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Phase-Only Synthesis of Antenna Array Patterns Having Gaussian Shaped Nulls

Giulia Buttazzoni*, Fulvio Babich, Francesca Vatta and Massimiliano Comisso
Department of Engineering and Architecture, University of Trieste, Via A. Valerio, 10, 34127, Trieste, Italy

Abstract

The potentialities offered by antenna arrays make them very attractive for future communication systems. Recent channel measurement at millimeter-waves (mmWaves) have in fact shown that the angular dispersion of the interference follows specific distributions, which are often Gaussian. Accordingly, this paper proposes a phase-only synthesis algorithm for antenna arrays, able to generate patterns having Gaussian shaped nulls to properly suppress the interference. The developed method is suitable for arrays of arbitrary geometry and can operate in the three-dimensional (3D) scenario, thus performing zenith-azimuth synthesis. To prove the effectiveness of the proposed solutions, numerical results involving multi-ring arrays are presented.

1 Introduction

One of the most recognized advantages provided by antenna arrays with respect to isolated radiators is their capability to properly shape the pattern, so as to meet different requirements [1]. Among the others, antenna arrays allow one to obtain the pattern reconfigurability and the electronic beam steering, which are of paramount importance in many applications, such as radar signal acquisition, cellular coverage, vehicular communication, and satellite link establishment [2–4]. In this regards, the advantages provided by the phase-only control approach are well-recognized. The scientific literature offers in fact many algorithms of pattern synthesis for antenna arrays with constraints on the element excitations [5–12], some of which also allow one to include a certain number of desired pattern nulls [5, 7, 12]. However, the future deployment of millimeter-waves (mmWaves)-based cellular systems has introduced a further requirement, consisting in the possibility to properly shape not only the main beam, but also the null region. This capability may be of great interest for forthcoming fifth and sixth generation (5G/6G) systems, since recent measurement campaigns have revealed that often the angular dispersion of the interference in the mmWave spectrum is Gaussian-shaped [13–15].

To address the above introduced issue, this paper proposes an effective phase-only control algorithm for the synthesis of arbitrary antenna arrays with patterns having a main beam with an arbitrary shape and a Gaussian shaped null region. The algorithm is developed considering a three-dimensional (3D) scenario, thus involving both the zenith and azimuth domains, with the aim of better exploiting the available degrees of freedom by following the actual shape of the angular dispersion.

The remaining of this paper is organized as follows. In the next section, the problem is formulated and the synthesis procedure is explained. In Section 3, the numerical examples are presented and discussed. Finally, in Section 4, the most relevant conclusions are summarized.

2 Problem and synthesis algorithm

Let $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ denote the unit vectors of the coordinate axes of a reference Cartesian system $O(x,y,z)$, and let N represent the number of elements of an antenna array, which are located at the positions $\bar{\mathbf{r}}_n = x_n\hat{\mathbf{x}} + y_n\hat{\mathbf{y}} + z_n\hat{\mathbf{z}}, n = 1,\ldots,N$. The radiation pattern of such array at the generic direction $\hat{\mathbf{r}}$ may be expressed as:

$$F(\mathbf{a};\hat{\mathbf{r}}) = \sum_{n=1}^{N} a_n p_n(\hat{\mathbf{r}}) \exp(jk\bar{\mathbf{r}}_n \cdot \hat{\mathbf{r}}), \tag{1}$$

where $\mathbf{a} = [a_1, ..., a_N]^T$ is the excitation vector; $p_n(\hat{\mathbf{r}})$ is the embedded element pattern of the nth array element at the direction $\hat{\mathbf{r}} = \sin\theta\cos\phi\ \hat{\mathbf{x}} + \sin\theta\sin\phi\ \hat{\mathbf{y}} + \cos\theta\ \hat{\mathbf{z}}$, being $\phi \in [0, 2\pi[$ the azimuth angle and $\theta \in [0, \pi]$ the zenith one; and $k = 2\pi/\lambda$ is the wave number, being λ the wavelength. Besides, identify as $\mathbf{a}_0 = [a_{01}, ..., a_{0N}]^T$ the excitation vector that generates a desired pattern $F_0(\mathbf{a}_0;\hat{\mathbf{r}})$, having arbitrary shape, and as $F_{\mathrm{G}}(\hat{\mathbf{r}})$ the probability density function of the angle of arrival (AoA) of the interference. This function is assumed bivariate Gaussian, thus it can be expressed as:

