.g., [2, 10, 25]. An interesting difference that these works have with ours is that they use the labels acquired with active learning within the (or an, if multiple) objective function of GP *directly*, whereas we use these labels to *update* an objective function, namely the estimator of the interpretability of the user.

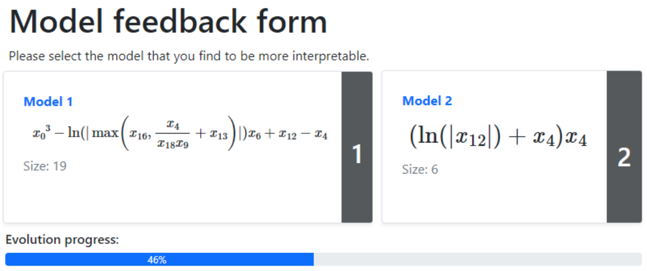
3 ML-PIE OVERVIEW

Problem statement. We consider the case of a user interested in obtaining a *model* by means of some model synthesis process. For the sake of simplicity, we assume the latter to be a supervised learning algorithm (but needs not be). We assume that some data are available to synthesize (and validate) the model, e.g., $\{(\boldsymbol{x}_i, y_i)\}_{i=1}^{n}$ with $\boldsymbol{x}_i \in \mathbb{R}^d$ a vector of feature values and $y_i \in \mathbb{R}$ a label value, and that the user desires the model to be (a) accurate in terms of estimating a y_i when given an $\boldsymbol{x}_i$ as input, and (b) interpretable. We do not enforce a precise definition of interpretabilty. Instead, we only assume that the user, when asked to compare two models, is able to choose the most interpretable one.

Use case. From the point of view of the user, ML-PIE works as follows: once the user has provided the data to the system, the model synthesis process starts. In ML-PIE, the model synthesis process takes into account both the accuracy and the interpretability of the model being synthesized: the accuracy is measured with an appropriate index (e.g., accuracy for classification or negative mean squared error for regression), the interpretability is estimated according to a dedicated *estimator* that is updated during the process—in fact, both the model under synthesis and the estimator of interpretability can be called "model" or "estimator", and both are "synthesized", "trained", or "learned" according to some machine learning algorithm; for the sake of clarity, we choose to use a different terminology to distinguish them.

While synthesizing the model, the system shows the user, through a suitable user interface, a *query*: given a pair of models, the user is asked to tell which one is the most interpretable according to her or his subjective judgment. As soon as the user answers the query (by a single click or tap), ML-PIE shows a new query; internally, it also uses the feedback to update the interpretability estimator. Query-answer interactions continue until the model synthesis process stops because a termination criterion is met. We remark that query-answer interactions and the model synthesis process proceed concurrently and asynchronously (i.e., we do not halt the model synthesis process at any point in time, see Sec. 7). The interpretability estimator acts as point of contact between the two, as it is updated right after a user's feedback and is used by the model synthesis process to assess model quality. Fig. 2 shows the user interface we used: the user answers the query by simply clicking on the preferred model (in our case, models are formulae).

Design goals. While designing ML-PIE we were driven by two goals. First, we wanted to keep low the annotation effort: we do not ask the users to provide detailed descriptions of what they mean by interpretability, neither do we ask for detailed, informative feedback about the models. Rather, we just ask to compare pairs of models and answer by performing an immediate action (single click or tap). Second, we aimed at obtaining a system that can learn models that are interpretable for a specific user on a specific problem, i.e., we wanted a *personalized* notion of interpretability. To achieve this goal, we decided to update the interpretability estimator concurrently to the model synthesis process. Note that, this way, the models undergoing user assessment necessarily come from the distribution of models under synthesis. In contrast, performing an off-line annotation approach prior to model synthesis would require to decide a sample of possible models beforehand, some of which may turn out to be unrepresentative of those under synthesis, ultimately wasting part of the user's effort. Even if ML-PIE updates the estimator while the model synthesis process takes place, we highlight that ML-PIE does not have to start from scratch: a partly pre-trained estimator can be used, as well as one that was previously built for other users, or a similar domain (e.g., healthcare).



Model feedback form

Please select the model that you find to be more interpretable.

Model 1

$${x_0}^3 - \ln(|\max\left(x_{16}, \frac{x_4}{x_{18}x_9} + x_{13}\right)|)x_6 + x_{12} - x_4$$

Size: 19

1

Model 2

$$(\ln(|x_{12}|) + x_4)x_4$$

Size: 6

2

Evolution progress:

46%

Figure 2: Screenshot of the interface used to collect the feedback of the user. The user clicks (or taps) on the model he or she deems more interpretable. Two new models are shown next. On the bottom, the progress bar displays the progress of the model synthesis process.

