Impossibility of extending the Ghirardi-Rimini-Weber model to relativistic particles

C. Jones*
Department of Physics, University of Trieste, Strada Costiera 11, 34151 Trieste, Italy
and Istituto Nazionale di Fisica Nucleare, Trieste Section, Via Valerio 2, 34127 Trieste, Italy

T. Guaita†
Max Planck Institute of Quantum Optics, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany

A. Bassi‡
Department of Physics, University of Trieste, Strada Costiera 11, 34151 Trieste, Italy
and Istituto Nazionale di Fisica Nucleare, Trieste Section, Via Valerio 2, 34127 Trieste, Italy

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Spontaneous collapse models are proposed modifications to quantum mechanics which aim to solve the measurement problem. In this article, we will consider models which attempt to extend a specific spontaneous collapse model, the Ghirardi-Rimini-Weber model (GRW), to be consistent with special relativity. We will present a condition that a relativistic GRW model must meet for three cases: for a single particle, for N distinguishable particles, and for indistinguishable particles. We will then show that this relativistic condition implies that one can have a relativistic GRW model for a single particles or for distinguishable noninteracting, nonentangled particles but not otherwise.

I. INTRODUCTION

In quantum mechanics, there are two forms of dynamics: (1) unitary evolution, which is time reversible and preserves superpositions, which describes the evolution of isolated systems, and (2) evolution described by positive operator valued measures (POVMs), which describes a system undergoing a measurement. The measurement problem is the fact that quantum mechanics fails to provide a precise description of which form of evolution describes any one situation. From observation, limits can be placed on which regimes may be described with unitary evolution or POVMs, but the theory itself does not provide these.

Quantum field theory, the version of quantum mechanics consistent with special relativity, suffers from the same conceptual issue as nonrelativistic quantum mechanics. It is a mathematical tool for calculating the probability of an outcome of a measurement given a initial condition, but it does not have prescription for when to use unitary or nonunitary dynamics.

Since its discovery, there have been many attempts to solve the measurement problem; some of the most famous suggestions include the many worlds interpretation [1,2] and Bohmian mechanics [3–5]. Both of these suggestions are experimentally indistinguishable from standard quantum mechanics.

Spontaneous collapse models, first introduced by Ghirardi-Rimini-Weber [6] and Pearle [7], solve the measurement problem by giving a new dynamics which completely describes the time evolution of the system at a nonrelativistic level. They offer different experimental predictions than standard QM, and there are currently experiments investigating these effects [8–11]. The new dynamics is defined by introducing additional stochastic nonlinear terms to the Schrödinger equation. These terms alter the form of unitary evolution such that there is a nonzero probability of the wave function describing a particle undergoing a spontaneous spatial localization. This rate is proposed to be extremely low, such that a single particle may remain in a superposition for a long period of time, in line with what is seen experimentally. However, for multiple particles which are entangled, any single particle spontaneously collapsing collapses all particles it is entangled with. This effectively increases the rate of collapse for systems with high numbers of particles, such that macroscopic bodies are localized on extremely short timescales. This is often called the amplification mechanism and it ensures macroscopic classicality. This removes the need for the theory to include a description of an external observer, as macroscopic measuring apparatus interacting with a microscopic system causes the microscopic system to become entangled and hence collapse, via the amplification mechanism. For a full review of spontaneous collapse models, see Ref. [12].

In order for a spontaneous collapse model to be a successful description of the underlying physics, it must be consistent with special relativity. There is a tension between quantum mechanics and special relativity as quantum mechanics is

*caitlinisobel.jones@phd.units.it
†tommaso.guaita@mpq.mpg.de
‡bassi@ts.infn.it

nonlocal because spacelike separated measurements of entangled systems must be correlated (as argued by Einstein, Podolsky, and Rosen in Ref. [13]). A spontaneous collapse model should predict nonlocal correlations in order to remain consistent with experiments.

This paper is concerned with the collapse model's consistency only with special relativity. From now on in this article, we will use *relativistic* to mean consistent with special relativity.

In its original formulation, the GRW model was not relativistic and described distinguishable particles with discrete points of localization. Continuous-time collapse models have also been developed, for instance, in Refs. [14,15]. There are various proposed models for relativistic collapse models: Ref. [16], where a prescription for the probability distribution of a matter density operator is Lorentz invariant; Ref. [17], which introduces a mediating pointer field; Ref. [18], in which collapse dynamics emerge by tracing out an environment from a relativistic quantum field theory; and Ref. [19], which proposes that the terms modifying the conventional Schrödinger equation are functions of the stress-energy tensor. Pearle suggested a model [20] and a proposed alteration of this in Ref. [21], where energy is conserved by considering relational collapses. In Ref. [22], a collapse model on a $1+1$ lattice is presented and the authors suggest that it may be relativistic in the continuum limit.

In this paper, we will ask if it is possible for GRW to be made consistent with relativity while retaining its characteristic features.

For single particles, distinguishable and indistinguishable particles we will apply the conditions for consistency with relativity to the case of GRW and consider if such conditions permit models which give rise to collapses which localize the wave function and cause classical behavior to emerge at large scales. We discuss where an existing model fits into this framework [23]. Something that has been under analyzed in the relativistic collapse model literature is the fact that in order to be consistent with special relativity is it not sufficient to only ask that the dynamics are Lorentz covariant, it is also required that initial conditions between two inertial frames can be compared. In this article, we pay special attention to this fact and show how this limits the possible extensions of GRW.

This paper is organized as follows: In Sec. II, the relationship between special relativity and quantum mechanics is reviewed and the Tomogana-Schwinger formalism is discussed. In Sec. III, the original GRW model is introduced in the Tomogana-Schwinger formalism. In Sec. IV, relativistic conditions for single-particle distinguishable and indistinguishable particle GRW models are given. For the indistinguishable case, it is shown that a such a model is either not relativistic or does not achieve macroscopic classicality.

II. QUANTUM MECHANICS AND SPECIAL RELATIVITY

Standard quantum mechanics (ignoring the measurement problem) provides probability distributions for the values of observables that are measured. A relativistic quantum mechanics must predict that observers in any two inertial frames have the same measurement statistics for the outcome of any experiment they can perform. This is the conclusion reached in Ref. [24] by Aharonov and Albert. They state that for a system with observables $A, B, C...$ each with potential values $a, b, c...$ where observable A is measured at time t_a and found to have the value a, and other variables respectively, then agreement with special relativity implies that there is a covariant way of calculating the probability P of $P_{t_a,t_b}(a, b, ...|c, t_c, d, t_d...)$, i.e.,

$$P_{t_a,t_b}(a, b, ...|c, t_c, d, t_d...) = P_{t'_a,t'_b}(a', b', ...|c', t'_c, d', t'_d...). \tag{1}$$

where a', b', etc. are the values of the observables in the coordinates of a different inertial frame. This condition is *stronger* than only requiring that the equations of motion transform covariantly, as in order to check this condition *one must be able to compare the initial conditions in each frame*.[1] This requirement is inline with the usual definition of Lorentz covariance for quantum field theory [25], Chapter 3], where is stated that for Lorentz covariance of a theory there must be an explicit rule for one observer to find their state of the system given the state of the same system in a different inertial frame (and, of course, the dynamics must be Lorentz invariant).

We choose to use this definition instead of other ways of characterising *relativistic* as it ensures that observers in any two frames are guaranteed to obtain the same result of an experimental run.

Probability distributions in quantum mechanics are found from the wave function via the Born rule. For nonrelativistic quantum mechanics, single-particle wave functions are functions over every point in spacetime. However, if one wishes to have a relativistic quantum mechanics where the wave function undergoes instantaneous collapses triggered by measurements, then this is not possible, as this implies that the wave function will not be normalized on a constant time hyperplane in some inertial frames; see Fig. 1. Also, a preferred frame is selected, the one where the collapse occurs instantaneously. On the other hand, instantaneous collapses are required to ensure that nonlocal observables (for example, momentum or total charge) are conserved [24,26]. Additionally, instantaneous collapse of the state vector is required to ensure that Bell's inequalities are violated. This is because in order for the outcomes of the Bell experiment to be perfectly correlated even though the results are from spacelike measurements, the two wings of the experiment must effect each other instantly, and hence the wave function must collapse instantly.[2]

In order to offer a frame-independent description of instantaneous collapse of the wave function, Aharonov and Albert proposed an alternative way of describing the collapse when

[1]This requirement for consistency with Special Relativity (SR) in the active transformation rather that the passive viewpoint, because to us it seems the most natural viewpoint when considering stochastic dynamics.

[2]Hence, proposals like in Ref. [27] where collapse only effects the future light cone fail as they do not predict nonlocal correlations between outcomes of experiments. Since these nonlocal correlations are observed in nature, then any successful theory must predict them.