$$F_{\mathrm{G}}(\hat{\mathbf{r}}) = \frac{Q_{\mathrm{G}}}{2\pi\sigma_\theta\sigma_\phi} \exp\left[-\frac{(\theta-\bar{\theta})^2}{2\sigma_\theta^2} - \frac{(\phi-\bar{\phi})^2}{2\sigma_\phi^2}\right], \tag{2}$$

where $\bar{\theta}$ and $\bar{\phi}$ are the mean AoAs in the elevation and azimuth domains, respectively, σ_θ and σ_ϕ are the standard deviations (available from measurements [16]) of the AoAs in the elevation and azimuth domains, respectively, and Q_{G} is the normalization constant, ensuring the satisfaction of the condition $\int_0^{2\pi}\int_0^{\pi} F_{\mathrm{G}}(\theta,\phi)\,d\theta d\phi = 1$.

The problem of phase-only synthesis of arbitrary power patterns having Gaussian shaped nulls can be formulated as follows:

$$\min_{\mathbf{a}} \left\{ \| |F(\mathbf{a};\hat{\mathbf{r}})| - F_0(\mathbf{a}_0;\hat{\mathbf{r}}) \|^2 \right\} \quad (3)$$

$$\text{subject to } |a_n| = a_{0n}, n = 1,\ldots,N \quad (4)$$

$$|F(\mathbf{a};\hat{\mathbf{r}})| \leq K F_{\mathrm{G}}(\hat{\mathbf{r}}), \hat{\mathbf{r}} \in \Omega_i. \quad (5)$$

where K is a constant depending on the interference level and Ω_i denotes the angular region of the Gaussian interference. In the above minimization problem, the objective function (3) allows one to make the radiation pattern as close as possible to the desired one. Constraint (4) imposes the phase-only control requirement, while constraint (5) is responsible for the interference suppression.

The minimization problem (3)-(5) is neither linear, nor convex, thus it is very difficult to solve in its present form. For these reasons, a procedure is proposed to formulate an equivalent problem, which can be solved with available methods. The proposed approach has been previously successfully applied to a two-dimensional (2D) scenario in [17], and is here extended to the complete 3D space. The procedure consists in substituting the constraint (5) with the following one:

$$F(\mathbf{a};\hat{\mathbf{r}}_m) = 0, \qquad m = 1,\ldots,M. \quad (6)$$

which imposes a discrete number M of pattern nulls that are suitably located at the directions $\hat{\mathbf{r}}_m$. This defines a new minimization problem, having the objective function in (3) and the constraints in (4) and (6). The next steps for solving the problem consist in properly defining the positions $\hat{\mathbf{r}}_m$ for $m = 1,...,M$ and then applying a suitable phase-only null-synthesis algorithm for arbitrary antenna arrays [5]. Thus, the aim of this paper is the development of a proper procedure to locate the pattern nulls so as to make the new problem equivalent to the original one.