Generality and requirements. In principle, our approach is agnostic with respect to the algorithm employed for the model synthesis process and the nature of the models being synthesized, provided that: (a) the former is able to pursue accuracy and interpretability at the same time, either separately with a multi-objective algorithm or jointly with a single-objective one (e.g., to maximize an overall index of *trust*, we expand on this in Sec. 7); (b) the latter allows for a visual representation on a user interface (e.g., formulae or decision trees). A further, more practical requirement, is that the speed of the model synthesis process is compatible with allowing the user to provide a sufficient amount of feedback. Our approach is, in principle, also agnostic with respect to the algorithm employed for updating the estimator and the nature of the estimator, provided that these allow for active learning. This last requirement can be met if, e.g., the estimator can produce a measure of uncertainty or confidence when estimating the interpretability of the models [10].

In this paper, we instantiate ML-PIE for symbolic regression, i.e., our models are symbolic functions expressed as formulae. We use multi-objective GP as model synthesis process. For the estimator, we use a neural network and train it with stochastic gradient descent; to make the network produce predictions with uncertainty, we use dropout at prediction time. In the next section, we describe in detail these key internals of ML-PIE.

4 ML-PIE INTERNALS

We choose to use GP in a multi-objective setting because its outcome is a *collection* of models with different trade-offs between accuracy and interpretability, instead of a single model. This gives the user the possibility to inspect multiple options at the end of the run, and make a final call in terms of overall *trustworthiness*, i.e., what model meets *both* acceptable accuracy and interpretability.

Given a supervised learning problem, GP evolves a population of models, here expressed as symbolic functions $f : \mathbb{R}^d \to \mathbb{R}$, such that $f(\boldsymbol{x}_i) \simeq y_i, \forall(\boldsymbol{x}_i, y_i)$ in the data. The search of GP is driven by two objectives: accuracy, measured on (part of the) data (another being reserved for validation), and interpretability, measured by the interpretability estimator. We describe GP further in Sec. 4.2. Before that, we describe the neural network used as intepretability estimator (Sec. 4.1). This estimator takes as input a vector of six features that describe a model f, and produces an output in $\mathbb{R}^2$ that represents the interpretability of f and the respective uncertainty.

Following [58], the six features of f we use for interpretability estimation are: the total size of f (number of variables plus constants plus operations), the number of operations, the number of operations that are non-arithmetic, the maximum length of consecutive compositions of non-arithmetic operations, the number of constants, and the dimensionality of f (i.e., the number of unique variables)—the latter two being an addition with respect to [58].

4.1 The estimator of interpretability

We use a neural network as interpretability estimator because neural networks have excellent fitting capabilities, can produce uncertainty estimations similarly to Gaussian processes by simply making use of dropout at prediction time [19], and their training scales better than that of Gaussian processes [39].

The neural network takes the features of a model f as inputs, processes them with three fully-connected hidden layers with 100 ReLU activations [41] and 0.25 dropout rate [51], and finally applies the hyperbolic tangent. Importantly, we set the network to be trained using binary feedback (i.e., the one provided by the user). Our training signal is constituted by the features of two models f_1, f_2 and a label telling which of the two models is more interpretable for the user. Given this training signal, we feed the features of f_1 and f_2 as input to the network, and observe how the label compares to the respective predictions of interpretability $\hat{\psi}_1, \hat{\psi}_2$. Next, we compute a loss that can be seen as a two-points version of the Wasserstein loss commonly used to train generative adversarial networks [21]:

$$\mathcal{L}(\hat{\psi}_1, \hat{\psi}_2, l) := l\left(\hat{\psi}_1 - \hat{\psi}_2\right). \tag{1}$$

Here, the value of l depends on the label from the user: $l = -1$ if the first model is deemed more interpretable than the second, else $l = 1$. If, e.g., the network predicts that $\hat{\psi}_1$ is smaller than $\hat{\psi}_2$ but the user prefers f_1, then the loss will be positive. We remark that the proposed loss does not provide an explicit signal on what values $\hat{\psi}$ should assume: this is not an issue because for our model synthesis process, i.e., GP, relative values suffice. As optimizer we use stochastic gradient descent (learning rate = 0.001, momentum = 0.9).