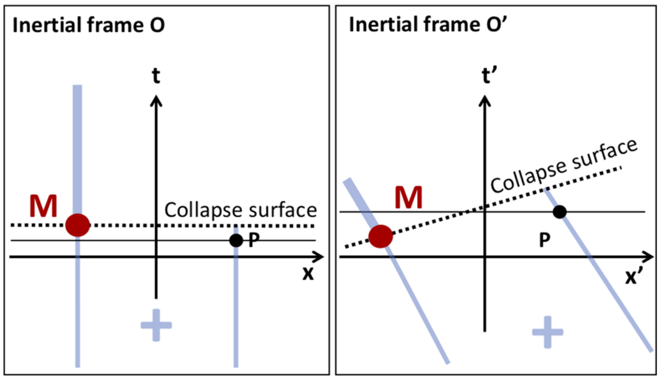

FIG. 1. A spacetime diagram showing the support of the wave function before and after a measurement M where the wave function is a function over all of spacetime. The support is the shaded line with the amplitude proportional to the thickness of the line. The point P is a spacetime point of interest. Suppose in one frame (left figure) the wave function is initially in a spatial superposition (as seen in that the support is present in two places and as denoted with the plus); then M occurs and the wave function collapses along a specific constant time hypersurface (dotted line). The wave function on the surface intersecting point P (thin black line) is normalized. However, in a different inertial frame (right figure) if the collapse occurs along the *same* hypersurface (dotted line), then the wave function on the constant time hyperplane intersecting point P in the new frame (thin black line) is not normalized.

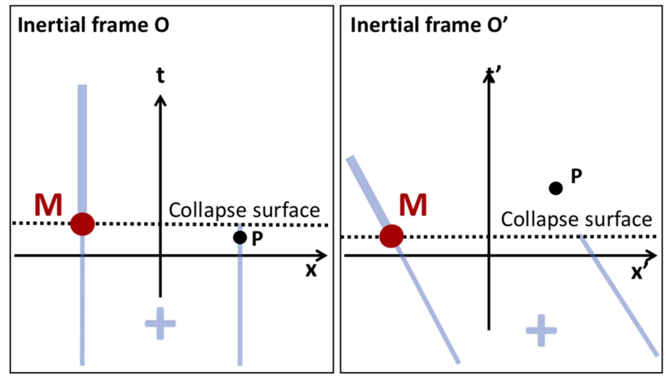

FIG. 2. A spacetime diagram showing the support of the wave function before and after a measurement M. Here, in every inertial frame the wave function collapses on a constant time hypersurface (dotted line) so that the wave function is always normalized for all observers. Note that the amplitude of the wave function at P in different frames differs; this is a consequence of treating the wave function as a function on a 3D hypersurface.

a measurement is performed [28], in which the wave function collapses instantaneously in *every* inertial frame. To allow the wave function to collapse instantaneously in every frame, it must be defined not on the four-dimensional (4D) manifold but on spacelike three-dimensional (3D) hypersurfaces which make up the manifold. Then the wave function and hence normalized state in a Hilbert space can be defined on each hypersurface.

Wave functions are defined on spacelike hypersurfaces; if we label a hypersurface as ω, then we can write a wave function on it as $\psi_\omega(x)$. The coordinate x here labels the coordinates of the 3D surface ω but is a four-vector $x \in \mathbb{M}^4$ as ω is understood to be embedded in 4D spacetime. So then every inertial observer has a wave function defined on their constant time 3D hypersurface. However, each state may have different values at the same specific space-time point X so that $\psi_\omega(X) \neq \psi_{\omega'}(X')$, where X and X' are the same point in two different inertial frames. This allows wave functions to be normalized in every frame; see Fig. 2. This is acceptable because the wave function in this framework has no ontological meaning; it is simply a tool for calculating probabilities for the value of observables.

In this framework, every inertial observer can describe the time evolution of their system in terms of wave functions on parallel constant-time hypersurfaces within their frame using the Tomogana-Schwinger formalism. We will introduce this formalism and show that if collapses are excluded, then this description is Lorentz covariant if it is integrable. Then for the case of quantum mechanics with measurements we will derive a condition on the measurement operator for Lorentz covariance in this framework.

A. The Tomogana-Schwinger formalism

The Tomogana-Schwinger formalism [29,30] describes unitary evolution as maps between wave functions defined on arbitrary spacelike hypersurfaces without collapses. First, we will introduce some additional notation for hypersurfaces. Let ω signify any generic spacelike 3D hypersurface, let σ_t denote a constant-time hyperplane at time t in a inertial frame $\mathcal{F}$, and hence $\sigma'_{t'}$ is a constant-time hyperplane in a different inertial frame $\mathcal{F}'$. Then suppose the wave function is defined on an ω in the manifold $\mathbb{M}^4$. In this article, we restrict ourselves to considering Minkowski spacetime $\mathbb{M}^4$ as it is sufficient to see the relevant Lorentz transformation properties of the probability distributions. Then in inertial frame $\mathcal{F}$ which has coordinates x on a hypersurface ω the wave function is $\psi_\omega(x)$. In another inertial frame $\mathcal{F}'$ with coordinates x' on the same hyperplane ω the wave function is written $\psi'_\omega(x')$. A wave function under a Lorentz boost transforms as

$$\psi_\omega(x) \to \psi'_\omega(x') = \Lambda\psi_\omega(x), \tag{2}$$

where Λ is a representation of the Lorentz group. In other words, on the same hypersurface the wave functions are equivalent up to a Lorentz transform.

Analogously to the Schrödinger equation, Tomogana and Schwinger defined the evolution of a wave function as it evolves between hypersurfaces, if there are no measurements between those surfaces:

$$\frac{\delta}{\delta\omega(x)}\psi_\omega(x) = -i\mathcal{H}(x)\psi_\omega(x), \tag{3}$$

where $\frac{\delta}{\delta\omega(x)}$ is the functional derivative with respect to ω and $\mathcal{H}(x)$ is the Hamiltonian density. The functional derivative can be understood to be the variation in $\psi_\omega(x)$ with respect to a infinitesimal variation of ω about point x; see Fig. 3. The integrability condition for this system is that $[\mathcal{H}(x), \mathcal{H}(y)] = 0$ if x and y are spacelike separated. Equation (3) gives rise to an unitary evolution operator which relates two hypersurfaces:

$$U_{\omega_1}^{\omega_2} = T\ \exp\left[-i\int_{\omega_1}^{\omega_2} d^4x\mathcal{H}(x)\right] \tag{4}$$

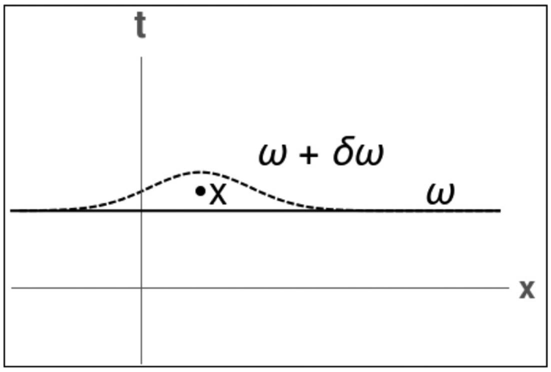

FIG. 3. A diagram showing the infinitesimal variation, $\delta\omega$, of the hypersurface ω about the point x.

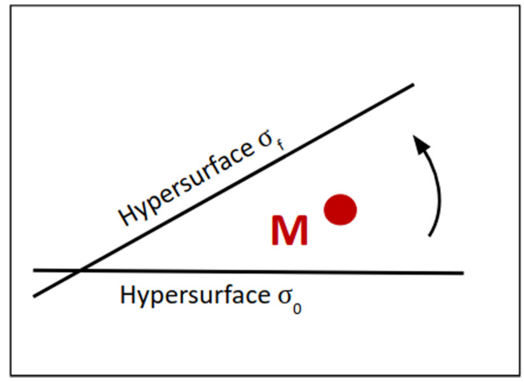

FIG. 4. A schematic showing a measurement at a point M between two hypersurfaces σ_0 and σ_f. It is not possible to relate wave functions on σ_0 and σ_f without knowing if there are measurements between them.

such that $\psi_{\omega_2}(x) = U_{\omega_1}^{\omega_2}\psi_{\omega_1}(x)$, where T means time ordering with respect to the frame $\mathcal{F}$. This operator is frame independent although $\mathcal{H}(x)$ is not Lorentz invariant; the only frame-dependant terms from the time ordering are zero due to the integrability condition [31,32]. Therefore, we have that for a frame $\mathcal{F}'$

$$U'^{\omega_2}_{\omega_1} = T' \exp\left[-i\int_{\omega_1}^{\omega_2} d^4x'\mathcal{H}'(x')\right] \tag{5a}$$

$$= \Lambda^\dagger U_{\omega_1}^{\omega_2}\Lambda. \tag{5b}$$

B. The Tomogana-Schwinger formalism with measurements

Now we wish to extend this formalism to describe collapses of the wave function due to measurements. In this article, we will consider only collapses in the spatial basis as this is sufficient to explain the values of any experiment performed, as any observable can be coupled to position [33].

In a frame $\mathcal{F}$, the spatial collapse of the wave function at $x \in \mathbb{M}^4$ is described though an operator $\hat{L}_\omega(x)$ defined on the Hilbert space on a spacelike hypersurface ω passing through x. Following Albert and Aharanov, we consider that the collapse occurs on the constant time hyperplane intersecting x, labeled σ_t, where $t = x_0$. This means that the collapse is described as occurring instantaneously in $\mathcal{F}$, as discussed in Sec. II.