The null localization in the 2D case has been carried out in [17] by introducing a density tapering technique that imposes an equi-area constraint. To generalize this approach to the 3D scenario, we here consider an equi-volume condition. The basic idea is to create, in the interference region Ω_i, a grid of points in such a way that the volume between the $\theta\phi$-plane and the desired upper bound $KF_{\mathrm{G}}(\hat{\mathbf{r}})$ be equally subdivided. To this aim, a rectangular grid is assumed, composed by $M = M_\theta \cdot M_\phi$ null points lying in directions (θ_p, ϕ_q), $p = 1,\ldots,M_\theta$, $q = 1,\ldots,M_\phi$. In order to find the null locations according to an equi-volume tapering, the equation:

$$\int_0^{\phi_q} \int_0^{\theta_p} F_{\mathrm{G}}(\theta,\phi)\, d\theta d\phi = \frac{p\, q}{(1+M_\theta)(1+M_\phi)}, \quad (7)$$

must be solved with respect to the unknown (θ_p, ϕ_q), which are the extrema of the integrals. Exploiting the Gaussian distribution of interference $F_{\mathrm{G}}(\theta,\phi)$ and inserting (2) into (7), one obtains, after some manipulations, the novel equation:

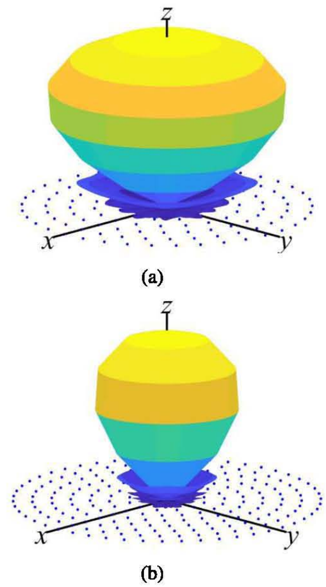

Figure 1. 3D polar plots of the synthesized patterns and of the corresponding adopted multi-ring arrays: (a) first example, (b) second example.

$$\frac{Q_G}{4}\left[\mathrm{erf}\left(\frac{\phi_q-\bar{\phi}}{\sqrt{2}\sigma_\phi}\right)+\mathrm{erf}\left(\frac{\bar{\phi}}{\sqrt{2}\sigma_\phi}\right)\right] \cdot \left[\mathrm{erf}\left(\frac{\theta_p-\bar{\theta}}{\sqrt{2}\sigma_\theta}\right)+\mathrm{erf}\left(\frac{\bar{\theta}}{\sqrt{2}\sigma_\theta}\right)\right] = \frac{p\,q}{(1+M_\theta)(1+M_\phi)}, \quad (8)$$

where $\mathrm{erf}(\cdot)$ denotes the error function. Now, (8) can be factorized in the following two equations:

$$\frac{\sqrt{Q_G}}{2}\left[\mathrm{erf}\left(\frac{\phi_q-\bar{\phi}}{\sqrt{2}\sigma_\phi}\right)+\mathrm{erf}\left(\frac{\bar{\phi}}{\sqrt{2}\sigma_\phi}\right)\right] = \frac{q}{(1+M_\phi)}, \quad (9)$$

$$\frac{\sqrt{Q_G}}{2}\left[\mathrm{erf}\left(\frac{\theta_p-\bar{\theta}}{\sqrt{2}\sigma_\theta}\right)+\mathrm{erf}\left(\frac{\bar{\theta}}{\sqrt{2}\sigma_\theta}\right)\right] = \frac{p}{(1+M_\theta)}, \quad (10)$$

which respectively have the closed-form solutions:

$$\phi_q = \bar{\phi}+\sqrt{2}\sigma_\phi \mathrm{erf}^{-1}\left[\frac{2q}{\sqrt{Q_G}(1+M_\phi)} - \mathrm{erf}\left(\frac{\bar{\phi}}{\sqrt{2}\sigma_\phi}\right)\right], \quad (11)$$

$$\theta_p = \bar{\theta}+\sqrt{2}\sigma_\theta \mathrm{erf}^{-1}\left[\frac{2p}{\sqrt{Q_G}(1+M_\theta)} - \mathrm{erf}\left(\frac{\bar{\theta}}{\sqrt{2}\sigma_\theta}\right)\right]. \quad (12)$$

These latter solutions determine the final null locations. Of course, they are not optimal, since an approximate problem is solved. Indeed, the proposed procedure is not univocal and better choices might exist also for the null localization strategy. However, the proposed strategy does not require cumbersome calculations and is hence of immediate application. Its ability to provide satisfactory results is checked in the next section.