Active learning with warm-up. As mentioned before, the models that are sent to the user for evaluation are those for which the network has registered the maximal prediction uncertainty so far, i.e., while being used to evaluate the models under synthesis. We measure this uncertainty by applying dropout at prediction time, specifically by taking the standard deviation over repeated predictions $\hat{\psi}^{(1)}, \dots, \hat{\psi}^{(k)}$ for a same input model f. We use $k = 10$ as a compromise between speed and fidelity, and set the interpretability estimation $\hat{\psi}$ to be the mean of the k predictions.

To avoid relying solely on the feedback of the user, we include an initial warm-up phase using interpretability estimates generated from the estimator of [58] for 100 random models (from now on, we refer to intepretability estimation according to [58] by ϕ). We do this because we have no guarantees that the user is responsive, and also so that the initial estimations of interpretability are not completely random. We heuristically halt the warm-up when the uncertainty of the network starts to decrease, specifically when the standard deviations of the predictions of the last two epochs is smaller than that of the previous two on average. Note that an increase of the uncertainty in the initial epochs is a normal consequence of the fact that initialized weights are small (we use Keras default initiliazer [7]), and as they increase with early gradients, so do the magnitudes of the (still semi-random) predictions. We found this scheme to result in around 3 to 6 epochs of warm-up.

Justification of adopting a ranking network. Before proceeding, we provide some evidence that the network we adopt, from now on called a *ranking network*, fares well compared to a more *classic network*, i.e., one trained as a regressor to predict specific values of interpretability. For the classic network we use the mean squared error loss and use a linear activation to obtain the output (instead of tanh). We create a toy scenario where we assume that $\psi := \phi$, i.e., we use the estimator from [58] in place of the unknown concept of interpretability of the user, and we create labels for a set of 1000 random symbolic models. Since we use ϕ as ground-truth, we change the warm-up phase to use model size (ℓ) instead of ϕ itself, again upon 1000 random models. For the ranking network, we assume that a feedback consists of a label that tells, given *two* symbolic models, which is best (as explained before). For the classic network, we have *regression* labels, i.e., the value of ϕ. Note that the number of models-per-feedback is twice for the ranking network than for the classic one. Moreover, providing a label for the classic network requires the user to specify a numeric value, which arguably takes more effort than only telling which of two models is better. For these reasons, we allow the ranking network to receive double the user's answers than the classic network.

We consider how well the networks are able to induce the same ranking of ϕ with increasing amount of feedback, using the Spearman footrule [13, 50]. This measures the misalignment between two rankings R and S (in our case, for r models $f_1, \dots, f_r$):

$$\textit{Spearman footrule}(R, S) = \frac{3}{q(r)} \sum_{i=1}^{r} |R(f_i) - S(f_i)|, \tag{2}$$

where $q(r) = r^2$ is r is even, otherwise $r^2 - 1$. The term $\frac{3}{q(r)}$ induces a normalization such that the expectation of the footrule is 1 when the order of one ranking is random with respect to the other, 0 for identical orders, and 1.5 for reversed orders. Fig. 3 shows results on this toy setting (with respect to a test set of 1000 random models). On the left panel, it can be seen that the ranking network learns quicker if we ask the user to provide feedback on the models for which the network is most uncertain. On the right panel, it can be seen that the ranking network outperforms the classic network. Note that the initial Spearman footrule is lower than 1 (value for random ranks) thanks to the warm-up phase on ℓ.

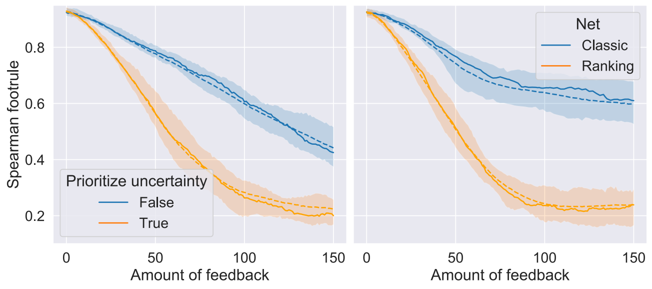


Figure 3: Capability of neural networks to learn the same ordering induced by ϕ as a function of the amount of feedback (median = solid, mean = dashed, interquartile range = shaded area, across 30 repetitions). A warm-up phase is applied using ℓ. *Left*: Prioritizing uncertainty to decide what models to submit for feedback improves the learning speed of the ranking network. *Right*: The ranking network adapts more quickly than the classic network.

