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Growth maximizing government size, social capital, and corruption

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Abstract

Our paper intersects two topics in growth theory: the growth maximizing government size and the role of Social Capital in development. We modify a simple overlapping generations framework by introducing two key features: a production function à la Barro together with the possibility that public officials steal a fraction of public resources under their own control. As underlined by the literature on corruption, Social Capital affects public officials' accountability through many channels which also affect the probability of being caught for embezzlement and misappropriation of public resources. Therefore, in our endogenous growth model such probability is taken as a proxy of Social Capital. We find that maximum growth rates are compatible with Big Government size, measured both in terms of expenditures and public officials, when associated with high levels of Social Capital.

1 | INTRODUCTION

The present paper intersects theoretically two topics in growth theory: the growth maximizing government size and the role of Social Capital in development. Decreasing marginal benefits of government expenditures and increasing distortions due to taxation typically leads to an inverted U relationship between growth and government size (Facchini & Melki, 2013), known in the literature as B.A.R.S. curve (Armey & Armey, 1995; Barro, 1989, 1990; Rahn & Fox, 1996; Scully, 1998, 2003). Although the theoretical approach appears sound and generally accepted the optimal point from a quantitative perspective is very debated. Several empirical works did not clear cut the point over the optimal government size, typically measured as government expenditure relative to gross domestic product (GDP). Often in this context the European Nordic Countries (ENC)[1] with large governments and significant growth rates are taken as outliers or as counter examples to dismiss the entire approach.

We believe that another stream of literature that goes under the vast and debated title of Social Capital could contain important contributions to this debate. The multidisciplinary concept of Social Capital, since its introduction by the sociologist Bourdieu (1985), has been declined in many acceptations, positive and negative, and referred to different social levels, individual, and collective. The political scientist Putnam speaks of a potentially negative bonding Social Capital as networks between homogeneous groups of people as opposed to bridging networks between socially heterogeneous groups capable to enhance community productivity. According to Putnam these "social networks and the norms of reciprocity and trustworthiness that arise from them" are fundamental for the well functioning of government as shown in his contribution about the Civic tradition of Italian local governments (Putnam et al., 1993).

Along these lines also Fukuyama (1995) defines social capital as rules that enable people to cooperate such as the norms of reciprocity. The dual nature of Social Capital, positive and negative, is also present in his thought where Social Capital can, from one hand, facilitate economic development by reducing transaction costs and increasing productivity, but on the other hand can distort democracy favoring special interest groups (Fukuyama, 1995). This intrinsically ambiguous, multifaced and multidimensional nature of Social Capital has spurred an immense literature both theoretical and empirical whose survey is beyond the scope of this paper.[2]

However, more recently, building on Putnam et al. (1993) and Fukuyama (1995), Guiso et al. (2011) have proposed a definition of social capital that can possibly overcome its former dualistic nature. They define Social Capital as "those persistent and shared beliefs and values that help a group overcome the free rider problem in the pursuit of socially valuable activities." The definition encompasses both values and beliefs but label different things: values govern people's behavior while beliefs affect expectations about the strategic choices of others (Persson & Tabellini, 2020). This definition of Social Capital, as civicness, is intrinsically positive and in line with the empirical literature where several proxy of civicness appear to be related with development and government efficiency. This literature shows that greater participation to civic life and high levels of moral stigma for uncivic behavior are important factors to explain corruption and government efficiency as well as tax evasion and corruption (Bethencourt & Kunze, 2019; Bjørnskov, 2003, 2009, 2011; Hauk & Saez-Marti, 2002; La Porta et al., 1997; Nannicini et al., 2013; Uslaner, 2004; Zak & Knack, 2001). Corruption is here broadly defined as the sale by bureaucrats of public property for purely private gains (Acemoglu & Verdier, 2000; Amir & Burr, 2015; Choi & Thum, 2003; Emerson, 2006; Shleifer & Vishny, 1993). Banerjee and Vaidya (2019) consider the interplay between corruption and the practice of harassment, by which tax officers threaten to overassess tax liabilities.

The literature on the evolution and dynamics of Social Capital unanimously underlines the extremely high degree of inertia of culture and beliefs and allows the scholars to assume as stable the stock of Social Capital in their modeling strategy (see, e.g., Bisin & Verdier, 2001;

[1] Sweden, Norway, and Denmark.

[2] For a survey see, for example, Alesina and Giuliano (2013) and Guiso et al. (2011).

Guiso et al., 2016; Tabellini, 2008, 2010).[3] Since Olson (1982) high Social Capital as understood as high trust in people and institutions as well as great level of participation in civic life is thought to exert pressure for well doing of bureaucrats increasing their accountability or to foster reporting of public officials' wrong doings by whistle blowers (Johnson, 2003; Knack, 2002). Both Knack (2002) and Harris (2007) highlight the importance of bonding (bridging) Social Capital on fostering (discouraging) corruption. In particular, Harris (2007) discusses how bridging Social Capital might increase the risk of the corrupt public officials of being caught.

We depart from the modeling approach that includes some utility costs in the utility function due to social stigma, as for example, in Guiso et al. (2004). This approach, implicitly or explicitly, emphasizes social values internalized with different degrees by individuals; we hereby want to explore the beliefs component of their definition of Social Capital. These beliefs are indirectly captured by the probability of being reported for malfeasances. Our idea is that a society with high levels of civicness is one where corruption and rent seeking behaviors are not tolerate easily. Since high Social Capital, as understood as high trust in people and institutions as well as great level of participation in civic life, foster reporting of public officials' wrong-doings to public authorities by citizens, the press or whistle blowers, we assume a positive relationship between the probability of being detected of dishonest public officials and the degree of civicness. We consider this approach as complementary to the utility one and somehow less exposed to criticism implied by an ad-hoc form of utility function.

In this paper, we modify a simple overlapping generations (OLG) framework (Barro & Sala-i-Martin, 2004, Chapter 3) by introducing two key features: a production function à la Barro (1990) and a role for public officials in monitoring the public expenditures for intermediate goods and services supplied to private firms. Specifically, there are two types of workers, private workers employed in a competitive production sector behaving in a standard fashion, and public officials who, thanks to their management and control activities, prevent entrepreneurs from stealing on public procurements. In fact, in Appendix A we will show that without the presence of public officials, competitive, and myopic risk-neutral entrepreneurs would maximize their expected profits by stealing the whole public procurement, thus annihilating from the start any attempt by the government to sustain growth. Therefore, in our model the crucial role of public officials is justified to prevent entrepreneurs from cheating on public procurements.[4] However, public officials have the opportunity to embezzle and/or misappropriate public resources under their own control, subject to a positive probability of being caught and pay a fine. As a consequence, not all the stock of tax revenues raised by the government reaches private firms as intermediate goods and services, since a fraction of it is being diverted by public officials, thus hampering growth.[5] As emphasized by Acemoglu and Verdier (2000), corruption of public officials may therefore be the lesser of *two evils*.

It is worth noticing that the public officials behavior as specified in the following sections requires a specific two-stage decision tree. For this reason we focus on a OLG model rather than on a standard infinite horizon model with a representative agent.

[3] A different line of research that emphasizes the network nature of Social Capital, defined at the individual and firm levels, looks at the optimal rate of investment by individuals/firms in Social Capital proxied by networking activities (see, e.g., Glaeser et al., 2002; Le Van et al., 2018). In the present paper instead we look at common beliefs in others' behavior that have consolidated in society.

[4] We assume for simplicity that public officials do not accept bribes from entrepreneurs.

[5] Also Marakbi and Villieu (2020) consider an endogenous growth model à la Barro (1990) with corruption but that hinges on tax evasion where households, to pay less taxes, are prompt to purchase bribery services produced by corrupt bureaucrats.

As expected, we find that the endogenous growth rate of the economy is affected by the probability of detection and the fine paid by those public officials who have been caught. This feature is in line with the empirical evidence on the connection between corruption and growth, as the magnitude of the negative effect of corruption on growth is found to be higher in those countries where political institutions are weak (and social capital is low); see, for example, Aidt et al. (2008) and, more recently, Gründler and Potrafke (2019) and references therein. Campos et al. (2010) and Ugur (2014) provide excellent surveys on the subject.

Moreover, as in Barro (1990), along the balanced growth path (BGP) the output growth rate turns out to be an Inverted U-shaped function of the tax rate, thus establishing uniqueness of the optimal tax rate with respect to growth. By performing comparative dynamics on the optimal tax rate, our main result shows that, under certain conditions, if the probability of detection or the fine charged on public officials who are caught stealing, or both, increase, then an increase of the optimal tax rate is required to keep the growth rate at its maximum level, provided that also the share of public workers on the total workforce is adjusted (increased) as well. As the probability of detection is assumed to be a linear function normalized to one of the Social Capital level,[6] in the proceeding we conclude that maximum growth rates are compatible with Big Government size (both in terms of expenditures and public officials) when associated with high levels of Social Capital. When Social Capital is low the growth maximizing government size shrinks and vice versa. Social Capital therefore could be the missing dimension accounting for the controversial empirical results on this issue as well as for the ENC case. According to the present model, the highest growth rates experienced by the ENC, despite their well above-the-average OECD countries' government size, could be explained by their highest level of Social Capital, which in turn affects the behavior of public officials and thus the efficiency of their governments as a whole.