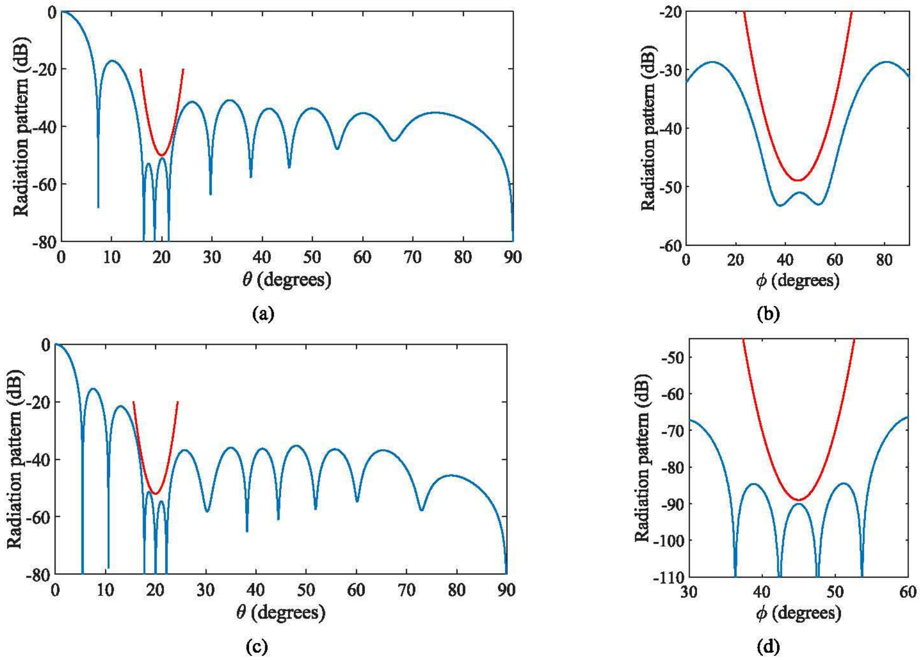

Figure 2. 2D cuts of the synthesized patterns (blue line) and of the upper bound imposed by (5) (red line): (a) cut at $\phi = \bar{\phi}$ for the first example in Fig. 1(a), (b) cut at $\theta = \bar{\theta}$ for the first example in Fig. 1(a), (c) cut at $\phi = \bar{\phi}$ for the second example in Fig. 1(b), (d) cut at $\theta = \bar{\theta}$ for the second example in Fig. 1(b).