4.2 Bi-objective evolution

We use the GP version of the well-known NSGA-II algorithm [11] implemented in [58][1]. The models evolved by GP, i.e., the symbolic functions, are represented with a traditional tree-based encoding [42]. At initialization, the trees are generated using the *ramped half-and-half* method, with tree depth between 1 and 3. The trees are rather small, and are allowed to have at most up to 25 components (nodes) at any point during the evolution, because larger trees quickly becomes incomprehensible. Trees that are selected with tournaments of size 2 undergo modification by means of traditional subtree crossover, subtree mutation, and one-point mutation, with equal probability.

Our GP uses two objectives. The first objective is maximizing accuracy (for regression, we actually minimize the mean squared error). We include *linear scaling* [28] to adapt the fitting of the evolving models, since it is often beneficial on real-world datasets [55]. The second objective is maximizing interpretability. In ML-PIE, this is estimated ($\hat{\psi}$) by our interpretability estimator. For comparison, we also consider using ϕ (estimation according to [58]) and ℓ (the model size, i.e., number of tree nodes). The outcome of GP is a collection of models that expresses a *trade-off front* of the objectives, i.e., taken any two models "from the front", if the first has better accuracy, then it must have worse interpretability (and vice versa).

The primitives used to evolve symbolic functions are (a) the variables of the problem at hand; (b) random constants between -5 and 5 with resolution of 0.25; (c) the functions $+$, $-$, $\times$, $\div^*$, $\cdot^3$, $\log^*$, max (*=protected). We use a relatively small population of 256 models and evolve it for 50 generations. We found this to be a good compromise between what performance the models can achieve, and how much time the users need to spend to terminate a run of ML-PIE (i.e., approx. 10 minutes). Since the population is not large, we include a simple method to oppose premature convergence that worked remarkably well in preliminary experiments. Specifically, for each model f_p in the population (parsed in a random order), we check whether another model f_q co-exists that has the same behavior (with respect to the data, i.e., $\forall \boldsymbol{x}_i, f_p(\boldsymbol{x}_i) = f_q(\boldsymbol{x}_i)$); for any such f_q, we mark it as duplicate (note, we do not mark the first f_p). Models marked as duplicates are set to have the lowest priority when GP selects parents and chooses the survivors of the generation.

[1]Code available at: https://github.com/marcovirgolin/pyNSGP

5 EXPERIMENTAL SETUP

We asked students enrolled at the engineering faculties of the University of Trieste, Italy and Chalmers University of Technology, Sweden to act as users for ML-PIE, which we disseminated as a web application on the cloud. Data collection lasted for approximately two weeks.

Experiments were run on two datasets, namely *Boston housing* (in short, Boston) and *German credit* (German), which concern regression of housing values and binary classification of credit approval, respectively. These datasets from the UCI repository [15] are well-known, of moderate-enough size to keep the users busy only for a reasonable time, and also interesting for assessing fairness in AI, because Boston includes a feature regarding race, while German has one regarding sex. Boston has 506 examples and 13 features, while German has 1000 examples and 20 features. With the parameter settings used for GP, it takes approximately 10 minutes to complete a run of ML-PIE; the estimation of interpretability by the neural network taking most computation time. Each run used a random split of the dataset in exam, namely into 70 % of the examples for training and 30 % for testing. The features of the datasets were standardized by z-scoring [14].

We also prepared an experiment to assess whether ML-PIE is any better than using pre-existing indices of interpretability, namely ϕ and ℓ. To this end, at the end of each ML-PIE run, the users were presented with a set of models from the trade-off front obtained during the run (i.e., by GP using $\hat{\psi}$) *paired* with models taken from runs where GP used ϕ or ℓ as indices of interpretability (100 precomputed runs for each). Models in each pair ($\hat{\psi}$ vs. ϕ or $\hat{\psi}$ vs. ℓ) had similar accuracy (pairing made by considering the minimal absolute difference). The users were asked to tell, for each pair, which of the two models was more interpretable. The users were not told which models were found during their session (i.e., with $\hat{\psi}$) and which were found using ϕ or ℓ, to prevent bias.

6 RESULTS

We show key results regarding the workings of ML-PIE and the performance it achieves. A total of 61 users answered the call, leading to 49 runs for Boston and 45 runs on German (some students did not run both). In the following, we report: how the users and the estimator of interpretability behaved during the runs (Sec. 6.1); the search performance of ML-PIE compared to that obtainable when ϕ or ℓ are used (Sec. 6.2); whether estimations of interpretability $\hat{\psi}$ obtained by different users behave differently (Sec. 6.3); and, last but not least, whether the users preferred models discovered using $\hat{\psi}$ over models found by ϕ or ℓ (Sec. 6.4).

