Gaussian approach versus Dolph-Chebyshev synthesis of pencil beams for linear antenna arrays

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A very simple and fast method for the synthesis of pencil beams with linear antenna arrays of equally spaced elements is presented. The proposed procedure starts selecting the desired pencil beam as a Gaussian function. This is very convenient for two reasons: first, the continuous line-source distribution that exactly produces the desired pencil beam (i.e. the Fourier transform of it) is in turn a Gaussian function and is immediately calculated. Second, a suitable weighted sampling of this distribution gives the excitations of the array elements in closed form. Two numerical examples reveal the good performances of the proposed approach, also in comparison with the classical method by Dolph-Chebyshev. It is shown that the synthesised array factors well approximate the desired pencil beams in real time, in particular ensuring a very good behaviour in the side lobe regions. Furthermore, the 'dynamic range ratio' of the excitations, defined as the ratio between the maximum and the minimum amplitude of the excitations, is very low and close to unity when the array length is sufficiently small.

Introduction: In many modern applications such as, for example, radar [1], satellite [2] and wireless communications [3], antennas are required to radiate narrow beams with low side lobes. In the last decades, several methods of synthesis have been developed for linear antenna arrays of equally [4, 5] or unequally [6, 7] spaced elements. This Letter proposes a method to synthesise pencil beams with linear antenna arrays of equally spaced isotropic elements. The innovative key idea consists in selecting the desired pencil beam as a Gaussian function. Then, the procedure develops as follows. First, a continuous line-source distribution is determined, whose array factor is exactly equal to the desired pattern. This distribution is the Fourier transform of the desired Gaussian pattern [8] (up to a multiplicative constant), so it is in turn a Gaussian function, and its calculation is immediate. Finally, a suitable weighted sampling of this distribution is performed, obtaining the excitations of the array elements in closed form.

In the sequel, the proposed algorithm is described in detail. Then, numerical comparisons with the classical method by Dolph-Chebyshev [8] are presented, which put into evidence some advantages of the proposed algorithm. Finally, some conclusions are summarised.

Problem and Gaussian approach: The radiation pattern of a linear array of length L, composed by N isotropic elements equally spaced on the z-axis of a Cartesian system $O(x, y, z)$, can be expressed as

$$F(\boldsymbol{a}; u) = \sum_{n=1}^{N} a_n \exp(\mathrm{j}uz_n) \tag{1}$$

where $u = k\sin\theta$, with $k = 2\pi/\lambda$ (being λ the wavelength) and θ the angle from broadside, $\boldsymbol{a} = [a_1, \ldots, a_N]^{\mathrm{T}}$ is the column vector of the N array excitations and

$$z_n = -L/2 + \Delta(n-1), \qquad n = 1, \ldots, N \tag{2}$$

are the positions of the array elements, with $\Delta = L/(N-1)$ the inter-element spacing. Given L and N, the problem here posed consists in finding the N excitations a_n that produce a pencil beam having a desired beamwidth. The idea is that of choosing the desired pattern $F_d(u)$ as the normalised Gaussian function

$$F_d(u) = \exp\left(-\frac{u^2}{2\sigma^2}\right) \tag{3}$$

where σ is the mean-square deviation. In order to control the pattern width, $F_d(u)$ is imposed to have a desired b dB beamwidth, $\mathrm{BW^{des}}$ (degrees). By (3), this requires to select σ as

$$\sigma = k\sqrt{\frac{10}{b\ln 10}}\sin\left(\frac{\pi\mathrm{BW^{des}}}{360}\right). \tag{4}$$

Now, it is well known that a continuous line-source distribution $a(z)$, $-\infty < z < +\infty$, yields the pattern

$$F(u) = \int_{-\infty}^{+\infty} a(z)\exp(\mathrm{j}uz)\,\mathrm{d}z \tag{5}$$

Hence, in order to obtain $F(u) = F_d(u)$ in (3), $2\pi a(z)$ must be the Fourier transform of $F_d(u)$ (see also [8]). Recalling the integral properties of the Gaussian function, it results

$$a(z) = \frac{\sigma}{\sqrt{2\pi}}\exp\left(-\frac{\sigma^2 z^2}{2}\right), \qquad \text{for} -\infty < z < +\infty, \tag{6}$$

which is a Gaussian function with standard deviation $1/\sigma$. Assuming a finite length L of the array, (5) is replaced by