This paper proceeds as follows. In Section 2 we formally introduce the OLG framework by describing in detail the competitive firms' optimal strategies, the optimal behavior of both private and public workers as well as their welfare heterogeneity, and the static general equilibrium with government transfers to the firms. In Section 3 we define the optimal dynamics of capital that take into account the cheating behavior of public officials, characterize the BGP, and establish the main result. In the same section we determine the positive monotonicity relationship between the probability of detection, viewed as a proxy of Social Capital, and government size, expressed both in terms of taxation level and share of public workers, necessary to keep growth at its maximum rate. In Section 4 we discuss a numerical example that illustrates our main result. Section 5 as usual concludes, while all mathematical proofs are gathered in Appendix B.

2 | THE MODEL

We consider a OLG model where each individual belonging to the tth cohort lives for two periods: in the first period, when she is young, she works, consumes and saves part of her wage, net of taxes. In the second period, when she is old, she does not work but consumes what she saved in the first period plus interests net of taxes. We assume that in the economy the population is constant over time. In each period we have L young workers, with L a large

[6]There are many measures of civicness in use (Guiso et al., 2004), each can be thought to be normalized and linked to a probability measure.

number. Moreover, in each period the economy is populated by L old individuals belonging to the previous $t-1$ cohort, so that, at each time t, the total population is $2L$. Following Barro (1990) we assume that in this growth model the government supplies intermediate goods and productive services G to private firms without user charges, as it is financed through a distortionary tax, with rate $0 < \tau < 1$, on the total national income (i.e., total gross wages plus total gross returns on capital). More specifically as in Barro (1990), we assume that G is homogeneous to the output and that the government owns no capital and performs no production activities, it just buys a flow of output from the private sector which is being used as input by the firms (Barro, 1990, pp. S106 and S107). Moreover, productive public services are assumed to be rival and non excludable, therefore subject to congestion; this is captured by public service intensity in the production function, following one of the options suggested by Eicher and Turnovsky (2000). As the production function can be parametrized, without loss of generality we assume that public service congestion implies that only per worker (public and private) public service G/L enters in the production function à la Barro. For simplicity Barro (1990) assumed that government sustains zero transaction costs for supplying public services, but such a shortcut prevents any study on the role of corruption, as it omits the public officials' activity: namely the management and control of public procurement for the purchase of goods and services that forms the productive public expenditures.

Instead, in our model bureaucracy allows the government to achieve a second-best equilibrium otherwise unreachable. To justify the existence of public officials' intermediation between the government and private firms receiving public goods and services as inputs we assume on the one hand that the government is aware of the possibility of corrupt behavior occurring at some stage in the process of transferring resources, raised by tax, to private firms in the form of productive factor, and on the other hand that all the firms involved in productive activities are risk neutral and their unique objective is to maximize expected profits by covering their costs through (net) revenues. As it is briefly shown in Appendix A, the only alternative to the employment of public officials available to the government is the purchase of public goods and services directly from the firms which are being paid by tax revenues. Under risk neutrality assumption, however, the firms would steal the whole revenues and provide zero public goods and services, thus frustrating the rise of endogenous growth à la Barro from the start. In this model, thus, public officials exercise the crucial role of preventing entrepreneurs from cheating on public procurements.

Therefore, we depart from Barro (1990) and assume that in the economy a share of young workers are employed in the public sector as public officials to monitor public expenditures. However, public officials, while preventing entrepreneurs from stealing the whole public procurements, have themselves the opportunity to steal a fraction of public resources under their own control, subject to a positive probability of being caught and pay a fine. Thus, given the presence of corrupt public officials,[7] there exists the possibility that not all the whole amount of tax revenues raised by the government reaches private firms. In the following we will show what remarkable consequences the necessity of bureaucracy has for the growth of the economy.[8] We assume that all young individuals inelastically supply one unit of labor either to

[7]As in Amir and Burr (2015), corruption is here broadly defined as the sale by bureaucrats of public property for purely private gains.

[8]On the choice between market failures and corruption, Acemoglu and Verdier (2000) argue that corruption of public officials may be seen as the lesser of *two evils*. In a different context Amir and Burr (2015) find that even the behavior of corrupt public officials might be justified to reach a second best in the presence of excess entry of the type studied by Mankiw and Whinston (1986). Banerjee and Vaidya (2019) show that the threat of harassment by the tax officer, even in the absence of any pecuniary rewards for reporting tax evasion, can boost the impact of anticorruption reforms, such as a moderate increase in the anticorruption penalty.

private firms or to the government. The share of workers employed in the public sector is denoted by $\lambda = L_1/L$, where L_1 denotes the number of public officials; whereas the share employed in the private sector is again constant and equal to $1 - \lambda = L_2/L$, with $0 < \lambda < 1$, where L_2 denotes the number of private workers, with $L = L_1 + L_2$. We assume that the government has a balanced budget. In Section 3 it will be shown that parameter λ provides the government with a policy tool that, together with the tax rate, affects the (optimal) growth rate.

2.1 | Firms

The representative private firm behaves competitively and produces a composite consumption good according to a Cobb–Douglas technology, so that the firm i output is given by

$$Y_i = \theta K_i^{\alpha} L_i^{1-\alpha} (G/L)^{1-\alpha},$$

where θ is some positive constant indicating the (exogenous) technological level, K_i is capital, L_i is the number of workers employed, $0 < \alpha < 1$ is the capital factor share, and, due to congestion effects, only the share $g = G/L$ of intermediate goods and productive services G provided by the government is available to firm i.

Assuming, for simplicity, that capital fully depreciates after use (see section 9.2 in Acemoglu, 2009), for given K_i, L_i, and g firms maximize profit when

$$\frac{\partial Y_i}{\partial K_i} = \alpha\theta\left(\frac{g}{k_i}\right)^{1-\alpha} = R, \tag{1}$$

$$\frac{\partial Y_i}{\partial L_i} = (1-\alpha)\theta k_i^{\alpha} g^{1-\alpha} = w, \tag{2}$$

where $k_i = K_i/L_i$ is the firm i capital-labor ratio, R is the market (gross) return to capital, and w is the market (gross) wage. As all firms are equal, they choose the same capital-labor ratio, $k_i \equiv K/L_2$ for any i, where K denotes aggregate capital; the production function can thus be aggregated

$$Y = \theta L_2\left(\frac{K}{L_2}\right)^{\alpha} g^{1-\alpha}, \tag{3}$$

which, in per worker terms (private plus public workers), becomes

$$y = \theta\frac{L_2}{L}\left(\frac{K}{L_2}\right)^{\alpha} g^{1-\alpha} = \theta(1-\lambda)\left(\frac{L}{L_2}\frac{K}{L}\right)^{\alpha} g^{1-\alpha} = \theta(1-\lambda)^{1-\alpha}k^{\alpha}g^{1-\alpha}, \tag{4}$$

where $y = Y/L$, $k = K/L$, and $(1-\lambda) = L_2/L$.

It's worth noticing that in our setting as well as in Barro (1990) capital is viewed as a broad concept encompassing rewardable physical and human capital.

In equilibrium, the net return on capital, $\bar{R}$, is equal to $\bar{R} = (1-\tau)R$, where R is given by (1) and, in view of (4), can be rewritten in per worker terms as follows:

$$R = \frac{\partial y}{\partial k} = \alpha\theta(1-\lambda)^{1-\alpha}k^{\alpha-1}g^{1-\alpha} = \alpha\frac{y}{k}, \tag{5}$$

while the net wage of a private employee (and public official[9]) is equal to $\bar{w} = (1 - \tau)w$, where w is given by (2) and, in view of (4), can be rewritten in per worker terms as follows:

$$w = \frac{\partial y}{\partial(1-\lambda)} = (1-\alpha)\theta(1-\lambda)^{-\alpha}k^{\alpha}g^{1-\alpha} = (1-\alpha)\frac{y}{1-\lambda} \tag{6}$$

so that the per worker gross private output is given by $Rk + w(1 - \lambda) = y$.

It is worth noticing that, given the share of private workers, the wage w is a linear function of the per worker gross private output y. Then the wage will grow at the output rate. Moreover, if y and k grow at the same rate, then R is constant over time; we shall see in Section 2.4 that in equilibrium this is actually the case (see Equation (20) and Proposition 1).

2.2 | Private employees

We assume that each private and public employee has the same logarithmic instantaneous utility function, $u(c) = \ln c$, the same (constant) pure rate of time preference, $0 < \beta < 1$, and all earn the same equilibrium gross wage w defined in (6). Such a wage is being taxed at a flat rate $0 < \tau < 1$ by the government, who runs a balanced fiscal rule by applying the same flat tax on both labor and capital income.