6.1 Behavior of the users and of the interpretability estimator

Fig. 4 summarizes how the users and the estimator behaved over time. In the top panel, we see that the amount of feedback given per generation tends to be slightly smaller for early generations. We remark that this is because, in GP, the average model size tends to grow over time, causing early generations to be computed more quickly than later ones, and the users to have less time to provide feedback. We also measured the rate at which the estimator made mispredictions, by checking whether the estimates $\hat{\psi}_1, \hat{\psi}_2$ for the respective models f_1 and f_2 *before the user's answer* turned out to be in disagreement with the user's answer (e.g., $\hat{\psi}_1 < \hat{\psi}_2$ but the user picks f_1). Mispredictions were quite rare. This result is in line with what can be seen on the bottom panel, i.e., that the uncertainty of the estimator (the standard deviation of its estimations $\sigma(\hat{\psi})$) dropped rather quickly and remained low over time. We also see that the cumulative amount of feedback had a steady, linear growth.

Table 1 shows examples of models[2] extracted from the trade-off fronts obtained by the user-guided evolutions, at different trade-off levels between accuracy and interpretability (interpretability percentile τ). Note that we actually report errors: the training and test errors for Boston are mean squared errors, while those for German are inaccuracies (1 – accuracy). It can clearly be seen that, the more we move from more accurate models to more interpretable ones (i.e., increasing τ), the more the models become simpler under several aspects, such as total size, number of dimensions, and presence of non-arithmetic operations. This confirms that the estimator was capable of capturing a sensible notion of interpretability. A further confirmation of this is provided later in Sec. 6.3.

Take-home. Overall, the users provided good amounts of feedback through the process, and the estimator of interpretetability responded in a sensible manner. Observed uncertanties, mispredictions, and model ranking behaviors, are reasonable.

6.2 Search performance

Table 2 summarizes the search performance of GP using $\hat{\psi}$ (over the runs performed during the feedback sessions with the users, 49 for Boston and 45 for German), ϕ, and ℓ (100 runs per dataset for each). Before delving into $\hat{\psi}$, we remark that the results obtained by ϕ and ℓ are similar to those reported in literature (see, e.g., [58] for Boston[3] and [40, 48] for German), despite the fact that our GP used limited resources (population size = 256, generations = 50).

What can be mainly drawn from Table 2 is that, error-wise, results with $\hat{\psi}$ can be slightly worse than those obtained with ϕ and ℓ. However, this is rarely important (e.g., only statistically significant in two cases at test time, for $\tau = 50$). Likely, this is a consequence of the fact that the estimator of $\hat{\psi}$ is stochastic (due to dropout) and is only refined over time. Noisy objectives are known to hinder search algorithms, including evolutionary ones [4, 44]. This fact also justifies larger front sizes being produced by $\hat{\psi}$. We

[2]We do not perform model simplification because too costly to perform upon each model being evolved, it might harm evolvability, and, if done only for the models displayed to the user, it may create a perception mismatch with respect to the features of the models (size, no. operators, etc.), used by the interpretability estimator.

[3]Besides front sizes, this can be checked by de-normalizing the errors reported in Table 2 of [58] by multiplying them with the variance of the label, i.e., ≈ 84.6.

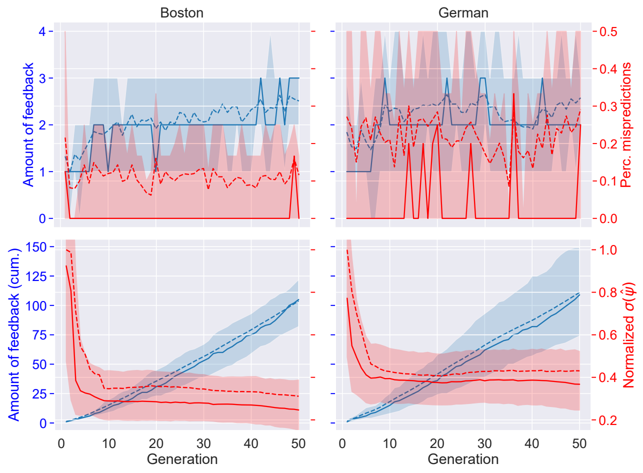


Figure 4: User and framework behavior over the number of generations (median = solid, mean = dashed, interquartile range = shaded area, across all users). *Top*: Amount of feedbacks given and percentage of mispredictions made by the intepretability estimator, per generation. *Bottom*: Cumulative amount of feedback and estimator uncertainty $\sigma(\hat{\psi})$ over the generations. Uncertainty is normalized by dividing by the average uncertainty observed at the first generation.

note that if we restrict the results with $\hat{\psi}$ to only account for good amounts of feedback being given, e.g., ≥ 100 (approximately 50 %), results do not change substantially.