$\hat{L}_{\sigma_t}(x)$ localizes the particle it acts on about x (if the wave function is not already localized). The properties of this operator are model dependant; however, in general it is not unitary. In a different frame $\mathcal{F}'$, the collapse operator $\hat{L}'_{\sigma'_{t'}}(x')$ is defined on a constant time hypersurface $\sigma'_{t'}$.

To illustrate evolution with collapses consider in a frame $\mathcal{F}$ two hypersurfaces σ_0 and σ_f before and after a collapse at a point x; see Fig. 4. The wave function ψ_{σ_f} is found by evolving the wave function to a hyperplane of collapse, applying the collapse operator and normalizing, then evolving to σ_f:

$$\psi_{\sigma_0} \to \psi_{\sigma_f} = \frac{U_{\sigma_t}^{\sigma_f}\hat{L}_{\sigma_t}(x)U_{\sigma_0}^{\sigma_t}\psi_{\sigma_0}}{\|\hat{L}_{\sigma_t}(x)U_{\sigma_0}^{\sigma_t}\psi_{\sigma_0}\|}. \tag{6}$$

It is necessary that *all points of collapse between σ_0 and σ_f are known in order to construct such a map between them*, as in general $\hat{L}_{\sigma_t}(x)\psi_\omega \neq \psi_\omega$ for any ω. Therefore, in order to relate wave functions in different frames on their respective constant time hypersurfaces, all collapses between those hypersurfaces must be known.

To find the condition on $\hat{L}_\sigma(x)$ for consistency with relativity, we consider the probability $P(x_1, x_2|\psi_{\sigma_1})$, which is Eq. (1) applied to the case of two measurements at space-time points x_1 and x_2 given an initial wave function ψ_{σ_1}[3]. σ_1 is a constant-time hypersurface intersecting the point x_1 in $\mathcal{F}$. For quantum mechanics with measurements, the wave function ψ_{σ_1} can be assumed to be specified by measurements in the past of σ_1. Then SR implies that

$$P(x_1, x_2|\psi_{\sigma_1}) = P(x'_1, x'_2|\psi'_{\sigma'_{1'}}). \tag{7}$$

If the points x_1 and x_2 are timelike to each other and x_1 occurs before x_2 in all frames, the conditional probability for one frame is given by

$$P(x_1, x_2|\psi_{\sigma_1}) = \left\|\hat{L}_{\sigma_2}(x_2)U_{\sigma_1}^{\sigma_2}\hat{L}_{\sigma_1}(x_1)\psi_{\sigma_1}\right\|^2. \tag{8}$$

To compare the two sides of Eq. (7), the relationship between the wave functions ψ_{σ_1} and $\psi'_{\sigma'_{1'}}$ must be specified. If there are no measurements (hence no collapses) between the two hypersurfaces, then they can be related by

$$\psi'_{\sigma'_{1'}} = \Lambda^\dagger U_{\sigma_1}^{\sigma'_{1'}}\psi_{\sigma_1}. \tag{9}$$

If there are measurements between ψ_{σ_1} and $\psi'_{\sigma'_{1'}}$, then the wave functions can be related with Eq. (6) when measurement occurs and Eq. (9) for subsequent evolution, using the appropriate positions and outcomes of measurements. In standard quantum mechanics, this is acceptable as it includes the concept of observers performing measurements and recording the results. So all measurements between the two surfaces can be compared between two frames. Assuming that the

[3] To keep notation simple and to highlight the invariance requirements, we write $P_{t_1}(\mathbf{x}_1, |\psi_{\sigma_1})$ as $P(x_1, |\psi_{\sigma_1})$; however, as x_1 is a space-time point of measurement, the equation below should be understood in the same way as Eq. (1).

Hamiltonian is covariant so that Eq. (5b) holds, then the right-hand side of Eq. (7) can be written as

$$P(x'_1, x'_2|\psi'_{\sigma'_{1'}}) \tag{10a}$$

$$= \left\| \hat{L}'_{\sigma'_{2'}}(x'_2) U'^{\sigma'_{2'}}_{\sigma'_{1'}} \hat{L}'_{\sigma'_{1'}}(x'_1) \psi'_{\sigma'_{1'}} \right\|^2 \tag{10b}$$

$$= \left\| \hat{L}'_{\sigma'_{2'}}(x'_2) \Lambda^\dagger U^{\sigma'_{2'}}_{\sigma_2} U^{\sigma_2}_{\sigma_1} U^{\sigma_1}_{\sigma'_{1'}} \Lambda \hat{L}'_{\sigma'_{1'}}(x'_1) \Lambda^\dagger U^{\sigma'_1}_{\sigma_1} \psi_{\sigma'_1} \right\|^2, \tag{10c}$$

where Eq. (5b) has been used to transform the unitary operators, $\sigma'_{1'}$ and $\sigma'_{2'}$ are hypersurfaces of collapse intersecting x_1 and x_2 in frame $\mathcal{F}'$, and x' is the same spacetime point in a different coordinate system. So, by inspection, the condition for Eq. (7) to hold is

$$\hat{L}'_{\sigma'_{i'}}(x') = \Lambda^\dagger U^{\sigma'_{i'}}_{\sigma_i} \hat{L}_{\sigma_i}(x) U^{\sigma_i}_{\sigma'_{i'}} \Lambda. \tag{11}$$

Equation (11) requires that the collapse operator transforms covariantly and that the collapse can be described by an operator acting on any spacelike hypersurface intersecting x. This is equivalent to requiring that the collapse happens instantaneously in all inertial frames.

If instead x_1 and x_2 are spacelike to each other, then in some frames their time ordering may be reversed. In this case, if in $\mathcal{F}'$ x_1 precedes x_2, then Eq. (8) holds and in the primed frame we have

$$P(x'_1, x'_2|\psi'_{\sigma'_{1'}}) = \left\| \hat{L}'_{\sigma'_{1'}}(x'_1) U'^{\sigma'_{1'}}_{\sigma'_{2'}} \hat{L}'_{\sigma'_{2'}}(x'_2) U'^{\sigma'_{2'}}_{\sigma'_{1'}} \psi'_{\sigma'_{1'}} \right\|^2. \tag{12}$$

Substituting in Eqs. (9) and (11), it is found that for Eq. (7) to be satisfied

$$[\hat{L}_{\sigma_1}(x_1), U^{\sigma_1}_{\sigma_2} \hat{L}_{\sigma_2}(x_2) U^{\sigma_2}_{\sigma_1}] = 0, \tag{13}$$

which is met if $\hat{L}_\sigma(x)$ satisfies the microcausality condition.

As discussed in Sec. II, the wave function is a tool to calculate probabilities and in order to be consistent with special relativity the wave function must collapse instantaneously in every inertial frame. Therefore, although we have written the collapse operator as acting on a constant time hypersurface in a particular frame, it could be written as a collapse operator acting on the Hilbert space of any spacelike surface using the relationship

$$\hat{L}_{\sigma_t}(x) = U^{\sigma_t}_{\omega} \hat{Q}_\omega(x) U^{\omega}_{\sigma_t}, \tag{14}$$

where ω is any arbitrary spacelike hypersurface intersecting the point x and $\hat{Q}_\omega(x)$ is a collapse operator like $\hat{L}_\omega(x)$ that also satisfies Eqs. (11) and (13).

In order to check that Eq. (7) is satisfied, it has been implicitly assumed that in any one frame the time ordering between x_1 and x_2 is known. Otherwise, it would not have been possible to write the explicit expressions of Eqs. (8), (10), and (12).

As mentioned already, standard quantum mechanics has the concept of observers comparing results, which means that the order of measurements can be known between frames. If in one frame observer A measures x_1 to be before x_2 and in another frame observer B measures the inverse, then A and B can reconcile their conditional probability distributions and check consistency with SR. In this section, we have found the condition for relativistic collapse using the conditional probability for two collapses; however, it can be easily shown that this applies to any number of collapses.

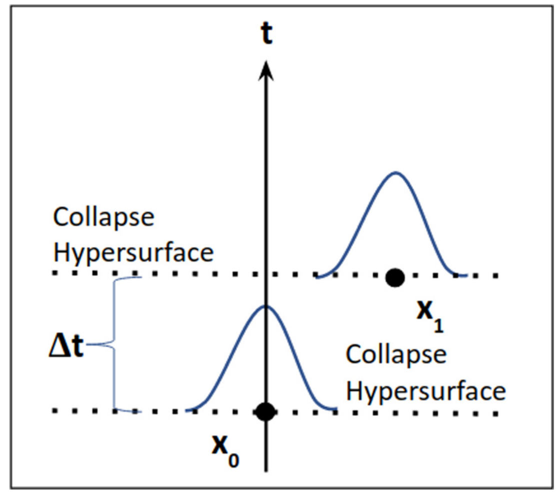

FIG. 5. Surfaces of collapse for the GRW model. The dotted black lines show the hypersurfaces where collapses occur. The solid lines show the amplitude of the wave function immediately after collapse. The initial collapse, x_0, is assumed to be at the origin; the next collapse will occur on a flat hypersurface after Δt.