At each given time t all young private employees in the t-cohort solve the same deterministic two-period maximization problem:[10]

$$\max_{\{x_t\}} (\ln c_{1,t} + \beta \ln c_{2,t+1}) \tag{7}$$

$$\text{s.t.} \begin{cases} c_{1,t} = \bar{w}_t - x_t \\ c_{2,t+1} = \bar{R}_{t+1}x_t, \end{cases} \tag{8}$$

where $c_{1,t}$ and $c_{2,t+1}$ denote consumption in the first and second period respectively, x_t denotes the asset amount (saving) to be chosen, while $\bar{R}_{t+1}$ is the net of taxes rate of return to saving ($\bar{R}_{t+1} = (1-\tau)(1 + r_{t+1})$ where $r_{t+1} > 0$ is the interest rate), and $\bar{w}_t > 0$ is the net of taxes wage earned. They are defined as $\bar{R} = (1 - \tau)R$ and $\bar{w} = (1 - \tau)w$, where the tax rate $0 < \tau < 1$, as well as the gross rate of return to saving R and gross wage w, are taken as exogenously given.

After replacing $c_{1,t}$ and $c_{2,t+1}$ according to the constraints (8) into the objective function (7), the first order conditions (FOC) with respect to the asset x_t yields the optimal individual saving as a fraction of the wage $\bar{w}_t$:

$$x_t = \frac{\beta}{1+\beta}\bar{w}_t = \frac{\beta(1-\tau)}{1+\beta}w_t. \tag{9}$$

It is well known that the "canonical" OLG model with logarithmic utility yields an optimal saving amount which is independent of the (net) rate of return to saving $\bar{R}_{t+1}$ (see section 9.3 in Acemoglu, 2009).

[9] In the next subsection on public officials we will give a justification to this assumption.

[10] As all individuals are the same, we drop the index indicating each of them.

2.3 | Public officials

Unlike private workers, each corrupt public official has the opportunity to divert a fraction $0 \leq s \leq 1$ of the amount q of given public resources under her own control, and add such amount to her individual asset when she is young at time t. During the same initial period in her life, but after she took her optimal decision on how much to steal, she may get caught by the authorities, in which case she must give back the whole amount stolen and pay a fine $\varphi > 0$ per unit of resource stolen. The probability of being caught is $0 < p < 1$, constant through time.

At each given time t all young public officials solve the same stochastic two-period maximization problem:[11]

$$\max_{\{x_t, s_t\}} \mathbb{E}\ (\ln c_{1,t} + \beta \ln c_{2,t+1}), \tag{10}$$

$$\text{s.t.} \begin{cases} c_{1,t} = \bar{w}_t + (1 - z_t) q_t s_t - z_t f q_t s_t - x_t \\ c_{2,t+1} = \bar{R}_{t+1} x_t \\ 0 \leq s_t < 1, \end{cases} \tag{11}$$

where $\mathbb{E}$ denotes time t expectation, $c_{1,t}$ and $c_{2,t+1}$ denote consumption in the first and second period, respectively, x_t denotes the asset amount (saving) to be chosen, s_t the share of public resources under control, q_t, that will be stolen at time t, $f = 1 + \varphi > 1$ is the amount that must be returned to the government in the event of being caught, while again $\bar{R}_{t+1}$ is the net of taxes rate of return to saving, and $\bar{w}_t > 0$ is the net of taxes wage earned. The latter are the same as those of private employees and are taken as exogenously given. Although public officials have the opportunity to steal some fraction of the tax revenues that private workers do not have, and thus they should be ready to apply for this job at a lower wage, we can justify the assumption of equality of wages in both sectors by arguing that the government pursues payment parity according to a legal principle of equality, as it cannot admit the possibility of stealing. We assume that, to keep the labour market in equilibrium, the government adopts a lottery mechanism that allows the selection of exactly the number of public officials it needs out of the whole working population eager to apply for employment in the public sector, which, as it will be shown in Section 2.5, offers higher expected illicit earnings than the private sector.

The indicator function z_t, associated to probability p of being caught at time t, is defined as

$$z_t = \begin{cases} 1 & \text{with probability} \quad p, \\ 0 & \text{with probability} \quad 1 - p, \end{cases} \tag{12}$$

and it is unknown (i.e., it is a random variable) at the time in which the (optimal) decision is taken upon x_t and s_t, but it is revealed before the instant in which the amount $c_{1,t}$ is being consumed; therefore, the first constraint in (11) is truly random but affects only the consumption $c_{1,t}$ of young public officials, as the consumption $c_{2,t+1}$ in the old age is fully determined by the rate of return to saving $\bar{R}_{t+1}$, which is deterministic and exogenously given, and by the choice on savings x_t, which has already been taken. $\{z_t\}_{t=0}^{\infty}$ is a process of i.i.d. Bernoulli random variables such that $\Pr(z_t = 1) = p$, where $0 < p < 1$ corresponds to the probability that each public official will be caught to steal in the period between her optimal decisions and her consumption. In other words, the amount of consumption in the old age, $c_{2,t+1}$, is not being

[11] As all individuals are the same, we drop the index indicating each of them.

affected by the realization of the random variable z_t one period before (in the young age), as the decision on the optimal saving x_t has been taken before the administration controls take place, and cannot be modified. This somewhat peculiar assumption is based on the presumption that agents may prefer to concentrate all risk originating from their illegal behavior on their young age together with the observation that it seems unrealistic that each public official must wait until retirement (or any time after consumption occurred in the young age) to know whether she has being caught or she can get it free. Moreover, and perhaps more importantly, it is crucial in simplifying the analysis.[12]

We assume that q_t is exogenously given and that public officials maximize their total expected utility independently from each other. Moreover, the Bernoulli process is assumed to be i.i.d. both over time and across public officials.

After replacing $c_{1,t}$ and $c_{2,t+1}$ according to the constraints (11) into the objective function (10), the problem can be rewritten as follows:

$$\begin{aligned} &\max_{\{x_t,s_t\}}[(1-p)\ln(\bar{w}_t - x_t + q_t s_t) + p\ln(\bar{w}_t - x_t - f q_t s_t) + \beta\ln(\bar{R}_{t+1}x_t)] \\ &\text{s.t.} \quad 0 \le s_t \le 1. \end{aligned} \tag{13}$$

Assuming an interior solution,[13] $x_t > 0, 0 < s_t < 1$, FOC on (13) yield the following optimal individual saving, which turns out to be the same as that in (9) for private workers:

$$x_t = \frac{\beta}{1+\beta}\bar{w}_t = \frac{\beta(1-\tau)}{1+\beta}w_t, \tag{14}$$

while the optimal individual stealing choice s_t turns out to be a fraction of the ratio $\bar{w}_t/q_t$:

$$s_t = \frac{1-p(f+1)}{(1+\beta)f}\frac{\bar{w}_t}{q_t} = \frac{[1-p(f+1)](1-\tau)}{(1+\beta)f}\frac{w_t}{q_t}. \tag{15}$$

For $s_t > 0$, the results obtained imply that those public officials who are not caught stealing will consume all the illicit earnings $s_t q_t$ during their working age and will save exactly the same amount of wage as the private workers for the old age. Thus, while consumption is the same both for private workers and public officials in the old age, this pattern introduces heterogeneity in consumption between them in the young age, which will be discussed in the next Section 2.5, after introducing conditions for a nontrivial general equilibrium to hold.

2.4 | Government and the static general equilibrium

As discussed above, the government employs $L_1 = L - L_2$ public officials to monitor the public expenditures for intermediate goods and services used by the private firms as input of their production process. Moreover, public official's wage is the same of the private worker's one and

[12]We considered an alternative model formulation based on the assumption that public officials are allowed to perform some degree of "consumption smoothing" by deferring to their old age part of their income from corruption, possibly after the realization of the random variable z_t, that is, after assessing whether they have been caught. Denoting by $\sigma(z_t)q_t s_t$ the portion of illegal income to be allocated into the old age (possibly with $\sigma(z_t) < 0$ if $z_t = 1$), the (expected) utility term in the old age in the reduced problem equivalent to (13) would be written as $\beta\mathbb{E}\ln[\bar{R}_{t+1}x_t + \sigma(z_t)q_t s_t]$, that is, with a sum containing all decision variables as argument, which implies that FOC for both s_t and x_t would depend on the rate of return to saving $\bar{R}_{t+1}$. By looking at the expression of $\bar{R}_{t+1}$ as in (20) it is apparent that such an approach would lead to a totally intractable analysis, thus explaining why we chose to rule out consumption smoothing in public officials' behavior.

[13]In the Appendix B, the proof of Proposition 1 shows that this is actually the case under straightforward conditions.

it is paid by the government using taxes. We assume that the government has a balanced budget. Total tax revenues are equal to $T = \tau\tilde{Y}$, where $\tilde{Y} = Y + L_1 w$ is the total taxable national income.[14] The public administration spends such amount in intermediate goods and services to private firms and in public officials' wages, that is,

$$T = \tau\tilde{Y} = \tau(Y + L_1 w) = \tilde{G} + L_1 w,$$

where $\tilde{G}$ denotes the *potential* amount of resources to be devoted to the firms as intermediate goods and services, which is given by

$$\tilde{G} = \tau Y + \tau L_1 w - L_1 w = \tau Y - (1 - \tau)L_1 w. \tag{16}$$

However, intermediate goods and services that actually reach firms are not $\tilde{G}$ but $G = [1 - \mathbb{E}(s)]\tilde{G}$, as public officials will steal on average the share $\mathbb{E}(s)$ of public resources committed to that scope. Moreover, taking into account the effect of congestion, only the share

$$g = \frac{G}{L} = \frac{[1 - \mathbb{E}(s)]\tilde{G}}{L} = [1 - \mathbb{E}(s)][\tau y - (1 - \tau)\lambda w] \tag{17}$$

will eventually enter the production function of each firm as input.