$$F(u) = \int_{-L/2}^{+L/2} a(z)\exp(\mathrm{j}uz)\,\mathrm{d}z \tag{7}$$

so $F(u)$ only approximates $F_d(u)$, and the approximation is good if L is chosen sufficiently large. Furthermore, the N excitations a_n must be chosen in such a way that the pattern in (1) be a good approximation of the pattern in (7). In the proposed synthesis procedure, the excitation a_n is selected as

$$a_n = \int_{z_n-\Delta/2}^{z_n+\Delta/2} a(z)\,\mathrm{d}z \tag{8}$$

as is depicted in Fig. 1. Substituting (6) into (8) and manipulating yields

$$a_n = \frac{1}{2}\left\{\mathrm{erf}\left[\frac{\sigma(z_n+\Delta/2)}{\sqrt{2}}\right] - \mathrm{erf}\left[\frac{\sigma(z_n-\Delta/2)}{\sqrt{2}}\right]\right\} \tag{9}$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x \exp(-t^2)\,\mathrm{d}t$ is the error function. Equation (9) provides the excitations a_n in closed form, thus simplifying the procedure and obtaining the required beam pattern in real time. As an important remark, note that the narrower is the desired pattern in (3), that is, the lower is σ, the wider is the line source $a(z)$ in (6). As a consequence, for a given L the minimum value of $a(z)$ in $[-L/2, L/2]$ increases when reducing σ. Therefore, the dynamic range ratio (DRR) values tend to reduce because by (8) the excitations can be approximated by samples of $a(z)$. In particular, DRR is close to unity if L is sufficiently small. The numerical results confirm that low DRR values are obtained. This is important as low values of DRR allow to use simpler feeding networks.

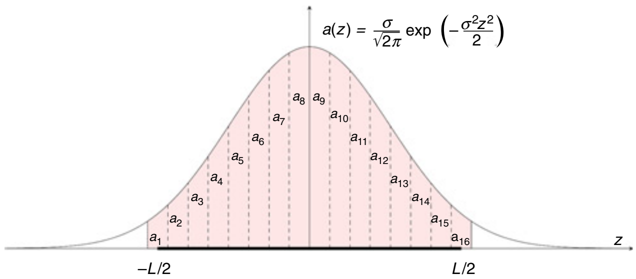

Fig. 1 *Graphical representation of choice of excitations: each excitation a_n is chosen as area between diagram of $a(z)$ and z-axis in corresponding interval $[z_n - \Delta/2, z_n + \Delta/2]$*

Numerical examples: Let us consider an antenna array of length $L = 20\lambda$, composed by $N = 41$ elements equally spaced on the z-axis at the positions in (2), where $\Delta = \lambda/2$. We want to find N excitations a_n that generate a pencil beam with a first null beamwidth of 5°. To this aim, we first calculate σ imposing $\mathrm{BW^{des}} = 5°$ and $b = 100$ dB in (4), obtaining $\sigma = 0.0571$ rad/m. Then, substituting σ into (9) yields the excitations listed in Table 1, which by (1) provide the array factor shown in Fig. 2. The same problem is solved also with the Dolph-Chebyshev algorithm [8], which was demonstrated to give an array factor with minimum beamwidth for a given (uniform) maximum side lobe level, and vice versa. The Dolph-Chebyshev array factor is shown in Fig. 2, and has all the side lobes at the same maximum level of −13.47 dB and, as required, a first null beamwidth of 5°. The array factor obtained with the proposed algorithm has a bit lower maximum side lobe level (−14.27 dB), and a considerably better side lobe pattern, at the expense of an increased first null beamwidth of 5.7°. Nevertheless, the maximum directivity is 16.12 dB with the proposed approach and 13.91 dB with the Dolph-Chebyshev algorithm. As a further advantage, the percentage of power radiated in the side lobe region with respect to the total radiated power, is considerably lower with the proposed approach (7.76 versus 51.45% of

Dolph-Chebyshev). Furthermore, the excitations obtained with the Dolph-Chebyshev algorithm (listed in Table 1) have a DRR = 7.93, while those obtained with the presented method have DRR = 1.18, which is considerably lower. Finally, the results are obtained in real time by both methods (<1 ms).

