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The Corner Tower of Anagni Cathedral: Geometry and Equilibrium

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Abstract

This paper explores the corner tower of the Anagni Cathedral, a Romanesque structure built in the eleventh and thirteenth centuries. The tower, located beneath the baptistery, was likely constructed to support a small chapel with a font. Through digital surveying and geometric analysis, this study examines the tower's structural element and speculates on the ideal stereotomic apparatus and reference models. The paper also delves into the mechanism responsible for maintaining the cantilevering structure's equilibrium. The tower and baptistery exemplify the role of stereotomy and friction in maintaining equilibrium, with internal tensile forces and unilateral contact between the structure's blocks. This study provides valuable insights into the Anagni Cathedral's structural elements and highlights the importance of understanding stereotomy and friction principles.

Keywords Structural systems · Geometric analysis · Statics · Stereotomy · Masonry structures

Introduction

The Anagni Cathedral is a Romanesque structure built between 1072 and 1104. Additional chapels were added in the thirteenth century. This paper focuses on the corner tower (Fig. 1) located beneath the baptistery, which was likely constructed to support the small chapel with the font. Using an accurate digital survey and geometric analysis, this study examines the structural element and speculates on the ideal conformation of the stereotomic apparatus and potential reference models. This paper also interprets and explains the primary mechanism responsible for

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maintaining the equilibrium of the cantilevering structure. The tower and baptistery are excellent examples of the role of stereotomy and friction in maintaining the state of equilibrium, where internal tensile forces are necessary, and the blocks composing the structure interact with each other via unilateral contact. Overall, this paper provides valuable insights into the structural elements of the Anagni Cathedral and highlights their significance in understanding the principles of stereotomy and friction.

The Research

Historical Notes

There are not many ancient documents related to Anagni Cathedral that can ensure an accurate reconstruction of the historical phases that led to the current layout of this building. We know that the construction of this complex was commissioned by bishop Pietro (?-1105), prince of Salerno, between 1072 and 1104, and it was carried out starting from the foundations of a previous church dedicated to St. Magnus. In this first phase, the building had a typical Romanesque three-nave structure with alternating quadrangular and circular pillars, a transept, and three apses (Matthiae 1942; Palandri 2006).

Two coeval crypts are located below the transept and part of the nave to the west: the first crypt – entirely frescoed and defined as the 'Sistine Chapel of the Middle Ages' – houses the body of St. Magnus. The second chapel was dedicated to St. Thomas Becket by Pope Alexander III. Both of these underground places were originally accessible from outside the church through an arched passage located on the west side. Today's access is guaranteed by two stairways that lead from the church's aisles to the level below. The building was probably constructed following the Abbey of Monte Cassino model, which was erected under abbot Desiderio's guidance between 1066 and 1072, as confirmed by spatiotemporal proximity and the friendship between Desiderio and the bishop Pietro (Urcioli 2006: 191).

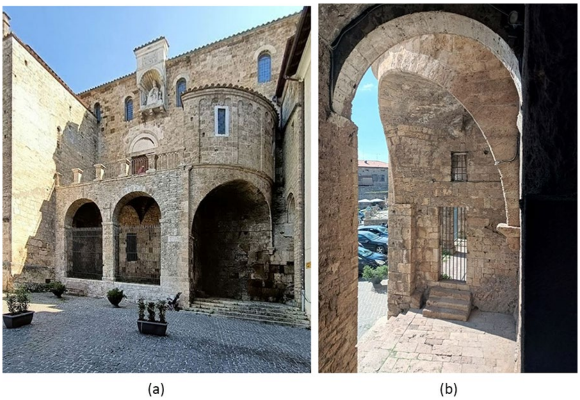
(a) (b)

Fig. 1 The corner tower of Anagni Cathedral: **a** view of the western façade with the corner tower and "the blessing Loggia"; **b** corner tower observed from the staircase that gives access to the main entrance of the Cathedral