3 Numerical results

Two numerical examples are presented. Both involve planar circular arrays with multiple rings and a minimum element distance not lower than $3\lambda/4$ [18], in such a way that mutual coupling effects can be reasonably neglected. In both cases, the reference pattern $F_0(\mathbf{a}_0;\hat{\mathbf{r}})$ is given by (1) with $p_n(\hat{\mathbf{r}}) = \cos\theta$ and $a_n = 1$ for $n = 1,\dots,N$. The first example considers an antenna array with $N = 128$ elements, which are uniformly placed on five concentric rings lying on the xy-plane and centered at the origin of the reference Cartesian system. The radii, normalized to λ, are $[3/4, 3/2, 9/4, 3, 15/4, 9/2]$ and the numbers of elements on each ring are $[5, 12, 18, 25, 31, 37]$. The mean AoAs and standard deviations are: $\bar{\theta} = 20°$, $\bar{\phi} = 45°$, $\sigma_\theta = 3.3°$, and $\sigma_\phi = 20.3°$ [16]. The rectangular grid for the null localization procedure is assumed composed by $M_\theta = 2$ and $M_\phi = 3$ points. By inserting these values in (11) and (12), one obtains the following azimuth and zenith null positions: $\phi_1 = 32.0°$, $\phi_2 = 45.5°$, $\phi_3 = 59.2°$, $\theta_1 = 18.6°$, $\theta_2 = 21.4°$.
Fig. 1(a) shows the 3D polar plot of the synthesized pattern and the geometry of the selected multi-ring array. The effectiveness of the developed null-placement strategy can be better appreciated in Figs. 2(a) and 2(b), where the 2D pattern cuts are shown in correspondence of $\bar{\theta} = 20°$ and $\bar{\phi} = 45°$, respectively. These cuts reveal that the synthesized pattern has deep nulls, which guarantee that the original constraint in (5) is properly satisfied.
The second example considers a narrower Gaussian interference. To properly satisfy this stronger requirement, two additional rings are added to the array of the previous example. The two novel rings have radii equal to $[21/4, 6]$ (still normalized to λ), and consist of $[43, 50]$ elements. These choices imply that the novel array has $N = 221$ elements. The mean AoAs and the standard deviation σ_θ are selected identical to those of the first example, while a different standard deviation $\sigma_\phi = 10.3°$ is adopted to model a narrower interference. Besides, a denser rectangular grid for the null localization procedure is assumed by adopting $M_\theta = 3$ and $M_\phi = 4$ points. By inserting these values in (11) and (12), one obtains the following azimuth and zenith null positions: $\phi_1 = 36.3°$, $\phi_2 = 42.4°$, $\phi_3 = 47.6°$, $\phi_4 = 53.7°$, $\theta_1 = 17.8°$, $\theta_2 = 20.0°$, $\theta_2 = 22.2°$.
The 3D polar plot of the obtained pattern, along with the array geometry is shown in Fig. 1(b). This second example further confirms that the developed null-placement strategy is capable of producing a pattern compliant with the original constraint in (5). In particular, the satisfactory performance can be properly observed from the cuts corresponding to $\bar{\theta} = 20°$ and $\bar{\phi} = 45°$ in Figs. 2(c) and 2(d). Hence, even in the presence of a narrower interference (note that Fig. 2(b) and Fig. 2(d) use different scales for the ϕ-axis), the null region can be properly shaped.

4 Conclusion

A deterministic procedure has been presented for solving the phase-only 3D power pattern synthesis of arbitrary antenna arrays in the presence of Gaussian interference. The original minimization problem has been replaced by an equivalent one in which a density tapering with equivolume constraints has been used for determining suitable pattern nulls capable to map the Gaussian shape of interference. The effectiveness of the method has been proved by numerical simulations for both wide and narrow nulls. The overall synthesis procedure is versatile, since it can be combined with any 3D phase-only null-synthesis algorithm and can be easily implemented with a personal laptop in quite acceptable CPU times.

References

[1] R. Hansen, "Array pattern control and synthesis," *Proceedings of the IEEE*, vol. 80, no. 1, pp. 141–151, 1992. [Online]. Available: http://ieeexplore.ieee.org/document/119573/

[2] S. Gao, Y. Rahmat-Samii, R. E. Hodges, and X.-X. Yang, "Advanced antennas for small satellites," *Proceedings of the IEEE*, vol. 106, no. 3, pp. 391–403, mar 2018. [Online]. Available: http://ieeexplore.ieee.org/document/8303877/

[3] T. Varum, J. N. Matos, P. Pinho, and R. Abreu, "Nonuniform Broadband Circularly Polarized Antenna Array for Vehicular Communications," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 9, pp. 7219–7227, sep 2016. [Online]. Available: http://ieeexplore.ieee.org/document/7328334/

[4] M. Stanley, Y. Huang, T. Loh, Q. Xu, H. Wang, and H. Zhou, "A high gain steerable millimeter-wave antenna array for 5G smartphone applications," in *2017 11th European Conference on Antennas and Propagation, EUCAP 2017*. Institute of Electrical and Electronics Engineers Inc., may 2017, pp. 1311–1314. [Online]. Available: https://ieeexplore.ieee.org/document/7928542