III. THE ORIGINAL GRW MODEL

We will now describe the original nonrelativistic GRW model. The original GRW model is a model for N distinguishable particles. For each particle, the initial condition is the first point of collapse x_0 and wave function ψ_0 at that time of collapse. The model then gives the probability distribution for the next point of collapse given the previous point. Hence, the model is Markovian and each particle has a series of collapses. For simplicity, here we describe the model for a single particle, using relativistic language to make clearer the similarities and differences between the original GRW model and its relativistic generalization.

Collapses occur randomly in time and are a realization of a Poisson point process with mean time τ. Let ΔT_i be the time interval between the $(i-1)$th and the ith collapse. Since time intervals are absolute in Galilean relativity, there is no need to specify with respect to which frame they are defined. This situation will change when we will consider relativistic generalizations.

Consider a one-particle wave function ψ_{σ_0} defined on some initial hyperplane σ_0, where the first collapse has occurred; we are now specializing the description to a frame where σ_0 refers to time $t = 0$.

The next collapse will occur on the hypersurface $\sigma_{\Delta t}$, where $\Delta t = \Delta T_1$ as shown in Fig. 5. The probability distribution for a collapse to occur at a point $\mathbf{x}_1 \in \mathbb{R}^3$ on this surface is

$$P(\mathbf{x}_1|x_0, \Delta t, \psi_{\sigma_0}) = \left\| \hat{L}_{\sigma_{\Delta t}}(\mathbf{x}_1) U^{\sigma_{\Delta t}}_{\sigma_0} \psi_{\sigma_0} \right\|^2, \tag{15}$$

where x_0 is the spacetime coordinates of the first point of collapse, $U^{\sigma_{\Delta t}}_{\sigma_0}$ is the standard unitary evolution between time 0 and time Δt, generated by the Schrödinger equation, and

$$\hat{L}_{\sigma_x}(\mathbf{x}) := \left(\frac{\alpha}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{\alpha(\mathbf{x}-\hat{\mathbf{q}})^2}{2}\right], \tag{16}$$

where α is a free parameter of the model, σ_x is the hyperplane intersecting the point $(\mathbf{x}, t)$, and $\hat{\mathbf{q}}$ is the position operator of the particle. This distribution is normalized such that

$$\int_{\mathbb{R}^3} d^3\underline{x}\, P(\mathbf{x}_1|x_0, \Delta t, \psi_{\sigma_0}) = 1. \tag{17}$$

Immediately after a collapse, the wave function is localized about the point of collapse, as given by Eq. (6), which in this case reads

$$\psi_{\sigma_{\Delta t}} \to \psi^{(c)}_{\sigma_{\Delta t}} = \frac{\hat{L}_{\sigma_{\Delta t}}(\mathbf{x}) U^{\sigma_{\Delta t}}_{\sigma_0} \psi_{\sigma_0}}{\|\hat{L}_{\sigma_{\Delta t}}(\mathbf{x}) U^{\sigma_{\Delta t}}_{\sigma_0} \psi_{\sigma_0}\|}. \tag{18}$$

Through this spontaneous collapse, spatial superpositions are destroyed and hence classicality can emerge. In between collapses, the wave function evolves according to the Scrödinger equation.

The above rules define *when*, *where,* and *how* the collapses occur. We will use the same logic to define the relativistic generalization of the GRW model.

IV. GRW AND SPECIAL RELATIVITY

The GRW model is a discrete time model which provides a conditional probability distribution for the position of a spontaneous collapse given the position of previous collapses. As the model is Markovian, the conditional probability for a collapse only depends on the most recent collapse, not the whole prior series of collapses. We claim that any extension of GRW to the relativistic regime must provide a prescription for calculating the probability of the next point of collapse given the position of the previous point and that this probability distribution must be Lorentz invariant. Note that this definition of Markovianity assumes that there is a time ordering for the points of collapse.

Additionally, a relativistic GRW model must cause spatial superpositions of particles to collapse, and for $N > 1$ particles it must have an amplification mechanistic to ensure emergence of macroscopic classicality.

We note that for a relativistic GRW model, an initial seed point of collapse must be given to define the model. This initial point breaks the Poincaré covariance, and hence the appropriate symmetry group is the Lorentz group.

As already remarked on in Sec. II, for a spontaneous collapse model to be relativistic *both* the initial conditions between inertial frames must be comparable *and* the dynamics must be Lorentz covariant. The literature focused in ensuring the second request to be satisfied, while here we will show that the first one in general is not, apart from specific situations. More specifically, in this section we will consider what these requirements imply for the form of a relativistic spontaneous collapse model for a single particle, distinguishable particles, and indistinguishable particles.

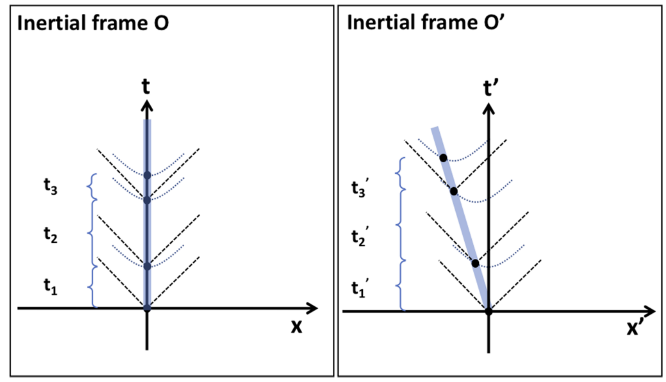

FIG. 6. A schematic diagram showing how the stochastic process defines intervals between collapses. The shaded area shows the maximum of the wave function's density, straight dotted lines show the future light cone of each point of collapse, and the curved dotted lines show the surfaces of constant four-distance from the previous point of collapse. The left diagram shows a frame where each collapse occurs at the same spatial point so the coordinate time and the four-distance coincide, $\Delta T_i = t_i$. The right diagram shows a different inertial frame where the four-distance between each point of collapse is still ΔT_i but the coordinate t_i' time is different.

A. Relativistic condition for a single particle

We consider a relativistic GRW model for a single particle. For a single particle, there is a single series of collapses. In analogy with the original GRW model, one is tempted to define the times at which, in a given frame, collapses occur via a Poissonian distribution with average time τ, but then one is faced with the fact that due to time dilation this prescription is not Lorentz invariant. In order to overcome this difficulty, the time intervals between collapses have to be defined in terms of Lorentz-invariant timelike four-distances; this seems to be the only way to ensure that the time intervals are defined in a frame-independent way. The four-distances have to be timelike not only because we are seeking a sequences of time intervals, but also because this prescription allows to define a time-ordered sequence of collapses. This is done, for example, in Ref. [23].

Consider then a Poissonian point process with average τ, with initial value 0. Let ΔT_i be the distance between the ith and $(i-1)$th point of the process. Then define the times at which collapses occur as follows. Given the initial point of collapse $x_0 = (\mathbf{x}_0, x_0^0)$, the next point of collapse $x_1 = (\mathbf{x}_1, x_1^0)$ will occur at four-distance ΔT_1 from x_0, and therefore x_1 will be on the future hyperboloid defined by all points with same *timelike* four-distance $\Delta T_1 = |x_1 - x_0|$ from x_0; the following point of collapse $x_2 = (\mathbf{x}_2, x_2^0)$ will lie in the future hyperboloid defined by all points with same timelike four-distance $\Delta T_2 = |x_2 - x_1|$ from x_1, and so on. See Fig. 6.

The four-distances among consecutive collapses have an interesting physical interpretation. Consider a particle whose wave function is well localized in an inertial reference frame O where the particle is at rest, for simplicity in the origin. In that frame, collapses are likely to occur only about the origin (where the wave function is nonzero), and the four-distances ΔT_i between consecutive collapses corresponds to the *coordinate* time intervals Δt_i between collapses. In a different inertial frame O', the particle will be moving, and while the four-distances among the collapses do not change, the coordinate time intervals $\Delta t_i'$ are dilated. The opposite would be true for a well-localized particle at rest with respect to O', thus in motion with respect to O. Therefore, the four-distances ΔT_i roughly correspond to the coordinate time intervals in the frame at rest with respect to the particle; in all other frames, the coordinate time intervals between collapses undergo time dilation. So observers measure different rates of collapse in different frames due to time dilation, but the overall prescription of the rate of collapse is frame independent. This is analogous to the situation in particle physics where a particle with a half-life λ, for example, a muon, appears to have a longer half-life when it is traveling at a high velocity inside a particle accelerator.

The prescription above defines, in a relativistic invariant way, *when* a collapse occurs. The model must also define *where* on the hyperboloid the collapse occurs, i.e., give a normalized probability distribution for the position of the collapse on that hypersurface, such as done in Ref. [23], for example. In the spirit of GRW, this probability distribution must be equal to $P_\Sigma(x) = \|\hat{L}_\Sigma(x)\psi_\Sigma\|^2$ in order to avoid superluminal signaling, where $\hat{L}_\Sigma(x)$ is the collapse operator centered around the point of collapse x, defined on the hyperboloid Σ. We can leave $\hat{L}_\Sigma(x)$ unspecified, but it has to be chosen in such a way that it localizes the wave function, it is Lorentz covariant, and that the probability is correctly normalized. However, once it is specified, it defines the collapse operator on all spacelike hypersurfaces through Eq. (14).