Fig. 2 Volumetric 3d reconstruction of the historical phases: in dark grey the romanesque church, in light grey the chapels built during the XIII century with the hypothetical staircase semitransparent

Side chapels were added in the thirteenth century (Lauri, Caetani, and the one hosting the baptismal font) on the west area, as well as a monumental staircase (later demolished in 1830) that used to give access to the church from the underlying Piazza Innocenzo III through a new portal walking on a terrace called 'the blessing Loggia.' As mentioned above, the room of the baptistery was also built between these chapels, under which there was a corner supporting vault, placed in continuity with the portico below the blessing Loggia (Fig. 2). The vaults of the portico have incorporated a system of hanging arches on zoomorphic shelves; to this day, they protrude out the intrados surfaces, but at the time they were used to support the above-elements (Matthiae 1942; Piacentini 2006).

Digital Survey

This research has employed laser scanners and photogrammetric techniques to develop the morphological genesis of the vaulted surface of Anagni Cathedral's corner tower. The survey practice involved the placement of targets on the vertical walls supporting the vault and performing four scans with a Leica BLK 360 laser scanner (Fig. 3), followed by a photogrammetric survey via a Fujifilm X-T20 camera. The resulting textured mesh model was scaled and oriented using data from the laser scanner and imported into 3D modelling software for geometric analysis. This study found that the intrados surface of the vault closely resembled an abstract geometric surface arising from the intersection between two cylinders (Fig. 4), demonstrating significant precision of the constructive method despite the monument's age.

The study also examined the similarities and differences between the Anagni vault and a complex architectural element called the Trompe (De l'Orme 1567; Calvo-Lopez 2020), created using stereotomic techniques during the Renaissance, particularly in France. The Trompe is a suspended corner tower that connects interior spaces and two converging walls, typically positioned outside in the corner of the facade. However, the Anagni vault differs from the Trompe because the latter has an intrados surface resembling a conoid rather than a cylindrical surface. The

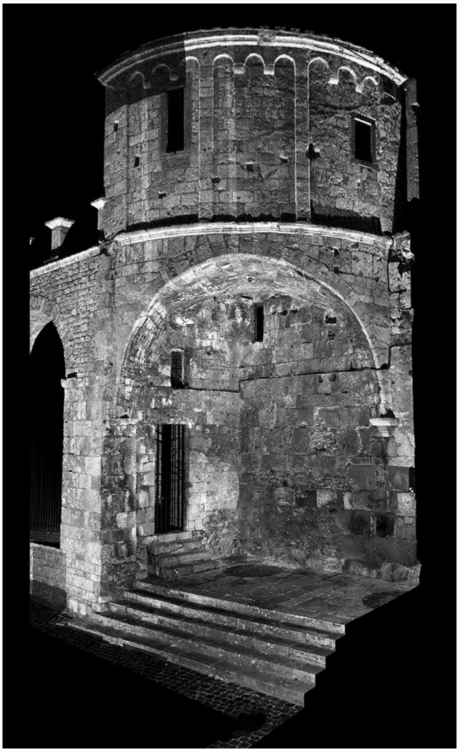

Fig. 3 The point cloud of the corner tower obtained with a 3d laser scanning

most famous example of the Trompe—created by the architect Philibert de L'Orme (1514–1570) at the Castle of Anet—has disappeared since then.

This research sheds light on using stereotomic techniques in architecture and implies a better understanding of the similarities and differences between the Anagni vault and the Trompe. The use of laser scanners and photogrammetric techniques in this research demonstrates the potential of modern technology to investigate and analyze historical architecture.