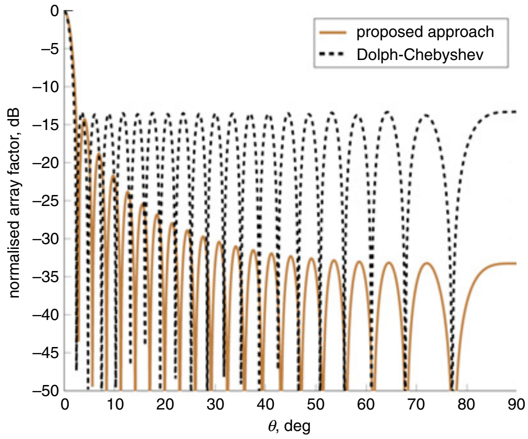

Fig. 2 *Example 1: array factor obtained with proposed approach (solid line) and with Dolph-Chebyshev algorithm (dotted line)*

Table 1: List of excitations obtained with proposed approach (PA) and with Dolph-Chebyshev method (DC)

n	a_n PA	a_n DC	n	a_n PA	a_n DC	n	a_n PA	a_n DC
1	1.0000	1.0000	8	1.0988	0.1674	15	1.1599	0.2010
2	1.0160	0.1260	9	1.1091	0.1733	16	1.1652	0.2040
3	1.0315	0.1334	10	1.1205	0.1790	17	1.1694	0.2064
4	1.0463	0.1406	11	1.1301	0.1843	18	1.1729	0.2084
5	1.0604	0.1476	12	1.1389	0.1891	19	1.1752	0.2098
6	1.0740	0.1545	13	1.1468	0.1935	20	1.1767	0.2106
7	1.0867	0.1610	14	1.1538	0.1975	21	1.1771	0.2109

Due to symmetry, only values corresponding to $n = 1, \ldots, 21$ are listed.

As a second example, Fig. 3 shows the array factors obtained with the two methods for an array of length $L = 30\lambda$, composed by $N = 61$ elements, with an inter-element distance $\Delta = \lambda/2$. The desired pattern has a 35 dB beamwidth of 5° (i.e. σ is calculated imposing $BW^{des} = 5°$ and $b = 35$ dB in (4), and is 0.0965 rad/m). Both algorithms require only 1 ms to evaluate the optimal excitations (which are not reported here for reasons of space). The results, which are consistent with those of the first example, are summarised in Table 2.

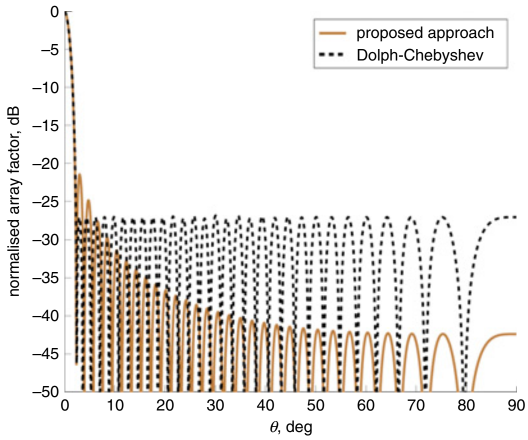

Fig. 3 *Example 2: array factor obtained with proposed approach (solid line) and with Dolph-Chebyshev algorithm (dotted line)*

Table 2: Comparison of performances of two algorithms

	Example 1		Example 2	
	PA	DC	PA	DC
beamwidth	5.7°	5°	4.82°	5°
directivity	16.12 dB	13.91 dB	17.50 dB	17.36 dB
SLL_{max}	−14.27 dB	−13.47 dB	−21.51 dB	−27.01 dB
$P_{SL\%}$	7.76%	51.45%	1.47%	5.19%
DRR	1.18	7.93	2.85	4.16

Conclusion: In this Letter, the problem of pencil beam synthesis for linear arrays of equally spaced elements has been addressed. It has been shown that the innovative choice of a Gaussian function to represent the desired pencil beam, in conjunction with the proposed synthesis procedure, allows to solve the problem easily, in real time, in closed form and with very satisfactory results. Two numerical comparisons between the proposed approach and the Dolph-Chebyshev method, show that array factors with comparable beamwidths have been obtained, and that the proposed approach produced side lobe patterns considerably better than those obtained by the Dolph-Chebyshev algorithm. In particular, with the presented approach the directivity resulted to be higher, and the percentage of power radiated in the side lobe region was much lower. Finally, the proposed method gave DRR values very low in both the examples, and considerably lower than those obtained by the Dolph-Chebyshev method.

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One or more of the Figures in this Letter are available in colour online.

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