Take-home. Model accuracy is slightly worse when $\hat{\psi}$ is used instead of ϕ or ℓ. This is likely because the estimator is noisy. A simple action that can be taken is to increase GP search resources.

6.3 Level of personalization

Fig. 5 shows comparisons of rankings induced by different users with respect to each other and with respect to ϕ and ℓ. We now use the notation $\hat{\psi}_i$ to refer to the interpretability estimates for a certain i-th user. The rankings are computed by ordering, according to ϕ, ℓ, and each $\hat{\psi}_i$, all models from the fronts ever discovered by all the users (excluding syntactic duplicates and totalling ≈ 9000 models).

We begin by observing that the distributions of Spearman footrule obtained when comparing user induced-rankings with one another ($\hat{\psi}_i$ vs. $\hat{\psi}_j$) can have heavy or long tails. Overall, this corroborates the hypothesis that some users may have non-negligible disagreements regarding their notions of interpretability, and thus that seeking personalized interpretability estimations is needed.

Average behaviors are also interesting. On average, user induced-rankings are more similar to one another (footrule of $\hat{\psi}_i$ vs. $\hat{\psi}_j$ has average value of ≈ 0.13) than to those induced by ϕ or ℓ (respectively $\hat{\psi}_i$ vs. ϕ with average value ≈ 0.2 and $\hat{\psi}_i$ vs. ℓ with average value ≈ 0.25). This means that, on average, users provided similar feedback, i.e., they agreed on a similar notion of interpretability ψ (at least, for the experimental setting we proposed and for the users involved). Furthermore, note that user induced-rankings are more similar to those of ϕ and ℓ than how the ranking of ϕ is similar to that of ℓ.

Table 1: Examples of models discovered by evolutions guided by the users. The models are taken at random from the evolved trade-off fronts (training error vs. $\hat{\psi}$) at different interpretability percentiles (τ). The test error is also reported, and some amount of overfitting can be observed (most notably, the first model at $\tau = 10$ for Boston). Our approach is capable of capturing sensible trends of model intepretability: models become simpler as τ increases.

Dataset	τ	Train	Test	Model
Boston	10	18.43	37.53	$x_{12} - \max(2.5, x_8 + x_9 + x_5 - x_8 - x_2)(-0.25 + x_{12}) + \max(x_5, x_{12})$
		19.86	18.57	$4.5 \times x_{12} + x_5\left(x_{10} + \frac{x_9}{x_5}\right) + (x_{12} + x_{10})x_5 + x_{12} + x_5(-3.25 - x_5)$
		21.24	20.07	$\max(x_{11}x_{11}x_{11}, \ln(\lvert x_5(x_5 - x_{12})\rvert)) - x_{12} + x_5 - \max(x_0, 2.5) + x_{12} + x_{10}$
	50	25.64	30.16	$x_{12} - \max(x_{12}, x_5)$
		26.05	35.42	$\frac{-1.0+x_4}{4.25} - 4.75 - 1.75 - (x_{12} + x_{10} - x_5) \times 4.75$
		32.13	29.09	$\frac{x_{12}}{x_{12}}(x_{12} + x_{12} - x_5)$
	90	35.47	45.54	$\frac{x_{12}}{-2.75} \times -0.25$
		57.82	61.84	$x_{10} - x_{11}$
		71.97	62.50	$3.0 - x_4$
German	10	0.24	0.26	$x_1 - x_0 - x_6 + x_7 + x_{11} - x_6 + \frac{x_1}{x_{17}} - x_2 - x_0$
		0.25	0.24	$x_4 + x_{19} - x_{10} + \frac{x_0}{x_{16}} - x_1 + x_{18} - x_1 - x_1 + x_5 - x_6 - x_6$
		0.25	0.25	$x_2 + x_2 + x_8 + x_0 - x_1$
	50	0.25	0.24	$(x_2 - x_4x_4 + x_1 - x_4 - 4.5)x_0$
		0.25	0.27	$x_{11} - 2.5 - x_2 - x_0 + 3.25$
		0.26	0.30	$x_4 - x_0 - x_0$
	90	0.26	0.29	$x_{11} - x_0$
		0.26	0.30	$x_0 - x_4 + x_5$
		0.28	0.26	$x_2 + x_0$