Substituting g as in (17) into (4) and using (6) yields

$$\begin{aligned}
y &= \theta(1 - \lambda)^{1-\alpha}k^{\alpha}g^{1-\alpha} = \theta(1 - \lambda)^{1-\alpha}k^{\alpha}[1 - \mathbb{E}(s)]^{1-\alpha}[\tau y - (1 - \tau)\lambda w]^{1-\alpha} \\
&= \theta(1 - \lambda)^{1-\alpha}k^{\alpha}[1 - \mathbb{E}(s)]^{1-\alpha}\left[\tau y - (1 - \tau)\lambda(1 - \alpha)\frac{y}{1 - \lambda}\right]^{1-\alpha} \\
&= \theta(1 - \lambda)^{1-\alpha}k^{\alpha}y^{1-\alpha}[1 - \mathbb{E}(s)]^{1-\alpha}\left[\frac{\tau(1 - \lambda) - (1 - \alpha)(1 - \tau)\lambda}{1 - \lambda}\right]^{1-\alpha} \\
&= \theta k^{\alpha}y^{1-\alpha}[1 - \mathbb{E}(s)]^{1-\alpha}[\tau(1 - \lambda) - (1 - \alpha)(1 - \tau)\lambda]^{1-\alpha},
\end{aligned}$$

from which it turns out that per worker private output is a linear function of per worker capital:[15]

$$y = \theta^{\frac{1}{\alpha}}[1 - \mathbb{E}(s)]^{\frac{1-\alpha}{\alpha}}[\tau(1 - \lambda) - (1 - \alpha)(1 - \tau)\lambda]^{\frac{1-\alpha}{\alpha}}k, \tag{18}$$

that is, our economy resembles the features of a typical "AK" model. To be defined, the right-hand side of (18) requires the following assumption.

Assumption 1. Parameters α, λ, and τ must satisfy $\tau(1 - \lambda) - (1 - \alpha)(1 - \tau)\lambda > 0$, which may be conveniently rewritten as $(1 - \alpha\lambda)\tau - (1 - \alpha)\lambda > 0$, that is, the following condition must hold:

$$\tau > \frac{(1 - \alpha)\lambda}{1 - \alpha\lambda}.$$

Note that the threshold for τ in Assumption 1 is increasing in the share λ of public officials. It is 0 for $\lambda = 0$, corresponding to zero transfers of public resources to the firms, and 1 for $\lambda = 1$, corresponding to zero workers employed in the private sector. We do not consider the extreme scenarios $\lambda = 0$ and $\lambda = 1$ because the former envisages no sustained growth à la

[14] The tax revenues stolen by the public officials who are caught and the fines they pay should be added to the budget of the government. For the sake of simplicity, we assume that such revenues are entirely devoted to cover the costs of investigative activity in pursuing corrupt public officials and thus we have omitted the two budget items.

[15] Notice that in turn this result is due to the linear relationship existing between wage and output.

Barro (1990) (the model would boil down to a standard Ramsey model converging to some steady state) while the latter implies the extinction of the economy as no output would be available under Cobb–Douglas production.

Substituting y as in (18) into (17) and using again (6) shows that g turns out to be linear in k as well:

$$\begin{aligned} g &= [1 - \mathbb{E}(s)]\left[\tau y - (1-\alpha)(1-\tau)\lambda \frac{y}{1-\lambda}\right] \\ &= [1 - \mathbb{E}(s)]\frac{\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda}{1-\lambda} y \\ &= \theta^{\frac{1}{\alpha}}[1 - \mathbb{E}(s)]^{\frac{1}{\alpha}}[(1-\lambda) - (1-\alpha)(1-\tau)\lambda]^{\frac{1}{\alpha}}(1-\lambda)^{-1}k. \end{aligned} \tag{19}$$

Similarly, from (5) and (18) it is immediately seen that the return to capital is given by

$$R = \alpha\frac{y}{k} = \alpha\theta^{\frac{1}{\alpha}}[1 - \mathbb{E}(s)]^{\frac{1-\alpha}{\alpha}}[\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda]^{\frac{1-\alpha}{\alpha}}, \tag{20}$$

while, from (6) and (18) it is easily seen that the gross market wage is given by

$$w = \frac{(1-\alpha)y}{1-\lambda} = (1-\alpha)\theta^{\frac{1}{\alpha}}[1 - \mathbb{E}(s)]^{\frac{1-\alpha}{\alpha}}\frac{[\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda]^{\frac{1-\alpha}{\alpha}}}{1-\lambda}k. \tag{21}$$

Note that, if the average share of public resources stolen by public officials, $\mathbb{E}(s)$, is constant, then the return to capital R in (20) and the gross wage in (21) turn out to be constant and a linear function of per worker capital, respectively. We shall see in Proposition 1 that this is actually the case.

Let us denote by $\tilde{g} = \tilde{G}/L$ the per worker supply of intermediate goods and services potentially available to firms, and by q the amount of public resources under the control of each public official that enter her intertemporal budget constraint, that is, before the public official takes a decision on what portion of it she is ready to steal. Then, by (16), (6), and (18),

$$\begin{aligned} q &= \frac{\tilde{G}}{L_1} = \frac{\tilde{g}}{\lambda} = \frac{\tau y}{\lambda} - (1-\tau)w = \left[\frac{\tau}{\lambda} - \frac{(1-\alpha)(1-\tau)}{1-\lambda}\right]y \\ &= \theta^{\frac{1}{\alpha}}[1 - \mathbb{E}(s)]^{\frac{1-\alpha}{\alpha}}\frac{[\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda]^{\frac{1}{\alpha}}}{\lambda(1-\lambda)}k \end{aligned} \tag{22}$$

Clearly, as y, g, w and q are all linear functions of per worker capital, k, if $\mathbb{E}(s)$ is constant through time they all will grow at the same rate. Moreover, note that the ratio w/q is always a constant:

$$\frac{w}{q} = \frac{(1-\alpha)\lambda}{\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda} \tag{23}$$

Hence, as public officials are all equal and s_t in (15) depends only on parameters β, τ, p, f plus the exogenous variables w_t and q_t, we have just established the next result, that will be crucial in the following analysis

Proposition 1. *Under Assumption 1, if*

$$p < \frac{1}{1+f} \quad \text{and} \quad \tau > \frac{[1 - p(f+1) + (1+\beta)f](1-\alpha)\lambda}{[1 - p(f+1)](1-\alpha)\lambda + (1+\beta)f(1-\alpha\lambda)}, \tag{24}$$

then the following hold:

(i) At each time t, all public officials steal the same amount $s_t \equiv s$ constant through time, which, according to (15) and (23) is given by

$$s = \frac{[1 - p(f + 1)](1 - \alpha)(1 - \tau)\lambda}{(1 + \beta)f\,[\tau(1 - \lambda) - (1 - \alpha)(1 - \tau)\lambda]}. \tag{25}$$

(ii) *Therefore, also the average theft is constant through time, $\mathbb{E}(s_t) \equiv s$, and y_t, g_t, w_t, q_t, being all linear functions of per worker capital, k_t, grow at the same constant rate whenever the economy features sustained growth.*

(iii) The optimal theft s defined in (25) is decreasing in p, f, and τ.

In the proof of Proposition 1 in Appendix B it is shown that the lower bound on τ in the second condition of (24) is always stronger than Assumption 1.

2.5 | Welfare heterogeneity among private and public employees in the young age

By replacing (9) into the budget constraint (8) one obtains the young private employees' consumption as follows:

$$c_{1,t}^{pe} = \bar{w}_t - x_t = \bar{w}_t - \frac{\beta}{1 + \beta}\bar{w}_t = \frac{\bar{w}_t}{1 + \beta},$$

while by replacing x_t as in (14) and s_t as in (15) into the budget constraint (11) the young public officials' (random) consumption turns out to be

$$\begin{aligned} c_{1,t}^{po} &= \bar{w}_t + (1 - z_t)q_t s_t - z_t f q_t s_t - x_t = \frac{\bar{w}_t}{1 + \beta} + [1 - z_t(f + 1)]q_t s_t \\ &= \frac{\bar{w}_t}{1 + \beta} + [1 - z_t(f + 1)]q_t \frac{1 - p(f + 1)}{(1 + \beta)f}\frac{\bar{w}_t}{q_t} \\ &= \left\{1 + \frac{[1 - z_t(f + 1)][1 - p(f + 1)]}{f}\right\}\frac{\bar{w}_t}{1 + \beta}, \end{aligned}$$

whose realizations in case of getting away with the theft ($z_t = 0$) and in case of being caught ($z_t = 1$) are respectively,

$$\begin{aligned} c_{1,t}^{po}(z_t = 0) &= \left\{1 + \frac{[1 - p(f + 1)]}{f}\right\}\frac{\bar{w}_t}{1 + \beta} \\ c_{1,t}^{po}(z_t = 1) &= \left\{1 - \frac{f\,[1 - p(f + 1)]}{f}\right\}\frac{\bar{w}_t}{1 + \beta} = p(f + 1)\frac{\bar{w}_t}{1 + \beta}. \end{aligned}$$

The first condition in (24) is equivalent to $[1 - p(f + 1)]/f > 0$ and $p(f + 1) < 1$: the former implies that public official are strictly better off than private employees in case of getting away with the theft, $c_{1,t}^{po}(z_t = 0) > c_{1,t}^{pe}$, while the latter implies that they are strictly worse off when they are getting caught, $c_{1,t}^{po}(z_t = 1) < c_{1,t}^{pe}$.