The last ingredient is *how* a collapse occurs, i.e., how the wave function changes due to a sudden collapse at x. Reference [23] assumes that the wave function collapses along the hyperboloid previously introduced; this is mathematically implemented by applying $\hat{L}_\Sigma(x)$ to ψ_Σ, and then normalizing the collapsed wave function. In fact, the collapses can be carried out with respect to *any* spacelike hypersurface containing the point of collapse as the two prescriptions can be related by a unitary transformation. Specifically, suppose the collapse is defined to occur along a spacelike hypersurface ω_1 according to the prescription

$$\psi_{\omega_1} \to \psi_{\omega_1}^{(c)} = \frac{\hat{L}_{\omega_1}(x)\psi_{\omega_1}}{\|\hat{L}_{\omega_1}(x)\psi_{\omega_1}\|}, \tag{19}$$

where x is the point of collapse. Given a second spacelike hypersurface ω_2 containing the point of collapse x, since $\psi_{\omega_2} = U_{\omega_1}^{\omega_2}\psi_{\omega_1}$ for the wave function prior to the collapse, and $\psi_{\omega_2}^{(c)} = U_{\omega_1}^{\omega_2}\psi_{\omega_1}^{(c)}$ for the wave function after the collapse (because by construction there are no collapses in between ω_1 and ω_2 apart from x, since all collapses are assumed to be timelike separated with respect to each other), then Eq. (19) can be equivalently rewritten as

$$\psi_{\omega_2} \to \psi_{\omega_2}^{(c)} = \frac{\hat{L}_{\omega_2}(x)\psi_{\omega_2}}{\|\hat{L}_{\omega_2}(x)\psi_{\omega_2}\|}, \tag{20}$$

with $\hat{L}_{\omega_2}(x) = U_{\omega_1}^{\omega_2}\hat{L}_{\omega_1}(x)U_{\omega_2}^{\omega_1}$. See Fig. 7. Also the probability distribution $P_\Sigma(x)$ previously defined can be computed along any spacelike hypersurface passing through x, since

$$P_\Sigma(x) = \|\hat{L}_\Sigma(x)\psi_\Sigma\|^2 = P_\omega(x) = \|\hat{L}_\omega(x)\psi_\omega\|^2 \equiv P(x), \tag{21}$$

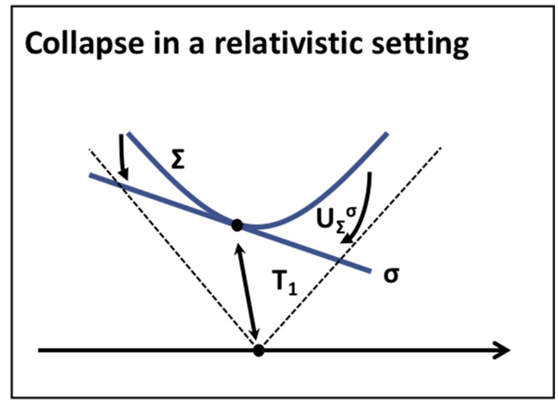

FIG. 7. Schematic diagram showing possible surfaces of collapse in relativistic GRW. The curved thick line labeled Σ is a hyperboloid of made up of points four-distance ΔT_1 from the previous point of collapse. The collapse operator can be defined on the surface Σ or equivalently on the hyperplane σ (the straight thick line labeled σ) via the operator U_Σ^σ.

as one can easily check. It is in this sense that we can say that the collapse can be described consistently in all frames.

Therefore, we are precisely in the same situations envisaged by Albert and Aharonov: *A collapse occurs instantaneously along all spacelike hypersurfaces intersecting the point of collapse*, with the only (important) difference that there the collapses are triggered by measurements, while here they are part of the dynamical law. As pointed out by Albert and Aharonov, this is necessary so that every inertial observer can provide a normalized wave function both before and after the collapse on their constant-time hyperplanes. Constant-time hyperplanes are important because these are the hypersurfaces where observers describe their physics.

It is for this reason that the model presented in Ref. [34] is not a successful relativistic model, as this model has the wave function collapse only in the future light cone of the point of collapse. This means that the state is not normalized along different constant time hyperplanes and hence the theory does not give normalized probability distributions for systems with entangled particles.

We argue that the only consistent way to understand the model in Ref. [23] is that the wave function collapses on every hypersurface intersecting the point of collapse; i.e., it collapses instantly in every frame. This is the only consistent way to interpret the model as otherwise the wave functions on hyperplanes after a point of collapse would be ill defined. This in agreement with Eq. (37) of Ref. [23], which gives the wave function on a constant-time hypersurface for given foliation of spacetime.

To see how an inconsistency would arise otherwise, consider a collapse at point x and three hypersurfaces of interest, a hyperboloid Σ intersecting x, a hyperplane σ_{t_1} intersecting the point x, and a hyperplane σ_{t_2} a short time in the future of σ_{t_1}. The state ψ_Σ will be the collapsed state. Suppose that collapses only occur on hyperboloids and then the state on

σ_{t_1} would be uncollapsed. Now the question is the following: What is the state on σ_{t_2}? As there is unitary dynamics everywhere except on the hyperboloid, then the state can either be written as $\psi_{\sigma_{t_2}} = U^{\sigma_2}_{\sigma_1}\psi_{\sigma_{t_1}}$, meaning $\psi_{\sigma_{t_2}}$ would be an uncollapsed state or as $\psi_{\sigma_{t_2}} = U^{\sigma_2}_{\Sigma}\psi_{\Sigma}$, meaning $\psi_{\sigma_{t_2}}$ would be collapsed. The only way to resolve this inconsistency and still have unitary dynamics is to have the collapse occur along σ_{t_1} as well.

Now we are in the position to assess whether this framework for a relativistic GRW model is consistent with special relativity. Given the initial wave function ψ_{σ_0} defined on a spacelike hyperplane σ_0 and the initial point of collapse x_0 on σ_0, the probability for the next collapse to occur at x is given by

$$P(x_1|x_0, \psi_{\sigma_0}) = \left\| \hat{L}_{\omega}(x_1) U^{\omega}_{\sigma_0} \psi_{\sigma_0} \right\|^2, \tag{22}$$

where ω is a surface intersecting x_1. This conditional probability distribution is analogous to Eq. (1), where ψ_{σ_0} gives all the possible information about the system at (x_0, t_0) based on the position of previous collapses.

For a Lorentz-transformed inertial frame $\mathcal{F}'$ with coordinates x', the initial conditions are the point of last collapse x_0' and the wave function on the hyperplane $\sigma'_{0'}$. Therefore, special relativity requires that

$$P(x_1|x_0, \psi_{\sigma_0}) = P(x_1'|x_0', \psi'_{\sigma'_{0'}}). \tag{23}$$

Here the condition of Eq. (1) is applied to spontaneous collapse models where the measurements replaced by points of spontaneous collapse.[4] For a single particle, the wave function is specified by the point of last collapse x_0. As described in Sec. II, in order to check this condition one must be able to *compare the initial conditions* (here the wave function and position of the previous collapse) *between different inertial frames*, as noted in Ref. [31]. This has consequences when considering collapse models for multiple particles, as we will see in Secs. IV B and IV C.

In order to verify Eq. (23), the map between ψ_{σ_0} defined on the constant-time hyperplane σ_0 for O and $\psi'_{\sigma'_{0'}}$ defined on the constant-time hyperplane $\sigma'_{0'}$ for O' must be known, and in order to do this, positions of all collapses between those surfaces must be known. For a series of *timelike* collapses, this condition is met as there can be no collapses between σ_0 and $\sigma'_{0'}$ (see Fig. 8); hence, the two hyperplanes are related by Eq. (9). By the same argument presented in Sec. II B, the collapse operator $\hat{L}_{\sigma_t}(x)$ must transform as in Eq. (11) and obey Eq. (13).

If these conditions are met, these spatial collapses which are timelike to one another may be described in a way that is consistent with special relativity for a single particle. The model proposed by Ref. [23] for a single particle meets these conditions.

In contrast, for a single-particle theory with collapses which are *spacelike* to each other (we do not discuss how such a model could be formulated), the initial wave function in different inertial frames can no longer be related to each other

[4]We will keep the notation of Sec. II B with the understanding that now the time coordinate of x is now probabilistic.