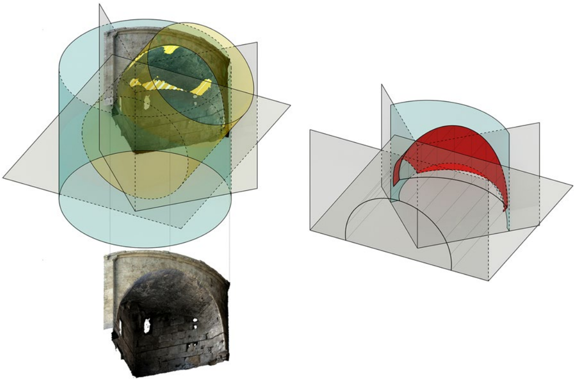

Fig. 4 Investigation about the geometrical genesis of the vault

Structural Analysis

The vault in Fig. 5 is more challenging since the Anagni vault is made of smaller and rougher stone elements. Hereafter, we will focus on a qualitative description of the mechanism that can be used to interpret and explain the equilibrium of such a cantilevering structure. This structure is a perfect example of the role of stereotomy and friction in maintaining equilibrium.

The stone blocks do not run from taut to compressed parts of the masonry (Fig. 6a). They are assumed to be in unilateral mutual contact and capable of exploiting small tensile forces in their "long" direction—orthogonal to the main thrust forces. The compressive thrust forces act transversely to the stone joints, allowing the employment of tangential friction forces that turn out to be pull forces inside the stones in the long joint directions. The equilibrium solution assumes that two spatial Linear Arches, called Γ_1 and Γ_2, exist inside the masonry (Fig. 6). These arches spring from the walls and produce thrusts, interacting with each other through tensile forces transmitted to them by the transverse stones. We consider these spatial Linear Arches—essentially spatial thrust lines—as 1D structures inside the masonry. Some authors recently introduced this kind of compressive line network (Angelillo et al. 2021), and it was also discussed in Carlo Olivieri's PhD thesis (Olivieri 2021).

We introduce a Cartesian reference system $\left\{O; x_1, x_2, x_3\right\}$ with the axis x_2 located along the cord of the arch from the springings, the origin O in the midpoint of the cord and the axis x_3 along the vertical (Fig. 6a). We start by assigning the shapes $\Gamma_1{}^\pi, \Gamma_2{}^\pi$ of the linear arches Γ_1, Γ_2 into the horizontal plane $\pi = \{x_1, x_2\}$ (Fig. 6b).