[5] R. Vescovo, "Null synthesis by phase control for antenna arrays," *Electronics Letters*, vol. 36, no. 3, p. 198, 2000. [Online]. Available: https://digital-library.theiet.org/content/journals/10.1049/el_20000255

[6] G. Buttazzoni and R. Vescovo, "Synthesis of co-polar and cross-polar patterns with dynamic range ratio reduction for phase-only reconfigurable arrays," in *Proceedings of 6th European Conference on Antennas and Propagation, EuCAP 2012*, Prague, Czech Republic, 2012, pp. 2623–2627. [Online]. Available: http://ieeexplore.ieee.org/document/6206586/

[7] A. A. Khan and A. K. Brown, "Null synthesis in irregularly spaced sparse antenna arrays," in *IEEE International Symposium on Phased Array Systems and Technology*, 2013, pp. 564–566.

[8] L. F. Yepes, D. H. Covarrubias, M. A. Alonso, and R. Ferrus, "Hybrid sparse linear array synthesis applied to phased antenna arrays," *IEEE Antennas and Wireless Propagation Letters*, vol. 13, pp. 185–188, 2014.

[9] G. Buttazzoni and R. Vescovo, "A deterministic approach to the synthesis of sparse arrays with far-field and near-field constraints," in *Proceedings of the 2015 IEEE-APS Topical Conference on Antennas and Propagation in Wireless Communications, IEEE APWC 2015*. Institute of Electrical and Electronics Engineers Inc., oct 2015, pp. 161–164.

[10] M. Comisso, G. Buttazzoni, and R. Vescovo, "Reconfigurable antenna arrays with multiple requirements: a versatile 3D approach," *International Journal of Antennas and Propagation*, vol. 2017, 2017.

[11] M. Khodier, "Comprehensive study of linear antenna array optimisation using the cuckoo search algorithm," *IET Microwaves, Antennas and Propagation*, vol. 13, no. 9, pp. 1325–1333, jul 2019.

[12] R. L. Haupt, "Array Pattern Synthesis with Prescribed Nulls," in *IEEE International Symposium on Phased Array Systems and Technology*, vol. 2019-Octob. Institute of Electrical and Electronics Engineers Inc., oct 2019.

[13] M.R. Akdeniz, Y. Liu, M.K. Samimi, S. Sun, S. Rangan, T.S. Rappaport, and E. Erkip, "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1164–1179, June 2014.

[14] M. Di Renzo, "Stochastic geometry modeling and analysis of multi-tier millimeter wave cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 9, pp. 5038–5057, Sep. 2015.

[15] M. Comisso and F. Babich, "Coverage analysis for 2D/3D millimeter wave peer-to-peer networks," *IEEE Transactions on Wireless Communications*, vol. 18, no. 7, pp. 3613–3627, July 2019.

[16] C. Ling, X. Yin, R. Muller, S. Hafner, D. Dupleich, C. Schneider, J. Luo, H. Yan, and R. Thoma, "Double-directional dual-polarimetric cluster-based characterization of 70âĂŞ77 GHz indoor channels," *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 2, pp. 857–870, feb 2018. [Online]. Available: http://ieeexplore.ieee.org/document/8103059/

[17] G. Buttazzoni, M. Comisso, F. Ruzzier, and R. Vescovo, "Phase-only antenna array reconfigurability with Gaussian-shaped nulls for 5G applications," *International Journal of Antennas and Propagation*, vol. 2019, pp. 1–8, feb 2019. [Online]. Available: https://www.hindawi.com/journals/ijap/2019/9120530/

[18] D. Pinchera, M. D. Migliore, F. Schettino, and G. Panariello, "Antenna arrays for line-of-sight massive MIMO: half wavelength is not enough," may 2017. [Online]. Available: http://arxiv.org/abs/1705.06804 http://dx.doi.org/10.3390/electronics6030057