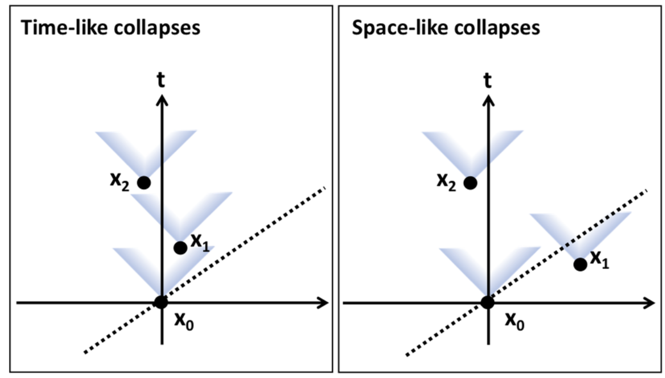

FIG. 8. For timelike collapses (left), initial conditions between two inertial frames can always be related by unitary evolution as, for an initial collapse x_1, there can be no collapses between constant time hypersurfaces intersecting x_1, σ_0, and $\sigma'_{0'}$. For spacelike collapses (right), then initial conditions between frames may not be related since there may be points of collapse between σ_0 and $\sigma'_{0'}$, e.g., x_1.

by Eq. (9), as there might be collapses in the region enclosed between σ_0 and $\sigma'_{0'}$, as shown in Fig. 8. Then to verify Eq. (23) the position of all collapses in this region must be known, since this region includes points which are in the future of x_0 in $\mathcal{F}$; this is not possible.

One should notice the difference between standard quantum mechanics and spontaneous collapse models. Standard quantum mechanics has spacelike collapses. However, as discussed in Sec. II B, this is consistent with relativity due to the position of collapses being given. In spontaneous collapse models, the position of collapses are probabilistic and are not know *a priori* and hence it cannot be taken for granted that initial conditions in two different inertial frames can be related. For spacelike spontaneous collapses, comparing initial conditions between two inertial frames is equivalent to requiring knowledge of future points of collapse in one of the inertial frames, as there might be collapses between two constant time hypersurfaces; see Fig. 8. Stochastic theories cannot meet this requirement, as the points of collapse are a single realization of a random process and hence cannot be determined with certainty.

The convention that the initial collapse occurs at the origin has been taken. Since this is just a choice of coordinate system, one would expect that the results discussed hold regardless of the choice of origin. Since in two different inertial frames $\mathcal{F}$ ($\mathcal{F}'$) the initial conditions are an initial point of collapse x (x') displaced from the origin and a wave function ψ_0 ($\psi_{0'}$) on the hypersurface intersecting it, then the same rules for relating the two initial conditions as in the case of collapse at the origin can be applied.

So single-particle spontaneous collapse models can meet condition Eq. (23) when collapses are timelike to each other, but for spacelike collapses the initial condition for observers in two frames cannot be compared so the condition is not satisfied. For the model proposed in Ref. [23] for a single particle, the collapses are timelike and hence it is a viable relativistic GRW model. Of course, one does not expect spacelike collapses for single particles as this would imply superluminal velocities; however, this observation is relevant for multiparticle spontaneous collapse models.

B. Relativistic condition for N distinguishable particles

The natural generalization of the previous model to the N-distinguishable particle case is to assume that there are N series of collapses and hence N realizations of the stochastic process. The ith realization is $S_i = \{T_{i1}, T_{i2}...\}$. For each realization, the construction of the collapse process—where they occur and how they change the wave function—is the same as for the single-particle case. Note that, in general, points of collapse associated to different particles can be spacelike separated, while points of collapse associated to the same particle are always timelike to each other. The Hilbert space for N distinguishable particles is given by

$$H = \underbrace{H_1 \otimes H_2 \otimes \cdots \otimes H_N}_{N}, \tag{24}$$

where H_i is a single-particle Hilbert space for the ith particle. The wave function on a hypersurface may be written as $\Psi_\sigma \in H$.

Then in this case, the condition for consistency with special relativity is that

$$\begin{aligned} &P(x_{11}, \ldots, x_{i1}, \ldots, x_{N1} | x_{10}, \ldots, x_{i0}, \ldots, x_{N0}, \Psi_{\sigma_0}) = \\ &\quad P(x'_{11}, \ldots, x'_{i1}, \ldots, x'_{N1} | x'_{10}, \ldots, x'_{i0}, \ldots, x'_{N0}, \Psi'_{\sigma'_{0'}}), \end{aligned} \tag{25}$$

where for the ith series of collapses the collapse at x_{i0} is followed by a collapse at x_{i1} and Ψ_{σ_0} is the multiparticle wave function on a constant time hyperplane at the initial time. In the single-particle case, the initial wave function was defined on a hyperplane intersecting the initial point of collapse. As for multiple particles, there are many initial points of collapse, and it is not immediately obvious which hypersurface the initial wave function should be defined on. The model can be defined consistently if in frame $\mathcal{O}$ the initial hypersurface intersects the earliest point of collapse in that frame for that generation of collapses, in this case the earliest point within the group $\{x_{10}, ..., x_{i0}, ..., x_{N0}\}$. So Ψ_{σ_0} is the wave function on the hyperplane intersecting the earliest x_{i0}. The relation Eq. (25) should hold true for every value of j, not only when $j = 1$; however, since the model is Markovian, the relation can be easily iterated and checked for any pair of consecutive collapses.

A necessary requirement for Lorentz invariance of the probability distribution is that the distance between each x_{ij} and x_{ij+1} is a timelike four-distance given by T_{ij}.

As in the case of the single-particle sector, in order for Eq. (25) to be satisfied it must be possible to relate the initial wave functions Ψ_{σ_0} and $\Psi'_{\sigma'_{0'}}$ in any two inertial frames. For a completely generic initial wave function Ψ_{σ_0}, which may be entangled, collapses for any particle may affect the probability for the collapse of another particle via entanglement. As collapses for different particles may be spacelike to each other, then the initial wave function in one frame Ψ_{σ_t} and cannot in general be related to $\Psi'_{\sigma'_{t'}}$, as is shown in Sec. IV of Ref. [24].

We will now give a simple illustrative example of an entangled initial state where two observers cannot compare

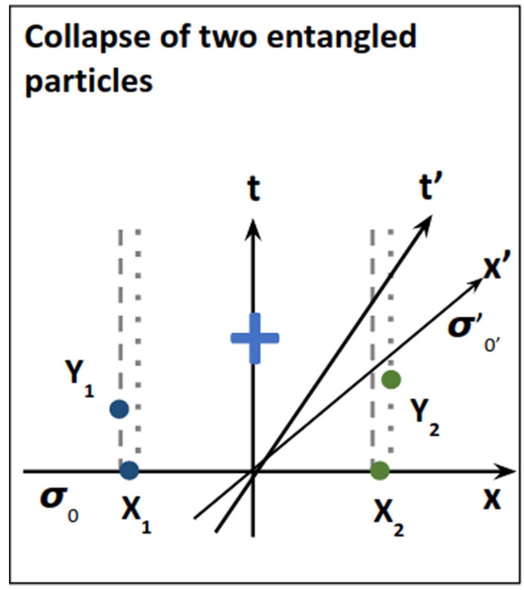

FIG. 9. Schematic diagram showing two entangled distinguishable particles. The time and space axis are shown for two different frames. The dashed and dotted lines show the support for each part of the wave function without collapses [see Eq. (26)]. Points X_1 and X_2 are the known initial points of collapse and Y_1 and Y_2 are possible future points of collapse.

initial conditions, but for a more rigorous explanation see Ref. [24]. Consider the situation shown in Fig. 9 with a system of two distinguishable particles. At time $t = 0$, there are two collapses, at X_1 for particle 1 and at X_2 for particle 2. We assume by fiat that immediately after this the system is in the entangled state:

$$|\psi_{\sigma_0}\rangle = \frac{1}{\sqrt{2}}\Big(|L\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2\Big), \tag{26}$$

where the subscripts refer to the particle number and $|L\rangle$ is a localized state centered on the left and $|R\rangle$ is a localized state centered on the right. We assume that their centers are sufficiently far apart such that $\langle L|R\rangle \approx 0$. The entanglement ensures that any further collapse will localize both particles; for example, if particle 2 collapses to $|R\rangle$ then particle 1 will also collapse to $|R\rangle$. This is a situation similar to the well-known Bell locality scenario.

For a GRW-type model in frame $\mathcal{F}$, the probability of particle 1 collapsing at Y_1 can be given by knowing the state $|\psi_{\sigma_0}\rangle$ and the position of the previous collapse X_1. However, in frame $\mathcal{F}'$ the initial state must be given on a constant time hypersurface $\sigma'_{0'}$ in that frame. As can be seen from Fig. 9, particle 2 may have already collapsed in $\mathcal{F}'$, for example, at Y_2, which would also affect particle 1.

So in order to compare the initial states on σ_0 and $\sigma'_{0'}$, the position of collapses between them (in this case Y_2) must be known, which would include collapse in the future for frame $\mathcal{F}$. Since in principle it is not possible to specify the position of future collapses for a stochastic theory, the two initial conditions cannot be compared.

In conclusion, there cannot be a special relativistic GRW model for entangled distinguishable particles. For the special case of noninteracting particles in a separable wave function, it is possible to have a relativistic GRW. We now discuss the two cases more in detail.