Fig. 5 Trompe of the Abbey of toussaint in angers

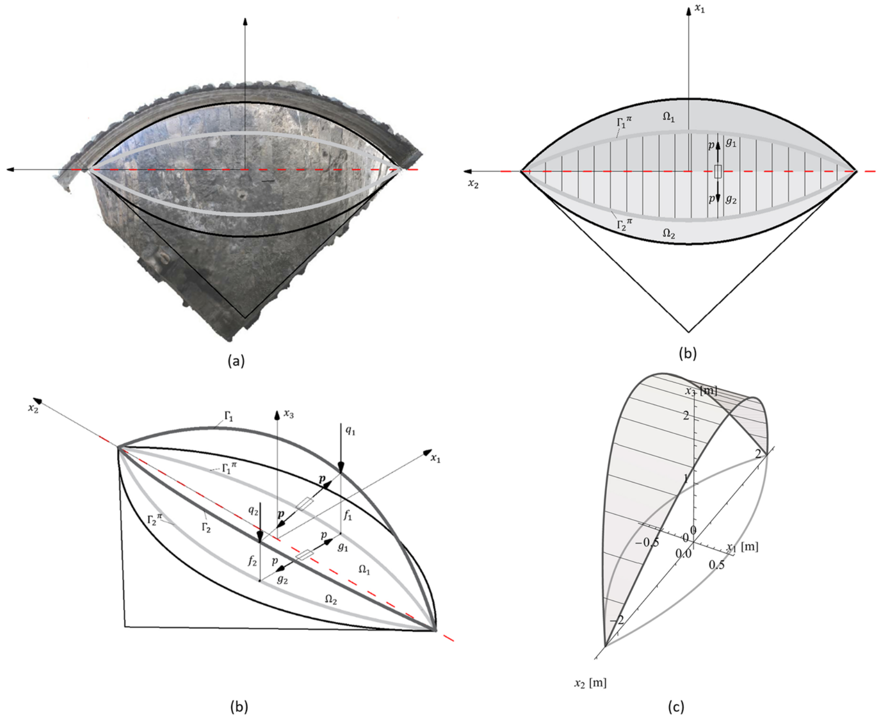

Fig. 6 Schematic view of the Linear Arches $\Gamma_1$andΓ_2 of their projections $\Gamma_1{}^\pi$and$\Gamma_2{}^\pi$ on the horizontal plane π, and their "influence areas" Ω_1 and Ω_2: **a** down-top view; **b** plan view; **c** axonometric view; **d** equilibrium solution

The weight of the superstructure, pavement, and filling is lumped in the two arches based on the "influence areas." $\Omega_1$andΩ_2 depicted in Fig. 6a. Known vertical forces represent this load $-q_1, -q_2$ applied as loads per unit projected length along x_2. The two arches are assumed to interact through mutually distributed tensile forces $\boldsymbol{p}, -\boldsymbol{p}$, which have zero components in the direction x_2: $\boldsymbol{p} = \{p, 0, q\}$ (Fig. 6c).

Therefore, the linear arches Γ_1, Γ_2 are acted on by vertical loads $-q_1 + q, -q_2 - q$ and by transverse forces acting in the directionx_1:$p, -p$. The equilibrium of the linear arches Γ_1, Γ_2 can be decomposed into two planar equilibrium analyses concerning the archesΓ_1, Γ_2, the first one in the horizontal plane$\pi = \{x_1, x_2\}$, and the second one in the vertical plane$\beta = \{x_2, x_3\}$. The equilibrium problem can be reduced to the following system of four ordinary differential equations.

$$g_1'' = \frac{p}{H_1}, g_2'' = \frac{-p}{H_2}, f_1'' = \frac{-q_1 + q}{H_1}, f_2'' = \frac{-q_2 - q}{H_2},$$

in which $g_1(x_2), g_2(x_2), f_1(x_2), f_2(x_2)$ are scalar functions describing the linear arches Γ_1, Γ_2, $q = p\frac{f_2 - f_1}{g_2 - g_1}$, and H_1, H_2 are two arbitrary constants representing the projection of the thrust along the x_2 axis. We note that this system of four odes has five unknowns $(g_1(x_2), g_2(x_2), f_1(x_2), f_2(x_2), p(x_2))$ and there exists a class of equilibrium solutions, among which the most favourable can be identified through optimization.

The solution shown graphically in Fig. 6d gives the following values for the thrust forces: $\mathrm{H}_1 = -48.0\mathrm{kN}$, $\mathrm{H}_2 = -32.1\mathrm{kN}$, and of the maximum pulling stress: $p = 9.6\mathrm{kN/m}$. On the safe side, on assuming as the least stone slenderness ratio the value $b°/l° = 0.217$ – $b°, l°$ being the two surface dimensions of the stones (Fig. 6a) –, we can estimate the maximum tangential stresses produced by p and acting on the long stone joints as $\tau = 2pb°/l° = 4.18\mathrm{kN/m}$, and for the normal stresses produced by the thrust H_2 acting on the same faces: $\sigma = \frac{\mathrm{H}_2}{\mathrm{L}} = -33.7\mathrm{kN/m}$. The ratio between these two stresses: $f = -\sigma/\tau = 8.06$ has to be compared with the friction coefficient between the stones, $\mu = \tan\varphi = 0.839$ (corresponding to a friction angle of 40°). The value of f is much greater than μ, indicating that the compressive forces are widely sufficient to sustain – through friction – the tangential stresses produced by the tensile stresses generated by the cantilevering arch section.

Conclusions

The vault covering the portico at the base of the corner tower is an impressive display of geometric complexity and morphological precision, given the time of its construction. It is possible that the builders took inspiration from previous solutions that had already been evaluated and tested in terms of static and stylistic efficiency. This is one of the few examples in which the role of tensile forces sustained by friction assumes (together with a clever and attentive cut of the stones) a crucial role in the equilibrium.