The opportunity available to public officials to steal a fraction of public resources, however, let them on average better off than private employees, thus forcing the government to introduce the lottery mechanism to select of a specific number of public officials out of the whole working population anticipated in Section 2.3. To see this, note that, by the law of large numbers, a fraction $(1-p)$ of the whole public officials will get away with the stolen public resources and a fraction p of them will be get caught. Therefore, the average per capita public officials' consumption in the young age is given by

$$\begin{aligned} c_{1,t}^{av} &= \left\{1 + (1-p)\frac{[1-p(f+1)]}{f} - p\frac{f\,[1-p(f+1)]}{f}\right\}\frac{\bar{w}_t}{1+\beta} \\ &= \left\{1 + \frac{[1-p(f+1)]^2}{f}\right\}\frac{\bar{w}_t}{1+\beta}, \end{aligned}$$

which, again because the first condition in (24) implies $[1-p(f+1)]^2/f > 0$, is clearly strictly larger than the per capita consumption of all private employees, $c_{1,t}^{av} > c_{1,t}^{pe}$.

3 | AGGREGATE EQUILIBRIUM AND GROWTH

Under the assumption that all agents have logarithmic utility, the optimal savings x_t of everybody, either private worker or public official, are the same and are given by (14). Assuming that savings of young agents at time t are employed as capital in time $t+1$ by private firms (see equation 3.105 in Barro and Sala-i-Martin, 2004), we can exploit the linear, "AK," structure of the production process in our economy discussed at the end of Section 2.4 to immediately compute the BGP growth rate:

$$\begin{aligned} k_{t+1} = x_t &= \frac{\beta(1-\tau)}{1+\beta}w_t \\ &= \frac{\beta(1-\alpha)(1-\tau)}{1+\beta}\theta^{\frac{1}{\alpha}}[1-\mathbb{E}(s)]^{\frac{1-\alpha}{\alpha}}[\tau(1-\lambda)-(1-\alpha)(1-\tau)\lambda]^{\frac{1-\alpha}{\alpha}}(1-\lambda)^{-1}k_t \\ &= \frac{\beta(1-\alpha)(1-\tau)}{1+\beta}\theta^{\frac{1}{\alpha}}(1-s)^{\frac{1-\alpha}{\alpha}}[\tau(1-\lambda)-(1-\alpha)(1-\tau)\lambda]^{\frac{1-\alpha}{\alpha}}(1-\lambda)^{-1}k_t \\ &= \Psi(\tau,\lambda)k_t, \end{aligned} \tag{26}$$

where in the second equality we used (14), in the third (21), in the fourth condition (25) of Proposition 1 establishing that $\mathbb{E}(s)$ is constant, $\mathbb{E}(s) \equiv s$, while in the last equality we emphasize the dependence on parameters τ and λ of the constant Ψ, defined as follows:

$$\begin{aligned} \Psi(\tau,\lambda) = &\frac{\beta(1-\alpha)\theta^{\frac{1}{\alpha}}(1-\tau)}{(1+\beta)(1-\lambda)} \\ &\times\left\{\tau(1-\lambda)-\left[1+\frac{1-p(f+1)}{(1+\beta)f}\right](1-\alpha)(1-\tau)\lambda\right\}^{\frac{1-\alpha}{\alpha}}, \end{aligned} \tag{27}$$

because in the sequel we will focus on comparative dynamics based on the tax rate τ and the share of workers employed in the public sector, $\lambda = L_1/L$, parameters that can be both interpreted as proxies of "government size." Moreover, parameter λ provides the government with a policy tool that, together with the tax rate, affects the (optimal) growth rate of the economy. Note that the term in curly brackets is certainly positive because it is the result of the product $(1-s)[\tau(1-\lambda)-(1-\alpha)(1-\tau)\lambda]$ under the same exponent, $(1-\alpha)/\alpha$, where

$[\tau(1-\lambda)-(1-\alpha)(1-\tau)\lambda]>0$ under Assumption 1 and $(1-s)>0$ under condition (24) of Proposition 1.

The linear difference equation of per worker physical capital defined in (26) immediately yields the positive growth rate of the economy,

$$\gamma(\tau,\lambda)=\Psi(\tau,\lambda)-1, \tag{28}$$

provided that parameters $\alpha,\beta,\theta,\lambda,\tau,f$, and p are such that $\Psi(\tau,\lambda)>1$. The growth rate $\gamma(\tau,\lambda)$ in (28) is constant and thus characterizes the only possible BGP on which the economy jumps immediately starting from $t=0$.

We are now ready to state our main result, Proposition 2, in which we establish an increasing relationship between the probability of catching corrupt public officials (viewed as proxy of social capital) or the fine charged to them in case of detection, or both, and the growth rate as defined in (28). Moreover, we find that such a relationship involves a positive correlation with the size of government—both in terms of tax rate and in the number of public officials—whenever the growth rate is to be kept at its maximum level; specifically, an increase in the capture probability and/or the fine requires a greater size of government to let the growth rate reach its maximum value.

Proposition 2. *Under Assumption 1 and the assumptions of Proposition 1 the following hold.*

(i) *For any fixed admissible pair (τ,λ) the economy growth rate $\gamma(\tau,\lambda)$ defined in (28) is an increasing function of both the probability of detection p and the fine f.*

(ii) *For any fixed $0<\lambda<1$, the economy growth rate $\gamma(\cdot,\lambda)$ defined in (28) is an inverted U-shaped function of the tax rate τ and admits one unique interior maximum point, $0<\tau^*(\lambda)<1$, which is itself function of all parameters according to*

$$\tau^*(\lambda)=\frac{[1-p(f+1)](1-\alpha)\lambda+(1+\beta)f(1-\alpha)}{[1-p(f+1)](1-\alpha)\lambda+(1+\beta)f(1-\alpha\lambda)}. \tag{29}$$

(iii) *Under the following further restrictions:*

$$\alpha>(\sqrt{5}-1)/2\simeq 0.618 \quad \text{and} \quad \frac{1-p(f+1)}{(1+\beta)f}<\frac{\alpha^2+\alpha-1}{\alpha(1-\alpha)} \tag{30}$$

the growth rate $\gamma(\tau,\lambda)$ defined in (28), considered as a function of both parameters τ and λ, admits one unique (interior) stationary point (τ^*,λ^*) with coordinates

$$\tau^*=\frac{(2\alpha-1)(1+\beta)f-[1-p(f+1)](1-\alpha)}{\alpha(1+\beta)f-[1-p(f+1)](1-\alpha)} \tag{31}$$

$$\lambda^*=\frac{(\alpha^2+\alpha-1)(1+\beta)f-[1-p(f+1)]\alpha(1-\alpha)}{(2\alpha-1)\{\alpha(1+\beta)f-[1-p(f+1)](1-\alpha)\}}. \tag{32}$$

(iv) *Both stationary values τ^* and λ^* in (31) and (32) are increasing functions of both the probability of detection p and the fine f.*

Clearly, assumptions (30) are necessary to have a positive numerator in the expression for λ^* in (32); in the proof it is shown that the same conditions imply that the denominator is positive as well. Note that the second condition in (30) holds if f and p are large enough, provided that they satisfy the first condition in (24) of Proposition 1.

Unfortunately, it is not possible to establish concavity of $\Psi(\tau,\lambda)$, and thus of $\gamma(\tau,\lambda) = \Psi(\tau,\lambda) - 1$, directly; however, graphic inspection shows that, for reasonable values of parameters in their admissible ranges, it should be (see Figure 1 in the next section). Nonetheless, we do not really need to know that the unique stationary point (τ^*, λ^*) with coordinates given by (31) and (32) is a global maximum point: assuming that the economy adopts the (optimal) tax rate τ^* as in (31) and the government employs a share λ^* as in (32) of total labour, part (iv) of Proposition 2 establishes that, if parameter p or f, or both, increase, then there exist a direction originating from (τ^*, λ^*) along which, to keep the growth rate $\gamma(\tau,\lambda)$ in (28) at its maximum level, an increase of the optimal tax rate τ^* is required, provided that also the share λ^* is adjusted (increased) as well according to (32). In other words, although we are not able to prove that for different values of p and f conditions (31) and (32) describe the upper envelope of the function $\gamma(\tau,\lambda)$ (but we conjecture they do, as it will be illustrated in the next section by means of a numerical example through Figures 1 and 2), part (iv) of Proposition 2 provides a criterion to follow (by adjusting the share of public employees, λ^*) to keep the (positive) monotonicity that links the optimal tax rate τ^* to increases in the Social Capital—represented by increases in p—that is found in empirical data. As p can be considered as a proxy of the level of Social Capital, we have thus shown that an increase of Social Capital requires larger levels of both the tax rate τ^* and the share of public employees λ^* to keep the growth rate $\gamma(\tau^*, \lambda^*)$ at its maximum.