Dataset		Boston				German			
	τ	0	10	30	50	0	10	30	50
$\hat{\psi}$	Front	12.4				10.9			
	Train	20.06	21.06	23.32	27.78	0.23	0.24	0.25	0.26
	Test	22.43	23.19	24.65	28.93	0.26	0.26	0.27	0.27
ϕ	Front	9.3				5.4			
	Train	19.09	**19.39**	**20.94**	**23.24**	0.24	0.24	0.24	0.25
	Test	22.62	21.92	22.44	**24.98**	0.27	0.27	0.27	0.28
ℓ	Front	9.1				6.9			
	Train	18.70	**19.03**	**20.54**	**22.35**	0.24	0.24	0.24	0.25
	Test	22.39	22.68	23.08	**24.24**	0.27	0.27	0.27	0.28

Table 2: Mean front sizes (Front), and mean train and test errors (respectively Train and Test), at different interpretability percentiles (τ) of the models in the best-found trade-off fronts, for GP using $\hat{\psi}$, ϕ, or ℓ as interpretablity indices. Errors in bold are significantly better than their $\hat{\psi}$ counterpart (Mann-Whitney U test p-value < 0.01 with Bonferroni correction).

This means that the average $\hat{\psi}_i$ has more in common with ϕ and ℓ than the latter two have with each other.

Take-home. The estimations can be personalized (long tails of $\hat{\psi}_i$ vs. $\hat{\psi}_j$) but are similar on average (footrule of ≈ 0.13), conditioned to our experimental settings and involved cohort (students).

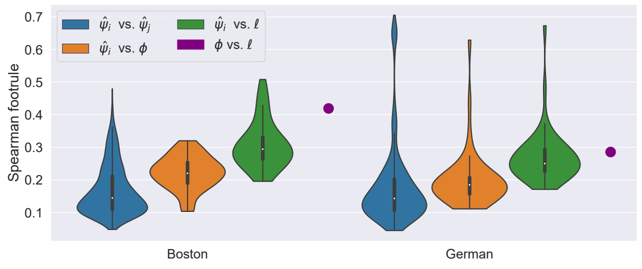


Figure 5: Violin plot of Spearman footrules from comparing rankings obtained with different interpretability estimators. On average, a user's estimated interpretability $\hat{\psi}_i$ is more similar to that of another user $\hat{\psi}_j$ than to ϕ and ℓ. However, high disagreements are also possible (fat and long tail for Boston, very long tail for German).

6.4 Final user judgment on $\hat{\psi}$, ϕ, and ℓ

Fig. 6 shows the preferences indicated by the users when presented, at the end of an ML-PIE run, with models found by the run itself at $\tau = 30$ and $\tau = 50$, and similarly accurate models obtained in pre-performed runs using ϕ and ℓ.

The first result we highlight is that, for sufficiently simple models (i.e., $\tau = 50$, cfr. Table 1), it does not seem to matter what interpretability index is used. Conversely, preferences start to emerge when more complex models are at play, i.e., at $\tau = 30$. In $\hat{\psi}$ vs. ϕ comparisons (top panels), we see that models discovered by our

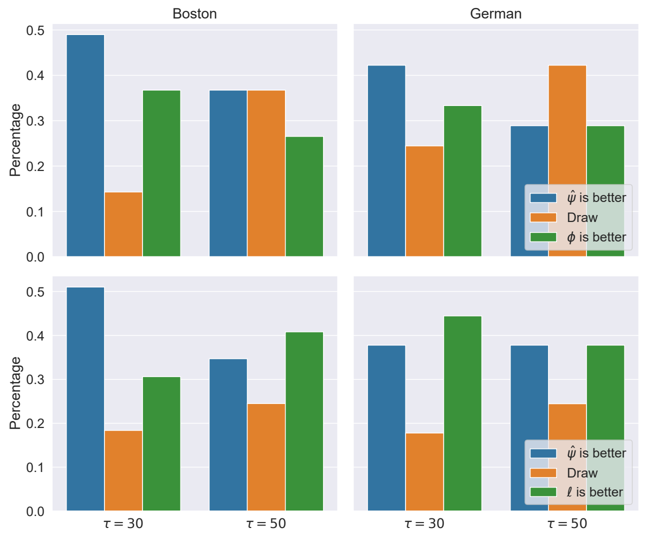


Figure 6: Outcome of the survey at the end of the evolution. For $\tau = 50$, it largely does not matter what index of interpretability is used. When the models start to be more complex, i.e., for $\tau = 30$, the users generally prefer $\hat{\psi}$ over ϕ and ℓ, except for $\hat{\psi}$ vs. ℓ on German.