1. Noninteracting separable particles

For noninteracting particles, the unitary evolution operator between two surfaces ω_1 ω_2 may be written

$$W_{\omega_1}^{\omega_2} = \underbrace{U_{\omega_1,1}^{\omega_2} \otimes U_{\omega_1,2}^{\omega_2} \otimes U_{\omega_1,i}^{\omega_2} \otimes \cdots}_{N \text{ terms}}, \tag{27}$$

where $U_{\omega_1,i}^{\omega_2}$ is the unitary operator for the ith particle. The collapse operator for the the ith particle is

$$\hat{L}_{\omega,i}(x) = \underbrace{\mathbb{I} \otimes \mathbb{I} \otimes \cdots}_{i \text{ terms}} \otimes \hat{L}_{\omega}(x) \otimes \underbrace{\cdots \otimes \mathbb{I} \otimes \mathbb{I}}_{N\text{-}i\text{-}1 \text{ terms}}, \tag{28}$$

where $\hat{L}_{\omega}(x)$ is the collapse operator for a single particle (here for simplicity we assume that the form of each collapse operator is the same for every particle). There are N of such operators.

For a *separable* initial condition,

$$\Psi_{\sigma_0} = \psi_{\sigma_0,1} \otimes \psi_{\sigma_0,2} \otimes \cdots \otimes \psi_{\sigma_0,N}, \tag{29}$$

then each side of Eq. (25) can be factorized into N distributions of the form

$$P(x_{i1}, |x_{i0}, \psi_{\sigma_0,i}) = \left\| \hat{L}_{\sigma_{i1}}(x_{i1}) U_{\sigma_0,i}^{\sigma_{i1}} \psi_{\sigma_0,i} \right\|^2, \tag{30}$$

where σ_{ij} is the hyperplane intersecting the point x_{ij}. For consistency with special relativity, each $P(x_{i1}, |x_{i0}, \psi_{\sigma_0,i})$ must satisfy Eq. (23). If each particle has a series of collapses that are timelike to each other and the collapse operator $\hat{L}_{\omega,i}(x)$ transforms as in Eq. (11), then the model is consistent with special relativity. The model presented in Ref. [23] with a separable initial wave function meets this condition.

If the initial wave function Ψ_{σ_0} is *not separable,* then Eq. (25) will not be factorable. If it is not factorable then the initial wave function Ψ_{σ_1} cannot be specified in a frame-independent way and hence the model cannot be consistent with special relativity.

2. Interacting particles

If the particles are interacting and the wave function is initially separable, then the unitary operator cannot be decomposed as in Eq. (27). In this case, the condition for Eq. (25) to be factorable is

$$[W_{\omega_1}^{\omega_2}, \hat{L}_{\omega,i}(x)] = 0 \Rightarrow [\hat{H}, \hat{L}_{\omega,i}(x)] = 0, \tag{31}$$

where $\hat{H}$ is the Hamiltonian for the system (both the free and interacting parts). As is well known, if an operator commutes with the Hamiltonian, then it corresponds to a globally conserved quantity. Therefore, if the condition of Eq. (31) holds, then $\hat{L}_{\omega,i}(x)$ is a global operator and hence cannot be a local function of the fields. In this case, it has been shown that the dynamics does not result in a successful collapse model [35]. For example, if the collapse operator is $\hat{H}$, then the collapse rate is proportional to the distance between the energy eigenvalues of the system; see Eq. (21) of Ref. [36]. For systems in spatial superpositions but with degenerate energy eigenstates, then the model would not predict any collapse. This would fail to solve the measurement problem as it would not lead to a reduction in the wave function for situations where we observe that wave function collapses.

Therefore, since Eq. (25) is not factorable, then one is faced with the problem as the nonseparable wave function had: The initial wave function Ψ_{σ_0} will be different in different frames due to the interaction. Hence, it is not possible to have a special relativistic GRW model for interacting distinguishable particles.

C. Relativistic condition for indistinguishable particles

A relativistic GRW model for indistinguishable particles must only have a single collapse operator $\hat{L}_{\sigma}(x)$ which acts over every particle to preserve the particle interchange symmetry or antisymmetry for bosons and fermions respectively. Due to this, indistinguishable particles have the same relativistic condition as a single particle, namely that the stochastic process gives the four-distance between points of collapse and Eq. (23), where ψ_{σ} is an element of an N-particle Fock space. From this, the requirements Eqs. (11) and (13) are derived. If the collapses are timelike to each other, then Eq. (9) can be used as the initial condition in one frame, and the position of last collapse x_0 and the wave function ψ_{σ_0} can be related to the initial condition in a different frame.

Conversely, if the collapses are spacelike to each other, then Eq. (9) does not hold and such a model is not consistent with special relativity.

D. Emergence of macroscopicality for indistinguishable particles

As has been discussed in Sec. IV C, a relativistic GRW model is possible for indistinguishable particles if each collapse is timelike to the previous one. However, such a model has an issue. If given a point of collapse x_0, the only region that the subsequent collapse can occur is in the future light cone of x_0, then macroscopic classicality is not recovered. This can be seen with a simple example, with two macroscopic objects. Suppose there is a system made up a large number of indistinguishable particles N, where N is an even number. The initial wave function of the system is two macroscopic objects, i.e., two areas with high densities of particles, with a large distance separation between the center of mass of these two areas, labeled $2d$; see Fig. 11. Assume that initially each object is in a spatial superposition, separated by a distance $2r$, where $r \ll d$. For simplicity, we will work in one dimension but the argument can be extended to three dimensions. We will work in the framework of second quantization.

The initial wave function of the system on a constant time hypersurface σ_0 is

$$|\Psi_{\sigma_0}\rangle = \frac{1}{2}(\hat{A}_1 + \hat{A}_2)(\hat{B}_1 + \hat{B}_2)|0\rangle, \tag{32}$$

where $|0\rangle$ is the vacuum of an N particle antisymmetric Fock space and

$$\hat{A}_1 = \prod_{n=0}^{N/2} \hat{g}(-d - r, n), \tag{33a}$$

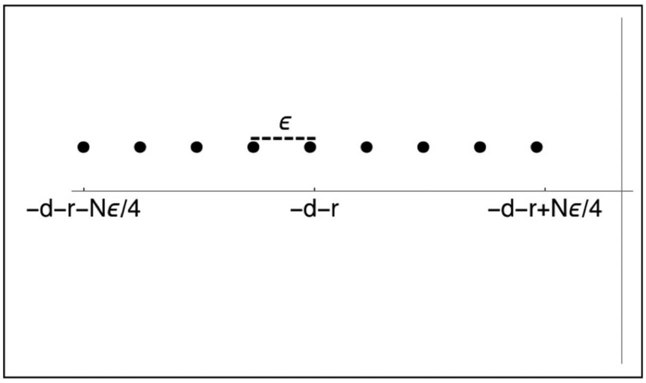

FIG. 10. Diagram showing the action of the operator $\hat{A}_1$ on the vacuum, where particles are created separated by distance ϵ.

$$\hat{A}_2 = \prod_{n=0}^{N/2} \hat{g}(-d+r, n), \tag{33b}$$

$$\hat{B}_1 = \prod_{n=0}^{N/2} \hat{g}(d-r, n), \tag{33c}$$

$$\hat{B}_2 = \prod_{n=0}^{N/2} \hat{g}(d+r, n), \tag{33d}$$

where

$$\hat{g}(x, n) = \hat{a}^\dagger(x - N\epsilon/4 + n\epsilon), \tag{34}$$

where ϵ is a distance such that $N\epsilon/2 \ll r \ll d$. Additionally assume that the distance scale of the collapse is much less than the size of the superposition: $1/\alpha \ll r$. So the operator $\hat{A}_1$ acting on the vacuum creates $N/2$ fermions, each displaced a

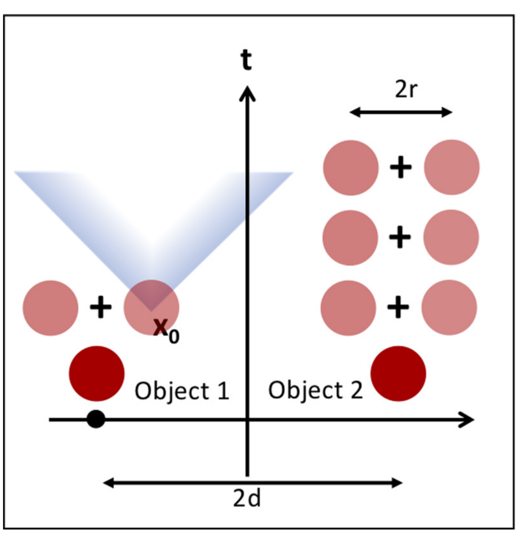

FIG. 11. Schematic spacetime diagram showing the evolution of a pair of spacelike separated macroscopic objects separated by distance $2d$. For timelike collapses, if there is a collapse at point x_0, the next collapse must occur in the future light cone of x_0 (shaded gray area), and therefore the object on the right will stay in a superposition.

distance ϵ from each other centered about the point $-(d+r)$ (see Fig. 10), and similarly for $\hat{A}_2$, $\hat{B}_1$, and $\hat{B}_2$.