4 | A NUMERICAL EXAMPLE

To carry out a numerical example we set the following parameters' values:

$$\beta = 0.3, \quad \alpha = 0.67, \quad \theta = 10.6.$$

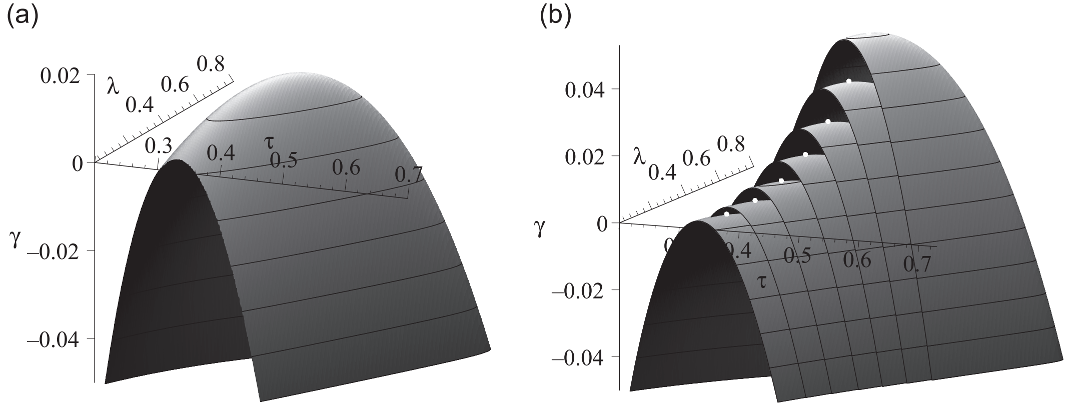

FIGURE 1 (a) Three-dimensional plot of the growth rate function $\gamma(\tau,\lambda)$ defined in (28) for $f = f_0 = 1.43$ and $p = 0.20$; (b) several three-dimensional plots of the same growth rate function—each uniformly higher then the other—for $f = f_0 = 1.43$ and the seven values of p listed in the first column of Table 1, the white dots denote the maximum value for each function

The individual discount rate value is compatible with some 30 years time-horizon for a generation to be employed either in the private or in the public sector, the capital factor share value clearly satisfies the first condition in (30), and the technology parameter value guarantees reasonable values of the growth rate defined in (28) around its maximum points, as it will be shown below. As previously discussed, the capital factor is to be understood in a broad sense as in Barro (1990) who, using the Cobb–Douglas production function in his Equation (10), sets a value for the elasticity of capital which would be equivalent to $\alpha = 0.75$ in our specification (see caption of Figure 1 on p. S110). Similarly, in their benchmark calibration, Marakbi and Villieu (2020) set $\alpha = 0.70$.

Note that the second condition in (30) of Proposition 2 can be reformulated as a lower bound for the probability of detection p given the fine f according to:

$$p > \frac{1}{1+f}\left[1 - \frac{(\alpha^2 + \alpha - 1)(1+\beta)f}{\alpha(1-\alpha)}\right].$$

Joining this condition with the first condition in (24) of Proposition 1 yields the following open interval as feasible range for the probability of detection p for any given value of the fine f:

$$p \in \left(\frac{1}{1+f}\left[1 - \frac{(\alpha^2 + \alpha - 1)(1+\beta)f}{\alpha(1-\alpha)}\right], \frac{1}{1+f}\right),$$

whose endpoints, when considered as functions of the fine f for given α and β values, are two hyperbolas, the former laying strictly below the latter whenever $f > 0$, while they collapse into the same point $p = 1$ when $f = 0$. The right endpoint is strictly positive for any $f > 0$, while the left endpoint intersects the horizontal axis on the point

$$f_0 = \frac{\alpha(1-\alpha)}{(\alpha^2 + \alpha - 1)(1+\beta)},$$

that is, on the value $f_0 = 1.43$ when $\alpha = 0.67$ and $\beta = 0.3$.

We focus on how changes in the probability of detection p affect the growth rate function $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$ defined in (28), paying special attention to its maximum point $\gamma(\tau^*, \lambda^*)$ with coordinates τ^* and λ^* given by (31) and (32). To this purpose, we fix the value of the fine at $f = f_0 = 1.43$, so to have the largest possible range for the p values, given by the interval $p \in (0, 0.41)$.[16] Similar results are obtained for increasing values of the fine f for a fixed probability p, or for increasing values of both f and p; we omit these types of illustration. Under these assumptions—that is, for $\beta = 0.3, \alpha = 0.67$, $\theta = 10.6$ and $f_0 = 1.43$ fixed—we consider seven values for the probability of detection p in the range $(0, 0.41)$ and compute the coordinates τ^* and λ^* according to (31) and (32), plus the corresponding maximum growth rate value $\gamma(\tau^*, \lambda^*)$, for each probability value considered. The results are reported in Table 1, where the monotonic increasing pattern of both the optimal tax rate τ^* and optimal share of public employees λ^*, as well as the corresponding maximum value of the growth rate $\gamma(\tau^*, \lambda^*)$, predicted by Proposition 2 is clearly evident as the probability of detection p increases.

Figure 1a shows the three-dimensional graph of the growth rate $\gamma(\tau, \lambda)$ defined in (28) as a function of τ and λ for $f_0 = 1.43$ and $p = 0.20$: it clearly exhibits "concavity" traits, with,

[16] Although we are not aware of empirical data on the public officials' probability of getting caught stealing, by considering the literature on tax evasion, it is well known that the latter can be estimated to be no larger than 0.01 (see, e.g., Alm, 2019, p. 355). Clearly tax evasion, which is considered over the whole population of taxpayers, is a quite different phenomenon than corruption among public officials, which involves only a fraction of taxpayers; however, it seems to us not unreasonable to believe that such probability be sufficiently below 0.5.

TABLE 1 Optimal values for the tax rate τ^* and the public employees share λ^* according to (31) and (32), and the corresponding maximum growth rate $\gamma(\tau^*, \lambda^*)$ according to (28), for seven feasible values of the probability of detection p; $\beta = .3$, $\alpha = .67$, $\theta = 10.6$, and $f = f_0 = 1.43$

p	τ^*	λ^*	$\gamma(\tau^*, \lambda^*)$
.05	0.3581	0.0827	0.0031
.10	0.3839	0.1587	0.0059
.15	0.4078	0.2288	0.0106
.20	0.4299	0.2937	0.0173
.25	0.4503	0.3539	0.0262
.30	0.4694	0.4100	0.0373
.35	0.4872	0.4623	0.0509

according to the fourth row of Table 1, a unique global interior maximum point reached on the pair $(\tau^*, \lambda^*) = (0.4299, 0.2937)$ with value $\gamma(\tau^*, \lambda^*) = 0.0173$, corresponding to an optimal growth rate of around 1.7%. Figure 1b presents an attempt to draw an imaginary upper envelope curve in the three-dimensional space (τ, λ, γ) of the seven maximum points defined by the three values listed in the last three columns of Table 1 by ideally joining the seven white dots in the figure, each corresponding to the maximum growth rate value for the graph of $\gamma(\tau, \lambda)$ determined by the seven values of the probability of detection p in the first column of Table 1. Note that, consistently with part (i) of Proposition 2, each graph of the growth rate function $\gamma(\tau, \lambda)$ plotted in Figure 1b is a surface (only partially reported for $p \geq 0.10$ so to emphasize their area close to their maximum points) that lays uniformly above the other surfaces corresponding to lower values of p and uniformly below the other surfaces corresponding to higher values of p. This feature renders difficult a three-dimensional graphical representation of the upper envelope of all the functions $\gamma(\tau, \lambda)$ as p increases and justifies the choice of plotting only partial sections of the surfaces (the graphs of the function $\gamma(\tau, \lambda)$) corresponding to probability values larger than 0.10.

While the increasing pattern of the single three-dimensional graphs of $\gamma(\tau, \lambda)$—further emphasized by their maximum points denoted by white dots, which provide a skeleton for drawing their upper envelope—clearly confirms part (i) of Proposition 2, the monotonic increasing pattern of both coordinates τ^* and λ^* established in part (iv) of Proposition 2 as p increases (our main result) can only be inferred from Figure 1b. Figure 2 provides an even better flavour of part (iv) of Proposition 2 by plotting separately the λ^* and τ^*-sections, respectively of the growth rate function $\gamma(\tau, \lambda)$ for the seven p values in the fist column of Table 1: specifically, Figure 2a plots the two-dimensional graphs of $\gamma(\tau, \lambda^*)$ as a function of the only variable τ for each optimal value λ^* reported in the third column of Table 1, while Figure 2b reports the two-dimensional graphs of $\gamma(\tau^*, \lambda)$ as a function of the only variable λ for each optimal value τ^* reported in the second column of Table 1. The increasing pattern of the maximum points, as well as the maximum values, of each curve as p increases is apparent in both figures. However, from Figure 1b we learn that each curve should be projected deeper and deeper in the third-dimensional orthogonal to the (τ, γ) and (λ, γ) spaces, respectively, as λ^* and τ^* increase for larger values of p; that is, each curve in Figure 2a corresponds to larger values of λ^*, while each curve in Figure 2b corresponds to larger values of τ^*, so that the upper envelope Figure 1b develops (increases) toward north-east in the (τ, λ) space.