approach were preferred over those found using ϕ almost 15 % of the times for Boston, and 10 % of the times for German. The biggest difference at $\tau = 30$ happens for $\hat{\psi}$ vs. ℓ for Boston (bottom left panel), with a 20 % preference gain for $\hat{\psi}$. For German at $\tau = 30$, however, ℓ was preferred over $\hat{\psi}$ by approximately 7 %. We can justify this result by looking back at Table 1, where it can be sees that the more complex models found for German ($\tau = 10$) are actually not that complex after all (e.g., they all lack non-arithmetic operations). On this dataset, decent accuracy can already be obtained when the models are linear, hence minimizing size seems to be sufficient. Note that, by contrast, the models found for Boston can be much more complex.

Take-home. Overall, for more complex models, the users indicated a preference for the models found with ML-PIE over those found using ϕ or ℓ. However, for simpler models, ℓ suffices.

7 DISCUSSION & CONCLUSION

We believe that our results, obtained on two datasets and involving 61 engineering students, are promising. The results show that ML-PIE behaves sensibly, the search performance of GP is not harmed excessively by $\hat{\psi}$ being noisy, different users can induce different $\hat{\psi}$, and the models obtained by ML-PIE tend to be preferred over those obtained when ϕ or ℓ are used instead.

Regarding the last point, preference for $\hat{\psi}$ over ϕ and ℓ is, however, only marginal. We believe that this result is a consequence of what can be considered the main limitation of this study: we did not involve actual domain experts and used a rather general set of model features to estimate model interpretability. Since our users (engineering students) were not experts in housing values (for Boston) nor credit approval risk (for German), we did not provide them with the meaning of the features of the datasets, nor we asked them to attempt to understand what the models in exam exactly mean. Yet, relying on students allowed us to have a decently-sized cohort with some general knowledge about formulae: we believe that this allowed us to gain sufficient evidence that ML-PIE works sensibly. We think that even if the preference gains obtained by $\hat{\psi}$ over ϕ and ℓ were moderate, they represent a very promising result: by enabling the interpretability estimator to use more specialized model features (e.g., what domain-specific features and functions appear, how many times, and in which context), and involving actual domain experts, it is reasonable to expect that the preference for $\hat{\psi}$ will increase substantially.

From a technical standpoint, there are a number of aspects worth studying to improve ML-PIE. For example, while we used a neural network with sensible settings to realize the interpretability estimator, we did not, e.g., optimize its architecture. Another point that deserves further investigation is concurrency aspects. We designed ML-PIE to make the user provide feedback while the model synthesis process is ongoing because, this way, the models that require user assessment come directly from the distribution of models under synthesis—and not from a potentially-unrepresentative sample, wasting (part of) the user's effort. However, the pace of feedback provided by the user may be too slow with respect to the speed of the model synthesis process, ultimately harming it. It may be beneficial to study whether the model synthesis process should be halted at times, e.g., based on statistics regarding the uncertainty of the interpretability estimator.

In the future, together with involving domain experts and adding representative power to the estimator of interpretability (e.g., enabling more model features to be accounted for), we plan to demonstrate that ML-PIE can be adapted to other settings than formulae-like models (e.g., decision trees) and other model synthesis processes than evolutionary ones (e.g., single-objective reinforcement learning or multi-objective gradient descent [12]). Regarding the last point, it would be interesting to study whether model synthesis can be guided exclusively by user feedback, to optimize some overall measure of *trust* [17], e.g., a measure that encapsulates, at once, accuracy, interpretability, and possibly other aspects such as soundness (e.g., for models that must satisfy physical constraints).

Concluding, we propose Model Learning with Personalized Interpretability Estimation (ML-PIE), a human-in-the-loop, active learning approach for the synthesis of models with user-tailored interpretability. ML-PIE answers the need for eXplainable AI (XAI) methods capable of synthesizing models that have a chance of being interpretable at a personalized level. Our experiments involving 61 students of engineering show that ML-PIE is capable of learning, from their feedback, user-specific estimations of interpretability. These estimations can be substantially different for different users, and are in general different from interpretability indices from literature. Last but not least, the users show a moderate preference for the models synthesized using their own feedback over models synthesized when other interpretability indices are used.

We believe that this work shows that XAI methods capable of model synthesis with personalized interpretability can be achieved. These methods may bring profound advantages for the responsible use of AI in high-stakes societal applications.

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