The number operator for the whole system is

$$\hat{N}_T = \int_{-\infty}^{\infty} dx\, \hat{a}^\dagger(x)\hat{a}(x). \tag{35}$$

The number operator for the left part of the system is

$$\hat{N}_A = \int_{-\infty}^{0} dx\, \hat{a}^\dagger(x)\hat{a}(x), \tag{36}$$

with the equivalent definition for $\hat{N}_B$. Finally, the number operator for the region to the left of $-d$ is

$$\hat{N}_{A_1} = \int_{-\infty}^{-d} dx\, \hat{a}^\dagger(x)\hat{a}(x). \tag{37}$$

The initial wave function $|\Psi_{\sigma_0}\rangle$ is an eigenstate of the number operator for the total system:

$$\begin{aligned}&\hat{N}_T|\Psi_{\sigma_0}\rangle \\ &= \int_{-\infty}^{\infty} dx\, \hat{a}^\dagger(x)\hat{a}(x)\frac{1}{2}(\hat{A}_1+\hat{A}_2)(\hat{B}_1+\hat{B}_2)|0\rangle \\ &= \frac{1}{2}(N\hat{A}_1\hat{B}_1 + N\hat{A}_1\hat{B}_2 + N\hat{A}_2\hat{B}_1 + N\hat{A}_2\hat{B}_2)|0\rangle \\ &= N|\Psi_{\sigma_0}\rangle.\end{aligned}$$

The initial wave function is also an eigenstate of the number operator for the left part of the system,

$$\begin{aligned}&\hat{N}_A|\Psi_{\sigma_0}\rangle \\ &= \int_{-\infty}^{0} dx\, \hat{a}^\dagger(x)\hat{a}(x)\frac{1}{2}(\hat{A}_1+\hat{A}_2)(\hat{B}_1+\hat{B}_2)|0\rangle \\ &= \frac{N}{2}|\Psi_{\sigma_0}\rangle,\end{aligned}$$

and similarly for the right part of the system, $\hat{N}_B|\Psi_{\sigma_0}\rangle = N/2|\Psi_{\sigma_0}\rangle$. However, the initial wave function is not in an eigenstate of the number operator for the region to the left of $-d$:

$$\begin{aligned}&\hat{N}_{A_1}|\Psi_{\sigma_0}\rangle \\ &= \int_{-\infty}^{-d} dx\, \hat{a}^\dagger(x)\hat{a}(x)(\hat{A}_1\hat{B}_1 + \hat{A}_1\hat{B}_2 + \hat{A}_2\hat{B}_1 + \hat{A}_2\hat{B}_2)|0\rangle \\ &= \frac{1}{2}(\frac{N}{2}\hat{A}_1\hat{B}_1 + \frac{N}{2}\hat{A}_1\hat{B}_2 + \mathbb{I} + \mathbb{I})|0\rangle,\end{aligned} \tag{40}$$

which is not proportional to $|\Psi_{\sigma_0}\rangle$. $|\Psi_{\sigma_0}\rangle$ is also not an eigenstate of $\hat{N}_{A_2}$, $\hat{N}_{B_1}$, and $\hat{N}_{B_2}$. This implies there are two objects each in a superposition over two areas, not one object in a superposition over four areas, nor four objects each in a localized position. The amplification mechanism will cause a collapse of one of the objects almost immediately. Suppose that the collapse is at spacetime point $(t, -d+r)$, where t is so small that $U_{\sigma_0}^{\sigma_t} \approx \mathbb{I}$. Then, following Eq. (6), we find the wave function immediately after the collapse, on constant time hypersurface σ_t to be

$$|\Psi_{\sigma_t}\rangle = \frac{\hat{J}_{\sigma_t}(-d+r)|\Psi_{\sigma_0}\rangle}{\|\hat{J}_{\sigma_t}(-d+r)|\Psi_{\sigma_0}\rangle\|^2}, \tag{41}$$

where $\hat{J}_{\sigma_t}(x)$ is an approximation for the form of a relativistic collapse operator $\hat{L}_{\sigma_t}(x)$ in the limit of low-velocity particles. The form of $\hat{J}_{\sigma_t}(x)$ is

$$\hat{J}_{\sigma_t}(x)|\Psi_{\sigma_0}\rangle = \int_{-\infty}^{\infty} dy\, K(y) f_\alpha(x-y)\hat{a}^\dagger(y)\hat{a}(y)|\Psi_{\sigma_0}\rangle, \tag{42}$$

where $f_\alpha(x)$ is a function sharply peaked about $x=0$ with a width proportional to $1/\alpha$ and $K(y)$ is a normalization function. This form ensures that particles are localized about the point of collapse. To evaluate Eq. (41), consider just the term

$$\hat{J}_{\sigma_t}(-d+r)\hat{A}_1\hat{B}_1|0\rangle = \frac{1}{2}\int_{-\infty}^{\infty} dy\, K(y) f_\alpha(-d+r-y) \times \hat{a}^\dagger(y)\hat{a}(y)\prod_{n=0}^{N/2}\prod_{m=0}^{N/2}\hat{g}(-d-r,n)\hat{g}(d-r,m)|0\rangle.$$

The contributions from the $\hat{g}(-d-r,n)$ and $\hat{g}(d-r,m)$ operators are weighted by factors of $f_\alpha(-2r+n\epsilon/4)$ and $f_\alpha(2d-2r+n\epsilon/4)$ respectively. As $-2r+n\epsilon/4 \gg 1/\alpha$ and $2d-2r+n\epsilon/4 \gg 1/\alpha$, then $f_\alpha(-2r+n\epsilon/4)\approx 0$ and $f_\alpha(2d-2r+n\epsilon/4)\approx 0$. Hence,

$$\hat{J}_{\sigma_t}(-d+r)\hat{A}_1\hat{B}_1|0\rangle \approx 0. \tag{44}$$

A similar suppression occurs for $\hat{J}_{\sigma_t}(-d+r)\hat{A}_1\hat{B}_2|0\rangle$. However, the terms $\hat{A}_2\hat{B}_1 + \hat{A}_2\hat{B}_2$ are not suppressed as the f_α is approximately 1 for the part of the wave function centered on $-d+r$. Therefore, we are left with

$$\hat{J}_{\sigma_t}(-d+r)|\Psi_{\sigma_0}\rangle \approx \frac{N}{4}\hat{A}_2(\hat{B}_1+\hat{B}_2)|0\rangle \tag{45}$$

and therefore

$$|\Psi_{\sigma_t}\rangle \approx \frac{1}{\sqrt{2}}\hat{A}_2(\hat{B}_1+\hat{B}_2)|0\rangle. \tag{46}$$

So object 1 has been collapsed but object 2 remains in a superposition. Object 2 will be left in a superposition for approximately $2d/c$ s, where c is the speed of light, as can be seen in Fig. 11. If d is sufficiently large, then one of the macroscopic objects will remain in a spatial superposition for an arbitrarily long time, in violation of what we observe in nature.

To avoid this problem for macroscopic objects, then a collapse model must permit spacelike points of collapse. If there are spacelike collapse points, then the position of the initial collapse does not limit the region of possible collapses. Hence, any region with a high average number density of particles is almost certain to have a collapse occur within it in a short time interval. However, as discussed in Sec. IV C, if one attempts to include spacelike collapses into the indistinguishable particle model suggested here, then the model is not consistent with special relativity.

V. CONCLUSION

In this work, we have considered the GRW model and its consistency with special relativity. We have emphasized that for a model to be consistent with special relativity the dynamics must be Lorentz covariant and initial conditions in different inertial frames must be able to be be related, and we have applied these requirements for the case of relativistic GRW models. For a relativistic quantum theory, this means that the initial wave function on a constant time hypersurface must be able to related between different frames. In Table I, we have summarized the conclusions of this work, showing for which cases a relativistic GRW model is possible.

TABLE I. A table showing the regimes where a relativistic GRW model is possible.

Particle type	Separable state	Entangled state
Single	Yes	N/A
N distinguishable noninteracting	Yes	No
N indistinguishable noninteracting	No	No
Interacting	No	No

We have shown that a relativistic GRW is possible for single particles and noninteracting, nonentangled distinguishable particles as due to the fact that the collapses for each particle are timelike to each other, the initial conditions can be related in different inertial frames. However, for entangled noninteracting distinguishable particles (as entanglement implies that spacelike collapses for one particle can effect the probability of collapse of another particle), the initial conditions in different inertial frames cannot be related. For indistinguishable particles, either the collapses are spacelike and hence not compatible with special relativity, or the collapses are timelike, the recovery of macroscopic classicality is not certain, and hence such a model is not a viable collapse model. For interacting particles, as the interactions can entangle the particles, the initial conditions in two frames cannot be related, and hence there is not a relativistic GRW model for interacting particles.

The question is then if *any* spontaneous collapse model can be relativistic, whether that model describes pointlike collapses such as in GRW or other models such as continuous spontaneous collapse. The two requirements that collapses for interacting particles must be timelike to each other to preserve frame-independent probability distributions and that collapses must be spacelike to ensure that macroscopic objects remain localized seem to imply a contradiction and therefore that such a model is not possible. Recent work by Adler [37] supports this idea. If collapse models are not consistent with special relativity, the effect of violations of the Lorentz symmetry should be investigated in order to be confronted by experiment.

One thing to note is that this work only considers the fixed particle sector, and a completely relativistic collapse model must also describe changes in particle number. This is something that should be considered for further work in this subject.

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