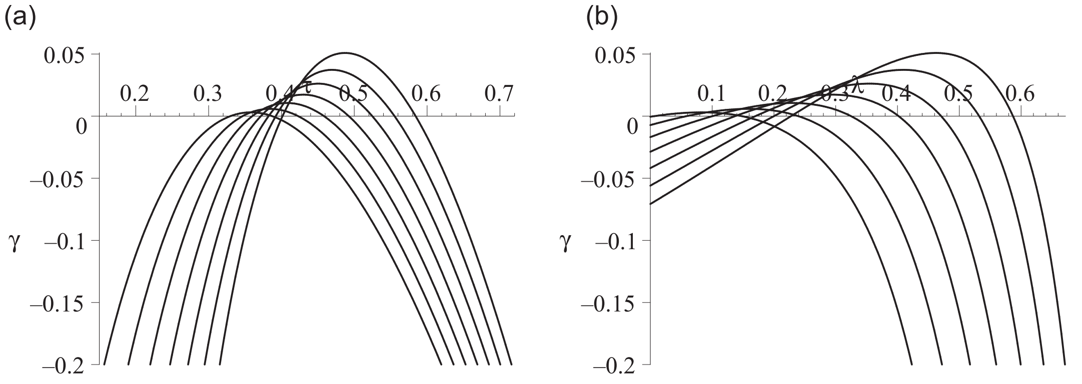

FIGURE 2 (a) Two-dimensional plots of the λ^*-sections of the growth rate function $\gamma(\tau, \lambda)$ defined in (28) for $f = f_0 = 1.43$ and the seven values of the pairs p, λ^* listed in the first and third columns of Table 1; (b) two-dimensional plots of the τ^*-sections of the same growth rate function for $f = f_0 = 1.43$ and the seven values of the pairs p, τ^* listed in the first and second columns of Table 1

5 | CONCLUSIONS

Using the OLG framework, we have shown that a simple modification of the Barro (1990)'s endogenous growth model, introduced to take into account the possibility that corrupt public officials will steal a fraction of public resources under their own control, is capable of theoretically explain the existence of high growth rates in the presence of Big Government size. Specifically, we have shown that, under realistic conditions on parameters' values, a monotonic increasing relationship exists among the level of Social Capital (expressed in terms of the probability to detect cheating public officials), the government size (expressed both in terms of the tax rate and the number of public employees) and the maximum achievable economic growth rate. Social Capital could thus be the missing dimension accounting for the controversial empirical results on this issue as well as for the ENC case. High levels of Social Capital affect the behavior of public officials monitoring the public expenditures for intermediate goods and services supplied to private firms, and thus the efficiency of government as a whole.

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APPENDIX A: ON THE NECESSITY OF A BUREAUCRACY

Assume that the government purchases public goods and services directly from the firms described in Section 2.1 instead of relying on the services of public officials. Let M be the (large) number of competitive firms and $G = \tau Y$ the amount of resources to be devoted to the firms as intermediate goods and services; then $Q = G/M = \tau Y/M < \tau Y$ is the quantity purchased from a single firm at the numeraire. By denoting such quantity as a share $0 \leq q \leq 1$ of each firm's net output Y_i, that is, $Q = qY_i$, and noting that it is priced at the numeraire, firm i's revenues can be decomposed as $Y_i = qY_i + (1-q)Y_i$, where $(1-q)Y_i$ denotes the share of output sold in the private market, and its profit is given by $\pi_i = qY_i + (1-q)Y_i - wL_i - RK_i$. Suppose that firm i has the opportunity of stealing a share $0 \leq s \leq 1$ of the revenue from the government, qY_i, and sell it to private households, while acknowledging that it may get caught with probability $0 < p < 1$, in which case it must give back the whole amount stolen sqY_i with the addition of a fine $\varphi > 0$ per unit of resource stolen, that is, it must return $fsqY_i$ with $f = 1 + \varphi > 1$. In this case, revenues from the government are still qY_i, but the true value (at the numeraire) of intermediate goods and services supplied is actually $(1 - s)qY_i$. Then, the expected profit of firm i can be rewritten as follows:

$$\begin{aligned}\mathbb{E}\pi_i &= qY_i + (1-p)sqY_i - pfsqY_i + (1-q)Y_i - wL_i - RK_i \\ &= \{1 + [1 - p(f+1)]qs\}Y_i - wL_i - RK_i.\end{aligned}$$

Recall that our main results in Section 2.4 (Proposition 1) and in Section 3 (Proposition 2) hold under condition (24), which implies that $[1 - p(f+1)] > 0$. Therefore, as $qY_i > 0$ must hold in a nontrivial economy with positive transfers of public goods and services to the firms,

$$\frac{\partial}{\partial s}\mathbb{E}\pi_i = [1 - p(f+1)]qY_i > 0,$$

that is, it would be optimal for each firm to set $s = 1$, so to steal all the revenues from the government and provide zero goods and services intended for the firms, independent of all other parameters (specifically, independent of the costs $wL_i + RK_i$). As this result would hold ceteris paribus in the economy studied in this paper, the government prefers to hire public officials to manage public expenditures, because, according to (25), such a choice can contain the loss from the potential resources to be transferred to the firms to a share $s < 1$.

APPENDIX B: PROOFS OF MAIN RESULTS

Proof of Proposition 1. To prove (i) we only must show that the optimal theft s in (25) is interior, that is it satisfies $0 < s < 1$, and that the argument of the utility in the second term in (13) is strictly positive, $\bar{w}_t - x_t - fq_t s_t > 0$, when x_t and s_t are defined according to (14) and (15) respectively, that is, the optimal choice on x_t and s_t must yield positive consumption in the unlucky event of being caught.

The former property is a consequence of Assumption 1 together with condition (24). First note that Assumption 1 requires the term $\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda$ in the denominator of (25) to be positive, so that, to have $s > 0$ in (25), as the term $(1-\alpha)(1-\tau)\lambda$ is positive by construction, the term $1 - p(f+1)$ in the numerator must be positive as well, that is, the first condition in (24) must hold. To have $s < 1$ as well we solve

$$\frac{[1 - p(f+1)](1-\alpha)(1-\tau)\lambda}{(1+\beta)f\,[\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda]} < 1,$$

which, under Assumption 1, is equivalent to

$$\{[1 - p(f+1)](1-\alpha)\lambda + (1+\beta)f(1-\alpha\lambda)\}\tau > [1 - p(f+1) + (1+\beta)f](1-\alpha)\lambda,$$

which immediately yields the second condition in (24). It is easily shown that the second condition in (24) is stronger than Assumption 1 on τ as, using the fact that the first condition in (24) is equivalent to $1 - p(f+1) > 0$, the inequality

$$\frac{[1 - p(f+1) + (1+\beta)f](1-\alpha)\lambda}{[1 - p(f+1)](1-\alpha)\lambda + (1+\beta)f(1-\alpha\lambda)} > \frac{(1-\alpha)\lambda}{1-\alpha\lambda}$$

boils down to $\lambda < 1$, which holds by construction.

The latter property follows directly from (14) and (15):

$$\bar{w}_t - x_t - fq_t s_t = \bar{w}_t - \frac{\beta}{1+\beta}\bar{w}_t - fq_t \frac{1 - p(f+1)}{(1+\beta)f}\frac{\bar{w}_t}{q_t} = \frac{p(f+1)}{1+\beta}\bar{w}_t,$$

where the last term is positive whenever $\bar{w}_t = (1-\tau)w_t > 0$.

Property (ii) is an immediate consequence of s in (25) being constant and all the discussion in Section 2.4.

Finally, to prove (iii), direct computation of $\partial s/\partial p$, $\partial s/\partial f$, and $\partial s/\partial \tau$ in (25) show that they are all negative under our assumptions on all parameters. □

Proof of Proposition 2. Property (i) is established by direct computation of the partial derivatives with respect to p and f of the function $\gamma(\tau,\lambda) = \Psi(\tau,\lambda) - 1$ defined in (28), which turn out to be both positive.