This study presented an accurate geometric restitution of the structure, and a simplified structural analysis was carried out using the equilibrium method. Preliminary findings indicate that the structure is in a stable equilibrium state with a large geometrical safety factor. Further work will provide this equilibrium analysis with a kinematical analysis based on the Distinct Element Method (DEM) and 3D analyses based on PRD (Olivieri et al. 2022). These efforts will involve a more comprehensive understanding of the vault's structural behaviour and contribute to our knowledge of the innovative construction techniques used in this period (Iannuzzo et al. 2021).

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Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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Maurizio Angelillo (Roma 1952) is Professor of Statics and Strength of Materials at the school of Engineering and Architecture of the "Università degli Studi di Salerno". Architect and structural expert with multi-disciplinary research interests including Masonry Mechanics, and Biomechanics, trained in Architecture at the University of Napoli and in Theoretical Mechanics at the University of Minnesota, he and his group are actively working on the kinematics and on the equilibrium of masonry buildings. He coordinates the activity of a research group within the laboratory of micromechanics and biomechanics of the University of Salerno. Angelillo works on unilateral models for masonry since the early 80s, being the author of numerous papers on the theory of masonry modelling and on the application of these models to real masonry structures and elements such as arches, domes, vaults and spiral stairs. Since 2018, with Gianmarco De Felice, Santiago Huerta and John Ochsendorf, and as a co-founder partner of KEIKO Cultural Association, he organizes and manages the Summer School on Historic Masonry Structures https://www.himass.org/.

Alessio Bortot (Treviso 1978), architect, is Doctor Europaeus since 2016 in Architecture, City and Design, with a specialization in Representation. He has been professor for the course of "Descriptive geometry", "Advanced technologies for representation" and "Digital 3D modelling" at the IUAV University of Venezia, at the faculty of Engineering of the University of Padova, at the IED of Venice and at the École National Supérieure des Travaux Publics in Yaoundé (Camerun). He has been research fellow working on topics about History of representation and advanced technologies for Architecture. He has lectured in conferences in academic institutions in Italy and abroad, and has participated to national (PRIN 2010-2011) and international research projects (James Turrell. Roden Crater Project – Florence 2007; Jean François Niceron. Prospettiva, catottrica e magia artificiale – Rome 2013). He is author of several publications, among which: Modelli digitali. Approcci multidisciplinari alla rappresentazione eidomatica (Venice 2010), La Geomatica per la documentazione e la tutela dell'architettura e del paesaggio Veneto (Venice 2012) and Emmanuel Maignan e Francesco Borromini. Il progetto di una villa scientifica nella Roma barocca, (Siracusa 2020). He became Associate Professor at the Department of Engineering and Architecture at the University of Trieste in 2021.

Carlo Olivieri (Avellino 1989) obtained his PhD in Structural Engineering in 2021 at the University of Salerno, where he is currently a Postdoctoral Researcher collaborating with the University of California, Berkeley. In 2016, after receiving his MSc in Building Engineering and Architecture at the University of Salerno, Carlo joined as Structural Engineer and Assistant Project Manager an International General Contractor, where, for two years, he was involved in relevant international projects. During his PhD, he developed new strategies for the assessment and for the form-finding of complex curved structures using the membrane theory and an extension of the classical Thrust Line method to special structures called Linear Arch Static Analysis. This new methodology opens new possibilities to look at the complex topic represented by curved masonry and concrete constructions. He also investigated the dynamics of masonry arches subjected to ground motion both from a theoretical and experimental perspective. His research is currently directed towards the definition of new optimization approaches for purely compressive shapes under seismic actions and new strategies for using low-carbon material blocks to construct these structures. As a co-founder partner of KEIKO Cultural Association, he has organized and participated as Teaching assistant at the International Summer School on Historic Masonry Structures since 2018. In 2022 he joined the Form Finding Lab of Princeton University.