To prove (ii) let

$$A = \frac{\beta(1-\alpha)\theta^{\frac{1}{\alpha}}}{(1+\beta)} \quad \text{and} \quad B = \left[1 + \frac{1-p(f+1)}{(1+\beta)f}\right](1-\alpha) \tag{B1}$$

so that we can rewrite $\Psi(\tau,\lambda)$ in (27) as

$$\Psi(\tau,\lambda) = A\left(\frac{1-\tau}{1-\lambda}\right)[\tau(1-\lambda) - (1-\tau)B\lambda]^{\frac{1-\alpha}{\alpha}}. \tag{B2}$$

Note that under all our assumptions, including the first condition in (24) which implies that $1 - p(f+1) > 0$, the constants in (B1) are positive: $A, B > 0$. The term in square brackets on the right-hand side (RHS) of (B2) is positive because it is the result of the product $(1-s)[\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda]$ with $[\tau(1-\lambda) - (1-\alpha)(1-\tau)\lambda] > 0$ under Assumption 1 and $(1-s) > 0$ under condition (24) of Proposition 1. Specifically, we have:

$$\tau(1-\lambda) - (1-\tau)B\lambda > 0 \quad \text{for all } 0 < \lambda < 1 \quad \text{and} \quad \tau_L(\lambda) < \tau < 1,$$

where $\tau_L(\lambda)$ is the lower bound in the admissible range for τ defined by the second condition in (24),

$$\tau_L(\lambda) = \frac{[1 - p(f+1) + (1+\beta)f](1-\alpha)\lambda}{[1-p(f+1)](1-\alpha)\lambda + (1+\beta)f(1-\alpha\lambda)}, \tag{B3}$$

where we stress its dependency on λ. For a given $0 < \lambda < 1$, the problem $\max\{\gamma(\tau) = \Psi(\tau) - 1 \colon \tau_L(\lambda) < \tau < 1\} = \max\{\ln[\Psi(\tau)] : \tau_L(\lambda) < \tau < 1\}$ can be written as

$$\max_{\tau_L(\lambda)<\tau<1}\left\{\ln A + \ln(1-\tau) - \ln(1-\lambda) + \frac{1-\alpha}{\alpha}\ln[\tau(1-\lambda) - (1-\tau)B\lambda]\right\}, \tag{B4}$$

where we used (B2). FOC on the RHS easily yields the optimal value

$$\tau^*(\lambda) = \frac{B\lambda + (1-\alpha)(1-\lambda)}{B\lambda + 1 - \lambda}. \tag{B5}$$

Using the definition of B in (B1), note that $[1 - p(f+1) + (1+\beta)f](1-\alpha) = (1+\beta)fB$ and $[1 - p(f+1)](1-\alpha) = (B + \alpha - 1)(1+\beta)f$, so that, substituting into (B3), we get the lower bound $\tau_L(\lambda)$ as a function of B and B as a function of $\tau_L(\lambda)$:

$$\tau_L(\lambda) = \frac{B\lambda}{B\lambda + 1 - \lambda}, \quad \text{and} \quad B = \frac{(1-\lambda)\tau_L(\lambda)}{\lambda[1 - \tau_L(\lambda)]}. \tag{B6}$$

Using the latter expression for B in in the expression (B5) we can write the optimal tax rate as a function of the lower bound $\tau_L(\lambda)$ recalled in (B3):

$$\tau^*(\lambda) = 1 - \alpha + \alpha\tau_L(\lambda), \tag{B7}$$

from which, as $\tau_L(\lambda) < 1$, it is apparent that $\tau^*(\lambda) > \tau_L(\lambda)$; while, as $\alpha < 1$, from (B5) it follows that $\tau^*(\lambda) < 1$ as well, establishing that the unique stationary point $\tau^*(\lambda)$ is interior. Substituting $\tau_L(\lambda)$ as in (B3) into (B7),

$$\tau^*(\lambda) = 1 - \alpha + \frac{\alpha[1 - p(f+1) + (1+\beta)f](1-\alpha)\lambda}{[1 - p(f+1)](1-\alpha)\lambda + (1+\beta)f(1-\alpha\lambda)},$$

the expression in (29) is immediately obtained.

To confirm that $\tau^*(\lambda)$ is the unique solution of (B4) for fixed λ, first note that, substituting B according to the second equation in (B6) in the term $\tau(1-\lambda) - (1-\tau)B\lambda$, for $\tau > \tau_L(\lambda)$ it holds

$$\tau(1-\lambda) - (1-\tau)B\lambda = \frac{1-\lambda}{1 - \tau_L(\lambda)}[\tau - \tau_L(\lambda)] > 0,$$

moreover, using the first equation in (B6) for $\tau_L(\lambda)$ in the term $\tau(1-\lambda) - (1-\tau)B\lambda$, for $\tau \to \tau_L^+(\lambda)$ one has

$$\tau(1-\lambda) - (1-\tau)B\lambda \to \frac{B\lambda(1-\lambda) - (1-\lambda)B\lambda}{B\lambda + 1 - \lambda} = 0^+,$$

so that,

$$\lim_{\tau \to \tau_L^+(\lambda)} \frac{\partial}{\partial\tau} \ln[\Psi(\tau)] = \lim_{\tau \to \tau_L^+(\lambda)} \left\{ -\frac{1}{1-\tau} + \frac{1-\alpha}{\alpha}\left[\frac{B\lambda + 1 - \lambda}{\tau(1-\lambda) - (1-\tau)B\lambda}\right] \right\} = +\infty,$$

implying that $(\partial/\partial\tau)\ln[\Psi(\tau)] > 0$ for all $\tau_L(\lambda) < \tau < \tau^*(\lambda)$. Next, it holds

$$\lim_{\tau \to 1^-} \frac{\partial}{\partial\tau} \ln[\Psi(\tau)] = \frac{1-\alpha}{\alpha}\left(\frac{B\lambda + 1 - \lambda}{1-\lambda}\right) + \lim_{\tau \to 1^-}\left(-\frac{1}{1-\tau}\right) = -\infty,$$

implying that $(\partial/\partial\tau)\ln[\Psi(\tau)] < 0$ for all $\tau^*(\lambda) < \tau < 1$. This establishes that $\ln[\Psi(\tau)]$ is inverted U-shaped functions and that $\tau^*(\lambda)$ in (29) is the unique solution of (B4); as $\ln[\Psi(\tau)]$ is a monotone transformation of $\Psi(\tau)$, the same holds true for $\Psi(\tau)$, and in turn, for the growth rate $\gamma(\tau) = \Psi(\tau) - 1$ defined in (28), for each fixed $0 < \lambda < 1$.

To establish (iii) we consider the objective function in (B4),

$$\begin{aligned} \ln\Psi(\tau,\lambda) = \ln A + \ln(1-\tau) - \ln(1-\lambda) \\ + \frac{1-\alpha}{\alpha}\ln[\tau(1-\lambda) - (1-\tau)B\lambda], \end{aligned} \tag{B8}$$

as a function of both variables τ and λ and study it over the open set $\{(\tau,\lambda): (0 < \lambda < 1) \wedge [\tau_L(\lambda) < \tau < 1]\}$. As, according to (B1), A and B do not depend on τ or λ, FOC with respect to λ on the RHS of (B8) easily yields the critical value

$$\lambda^* = \frac{(1-2\alpha)\tau + (1-\alpha)(1-\tau)B}{(1-2\alpha)[\tau + (1-\tau)B]},$$

and pairing it with equation (B5) leads to the unique stationary point with coordinates

$$\tau^* = \frac{\alpha - B}{1-B}, \quad \text{and} \quad \lambda^* = \frac{2\alpha - 1 - \alpha B}{(2\alpha-1)(1-B)}, \tag{B9}$$

which, after replacing B according to (B1) and through some algebra, establish (31) and (32). The two conditions in (30) clearly guarantee that $2\alpha - 1 - \alpha B > 0 \Leftrightarrow B < 2 - 1/\alpha$, which, as $0 < (1-\alpha)^2 = \alpha^2 - 2\alpha + 1 \Leftrightarrow 2 - 1/\alpha < \alpha$, in turn, implies $B < \alpha < 1$; as $\alpha > (\sqrt{5}-1)/2 > 1/2 \Rightarrow (2\alpha - 1) > 0$, all these conditions together establish that both τ^* and λ^* are positive. Moreover, $\tau^* < 1$ because $0 < \alpha - B < 1 - B$, while $\lambda^* < 1$ because $2\alpha - 1 - \alpha B < (2\alpha-1)(1-B) \Leftrightarrow (1-\alpha)B > 0$. To check that $\tau^* > \tau_L(\lambda^*)$, with $\tau_L(\lambda^*)$ defined in (B3) for $\lambda = \lambda^*$, we first use the expression of λ^* in (B9) in the expression for B as in (B6) to get

$$B = \frac{(1-\lambda^*)\tau_L(\lambda)}{\lambda^*[1-\tau_L(\lambda)]} = \frac{(1-\alpha)B\tau_L(\lambda)}{(2\alpha - 1 - \alpha B)[1-\tau_L(\lambda)]} \Leftrightarrow B = \frac{\alpha[2-\tau_L(\lambda)] - 1}{\alpha[1-\tau_L(\lambda)]}.$$

Next, we replace the last expression for B in the first equation in (B9) to get, after some algebra,

$$\tau^* = \frac{\alpha - B}{1-B} = \frac{1-\alpha(2-\alpha)}{1-\alpha} + \alpha\tau_L(\lambda);$$

as $[1-\alpha(2-\alpha)]/(1-\alpha) + \alpha\tau_L(\lambda) > \tau_L(\lambda)$ is equivalent to $1 > \tau_L(\lambda)$, which is definitely true, we have shown that $\tau^* > \tau_L(\lambda)$. Therefore, we conclude that the stationary point (τ^*, λ^*) with coordinates given by (31) and (32) is an interior point of the open set $\{(\tau,\lambda): (0 < \lambda < 1) \wedge [\tau_L(\lambda) < \tau < 1]\}$.

To prove (iv) we first verify that both $(\partial/\partial p)B(p,f)$ and $(\partial/\partial f)B(p,f)$ are strictly negative through direct differentiation of B as in (B1) with respect to p and f. Finally, using the fact that $(\partial/\partial p)B(p,f) < 0$ and $(\partial/\partial f)B(p,f) < 0$, by differentiating the expressions of both τ^* and λ^* as in (B9) with respect to p and f it is easily established that that they are all strictly positive, and the proof is complete. □